Evolutionary sequence of spacetime and intrinsic spacetime and associated sequence of geometries in metric force fields II

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Abstract

The geometry of curved 'three-dimensional' absolute intrinsic metric space (an absolute intrinsic Riemannian metric space) $\varnothing\hat{\mathbb{M}}^3$, which is curved (as a curved hyper-surface) toward the absolute time/absolute intrinsic time 'dimensions' (along the vertical), and projects a flat three-dimensional absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ and its outward manifestation namely, the flat absolute proper 3-space $\mathbb{E}'^3_{ab}$, both as flat hyper-surfaces along the horizontal, isolated in the first part of this paper, is subjected to graphical analysis. Two absolute intrinsic metric tensor equations, one of which is of the form of Einstein free space field equations and the other which is a tensorial statement of absolute intrinsic local Euclidean invariance (A$\varnothing$LEI) on $\varnothing\hat{\mathbb{M}}^3$, are derived. Simultaneous (algebraic) solution to the equations yields the absolute intrinsic metric tensor and the absolute intrinsic Ricci tensor of absolute intrinsic Riemann geometry on the curved $\varnothing\hat{\mathbb{M}}^3$, in terms of a derived absolute intrinsic curvature parameter. A superposition procedure that yields the resultant absolute intrinsic metric tensor and the resultant absolute intrinsic Ricci tensor, when two or a larger number of absolute intrinsic Riemannian metric spaces co-exist (or are superposed) is developed. The fact that a curved 'three-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ is perfectly isotropic and is consequently contracted to a 'one-dimensional' isotropic absolute intrinsic metric space, denoted by $\varnothing\hat{\rho}$, which is curved toward the absolute time/absolute intrinsic time 'dimensions' ($\hat{c}\hat{t}/\varnothing\hat{c}\varnothing\hat{t}$) along the vertical is shown.

Keywords: absolute intrinsic Riemann geometry, coexisting absolute intrinsic metric spaces, superposition proceure, resultant absolute intrinsic metric tensor, resultant absolute intrinsic Ricci tensor, contraction to curved 'one-dimensional' isotropic absolute intrinsic metric space

1 Introduction

This second part of this paper is continuation of the derivation of absolute intrinsic Riemann geometry of curved absolute intrinsic Riemannian metric space started

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in the first part [1]. The absolute intrinsic Riemann geometry of a curved 'three-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$, as a curved hyper-surface toward the absolute time/absolute intrinsic time dimensions along the vertical and its projective flat 'three-dimensional' absolute proper intrinsic metric space $\varnothing\mathbf{E}'^3_{ab}$ that underlies a flat relative proper metric space as a flat hyper-surface along the horizontal E'^3, in which the observers are located, isolated in the first part of this paper, is developed in this second part. The new geometry is more all-encompassing (or more complete) than the proposed curved four-dimensional spacetime solely in the gravitational field in the general theory of relativity GR).

The development shall be extended to curved 'two-dimensional' absolute intrinsic Riemannian metric spacetime with absolute intrinsic sub-Riemannian metric tensor $\varnothing\hat{g}_{ik}$ in long-range metric force fields in general in the third part of this paper. The curved two-dimensional absolute intrinsic metric spacetime coexist with flat four-dimensional metric spacetime in long-range metric force fields. The curved absolute intrinsic metric spacetime will support absolute intrinsic metric theory of gravity, while the flat four-dimensional spacetime will support a flat spacetime theory of gravity. Thus the new geometry is naturally equipped to support a two-theory approach to gravitation.

No work on absolute intrinsic Riemann geometry in physics or mathematics exists in the open literature, as far as can be found. This thereby limits the references in this paper to the previous papers of the author on which it is based essentially.

2 Derivation graphically of absolute intrinsic Riemann geometry on curved absolute intrinsic metric space

Let us start with a curved 'two-dimensional' absolute intrinsic metric space (an absolute intrinsic Riemannian metric space) $\varnothing\hat{\mathbb{M}}^2$ with extended curved absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$. The extended curved absolute intrinsic metric 'dimensions' of $\varnothing\hat{\mathbb{M}}^2$ originate from a point $\mathrm{O}(\varnothing x'^1_{ab(0)}, \varnothing\hat{x}'^2_{ab(0)})$ of the underlying flat two-dimensional absolute proper intrinsic metric space $\varnothing\mathbb{E}'_{ab}{}^2$, with extended straight line absolute proper intrinsic metric dimensions, $\varnothing x'^1_{ab}$ and $\varnothing x'^2_{ab}$, as illustrated in Figs. 1a and 1b.

Figures 1a and 1b are two-dimensional forms of Figs. 6a and 6b of the first part of this paper [1], with the absolute intrinsic metric dimensions shown explicitly. Figure. 5 of part one of that paper is reproduced as Fig. 2 of this paper.

The flat 'two-dimensional' absolute proper metric space $\mathbb{E}'^2_{ab}$ in Fig. 1b is the outward manifestation of the flat 'two-dimensional' absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ in Fig. 1a and the flat two-dimensional relative proper metric space $\mathbb{E}'^2$ in Fig. 1b is the outward manifestation of the flat two-dimensional relative proper intrinsic metric space $\varnothing\mathbb{E}'^2$ in Fig. 1a. Figures 1a and 1b are not separated in nature; their separation is done for clarity only. Figures 1a and 1b are valid with respect to

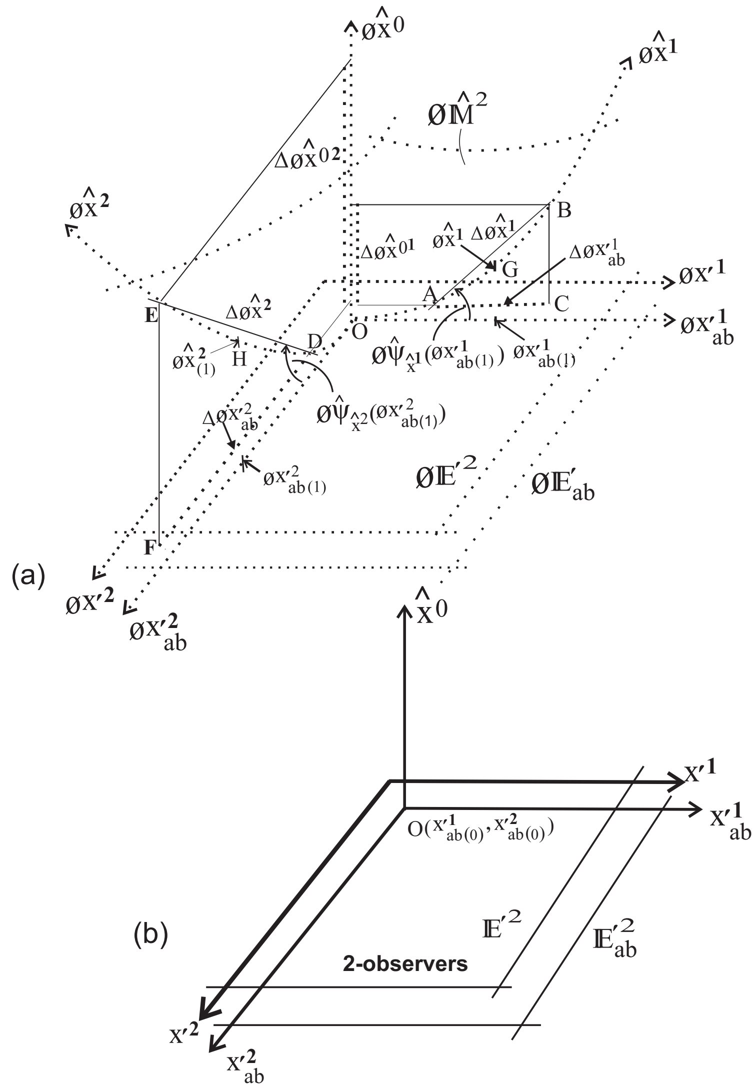

Figure 1: (a) A curved 'two-dimensional' absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^2$ and its projective flat '2-dimensional' absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ underlying a flat 2-dimensional relative proper intrinsic metric space $\varnothing\mathbb{E}'^2$ that automatically appears. (b) The flat absolute proper metric space $\mathbb{E}'^2_{ab}$ as outward manifestation of the flat $\varnothing\mathbb{E}'^2_{ab}$ underlying a flat relative proper metric space $\mathbb{E}'^2$ as outward manifestation of $\varnothing\mathbb{E}'^2$.

2-observers in the relative (or physical) proper metric space $\mathbb{E}'^2$, as indicated.

It is to be recalled from part one of this paper that a 'three-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ is curved (as a hyper-surface) onto the absolute intrinsic

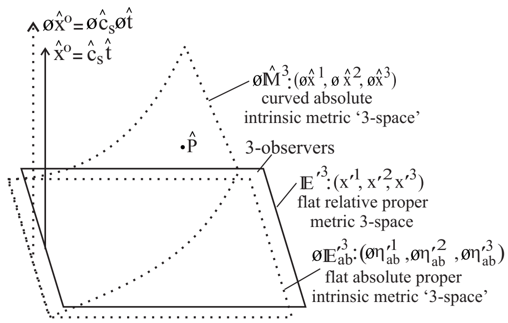

Figure 2: The '3-dimensional' absolute intrinsic metric space curving toward the absolute intrinsic time 'dimension' along the vertical, projects flat 3-dimensional absolute proper intrinsic metric space, which lies underneath (or is embedded in) the flat relative proper metric 3-space along the horizontal; Fig. 5 of [1]

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metric time 'dimension', $\varnothing\hat{x}^0$ $(= \varnothing\hat{c}_s\varnothing\hat{t})$, along the vertical and projects a 'three-dimensional' flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ (as a flat hypersurface) along the horizontal. The curved absolute intrinsic metric space 'dimensions', $\varnothing\hat{x}^1, \varnothing\hat{x}^2$ and $\varnothing\hat{x}^3$, of $\varnothing\hat{\mathbb{M}}^3$, are curved onto the absolute intrinsic metric time 'dimensions' $\varnothing\hat{x}^0$ along the vertical.

A feature of Fig. 1a to note is that an absolute intrinsic metric space 'dimension' $\varnothing\hat{x}^i$; $i = 1$, 2 or 3, of $\varnothing\hat{\mathbb{M}}^3$ that is curved onto $\varnothing\hat{x}^0$ along the vertical, is curved relative to and, hence, lies above (or spans) only the straight line absolute proper intrinsic metric space dimension $\varnothing x'^i_{ab}$ it projects along the horizontal. This is a peculiar feature of absolute intrinsic metric spaces. The situation where the curved $\varnothing\hat{x}^1$ spans $\varnothing x'^2_{ab}$ or/and $\varnothing x'^3_{ab}$ along the horizontal; the curved $\varnothing\hat{x}^2$ spans $\varnothing x'^1_{ab}$ or/and $\varnothing x'^3_{ab}$ along the horizontal; and the curved $\varnothing\hat{x}^3_{ab}$ spans $\varnothing x'^1_{ab}$ or/and $\varnothing x'^2_{ab}$ along the horizontal, does not arise in absolute intrinsic Riemann geometry (of curved absolute intrinsic metric spaces). The implication of this on the structure of the absolute intrinsic metric tensors on absolute intrinsic metric spaces (or in absolute intrinsic Riemann geometry) shall be seen shortly.

Let us take a short segment, AGB $\equiv \Delta\varnothing\hat{x}^1$, about point $\varnothing\hat{x}^1_{(1)}$ along the curved 'dimension' $\varnothing\hat{x}^1$. Then in the limit as $\Delta\varnothing\hat{x}^1$ becomes indefinitely short, that is, in the limit as A$\rightarrow$B, we must let $\Delta\varnothing\hat{x}^1 \rightarrow \mathrm{d}\varnothing\hat{x}^1$ and $\Delta\varnothing x'^1_{ab} \rightarrow \mathrm{d}\varnothing x'^1_{ab}$ in Fig. 1a. It is required in this limit that the length of the arc AGB be equal to the length of the hypotenuse AB of the triangle ABC. Then the absolute intrinsic angle $\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$ is single-valued, being equal to $\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1_{(1)})$ over the arc AGB in this limit.

Similarly by taking a short segment, DHE $\equiv \Delta\varnothing\hat{x}^2$, about point $\varnothing\hat{x}^2_{(1)}$ along the curved 'dimension' $\varnothing\hat{x}^2$ we have in the limit as $\Delta\varnothing\hat{x}^2$ becomes indefinitely short, that is, in the limit as D $\rightarrow$ E, $\Delta\varnothing\hat{x}^2 \rightarrow \mathrm{d}\varnothing\hat{x}^2$ and $\Delta\varnothing x'^2_{ab} \rightarrow \mathrm{d}x'^2_{ab}$ in Figs. 1a&b. It is also require in this limit that the length of the arc DHE be equal to the length of the hypotenuse DE of the triangle DEF. Then the absolute intrinsic angle $\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$ is single-valued, being equal to $\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2_{(1)})$ over the arc DHE.

Thus by displacing the limiting constant elementary straight line intervals, $d\varnothing\hat{x}^1$ and $d\varnothing\hat{x}^2$, defined above along the curved 'dimensions', $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$, respectively, one can attach a locally flat manifold of elementary 'dimensions', $d\varnothing\hat{x}^1$ and $d\varnothing\hat{x}^2$, to every point of the '2-dimensional' curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$. One can then construct geometry, that is, derive single absolute intrinsic metric tensor, single absolute intrinsic Ricci tensor, single absolute intrinsic Riemann scalar, etc (in a lumped parameter fashion), which are valid at every point within the local neighborhood with elementary straight line 'dimensions', $d\varnothing\hat{x}^1$ and $d\varnothing\hat{x}^2$, with respect to 2-observers in the underlying flat relative proper metric 2-space $\mathbb{E}'^2$, and repeat this about every point on the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$. This is the graphical approach to the absolute intrinsic Riemann geometry of a curved '2-dimensional' absolute intrinsic metric space, which has no counterpart in conventional Riemann geometry. The derivation can be easily extended to a curved '3-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ — a '3-dimensional' absolute intrinsic Riemannian metric space.

The elementary straight line intervals, $d\varnothing\hat{x}^1$ and $d\varnothing\hat{x}^2$, defined about point $(\varnothing\hat{x}^1_{(1)}, \varnothing\hat{x}^2_{(1)})$ of the '2-dimensional' absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^2$, project intervals of absolute proper intrinsic metric space 'dimensions', $d\varnothing x'^1_{ab}$ and $d\varnothing x'^2_{ab}$, respectively about the corresponding point $(\varnothing x'^1_{ab(1)}, \varnothing x'^2_{ab(1)})$ of the underlying flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ in Fig. 1a. One obtains the following from elementary coordinate geometry

$$d\varnothing x'^1_{ab} = d\hat{x}^1 \cos\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1_{(1)}) \;\text{ and }\; dx'^2_{ab} = d\varnothing\hat{x}^2 \cos\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2_{(1)}) \qquad (1)$$

Having established the, $\sec\varnothing\psi = \varnothing\gamma$, parametrization of the the intrinsic Lorentz boost in the context of intrinsic special theory of relativity ($\varnothing$SR), in which the rotations of intrinsic affine spacetime coordinates are expressed as trigonometric ratios, cosine and sine, of an intrinsic angle $\varnothing\psi$ in [2], which leads to intrinsic length contraction formulae, $\varnothing\tilde{x} = \varnothing\gamma\varnothing\tilde{x}'\cos\varnothing\psi$ (or $\varnothing\tilde{x}' = (1 - \varnothing v^2/\varnothing c^2)^{1/2}$), intrinsic spacetime coordinate rotations on the vertical intrinsic spacetime hyperplane have uniformly been expressed in terms of the trigonometric ratios of the intrinsic angle $\varnothing\psi$ in the subsequent articles. This is also done in system (1) for the rotation of absolute intrinsic metric space coordinates by an absolute intrinsic angle $\varnothing\hat{\psi}$ on the vertical absolute intrinsic spacetime hyperplane in Fig. 1a.

A 'Riemannian' observer at an arbitrary point $(\varnothing\hat{x}^1, \varnothing\hat{x}^2)$ on $\varnothing\hat{\mathbb{M}}^2$ (this is the proper Riemannian observer), 'observes' Euclidean metric tensor locally about his position. This is guaranteed by the peculiar feature of a curved absolute intrinsic metric space mentioned above that, $\varnothing\hat{x}^1$ is curved relative to (or spans) its projective $\varnothing x_{ab}{}'^1$ along the horizontal only and $\varnothing\hat{x}^2$ is curved relative to (or spans) its projective

$\varnothing x_{ab}{'^2}$ along the horizontal only. These make the curved $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$ locally orthogonal at every point of $\varnothing\hat{\mathbb{M}}^2$.

The proper Riemannian observer therefore derives Euclidean line element in terms of the orthogonal elementary intervals, $\mathrm{d}\varnothing\hat{x}^1$ and $\mathrm{d}\varnothing\hat{x}^2$, at his position as

$$(\mathrm{d}\varnothing\hat{l})^2 = (\mathrm{d}\varnothing\hat{x}^1)^2 + (\mathrm{d}\varnothing\hat{x}^2)^2 = \sum_{i,k=1}^{2} \delta_{ik}\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\hat{x}^k \; . \tag{2a}$$

This local Euclidean absolute intrinsic line element on $\varnothing\hat{\mathbb{M}}^2$ will be written in terms of the coordinate intervals of the underlying projective absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ by the Euclidean 2-observers in the relative proper metric space $\mathbb{E}'^2$, by virtue of system (1) as

$$(\mathrm{d}\varnothing\hat{l})^2 = (\mathrm{d}\varnothing\hat{x}^1)^2 + (\mathrm{d}\varnothing\hat{x}^2)^2 = (\mathrm{d}\varnothing x'^1_{ab})^2 \sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) + (\mathrm{d}\varnothing x'^2_{ab})^2 \sec\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) \tag{2b}$$

or

$$(\mathrm{d}\varnothing\hat{l})^2 = \sum_{i,k=1}^{2} \sec\varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i)\sec\varnothing\hat{\psi}_{\hat{x}^k}(\hat{x}^k)\delta_{ik}\mathrm{d}\varnothing x'^i_{ab}\mathrm{d}\varnothing x'^k_{ab} \; , \tag{2c}$$

or

$$(\mathrm{d}\varnothing\hat{l})^2 = \sum_{i,k=1}^{2} \varnothing\hat{g}_{ik}(\varnothing\hat{x}^1, \varnothing\hat{x}^2)\mathrm{d}\varnothing x'^i_{ab}\mathrm{d}\varnothing x'^k_{ab} \; . \tag{2d}$$

The absolute intrinsic metric tensor $\varnothing\hat{g}_{ik}$ on the curved $\varnothing\hat{\mathbb{M}}^2$ in Fig. 1b, given in terms of absolute intrinsic angles, $\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$ and $\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2_{(2)})$, which Eqs. (2c) and (2d) imply is purely diagonal. It is the following

$$\begin{aligned} \varnothing\hat{g}_{ik} &= \sec\varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i)\sec\varnothing\hat{\psi}_{\hat{x}^k}(\varnothing\hat{x}^k)\delta_{ik} \; ; \\ &= \begin{pmatrix} \sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) & 0 \\ 0 & \sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) \; . \end{pmatrix} \end{aligned} \tag{3}$$

Thus the locally flat region of the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$ bounded by the orthogonal elementary straight line coordinate intervals, $\mathrm{d}\varnothing\hat{x}^1$ and $\mathrm{d}\varnothing\hat{x}^2$, about an arbitrary point $(\varnothing\hat{x}^1, \varnothing\hat{x}^2)$ of $\varnothing\hat{\mathbb{M}}^2$, which possesses Euclidean metric tensor δ_{ik} with respect to a Riemannian observer located within this locally flat region of $\varnothing\hat{\mathbb{M}}^2$, possesses the absolute intrinsic sub-Riemannian metric tensor $\varnothing\hat{g}_{ik}$ with respect to all Euclidean observers in the underlying flat relative proper metric space $\mathbb{E}'^2$ in Figs. 1a and 1b. Figures 1a and 1b shall be referred to as Figs. 1a&b for brevity henceforth.

It is however inappropriate for the absolute proper intrinsic metric coordinate intervals, $\mathrm{d}\varnothing x'^1_{ab}$ and $\mathrm{d}\varnothing x'^2_{ab}$, of $\varnothing\mathbb{E}'^2_{ab}$ to appear in the absolute intrinsic line element on the curved $\varnothing\hat{\mathbb{M}}^2$, with respect to the Euclidean observers in $\mathbb{E}'^2$, as happens in

Eqs. (2b) – (2d). Rather the absolute intrinsic geodesic on $\varnothing\hat{\mathbb{M}}^2$ must be written in terms of the orthogonal locally straight elementary absolute intrinsic coordinate intervals, $d\varnothing\hat{x}^1$ and $d\varnothing\hat{x}^2$, on $\varnothing\hat{\mathbb{M}}^2$ and the absolute intrinsic metric tensor $\varnothing\hat{g}_{ik}$ of Eq. (3), with respect to all Euclidean observers in $\mathbb{E}'^2$).

In order to be able to write the absolute intrinsic line element (2c) or (2d) in terms of the elementary absolute intrinsic coordinate intervals, $d\varnothing\hat{x}^1$ and $d\varnothing\hat{x}^2$, on $\varnothing\hat{\mathbb{M}}^2$ with respect to the Euclidean observers (in $\mathbb{E}'^2$), the following invariance must obtain.

$$\sum_{i,k}^{2} \delta_{ik} d\varnothing x'^i_{ab} d\varnothing x'^k_{ab} = \sum_{i,k}^{2} \delta_{ik} d\varnothing\hat{x}^i d\varnothing\hat{x}^k \ . \tag{4}$$

This is the discrete version (in the graphical approach), of absolute intrinsic local Euclidean invariance (A∅LEI), at every point on $\varnothing\hat{\mathbb{M}}^2$ with respect to Euclidean observers in $\mathbb{E}'^2$ in Figs. 1a&b.

The required absolute intrinsic local Euclidean invariance (A∅LEI) (4) on $\varnothing\hat{\mathbb{M}}^2$ with respect to observers in $\mathbb{E}'^2$ obtains naturally (or is trivial). It arises from two facts:

1. The absolute intrinsic Euclidean line element (2a) exists at every point on $\varnothing\hat{\mathbb{M}}^2$, because of the peculiar feature of the curved absolute intrinsic 'dimensions', $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$, of $\varnothing\hat{\mathbb{M}}^2$ relative to their projective straight line absolute proper intrinsic dimensions, $\varnothing x'^1_{ab}$ and $\varnothing x'^2_{ab}$, respectively along the horizontal, mentioned above.

2. There is absolutism of the absolute intrinsic metric coordinates of $\varnothing\hat{\mathbb{M}}^2$ expressed by, $d\varnothing x'^1_{ab} = d\varnothing\hat{x}^1$ and $d\varnothing x'^2_{ab} = d\varnothing\hat{x}^2$, with respect to the Euclidean observers in $\mathbb{E}'^2$. These trivial (or invariant) intrinsic coordinate projection relations, which are possible because $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$ are absolute, along with Eq. (2a), establishes the absolute intrinsic Euclidean invariance (4) on $\varnothing\hat{\mathbb{M}}^2$ with respect to Euclidean observers on $\mathbb{E}'^2$ naturally in Figs. 1a&b.

Apart from the absolute intrinsic Euclidean invariance (A∅LEI) on $\varnothing\hat{\mathbb{M}}^2$ with respect to the Euclidean observers on $\mathbb{E}'^2$, stated by Eq. (4), the intrinsic coordinate projection relations of system (1), state the fact of definite curvatures of $\varnothing\hat{\mathbb{M}}^2$ (or of $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$) explicitly, which leads to the absolute intrinsic metric tensor (3) on $\varnothing\hat{\mathbb{M}}^2$, with respect to the Euclidean observers in $\mathbb{E}'^2$. In other words, both the the absolute intrinsic Euclidean invariance (A∅LEI) (4) and the absolute intrinsic sub-Riemannian metric tensor (3) obtain on the curved $\varnothing\hat{\mathbb{M}}^2$ with respect to Euclidean observers in $\mathbb{E}'^2$, in the context of the absolute intrinsic Riemann geometry.

Application of the absolute intrinsic Euclidean invariance (4) then allows us to replace $d\varnothing x'^i_{ab} d\varnothing x'^k_{ab}$ by $d\varnothing\hat{x}^i d\varnothing\hat{x}^k$ in Eqs. (2c) and (2d) yielding the following

$$(d\varnothing\hat{l})^2 = \sum_{i,k=1}^{2} \sec\varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i) \sec\varnothing\hat{\psi}_{\hat{x}^k}(\varnothing\hat{x}^k) \delta_{ik} d\varnothing\hat{x}^i d\varnothing\hat{x}^k \ , \tag{5a}$$

or

$$(\mathrm{d}\varnothing\hat{l})^2 = \sum_{i,k=1}^{2} \varnothing\hat{g}_{ik}(\varnothing\hat{x}^1, \varnothing\hat{x}^2)\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\varnothing\hat{x}^k \ . \tag{5b}$$

The absolute intrinsic line element (5a) or (5b) and the absolute intrinsic metric tensor (3) on $\varnothing\hat{\mathbb{M}}^2$ admit of easy generalizations to the case of '3-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$. The absolute intrinsic geodesic is given at an arbitrary point $(\varnothing\hat{x}^1, \varnothing\hat{x}^2, \varnothing\hat{x}^3)$ on $\varnothing\hat{\mathbb{M}}^3$, which corresponds to point $(\varnothing x'^1_{ab}, \varnothing x'^2_{ab}, \varnothing x'^3_{ab})$ on the underlying projective flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$, with respect to observers in the relative proper metric space $\mathbb{E}'^3$ as

$$(\mathrm{d}\varnothing\hat{l})^2 = \sum_{i,k=1}^{3} \sec\varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i)\sec\varnothing\hat{\psi}_{\hat{x}^k}(\varnothing\hat{x}^k)\delta_{ik}\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\varnothing\hat{x}^k \ ; \tag{6a}$$

$$= \sum_{i,k=1}^{3} \varnothing\hat{g}_{ik}(\varnothing\hat{x}^1, \varnothing\hat{x}^2, \varnothing\hat{x}^3)\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\hat{x}^k \ . \tag{6b}$$

The absolute intrinsic metric tensor is a 3×3 diagonal matrix with elements, $\sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$, $\sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$ and $\sec^2\varnothing\hat{\psi}_{\hat{x}^3}(\varnothing\hat{x}^3)$, in this case.

In the graphical approach to the absolute intrinsic Riemann geometry of curved absolute intrinsic metric spaces, once one measures the absolute intrinsic angles $\varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i)$ on $\varnothing\hat{\mathbb{M}}^2$ or $\varnothing\hat{\mathbb{M}}^3$, of the inclinations of the intervals $\mathrm{d}\varnothing\hat{x}^i$ of the curved absolute intrinsic 'dimension' $\varnothing\hat{x}^i$ to the respective underlying projective straight line absolute proper intrinsic dimensions $\varnothing x'^i_{ab}$ at a point along the curved $\varnothing\hat{x}^i$. One then obtains the absolute intrinsic metric tensor from Eqs. (6a) and (6b), shown more explicitly as Eq. (3), at that point. There is no corresponding graphical approach in conventional Riemann geometry, as far as I know.

The method of synthesizing the absolute intrinsic metric tensor by substituting the numerical values of, $\sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$, $\sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$ and $\sec^2\varnothing\hat{\psi}_{\hat{x}^3}(\varnothing\hat{x}^3)$, into the elements of the absolute intrinsic metric tensor in Eq. (3), within an elementary locally flat neighborhood about every point of $\varnothing\hat{\mathbb{M}}^2$ or $\varnothing\hat{\mathbb{M}}^3$, in the absolute intrinsic Riemann geometry, is obviously numerical.

The absolute intrinsic metric tensor of the absolute intrinsic Riemann geometry of a curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ is purely diagonal (or is sub-Riemannian) always. This is a consequence of the peculiar feature of the absolute intrinsic metric spaces described in the second and third paragraphs of this section namely, all the absolute intrinsic 'dimensions' of $\varnothing\hat{\mathbb{M}}^3$ are curved onto the absolute intrinsic time 'dimension', $\varnothing\hat{x}^0 \equiv \varnothing\hat{c}_s\varnothing\hat{t}$, along the vertical, such that each curved absolute intrinsic 'dimension' $\varnothing\hat{x}^k$ of $\varnothing\hat{\mathbb{M}}^3$ lies above its projective straight line absolute proper intrinsic dimension $\varnothing x'^k_{ab}$ in $\varnothing\mathbb{E}'^3_{ab}$ (along the horizontal). Consequently each curved absolute intrinsic 'dimension' $\varnothing\hat{x}^k$ is a plane curve on the vertical $\varnothing x'^k_{ab}\varnothing\hat{x}^0-$hyperplane. The cross terms, $\mathrm{d}\varnothing\hat{x}^1\mathrm{d}\varnothing\hat{x}^2$, $\mathrm{d}\varnothing\hat{x}^1\mathrm{d}\varnothing\hat{x}^3$ and $\mathrm{d}\varnothing\hat{x}^1\mathrm{d}\varnothing\hat{x}^3$ are therefore naturally precluded in the absolute intrinsic line elements (5a) or (5b) and (6a) or (6b),

2.1 The absolute intrinsic dimensionless curvature parameter of a curved 'one-dimensional' absolute intrinsic metric space on a vertical absolute proper intrinsic space - absolute intrinsic time hyperplane

Let us consider a curve **s** on the horizontal $\varnothing x'^1_{ab}\varnothing x'^2_{ab}-$ plane in $\varnothing\mathbb{E}'^3_{ab}$ shown in Fig. 3(a). The Eulerian curvature κ_{Eul} (in honor of Euler), of the curve s at point P in Fig. 3(a) is given from definition, see chapter 1 of [3], as

$$\frac{\mathrm{d}\phi}{\mathrm{d}s}\hat{n} = \frac{\mathrm{d}\hat{t}}{\mathrm{d}s} = \kappa_{\mathrm{Eul}}\hat{n} \ , \tag{7}$$

where $\hat{t}$ and $\hat{n}$ are unit tangent vector and unit normal vector respectively, to the curve s at P and the angle ϕ is the inclination of the curve s to the axis x'_{ab} of $\varnothing\mathbf{E}'^2_{ab}$ in Fig. 3a . Hence,

$$\frac{d\phi}{ds} = \mid \frac{\mathrm{d}\hat{t}}{ds} \mid = \mid \kappa_{\mathrm{Eul}}\hat{n} \mid = \kappa_{\mathrm{Eul}} \ . \tag{8}$$

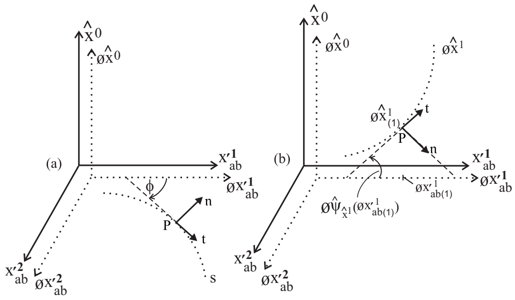

Figure 3: Deriving the absolute intrinsic curvature parameter of a curved 'one-dimensional' absolute intrinsic metric space on a vertical absolute proper intrinsic space - absolute intrinsic time plane.

Now let this same plane curve **s** be on the vertical $\varnothing x'^1_{ab}\varnothing\hat{x}^0-$plane as shown in Fig. 3b, where it has been re-denoted by $\varnothing\hat{x}^1$. It now carries a hat label since it is now a 'one-dimensional absolute intrinsic metric space (or an absolute intrinsic metric space 'dimension') on the $\varnothing x'^1_{ab}\varnothing\hat{x}^0-$plane. Again the curvature of $\varnothing\hat{x}^1$ is given by Eq. (7), except that the unit normal vector $\hat{n}$ projects components $\hat{n}\sin\varnothing\hat{\psi}(\hat{x}^1)$ into the absolute proper intrinsic metric space dimension $\varnothing x'^1_{ab}$ along the horizontal.

Hence the curvature of $\varnothing\hat{x}^1$ that is valid with respect to observers in (different 'frames' in) the underlying relative proper Euclidean space $\mathbb{E}'^2$ in Fig. 3b is

$$\frac{d\hat{t}}{ds} = \hat{n}\sin\varnothing\hat{\psi}(\varnothing\hat{x}^1)\kappa_{\text{Eul}} \; . \tag{9}$$

Let us define the absolute intrinsic Riemannian curvature $\varnothing\hat{\kappa}_{\text{Riem}}(\varnothing\hat{x}^1)$ of the plane curve $\varnothing\hat{x}^1$ (which is a one-dimensional absolute intrinsic Riemannian metric space), at an arbitrary point P (with arbitrary absolute intrinsic coordinate $\varnothing\hat{x}^1$) in Fig. 3b as

$$\varnothing\hat{\kappa}_{\text{Riem}}(\varnothing\hat{x}^1) \; = \mid \frac{d\hat{t}}{ds} \mid = \mid \hat{n} \mid \sin\varnothing\hat{\psi}(\varnothing\hat{x}^1)\kappa_{\text{Eul}} \; , \tag{10}$$

or

$$\varnothing\hat{\kappa}_{\text{Riem}}(\varnothing\hat{x}^1) = \sin\varnothing\hat{\psi}(\varnothing\hat{x}^1)\kappa_{\text{Eul}} \; . \tag{11}$$

The dimensionless intrinsic parameter $\sin\varnothing\hat{\psi}(\varnothing\hat{x}^1)$ shall be referred to as absolute intrinsic curvature parameter at an arbitrary point $\varnothing\hat{x}^1$ along the curved 'one-dimensional' absolute intrinsic metric space $\varnothing\hat{x}^1$ and denoted by $\varnothing\hat{k}(\varnothing\hat{x}^1)$. It is an absolute intrinsic parameter since $\varnothing\hat{x}^1$ is a 'one-dimensional' absolute intrinsic metric space (or a 'dimension' of 'three-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$). Hence Eq. (11) shall be re-written as follows

$$\varnothing\hat{\kappa}_{\text{Riem}}(\varnothing\hat{x}^1) = \varnothing\hat{k}(\varnothing\hat{x}^1)\kappa_{\text{Eul}} \; , \tag{12}$$

where

$$\varnothing\hat{k}(\varnothing\hat{x}^1) = \sin\varnothing\hat{\psi}(\varnothing\hat{x}^1) \; . \tag{13}$$

Since the absolute intrinsic angle $\varnothing\hat{\psi}$ has constant zero value along plane curves in the underlying flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$, the absolute intrinsic Riemannian curvature of a plane curve on $\varnothing\mathbb{E}'^3_{ab}$, such as in Fig. 3a is zero.

The absolute intrinsic curvature parameters, $\varnothing\hat{k}_{\hat{x}^1}$ and $\varnothing\hat{k}_{\hat{x}^2}$, at points $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$ of the curved absolute intrinsic 'dimensions', $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$, respectively in Figs. 1a&b, are given as follows by virtue of definition (13),

$$\left.\begin{array}{rcl} \varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1) & = & \sin\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) \\ \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2) & = & \sin\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) \end{array}\right\} \tag{14}$$

Since the absolute intrinsic angle $\varnothing\hat{\psi}_{\hat{x}^i}$ measures the inclination of the curved absolute intrinsic 'dimension' $\varnothing\hat{x}^{\,i}$ on the vertical $\varnothing x'^i_{ab}\varnothing\hat{x}^0-$hyperplane, relative to the underlying flat absolute proper intrinsic space $\varnothing\mathbb{E}'^3_{ab}$ (as a hyper-surface) along the horizontal, it has the same value with respect to all 'frames' (or all observers) in the underlying flat relative proper Euclidean 3-space $\mathbb{E}'^3$ (also as a hyper-surface) along the horizontal. Hence the absolute intrinsic curvature parameter $\varnothing\hat{k}_{\hat{x}^{\,i}}$ has the same value with respect to all 'frames' (or all observers) in the underlying flat relative (or physical) proper Euclidean 3-space $\mathbb{E}'^3$.

2.2 Expressing the components of the absolute intrinsic metric tensor in terms of absolute intrinsic curvature parameters in absolute intrinsic Riemann geometry

One obtains the following from the components of the absolute intrinsic metric tensor in Eq. (3),

$$\begin{aligned}
\varnothing\hat{g}_{11} &= \sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) = (1-\sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1))^{-1}\ ;\\
\varnothing\hat{g}_{22} &= \sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) = (1-\sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2))^{-1}\ ;\\
\varnothing\hat{g}_{12} &= \varnothing\hat{g}_{21} = 0\ .
\end{aligned} \tag{15}$$

Then by using system (14) in system (15), the components of the absolute intrinsic metric tensor are given in terms of absolute intrinsic curvature parameters as

$$\begin{aligned}
\varnothing\hat{g}_{11} &= (1-\varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2)^{-1}\ ;\\
\varnothing\hat{g}_{22} &= (1-\varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2)^{-1}\ ;\\
\varnothing\hat{g}_{21} &= \varnothing\hat{g}_{12} = 0\ .
\end{aligned} \tag{16}$$

Hence,

$$\varnothing\hat{g}_{ik} = \begin{pmatrix} \dfrac{1}{1-\varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x})^2} & 0 \\ 0 & \dfrac{1}{1-\varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2} \end{pmatrix} . \tag{17}$$

Extension to the case of a '3-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ is straight forward, in which case the 2×2 diagonal matrix of Eq. (17) becomes a 3×3 diagonal matrix.

Thus the absolute intrinsic metric tensor has parametric dependence on the square of the absolute intrinsic curvature parameters in absolute intrinsic Riemann geometry. The absolute intrinsic curvature parameters shall ultimately be related to the absolute intrinsic parameter(s) of the absolute intrinsic metric force field that establishes absolute intrinsic Riemann geometry with further development.

2.3 Recovering local intrinsic Euclidean line element on a curved absolute intrinsic metric space with respect to Euclidean observers

The flat 'two-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{E}}^2$ underlying the flat 'two-dimensional' absolute metric space $\hat{\mathbb{E}}^2$ (like Fig. 7 of part one of this paper [1], drawn for $\varnothing\hat{\mathbb{E}}^3$ and $\hat{\mathbb{E}}^3$), existed prior to the evolution of curved $\varnothing\hat{\mathbb{M}}^2$ and its underlying flat $\varnothing\mathbb{E}'^2_{ab}$ and $\mathbb{E}'^2$ in Figs. 1a&b of this article. The absolute intrinsic Euclidean line element (2a) obtains at every point of the flat $\varnothing\hat{\mathbb{E}}^2$ prior to the evolving into the curved $\varnothing\hat{\mathbb{M}}^2$ in the geometry of Figs. 1a&b. Figure 7 of [1] is reproduced as Fig. 4 of this article.

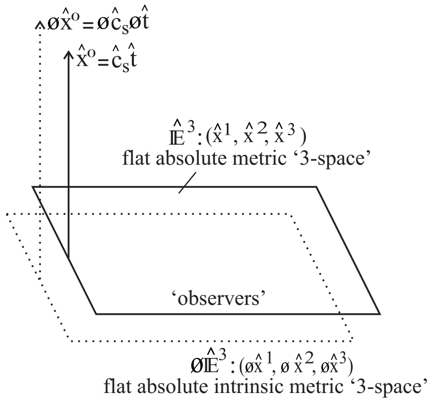

Figure 4: Flat '3-dimensional' absolute metric space - absolute metric time is underlay by flat '3-dimensional' absolute intrinsic metric space - absolute intrinsic metric time; Fig. 7 of [1].

On the other hand, the absolute intrinsic sub-Riem- annian line element (2c) and (2d) obtains on $\varnothing\hat{\mathbb{M}}^2$ with respect to Euclidean observers in the underlying flat relative proper metric space $\mathbb{E}'^2$ in Figs. 1a and 1b. An extra term shall be added to the right-hand side of Eq. (2c) or (2d) in order to recover the absolute intrinsic Euclidean line element locally on $\varnothing\hat{\mathbb{M}}^2$ with respect to observers in $\mathbb{E}'^2$ in Figs. 1a&b, and the result shall be extended to $\varnothing\hat{\mathbb{M}}^3$ in this sub-section.

One observes from Figs. 1a&b that the interval $d\varnothing\hat{x}^1$ of the curved absolute intrinsic 'dimension' $\varnothing\hat{x}^1$ projects component $d\varnothing x'^1_{ab}$ into the underlying absolute proper intrinsic metric space dimension $\varnothing x'^1_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$ and a component $d\varnothing\hat{x}^{01}$ (shown as $\Delta\varnothing\hat{x}^{01}$), into the vertical absolute intrinsic time 'dimension' $\varnothing\hat{x}^0$. Similarly the interval $d\varnothing\hat{x}^2$ of the curved absolute intrinsic metric space 'dimension' $\varnothing\hat{x}^2$ of $\varnothing\hat{\mathbb{M}}^2$, projects component $d\varnothing x'^2_{ab}$ into the underlying absolute proper intrinsic metric space dimension $\varnothing x'^2_{ab}$ and component $d\varnothing\hat{x}^{02}$ (shown as $\Delta\varnothing\hat{x}^{02}$), into the vertical absolute intrinsic metric time 'dimension' $\varnothing\hat{x}^0$.

The components, $d\varnothing x'^1_{ab}$ and $d\varnothing x'^2_{ab}$, projected into the underlying flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ have been made use of in deriving the absolute intrinsic metric line element (2c) or (2d) (which become converted to Eq. (5a) or (5b) by virtue of the absolute intrinsic Euclidean invariance (4)), with respect to observers in the relative proper metric space $\mathbb{E}'^2$, while the components $d\varnothing\hat{x}^{01}$ and $d\varnothing\hat{x}^{02}$ have been left out. This is done because the absolute time and absolute intrinsic time 'dimensions' along the vertical, being not curved from their vertical position, cannot appear in the absolute intrinsic line element on the curved $\varnothing\hat{\mathbb{M}}^2$. The absolute intrinsic time 'dimension' can at best appear in the Gaussian form of absolute intrinsic line element on $\varnothing\hat{\mathbb{M}}^2$, with respect to observers in the relative proper physical Euclidean 2-space $\mathbb{E}'^2$ as

$$d\varnothing\hat{s}^2 = (d\varnothing\hat{x}^0)^2 - \sum_{i,k=1}^{2} \varnothing\hat{g}_{ik} d\varnothing x'^i_{ab} d\varnothing x'^k_{ab} \ , \tag{18a}$$

which by virtue of A∅LEI of Eq. (4) becomes,

$$\mathrm{d}\varnothing\hat{s}^2 = (\mathrm{d}\varnothing\hat{x}^0)^2 - \sum_{i,k=1}^{2} \varnothing\hat{g}_{ik} \mathrm{d}\varnothing\hat{x}^i \mathrm{d}\varnothing\hat{x}^k \ . \tag{18b}$$

It is this Gaussian form that shall often be written on $\varnothing\hat{\mathbb{M}}^2$ with respect to observers in $\mathbb{E}'^2$ in Figs. 1a&b of this article.

The absolute intrinsic coordinate intervals $\mathrm{d}\varnothing\hat{x}^{01}$ and $\mathrm{d}\varnothing\hat{x}^{02}$ projected along $\varnothing\hat{x}^0$ cannot appear even in the Gaussian form (18b) with respect to observers in the relative proper Euclidean 2-space $\mathbb{E}'^2$. They are metrically elusive to these observers and they shall be referred to as 'non-metric' components consequently. On the other hand, the components $\mathrm{d}\varnothing x'^1_{ab}$ and $\mathrm{d}\varnothing x'^2_{ab}$ projected into the underlying flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$, which have been used in deriving the absolute intrinsic metric line element (2c) or (2d), shall be referred to as metric components with respect to observers in the relative proper metric Euclidean 2-space $\mathbb{E}'^2$.

Although the 'non-metric' absolute intrinsic coordinate intervals, $\mathrm{d}\varnothing\hat{x}^{01}$ and $\mathrm{d}\varnothing\hat{x}^{02}$, projected along the absolute intrinsic time 'dimension' $\varnothing\hat{x}^0$ are elusive and must be disregarded when deriving absolute intrinsic metric line element on the curved 'two-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$, with respect to observers in $\mathbb{E}'^2$ in Figs. 1a&b, as done in obtaining the absolute intrinsic line element (2c) and (2d) and the absolute intrinsic sub-Riemannian metric tensor (17), let us temporarily put both the metric components, $\mathrm{d}\varnothing x'^1_{ab}$ and $\mathrm{d}\varnothing x'^2_{ab}$, and the 'non-metric' components, $\mathrm{d}\varnothing\hat{x}^{01}$ and $\mathrm{d}\varnothing\hat{x}^{02}$, into consideration in order to recover the absolute intrinsic Euclidean line element and the absolute intrinsic Euclidean metric tensor on $\varnothing\hat{\mathbb{M}}^2$ with respect to observers in $\mathbb{E}'^2$. Thus let us apply the Pythagorean formula to triangles ABC and DEF in Figs. 1a&b to have the following

$$(\mathrm{d}\varnothing\hat{x}^1)^2 = (\mathrm{d}\varnothing x'^1_{ab})^2 + (\mathrm{d}\varnothing\hat{x}^{01})^2 \ \text{ and } \ (\mathrm{d}\varnothing\hat{x}^2)^2 = (\mathrm{d}\varnothing x'^2_{ab})^2 + (\mathrm{d}\varnothing\hat{x}^{02})^2 \ . \tag{19}$$

But $\mathrm{d}\varnothing\hat{x}^{01}$ and $\mathrm{d}\varnothing\hat{x}^{02}$ are given in terms of absolute intrinsic angles, $\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$ and $\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$, and intervals, $\mathrm{d}\varnothing\hat{x}^1$ and $\mathrm{d}\varnothing\hat{x}^2$, respectively as

$$\mathrm{d}\hat{x}^{01} = \mathrm{d}\varnothing\hat{x}^1 \sin\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) \ \text{ and } \ \mathrm{d}\varnothing\hat{x}^{02} = \mathrm{d}\varnothing\hat{x}^2 \sin\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) \ . \tag{20}$$

The following obtain from systems (19) and (20)

$$\begin{aligned} (\mathrm{d}\varnothing x'^1_{ab})^2 &= (\mathrm{d}\varnothing\hat{x}^1)^2 - (\mathrm{d}\varnothing\hat{x}^1)^2 \sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1); \\ (\mathrm{d}\varnothing x'^2_{ab})^2 &= (\mathrm{d}\varnothing\hat{x}^2)^2 - (\mathrm{d}\varnothing\hat{x}^2)^2 \sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) \ . \end{aligned} \tag{21}$$

And from system (21) Euclidean line element is constructed in terms of the components, $d\varnothing x'^1_{ab}$ and $d\varnothing x'^2_{ab}$, projected into $\varnothing\mathbb{E}'^2_{ab}$ as

$$\begin{aligned}(d\varnothing l')^2 &= (d\varnothing x'^1_{ab})^2 + (d\varnothing x'^2_{ab})^2 \\ &= (d\varnothing\hat{x}^1)^2 - (d\varnothing\hat{x}^1)^2 \sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) + (d\varnothing\hat{x}^2)^2 - \\ &\quad (d\varnothing\hat{x}^2)^2 \sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)\ . \end{aligned} \tag{22}$$

Then by using, $d\varnothing\hat{x}^i = d\varnothing x'^i_{ab} \sec\varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i); i = 1, 2$, which follows from system (1), Eq. (22) becomes the following

$$\begin{aligned}(d\varnothing l')^2 &= (d\varnothing x'^1_{ab})^2 \Big(\sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) - \tan^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)\Big) \\ &\quad + d\varnothing x'^2_{ab})^2 \Big(\sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) - \tan^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)\Big)\ , \end{aligned} \tag{23}$$

which upon using $\sec^2\varnothing\psi - \tan^2\varnothing\psi = 1$, gives $(d\varnothing l')^2 = (d\varnothing x'^1_{ab})^2 + (d\varnothing x'^2_{ab})^2$.

Thus by considering the 'non-metric' components, $d\varnothing\hat{x}^{01}$ and $d\varnothing\hat{x}^{02}$, projected into the absolute intrinsic time 'dimension' $\varnothing\hat{x}^0$ along the vertical along with the metric components, $d\varnothing x'^1_{ab}$ and $d\varnothing x'^2_{ab}$, projected into the absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ along the horizontal in Figs. 1a&b, in constructing absolute intrinsic line element on $\varnothing\hat{\mathbb{M}}^2$, with respect to observers in the relative proper metric space $\mathbb{E}'^2$, the absolute intrinsic Euclidean line element at every point on the flat absolute intrinsic metric space $\varnothing\hat{\mathbb{E}}^2$ that evolves into the curved $\varnothing\hat{\mathbb{M}}^2$ is recovered at every point on $\varnothing\hat{\mathbb{M}}^2$, with respect to observers in $\mathbb{E}'^2$.

The absolute intrinsic Euclidean invariance (A∅LEI), which obtains naturally on $\varnothing\hat{\mathbb{M}}^2$ with respect to observers in $\mathbb{E}'^2$, stated as Eq. (4) has thus been constructed (or recovered) by putting the projective 'non-metric' intrinsic coordinate intervals, $d\varnothing\hat{x}^{01}$ and $d\varnothing\hat{x}^{02}$, and the projective metric intrinsic coordinate intervals, $d\varnothing x'^1_{ab}$ and $d\varnothing x'^2_{ab}$, into consideration in constructing the absolute intrinsic line element on $\varnothing\hat{\mathbb{M}}^2$.

2.4 Deriving a tensorial statement for absolute intrinsic local Euclidean invariance on absolute intrinsic Riemannian metric spaces

Let us by virtue of absolute intrinsic local Euclidean invariance (A∅LEI) on $\varnothing\hat{\mathbb{M}}^2$, when the projective 'non-metric' intrinsic coordinate intervals $d\varnothing\hat{x}^{01}$ and $d\varnothing\hat{x}^{02}$ and the projective metric intrinsic coordinate intervals, $d\varnothing x'^1_{ab}$ and $d\varnothing x'^2_{ab}$, are put into consideration in constructing the absolute intrinsic line element on $\varnothing\hat{\mathbb{M}}^2$, established above, and written as Eq. (4) earlier, replace the elementary absolute proper intrinsic coordinate intervals, $d\varnothing x'^1_{ab}$ and $d\varnothing x'^2_{ab}$, by absolute intrinsic coordinate intervals, $d\varnothing\hat{x}^1$ and $d\varnothing\hat{x}^2$, respectively, at the right-hand side of Eq. (23) to have

$$\begin{aligned}(d\varnothing l')^2 &= (d\varnothing x'^1_{ab})^2 + (d\varnothing x'^2_{ab})^2 = (d\varnothing\hat{x}^1)^2 \Big(\sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) - \tan^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)\Big) \\ &\quad + (d\varnothing\hat{x}^2)^2 \Big(\sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) - \tan^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)\Big)\ . \end{aligned} \tag{24}$$

Equation (24) states formally the absolute intrinsic local Euclidean invariance,

$$(\mathrm{d}\varnothing x'^1_{ab})^2 + (\mathrm{d}\varnothing x'^2_{ab})^2 = (d\hat{x}^1)^2 + (d\hat{x}^2)^2 \ , \quad \text{or} \quad (\mathrm{d}\varnothing l'_{ab})^2 = d\varnothing\hat{l}^2 \ ,$$

on $\varnothing\hat{\mathbb{M}}^2$, which has also been stated by the invariance (4), in absolute intrinsic Riemann geometry. Thus the absolute intrinsic line element recovered at every point of $\varnothing\hat{\mathbb{M}}^2$ with respect to observers in $\mathbb{E}'^2$, when both the projective metric and 'non-metric' intrinsic coordinate intervals are put into consideration is the following

$$\begin{aligned}(\mathrm{d}\varnothing\hat{l})^2 \;\; = \;\; & (\mathrm{d}\varnothing\hat{x}^1)^2 \Big(\sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) \; - \tan^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)\Big) \\ & + \, (\mathrm{d}\varnothing\hat{x}^2)^2 \Big(\sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) \; - \tan^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)\Big) \ . \end{aligned} \tag{25}$$

It is for the purpose of recovering the absolute intrinsic Euclidean line element (25) on the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$ with respect to every observer in the underlying flat relative (or physical) proper metric space $\mathbb{E}'^2$ in Figs. 1a&b that the 'non-metric' intrinsic coordinate intervals, $\mathrm{d}\varnothing\hat{x}^{01}$ and $\mathrm{d}\varnothing\hat{x}^{02}$, projected along the absolute intrinsic time 'dimension' $\varnothing\hat{x}^0$ along the vertical have been considered along with the intrinsic metric coordinate intervals, $\mathrm{d}\varnothing x'^1_{ab}$ and $\mathrm{d}\varnothing x'^2_{ab}$, projected into $\varnothing\mathbb{E}'^2_{ab}$ along the horizontal in that figure, in deriving the absolute intrinsic line element in Eq. (19) – (23). However observers in the relative proper Euclidean space $\mathbb{E}'^2$ must actually make use of the metric intrinsic coordinate intervals, $\mathrm{d}\varnothing x'^1_{ab}$ and $\mathrm{d}\varnothing x'^2_{ab}$, projected into $\varnothing\mathbb{E}'^2_{ab}$ solely in deriving the absolute intrinsic sub-Riemannian metric line element (2b) or (5b) on $\varnothing\hat{\mathbb{M}}^2$ with respect to themselves, since the 'non-metric' intrinsic coordinate intervals are metrically elusive to these observers.

Now, by subtracting the absolute intrinsic metric line element (2b) (obtained by using the metric intrinsic coordinate intervals only) from the absolute intrinsic Euclidean line element (24), one obtains the absolute intrinsic line element $(\mathrm{d}\varnothing\hat{l}_{\mathrm{nm}})^2$ on the 'non-metric' sub-space formed by the 'non-metric' components $\mathrm{d}\varnothing\hat{x}^{01}$ and $\mathrm{d}\varnothing\hat{x}^{02}$ projected into the absolute intrinsic time 'dimension' $\varnothing\hat{x}^0$ along the vertical in Figs. 1a&b as

$$(\mathrm{d}\varnothing\hat{l}_{\mathrm{nm}})^2 = \tan^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)(\mathrm{d}\varnothing x'^1_{ab})^2 + \tan^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)(\mathrm{d}\varnothing x'^2_{ab})^2 \ . \tag{26}$$

Observe that $(\mathrm{d}\varnothing\hat{l}_{\mathrm{nm}})^2$ vanishes for, $\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) = \varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) = 0$, which will be the case if the absolute intrinsic 'dimensions' $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$ were along the horizontal in Figs. 1a&b. That is, if the $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$ were not curving onto the absolute intrinsic time 'dimension' $\varnothing\hat{x}^0$ along the vertical in that figure.

Let us rewrite the line element $(\mathrm{d}\varnothing\hat{l}_{\mathrm{nm}})^2$ of Eq. (26) as follows

$$(\mathrm{d}\varnothing\hat{l}_{\mathrm{nm}})^2 = \sum_{i,k=1}^{2} \tan^2\varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i)\tan^2\varnothing\hat{\psi}_{\hat{x}^k}(\varnothing\hat{x}^k)\delta_{ik}\mathrm{d}\varnothing x'^i_{ab}\mathrm{d}\varnothing x'^k_{ab} \ . \tag{27}$$

Equation (27) is the same as the following by virtue of the absolute intrinsic local Euclidean invariance (4) in absolute intrinsic Riemann geometry,

$$(\mathrm{d}\varnothing\hat{l}_{\mathrm{nm}})^2 = \sum_{i,k=1}^{2} \tan^2 \varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i) \tan^2 \varnothing\hat{\psi}_{\hat{x}^k}(\varnothing\hat{x}^k)\delta_{ik}\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\varnothing\hat{x}^k \ . \tag{28}$$

Then let us introduce another absolute intrinsic tensor to be denoted by $\varnothing\hat{R}_{ik}$ and rewrite Eq. (28) as

$$(\mathrm{d}\varnothing\hat{l}_{\mathrm{nm}})^2 = \sum_{i,k=1}^{2} \varnothing\hat{R}_{ik}\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\varnothing\hat{x}^k \ . \tag{29}$$

where

$$\varnothing\hat{R}_{ik} = \tan^2 \varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i) \tan^2 \varnothing\hat{\psi}_{\hat{x}^k}(\varnothing\hat{x}^k)\delta_{ik} \ , \tag{30}$$

or

$$\varnothing\hat{R}_{ik} = \begin{pmatrix} \tan^2 \varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) & 0 \\ 0 & \tan^2 \varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) \end{pmatrix} \ . \tag{31}$$

Again the absolute intrinsic tensor $\varnothing\hat{R}_{ik}$ vanishes for absolute intrinsic angles, $\varnothing\hat{\psi}_{\hat{x}^i}(\varnothing\hat{x}^i) = 0; i = 1,2$, which will be the case if none of the 'dimensions', $\varnothing\hat{x}^i; i = 1,2$, was curving toward the absolute intrinsic 'dimension' $\varnothing\hat{x}^0$ along the vertical in Figs. 1a&b. Certainly the absolute intrinsic tensor $\varnothing\hat{R}_{ik}$ conveys information about the absolute intrinsic curvature of the 'dimensions' of the absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^2$.

The absolute intrinsic local Euclidean line element (25) on the absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$ can then be written in terms of the absolute intrinsic metric tensor $\varnothing\hat{g}_{ik}$ and the new absolute intrinsic (curvature) tensor $\varnothing\hat{R}_{ik}$ as

$$(\mathrm{d}\varnothing\hat{l})^2 = \sum_{i,k=1}^{2} (\varnothing\hat{g}_{ik} - \varnothing\hat{R}_{ik})\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\varnothing\hat{x}^k = \sum_{i,k=1}^{2} \delta_{ik}\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\varnothing\hat{x}^k \ . \tag{32}$$

The absolute intrinsic Euclidean line element (32) obtains at every point on $\varnothing\hat{\mathbb{M}}^2$, once the projective 'non-metric' absolute intrinsic coordinate intervals, $\mathrm{d}\varnothing\hat{x}^{01}$ and $\mathrm{d}\varnothing\hat{x}^{02}$, and the projective absolute proper intrinsic metric coordinate intervals, $\mathrm{d}\varnothing x'^1_{ab}$ and $\mathrm{d}\varnothing x'^2_{ab}$, are put into consideration in constructing the absolute intrinsic metric line element on $\varnothing\hat{\mathbb{M}}^2$ with respect to observers in $\mathbb{E}'^2$ in Figs. 1a&b. Equation (32) can therefore be said to express absolute intrinsic local Euclidean invariance on $\varnothing\hat{\mathbb{M}}^2$ with this condition. Thus the tensorial statement of absolute intrinsic local Euclidean invariance (A∅LEI) on a curved 'two-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$ — a 'two-dimensional' absolute intrinsic Riemannian metric space

— in Figs. 1a&b, which is also valid for $\varnothing\hat{\mathbb{M}}^3$, with respect to Euclidean observers in the relative proper intrinsic metric space $\mathbb{E}'^2$ or $\mathbb{E}'^3$, is the following

$$\varnothing\hat{g}_{ik} - \varnothing\hat{R}_{ik} = \delta_{ik} \quad (\mathrm{A}\varnothing\mathrm{LEI}) \ . \tag{33}$$

This is a tensorial statement of absolute intrinsic local Euclidean invariance on the curved $\varnothing\hat{\mathbb{M}}^2$, with respect to Euclidean observers in $\mathbb{E}'^2$, stated by Eq. (4) earlier, which is required in order re-write the absolute intrinsic Euclidean line (2c) or (2d) as the the absolute intrinsic line element (5a) or (5b).

2.5 Deriving a second absolute intrinsic metric tensor equation ("the field equations") in absolute intrinsic Riemann geometry

Now let us introduce a 2×2 absolute intrinsic matrix (or scalar) $\varnothing\hat{\mathrm{C}}$ through the following relation,

$$\varnothing\hat{R}_{ik} - \varnothing\hat{\mathrm{C}}\varnothing\hat{g}_{ik} = 0 \ . \tag{34}$$

Then from the definitions of the absolute intrinsic tensors $\varnothing\hat{g}_{ik}$ and $\varnothing\hat{R}_{ik}$ in Eq. (3) and (31), the absolute intrinsic matrix $\varnothing\hat{\mathrm{C}}$ is given in the case of '2-dimensional' absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^2$ as

$$\varnothing\hat{\mathrm{C}} = \begin{pmatrix} \sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) & 0 \\ 0 & \sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) \end{pmatrix} \ . \tag{35}$$

And from system (14), the matrix $\varnothing\hat{\mathrm{C}}$ is given in terms of absolute intrinsic curvature parameters as

$$\varnothing\hat{\mathrm{C}} = \begin{pmatrix} \varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2 & 0 \\ 0 & \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2 \end{pmatrix} \ . \tag{36}$$

Equations (35) and (36) become the following respectively for '3-dimensional' absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^3$

$$\varnothing\hat{\mathrm{C}} \ = \ \begin{pmatrix} \sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) & 0 & 0 \\ 0 & \sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) & 0 \\ 0 & 0 & \sin^2\varnothing\hat{\psi}_{\hat{x}^3}(\varnothing\hat{x}^3) \end{pmatrix} \tag{37}$$

and

$$\varnothing\hat{\mathrm{C}} = \begin{pmatrix} \varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2 & 0 & 0 \\ 0 & \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2 & 0 \\ 0 & 0 & \varnothing\hat{k}_{\hat{x}^3}(\varnothing\hat{x}^3)^2 \end{pmatrix} \tag{38}$$

Now multiplying through Eq. (34) from the left by $\varnothing\hat{g}^{ik}$ one obtains the following

$$\varnothing\hat{g}^{ik}\varnothing\hat{R}_{ik} - \varnothing\hat{g}^{ik}\varnothing\hat{C}\varnothing\hat{g}_{ik} = 0 \ . \tag{39}$$

Then by applying the known rules for raising and lowering of the indices of a tensor in Riemann geometry namely, $g^{\alpha\beta}R_{\alpha\delta} = R^{\beta}_{\delta}$ and $g^{\alpha\beta}g_{\alpha\gamma} = \delta^{\beta}_{\gamma}$, so that $\varnothing\hat{g}^{ik}\varnothing\hat{R}_{ik} = \varnothing\hat{R}^{i}_{i}$ and $\varnothing\hat{g}^{ik}\varnothing\hat{g}_{ik} = \delta^{i}_{i}$, Eq. (39) simplifies as

$$\varnothing\hat{R}^{i}_{i} - \varnothing\hat{C} = 0 \ , \quad \text{or} \quad \varnothing\hat{C} = \varnothing\hat{R}^{i}_{i} \ . \tag{40}$$

Thus Eq. (34) can be re-written in terms of $\varnothing\hat{R}^{i}_{i}$ as

$$\varnothing\hat{R}_{ik} - \varnothing\hat{R}^{i}_{i}\varnothing\hat{g}_{ik} = 0 \ . \tag{41}$$

In a situation where

$$\sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) = \sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) = \sin^2\varnothing\hat{\psi}_{\hat{x}^3}(\varnothing\hat{x}^3) \equiv \sin^2\varnothing\hat{\psi} \ ,$$
$$\text{or} \quad \varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1) = \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2) = \varnothing\hat{k}_{\hat{x}^3}(\varnothing\hat{x}^3) \equiv \varnothing\hat{k} \ ,$$

in Eq. (38), as will be the case for an isotropic absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$ or $\varnothing\hat{\mathbb{M}}^3$, the purely diagonal matrix $\varnothing\hat{R}^{i}_{i}$ or $\varnothing\hat{C}$ can be replaced by a number namely, $\mathrm{Tr}\,\varnothing\hat{C}/n$ or $\mathrm{Tr}\,\varnothing\hat{R}^{i}_{i}/n$ in Eq. (34) or (41) to have

$$\varnothing\hat{R}_{ik} - \frac{1}{n}\mathrm{Tr}\,\varnothing\hat{C}\varnothing\hat{g}_{ik} = 0 \ , \tag{42}$$

or

$$\varnothing\hat{R}_{ik} - \frac{1}{n}\mathrm{Tr}\,\varnothing\hat{R}^{i}_{i}\varnothing\hat{g}_{ik} = 0 \ . \tag{43}$$

Equation (43) becomes its familiar form in conventional Riemann geometry for $n = 2$ namely,

$$\varnothing\hat{R}_{ik} - \frac{1}{2}\varnothing\hat{R}\varnothing\hat{g}_{ik} = 0 \ . \tag{44}$$

where, $\varnothing\hat{R}\,(= Tr\,\varnothing\hat{R}^{i}_{i})$, is the sum of the equal entries of the diagonal matrix $\varnothing\hat{R}^{i}_{i}$ or $\varnothing\hat{C}$. Obviously the absolute intrinsic tensor $\varnothing\hat{R}_{ik}$ defined by Eq. (31) (referred to as absolute intrinsic curvature tensor earlier), is the absolute intrinsic Ricci tensor in absolute intrinsic Riemann geometry (of curved absolute intrinsic metric spaces).

It is to be noted that Eqs. (42), (43) or (44) have been written for the restrictive situation in which all the curved absolute intrinsic 'dimensions' $\varnothing\hat{x}^q$ of $\varnothing\hat{\mathbb{M}}^3$ have identical absolute intrinsic curvatures or identical absolute intrinsic curvature parameters, $\varnothing\hat{k}_{\hat{x}^q}(\varnothing\hat{x}^q) = \varnothing\hat{k}$; $q = 1, 2, 3$, at each point of $\varnothing\hat{\mathbb{M}}^3$, as stated earlier. Interestingly it is this restrictive situation that pertains to isotropic absolute intrinsic metric spaces, which shall be of relevance in absolute intrinsic Riemann geometry ultimately. However situations where the matrix $\varnothing\hat{R}^{i}_{i}$ or $\varnothing\hat{C}$ is a diagonal matrix but, $\varnothing\hat{k}_{x^i} \neq \varnothing\hat{k}_{x^k}$, are admissible in general.

For the restrictive situation,

$$\varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1) = \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2) = \varnothing\hat{k}_{\hat{x}^3}(\varnothing\hat{x}^3) = \varnothing\hat{k} \ ,$$

in Eq. (38), let us re-write Eq. (42), (43) or (44) in the following final form in which it shall be found most useful for application later,

$$\varnothing\hat{R}_{ik} - \varnothing\hat{k}^2\varnothing\hat{g}_{ik} = 0 \; . \tag{45}$$

where $\varnothing\hat{k}$ is the identical absolute intrinsic curvature parameter of all the absolute intrinsic 'dimensions' of $\varnothing\hat{\mathbb{M}}^3$, whose value varies from point to point on $\varnothing\hat{\mathbb{M}}^3$.

What have been achieved to this point in this section is the formulation of the absolute intrinsic Riemann geometry of the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ — an absolute intrinsic Riemannian metric space — relative to 3-observers in the underlying flat relative proper (or physical) metric 3-space $\mathbb{E}'^3$. Two important absolute intrinsic metric tensor equations (33) and (45) have been derived the in the process. While Eq. (33) is a tensorial statement of absolute intrinsic local Euclidean invariance (A∅LEI) on $\varnothing\hat{\mathbb{M}}^3$ with respect to Euclidean observers in $\mathbb{E}'^3$, as stated earlier, the corresponding significance of Eq. (45), which shall sometimes be referred to as "field equations", shall be derived elsewhere with further development.

Equations (33) and (45) apply on the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ with respect to every Euclidean observer in $\mathbb{E}'^3$. They must be solved algebraically to obtain the absolute intrinsic metric tensor $\varnothing\hat{g}_{ik}$ and absolute intrinsic Ricci tensor $\varnothing\hat{R}_{ik}$ on $\varnothing\hat{\mathbb{M}}^3$ with respect to these 3-observers in $\mathbb{E}'^3$ respectively as

$$\varnothing\hat{g}_{ik} = \begin{pmatrix} (1-\varnothing\hat{k}^2)^{-1} & 0 & 0 \\ 0 & (1-\varnothing\hat{k}^2)^{-1} & 0 \\ 0 & 0 & (1-\varnothing\hat{k}^2)^{-1} \end{pmatrix} \tag{46}$$

and

$$\varnothing\hat{R}_{ik} = \begin{pmatrix} \dfrac{\varnothing\hat{k}^2}{1-\varnothing\hat{k}^2} & 0 & 0 \\ 0 & \dfrac{\varnothing\hat{k}^2}{1-\varnothing\hat{k}^2} & 0 \\ 0 & 0 & \dfrac{\varnothing\hat{k}}{1-\varnothing\hat{k}^2} \end{pmatrix} , \tag{47}$$

where it is to be noted that the situation in which all the absolute intrinsic 'dimensions' of $\varnothing\hat{\mathbb{M}}^3$ possess identical absolute intrinsic curvature parameter $\varnothing\hat{k}$ that varies from pint to point on $\varnothing\hat{\mathbb{M}}^3$, which pertains to isotropic absolute intrinsic metric spaces that shall be the only relevant situation in absolute intrinsic Riemann geometry, has been considered.

The 'non-metric' component $\varnothing\hat{R}_{ik}\mathrm{d}\varnothing\hat{x}^i\mathrm{d}\varnothing\hat{x}^k\delta_{ik}$ of the absolute intrinsic Euclidean line element (32) on $\varnothing\hat{\mathbb{M}}^2$ or $\varnothing\hat{\mathbb{M}}^3$ is metrically elusive to observers in the underlying $\mathbb{E}'^2$ or $\mathbb{E}'^3$. Consequently $\varnothing\hat{\mathbb{M}}^3$ possesses unique absolute intrinsic sub-Riemannian metric tensor $\varnothing\hat{g}_{ik}$ of Eq. (46) with respect to these Euclidean observers, from the point of view of absolute intrinsic metric theories of physics. Hence the Euclidean observers write absolute intrinsic line element on $\varnothing\hat{\mathbb{M}}^3$ in terms of $\varnothing\hat{g}_{ik}$ in the Gaussian form involving isotropic absolute intrinsic coordinate intervals as Eq. (18b), which is given explicitly as

$$d\varnothing\hat{s}^2 = (\mathrm{d}\varnothing\hat{x}^0)^2 - \frac{(\mathrm{d}\varnothing\hat{x}^1)^2 + (\mathrm{d}\varnothing\hat{x}^2)^2 + (\mathrm{d}\varnothing\hat{x}^3)^2}{1 - \varnothing\hat{k}^2} . \tag{48}$$

The two steps of formulation of the absolute intrinsic Riemann geometry of curved absolute intrinsic metric spaces (or of curved absolute metric nospaces), isolated in part one of this paper [1] have been accomplished in this section namely,

1. derivation of the projections of the curved absolute intrinsic 'dimensions' of a curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ into its underlying projective absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ and
2. formulation of absolute intrinsic Riemann geometry on the curved absolute intrinsic metric space from the projections.

The derived projection relations (1) for $\varnothing\hat{\mathbb{M}}^2$, which is directly extendable to $\varnothing\hat{\mathbb{M}}^3$, with respect to Euclidean observers in the underlying relative proper Euclidean metric 3-space $\mathbb{E}'^3$, is the accomplishment of step one. On the other hand, the derivation of the two absolute intrinsic tensor equations (33) and (45) by starting from system (19) and the derivations of the absolute intrinsic metric tensor (45), absolute intrinsic Ricci tensor (47) and the absolute intrinsic line element (48), by solving equations (33) and (45) simultaneously, is accomplishment of step two. Let us proceed to the accomplishment of the two steps of formulation of absolute intrinsic Riemann geometry on curved absolute intrinsic metric space, in a situation where two and a larger number of absolute intrinsic metric spaces co-exist, or are superposed, in the next sub-section.

2.6 Superposition of absolute intrinsic Riemannian metric spaces

Although superposition of Riemann spaces may be unknown or meaningless in conventional Riemann geometry, it is definitely of important relevance in absolute intrinsic Riemann geometry. The 'two-dimensional' absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^2$ in Figs. 1a&b, to be re-denoted by $\varnothing\hat{\mathbb{M}}^2_{(1)}$, with curved absolute intrinsic metric 'dimensions' $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$ in Figs. 1a&b, is curved relative to its underlying projective flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$. If another 'two-dimensional' absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^2_{(2)}$ with curved absolute intrinsic 'dimensions' $\varnothing\hat{y}^1$ and $\varnothing\hat{y}^2$, say, is brought to the location of $\varnothing\hat{\mathbb{M}}^2_{(1)}$, so that $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and $\varnothing\hat{\mathbb{M}}^2_{(2)}$ co-exist, then $\varnothing\hat{\mathbb{M}}^2_{(2)}$ will be curved relative to $\hat{\mathbb{M}}^2_{(1)}$.

The resultant absolute intrinsic curvature parameter $\varnothing\hat{\hat{k}}$ of the absolute intrinsic space $\hat{\mathbb{M}}^2_{(2)}$ relative to the underlying flat absolute proper intrinsic metric space

$\varnothing\mathbb{E}'^2_{ab}$ can then be derived, and the resultant absolute intrinsic metric tensor $\varnothing\hat{\hat{g}}_{ik}$, the resultant absolute intrinsic Ricci tensor $\varnothing\hat{\hat{R}}_{ik}$ and the resultant absolute intrinsic line element $d\varnothing\hat{\hat{s}}^2$ can be written straight away in terms of $\varnothing\hat{\hat{k}}$, by simply replacing $\varnothing\hat{k}$ by $\varnothing\hat{\hat{k}}$ in equations (45), (46), (47) and (48). The resultant projections into $\varnothing\mathbb{E}'^2_{ab}$ of the curved absolute intrinsic metric 'dimensions' of $\hat{\mathbb{M}}^2_{(2)}$ can also be derived. The procedure can be extended to situations where three, four and larger number of absolute intrinsic metric spaces coexist (or are superposed).

2.6.1 The resultant absolute intrinsic metric tensor and resultant absolute intrinsic Ricci tensor when two or a larger number of 'parallel' absolute intrinsic metric spaces coexist

Let us consider a pair of 'two-dimensional' absolute intrinsic metric spaces denoted by $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and $\varnothing\hat{\mathbb{M}}^2_{(2)}$, with absolute intrinsic 'dimensions', $\varnothing\hat{x}^1$, $\varnothing\hat{x}^2$ and $\hat{y}^1$, $\hat{y}^2$, respectively. Let these 'dimensions' of the two absolute intrinsic metric spaces be curved relative to the same absolute proper intrinsic metric dimensions, $\varnothing x'^1_{ab}$ and $\varnothing x'^2_{ab}$, respectively, of their underlying global flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ prior to their superposition. In other words, as the two absolute intrinsic metric spaces existed at their separate locations before superposing them, the following intrinsic coordinate transformations existed,

$$\left.\begin{aligned} \varnothing x'^1_{ab} &= f^1(\varnothing\hat{x}^1)\ ;\quad \varnothing x'^2_{ab} = f^2(\varnothing\hat{x}^2)\ ;\\ \varnothing x'^1_{ab} &= g^1(\varnothing\hat{y}^1)\ ;\quad \varnothing x'^2_{ab} = g^2(\varnothing\hat{y}^2)\ . \end{aligned}\right\} \tag{49}$$

The absolute intrinsic metric spaces, $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and $\varnothing\hat{\mathbb{M}}^2_{(2)}$, in the situation of system (49), in which $\varnothing\hat{x}^1$ of $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and $\hat{y}^1$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ are both curved relative to $\varnothing x'^1_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$ and $\varnothing\hat{x}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and $\varnothing\hat{y}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ are both curved relative to $\varnothing x'^2_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$ at their different locations, as illustrated in Figs. 5a and 5b, shall be referred to as parallel absolute intrinsic metric spaces (or parallel absolute intrinsic Riemannian metric spaces).

Now let us superpose the absolute intrinsic metric spaces $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and $\varnothing\hat{\mathbb{M}}^2_{(2)}$ in Figs. 5a and 5b by bringing $\varnothing\hat{\mathbb{M}}^2_{(2)}$ to the location of $\varnothing\hat{\mathbb{M}}^2_{(1)}$. The origin P of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ does not have to coincide with the origin O of $\varnothing\hat{\mathbb{M}}^2_{(1)}$ in doing this. Since the curved absolute intrinsic metric 'dimensions' $\varnothing\hat{x}^1$ of $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and $\varnothing\hat{y}^1$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ both lie above the same absolute proper intrinsic metric dimension $\varnothing x'^1_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$ (and dimension x'^1 of $\mathbb{E}'^2$), and the curved absolute intrinsic metric 'dimensions' $\varnothing\hat{x}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and $\varnothing\hat{y}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ both lie above the same absolute proper intrinsic metric dimension $\varnothing x'^2_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$ (and dimension x'^2 of $\mathbb{E}'^2$) prior to their superposition, the curved absolute intrinsic metric 'dimension' $\hat{y}^1$ will be naturally curved relative to the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^1$ on the vertical $\varnothing x'^1_{ab}\varnothing\hat{x}^0-$plane,

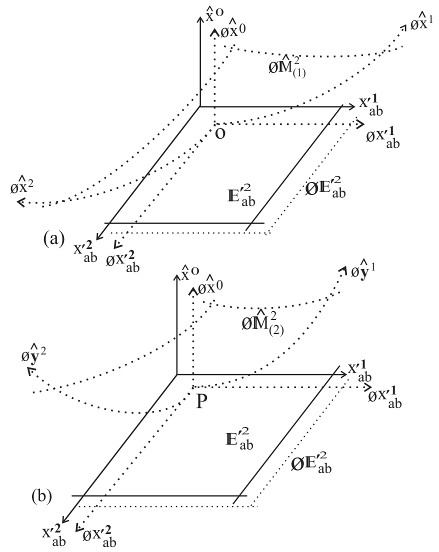

Figure 5: A pair of 'parallel' curved 'two-dimensional' absolute intrinsic metric spaces (or absolute intrinsic Riemannian metric spaces) are underlay by the global two-dimensional flat absolute proper intrinsic metric space and flat relative proper (or physical) metric 2-space (not shown) prior to their superposition.

and the curved absolute intrinsic metric 'dimension' $\hat{y}^2$ will be naturally curved relative to the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^2$ on the vertical $\varnothing x'^2_{ab}\varnothing\hat{x}^0-$plane, upon bringing them to the same location (or upon superposing them), as illustrated in Fig. 6. This case shall be referred to as superposition of parallel absolute intrinsic Riemannian metric spaces.

The point $\varnothing\hat{y}^1$ measured from point P of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ lies above point $\varnothing\hat{x}^1$ measured from point O of $\varnothing\hat{\mathbb{M}}^2_{(1)}$, where points P and O may not be coincident, and they both lie vertically above point $\varnothing x'^1_{ab\,(1)}$ of $\varnothing\mathbb{E}'^2_{ab}$ (and point x'^1 of $\mathbb{E}'^2$). Likewise point $\varnothing\hat{y}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ lies vertically above point $\varnothing\hat{x}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(1)}$, and they both lie vertically above point $\varnothing x'^2_{ab\,(1)}$ of $\varnothing\mathbb{E}'^2_{ab}$ (and point x'^2 of $\mathbb{E}'^2$).

The curved absolute intrinsic metric space 'dimension' $\varnothing\hat{y}^1$ has known absolute intrinsic curvature parameter $\varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1)$ at point $\varnothing\hat{y}^1$ relative to $\varnothing\mathbb{E}'^2_{ab}$, and the curved absolute intrinsic metric 'dimension' $\varnothing\hat{y}^2$ has known absolute intrinsic curvature parameter $\varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2)$ at point $\varnothing\hat{y}^2$ relative to $\varnothing\mathbb{E}'^2_{ab}$ from Fig. 5b.

Likewise the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^1$ has known absolute intrinsic curvature parameter $\varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)$ at point $\varnothing\hat{x}^1$ relative to $\varnothing\mathbb{E}'^2_{ab}$ and the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^2$ has known absolute intrinsic curvature parameter $\varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)$ at point $\varnothing\hat{x}^2$ relative to $\varnothing\mathbb{E}'^2_{ab}$ from Fig. 5a.

We wish to obtain the resultant absolute intrinsic curvature parameters of the

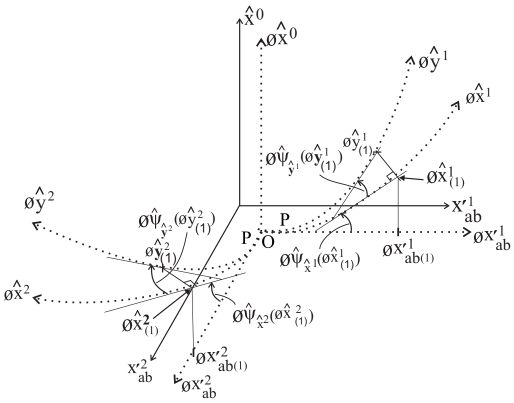

Figure 6: Two co-existing parallel absolute intrinsic Riemannian metric spaces.

curved absolute intrinsic metric 'dimension' $\varnothing\hat{y}^1$ at point $\varnothing\hat{y}^1$ and of the curved absolute intrinsic metric 'dimension' $\varnothing\hat{y}^2$ at point $\varnothing\hat{y}^2$ relative to relative to $\varnothing\mathbb{E}'^2_{ab}$, or with respect to observers in $\mathbb{E}'^2$, when the absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(2)}$ is curved relative to the absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(1)}$ as illustrated in Fig. 6, and then write the resultant absolute intrinsic metric tensor, resultant absolute intrinsic Ricci tensor and resultant absolute intrinsic line element at an arbitrary point $(\varnothing\hat{y}^1, \varnothing\hat{y}^2)$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$, in terms of the resultant absolute intrinsic curvature parameters relative to these observers in $\mathbb{E}'^2$.

Now the resultant absolute intrinsic metric tensor, $\varnothing\hat{\bar{g}}_{ik}$ at point $(\varnothing\hat{y}^1, \varnothing\hat{y}^2)$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ is given in terms of the absolute intrinsic angles, $\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)$ and $\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)$, of inclination of the curved absolute intrinsic metric 'dimension' $\varnothing\hat{y}^1$ relative to the straight line absolute proper intrinsic metric dimension $\varnothing x'^1_{ab}$ and of the curved absolute intrinsic metric 'dimension' $\varnothing\hat{y}^2$ relative to the straight line absolute proper intrinsic metric dimension $\varnothing x'^2_{ab}$ respectively in Fig. 5b as

$$\begin{aligned}
\varnothing\hat{g}^{(2)}_{ik} &= \begin{pmatrix} \frac{1}{1-\sin^2\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)} & 0 \\ 0 & \frac{1}{1-\sin^2\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)} \end{pmatrix} ; \\
&= \begin{pmatrix} \frac{1}{1-\varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1)^2} & 0 \\ 0 & \frac{1}{1-\varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2)^2} \end{pmatrix} . \qquad (50)
\end{aligned}$$

Likewise the absolute intrinsic metric tensor is given at point $(\varnothing\hat{x}^1, \varnothing\hat{x}^2)$ of $\varnothing\hat{\mathbb{M}}^2_{(1)}$

in Fig. 5a as

$$\varnothing\hat{g}^{(1)}_{ik} = \begin{pmatrix} \frac{1}{1-\sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)} & 0 \\ 0 & \frac{1}{1-\sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)} \end{pmatrix} ;$$

$$= \begin{pmatrix} \frac{1}{1-\varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2} & 0 \\ 0 & \frac{1}{1-\varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2} \end{pmatrix} . \quad (51)$$

When the two parallel absolute intrinsic Riemannian metric spaces coexist, as illustrated in Fig. 6, then the resultant absolute intrinsic metric tensor $\varnothing\hat{\bar{g}}_{ik}$ of the upper absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^{(2)}_2$ relative to its projective underlying absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ and, hence, relative to the relative proper physical Euclidean space $\mathbb{E}'^2$ overlying $\varnothing\mathbb{E}'^2_{ab}$, is given in terms of the resultant absolute intrinsic angles, $\varnothing\hat{\bar{\psi}}_1$ and $\varnothing\hat{\bar{\psi}}_2$, and in terms of the resultant absolute intrinsic curvature parameters, $\varnothing\hat{\bar{k}}_1$ and $\varnothing\hat{\bar{k}}_2$, respectively as

$$\varnothing\hat{\bar{g}}_{ik} = \begin{pmatrix} \dfrac{1}{1-\sin^2\varnothing\hat{\bar{\psi}}_1} & 0 \\ 0 & \dfrac{1}{1-\sin^2\varnothing\hat{\bar{\psi}}_2} \end{pmatrix} \quad (52)$$

and

$$\varnothing\hat{\bar{g}}_{ik} = \begin{pmatrix} \dfrac{1}{1-(\varnothing\hat{\bar{k}}_1)^2} & 0 \\ 0 & \dfrac{1}{1-(\varnothing\hat{\bar{k}}_2)^2} \end{pmatrix} , \quad (53)$$

where, as can be observed from Fig. 6,

$$\varnothing\hat{\bar{\psi}}_1 = \varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1) + \varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) \text{ and } \varnothing\hat{\bar{\psi}}_2 = \varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2) + \varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2).$$

Now the absolute intrinsic metric spaces, $\varnothing\hat{\mathbb{M}}^2_{(2)}$ and $\varnothing\hat{\mathbb{M}}^2_{(1)}$, are curved relative to their common projective flat relative proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ with the Euclidean metric δ_{ik} (in Figs. 5a and 5b), prior to their superposition. Hence the components of their absolute intrinsic metric tensors can be written in terms of the components of the Euclidean metric tensor prior to their superposition respectively as

$$\begin{aligned} (\varnothing\hat{g}^{(2)}_{11})^{-1} &= \delta_{11} - \sin^2\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1) = \delta_{11} - \varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1)^2 ; \\ (\varnothing\hat{g}^{(2)}_{22})^{-1} &= \delta_{22} - \sin^2\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2) = \delta_{22} - \varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2)^2 ; \\ (\varnothing\hat{g}^{(2)}_{12})^{-1} &= (\varnothing\hat{g}^{(2)}_{21})^{-1} = 0 ; \text{ for } \varnothing\hat{\mathbb{M}}^2_{(2)} \end{aligned} \quad (54)$$

and

$$\begin{aligned}
(\varnothing\hat{g}^{(1)}_{11})^{-1} &= \delta_{11} - \sin^2 \varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) = \delta_{11} - \varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2 \ ; \\
(\varnothing\hat{g}^{(1)}_{22})^{-1} &= \delta_{22} - \sin^2 \varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) = \delta_{22} - \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2 \ ; \\
(\varnothing\hat{g}^{(1)}_{12})^{-1} &= (\varnothing\hat{g}^{(1)}_{21})^{-1} = 0 \ ; \quad \text{for } \varnothing\hat{\mathbb{M}}^2_{(1)} \ .
\end{aligned} \tag{55}$$

Upon the two absolute intrinsic metric spaces co-existing as in Fig. 6, on the other hand, while $\varnothing\hat{\mathbb{M}}^2_{(1)}$ is still curved relative to the flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ with the Euclidean metric tensor δ_{ik}, such that the tangent to the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^1$ at point $\varnothing\hat{x}^1$ is inclined to the straight line absolute proper intrinsic metric dimension $\varnothing x'^1_{ab}$ at absolute intrinsic angle $\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$ and the tangent to the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^2$ at point $\varnothing\hat{x}^2$ is inclined to $\varnothing x'^2_{ab}$ at absolute angle intrinsic $\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$, the absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^2_{(2)}$ is curved relative to the absolute intrinsic Riemannian metric space $\varnothing\hat{\mathbb{M}}^2_{(1)}$ with absolute intrinsic metric tensor $\varnothing\hat{g}^{(1)}_{ik}$.

It follows as a consequence of the foregoing that the tangent to the curved absolute intrinsic metric 'dimension' $\varnothing\hat{y}^1$ at point $\varnothing\hat{y}^1$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ is now inclined at absolute intrinsic angle $\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)$ relative to the tangent to the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^1$ at point $\varnothing\hat{x}^1$ of $\varnothing\hat{\mathbb{M}}^2_{(1)}$ and the tangent to the absolute intrinsic metric 'dimension' $\varnothing\hat{y}^2$ at point $\varnothing\hat{y}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ is now inclined at absolute intrinsic angle $\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)$ relative to the tangent to the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^2$ at point $\varnothing\hat{x}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(1)}$. In the present situation, the absolute intrinsic metric tensor $\varnothing\hat{g}^{(1)}_{ik}$ of the absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(1)}$ serves as the foundation absolute intrinsic metric tensor upon which the absolute intrinsic metric tensor of absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(2)}$ must be constructed.

The components of the resultant absolute intrinsic metric tensor (i.e. of the upper curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(2)}$ relative to the flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ in Fig. 6), are therefore given in terms of the components of $\varnothing\hat{g}^{(1)}_{ik}$ (like system (54) or (55) is written relative to the Euclidean metric δ_{ik}) as

$$\begin{aligned}
(\varnothing\hat{\hat{g}}_{11})^{-1} &= \varnothing\hat{g}^{(1)}_{11} - \sin^2 \varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1) = \varnothing\hat{g}^{(1)}_{11} - \varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1)^2 \ ; \\
(\varnothing\hat{\hat{g}}_{22})^{-1} &= \varnothing\hat{g}^{(1)}_{22} - \sin^2 \varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2) = \varnothing\hat{g}^{(1)}_{22} - \varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2)^2 \ ; \\
(\varnothing\hat{\hat{g}}_{12})^{-1} &= (\varnothing\hat{\hat{g}}_{21})^{-1} = 0 \ .
\end{aligned} \tag{56}$$

It is appropriate to further elucidate system (56). The components $\varnothing\hat{g}^{(1)}_{11}$ and $\varnothing\hat{g}^{(1)}_{22}$ of the absolute intrinsic metric tensor $\varnothing\hat{g}^{(1)}_{ik}$ of the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(1)}$ have been written relative to the intrinsic Euclidean metric tensor reference in system (54) (by virtue of the appearance of the components δ_{11}

and δ_{22} of the Euclidean metric tensor in (54)), because $\varnothing\hat{\mathbb{M}}^2_{(1)}$ is curved relative to the absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ with intrinsic Euclidean metric tensor in Fig. 5a.

The components $\varnothing\hat{g}^{(2)}_{11}$ and $\varnothing\hat{g}^{(2)}_{22}$ of the absolute intrinsic metric tensor $\varnothing\hat{g}^{(2)}_{ik}$ of the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(2)}$ have likewise been written relative to the absolute intrinsic Euclidean metric tensor reference in system (55), because $\varnothing\hat{\mathbb{M}}^2_{(2)}$ is curved relative to the absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ in Fig. 5b. The components δ_{11} and δ_{22} of the intrinsic Euclidean metric tensor of $\varnothing\mathbb{E}'^2_{ab}$ that appear in systems (54) and (55) are related to the constant zero absolute intrinsic angle ($\varnothing\hat{\psi} = 0$) of inclination to the horizontal of the absolute proper intrinsic dimensions $\varnothing x'^1_{ab}$ and $\varnothing x'^2_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$ along the horizontal in Figs. 5a and 3b as, $\delta_{11} = \delta_{22} = \sec^2(\varnothing\hat{\psi} = 0) = 1$.

On the other hand, the components, $\varnothing\hat{\hat{g}}_{11}^{(2)}$ and $\varnothing\hat{\hat{g}}_{22}^{(2)}$, of the resultant absolute intrinsic metric tensor $\varnothing\hat{\hat{g}}_{ik}^{(2)}$ of the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(2)}$ relative to the flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ in Fig. 6, have been written relative to absolute intrinsic sub-Riemannian metric tensor reference in system (56). This is so because $\varnothing\hat{\mathbb{M}}^2_{(2)}$ is curved relative to the intermediate curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(1)}$, which, in turn, is curved relative to the flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ in Fig. 6.

The components $\varnothing\hat{g}^{(1)}_{11}$ and $\varnothing\hat{g}^{(1)}_{22}$ of the absolute intrinsic metric tensor on $\varnothing\hat{\mathbb{M}}^2_{(1)}$ that appear in system (56) are related to the varying absolute intrinsic angles, $\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$ and $\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$, of the inclinations of the curved absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$, of $\varnothing\hat{\mathbb{M}}^2_{(1)}$ relative to the flat absolute proper intrinsic metric dimensions, $\varnothing x'^1_{ab}$ and $\varnothing x'^2_{ab}$, of $\varnothing\mathbb{E}'^2_{ab}$ respectively at an arbitrary point on $\varnothing\hat{\mathbb{M}}^2_{(1)}$ as, $\varnothing\hat{g}^{(1)}_{11} = \sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$ and $\varnothing\hat{g}^{(1)}_{22} = \sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$.

In other words, the constant zero absolute intrinsic angle ($\varnothing\hat{\psi} = 0$) of inclination to the horizontal of the absolute proper intrinsic metric dimensions $\varnothing x'^1_{ab}$ and $\varnothing x'^2_{ab}$ along the horizontal, of the flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^2_{ab}$ in Figs. 5a and 3b, have been replaced by the varying absolute intrinsic angles, $\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$ and $\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$, of inclinations to the horizontal of the curved absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$, of the intermediate curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(1)}$ in Fig. 6. Consequently, $\delta_{11} = \delta_{22} = \sec^2(\varnothing\hat{\psi} = 0) = 1$ in systems (54) and (55) have been replaced by, $\varnothing\hat{g}^{(1)}_{11} = \sec^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)$ and $\varnothing\hat{g}^{(1)}_{22} = \sec^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)$, respectively in system (57). Substitution of system (55) into system (56) gives the following

$$\begin{aligned}(\varnothing\hat{\bar{g}}_{11})^{-1} &= 1-\sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)-\sin^2\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1) = \\ &\quad 1-\varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2-\varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1)^2; \\ (\varnothing\hat{\bar{g}}_{22})^{-1} &= 1-\sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)-\sin^2\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2) = \\ &\quad 1-\varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2-\varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2)^2; \\ (\varnothing\hat{\bar{g}}_{12})^{-1} &= (\varnothing\hat{\bar{g}}_{21})^{-1} = 0\ .\end{aligned}$$

The components of the resultant absolute intrinsic metric tensor in system (57) are the same as in Eqs. (52) and (53). Hence we obtain expressions for the resultant absolute intrinsic angles $\varnothing\hat{\bar{\psi}}$ and the resultant absolute intrinsic curvature parameter $\varnothing\hat{\bar{k}}$ in terms of the absolute intrinsic angles $\varnothing\hat{\psi}_{\hat{x}}$ and $\varnothing\hat{\psi}_{\hat{y}}$ and absolute intrinsic curvature parameters $\varnothing\hat{k}_{\hat{x}}$ and $\varnothing\hat{k}_{\hat{y}}$ of the individual absolute intrinsic metric spaces prior to their superposition respectively as follows

$$\varnothing\hat{\bar{g}}_{11} = \left(1-\sin^2\varnothing\hat{\bar{\psi}}_1\right)^{-1} = \left(1-\sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)-\sin^2\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)\right)^{-1}$$

hence,

$$\sin^2\varnothing\hat{\bar{\psi}}_1 = \sin^2\left(\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)+\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)\right)\ ; \quad \sin^2\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)+\sin^2\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)\ ; \tag{58a}$$

$$\varnothing\hat{\bar{g}}_{22} = \left(1-\sin^2\varnothing\hat{\bar{\psi}}_2\right)^{-1} = \left(1-\sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)-\sin^2\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)\right)^{-1}\ ,$$

hence,

$$\sin^2\varnothing\hat{\bar{\psi}}_2 = \sin^2\left(\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)+\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)\right) = \sin^2\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)+\sin^2\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)\ . \tag{58b}$$

Consequently,

$$(\varnothing\hat{\bar{k}}_1)^2 = \varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2+\varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1)^2\ ; \tag{58a}$$

$$(\varnothing\hat{\bar{k}}_2)^2 = \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2+\varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2)^2\ . \tag{58b}$$

Equations (58a) and (58b) give the rules for the composition of two absolute intrinsic angles, $\varnothing\hat{\psi}_{\hat{x}}$ and $\varnothing\hat{\psi}_{\hat{y}}$, while Eqs. (59a) and (59b) give the corresponding rule for the composition two absolute intrinsic curvature parameters, for the purpose of writing the resultant absolute intrinsic metric tensor and resultant absolute intrinsic Ricci tensor in absolute intrinsic Riemann geometry, for the situation where a pair of parallel 'two-dimensional' absolute intrinsic metric spaces coexist, as illustrated in Fig. 6. They can be extended to the situation where a pair of parallel 'three-dimensional' absolute intrinsic metric spaces coexist as follows

$$
\begin{aligned}
\sin^2 \varnothing\hat{\hat{\psi}}_1 &= \sin^2\left(\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) + \varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)\right) ;\\
&= \sin^2 \varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) + \sin^2 \varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1) ;\\
\sin^2 \varnothing\hat{\hat{\psi}}_2 &= \sin^2\left(\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) + \varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)\right) ;\\
&= \sin^2 \varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) + \sin^2 \varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2) ;\\
\sin^2 \varnothing\hat{\hat{\psi}}_3 &= \sin^2\left(\varnothing\hat{\psi}_{\hat{x}^3}(\varnothing\hat{x}^3) + \varnothing\hat{\psi}_{\hat{y}^3}(\varnothing\hat{y}^3)\right) ;\\
&= \sin^2 \varnothing\hat{\psi}_{\hat{x}^3}(\varnothing\hat{x}^3) + \sin^2 \varnothing\hat{\psi}_{\hat{y}^3}(\varnothing\hat{y}^3) ;
\end{aligned}
\tag{59}
$$

and

$$
\begin{aligned}
(\varnothing\hat{\hat{k}}_1)^2 &= \varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2 + \varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1)^2 ;\\
(\varnothing\hat{\hat{k}}_2)^2 &= \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2 + \varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2)^2 ;\\
(\varnothing\hat{\hat{k}}_3)^2 &= \varnothing\hat{k}_{\hat{x}^3}(\varnothing\hat{x}^3)^2 + \varnothing\hat{k}_{\hat{y}^3}(\varnothing\hat{y}^3)^2 .
\end{aligned}
\tag{60}
$$

Systems (60) and (61) admit of generalization to a situation where N parallel '3-dimensional' absolute intrinsic metric spaces coexist (where the Nth absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3_{(N)}$ has curved absolute intrinsic metric 'dimensions' $\varnothing\hat{w}^1$, $\varnothing\hat{w}^2$ and $\varnothing\hat{w}^3$) as follows

$$
\begin{aligned}
\sin^2 \varnothing\hat{\hat{\psi}}_1 &= \sin^2\left(\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) + \varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1) + \cdots + \varnothing\hat{\psi}_{\hat{w}^1}(\varnothing\hat{w}^1)\right) ;\\
&= \sin^2 \varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1) + \sin^2 \varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1) + \cdots + \sin^2 \varnothing\hat{\psi}_{\hat{w}^1}(\varnothing\hat{w}^1) ;\\
\sin^2 \varnothing\hat{\hat{\psi}}_2 &= \sin^2\left(\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) + \varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2) + \cdots + \varnothing\hat{\psi}_{\hat{w}^2}(\varnothing\hat{w}^2)\right) ;\\
&= \sin^2 \varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2) + \sin^2 \varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2) + \cdots + \sin^2 \varnothing\hat{\psi}_{\hat{w}^2}(\varnothing\hat{w}^2) ;\\
\sin^2 \varnothing\hat{\hat{\psi}}_3 &= \sin^2\left(\varnothing\hat{\psi}_{\hat{x}^3}(\varnothing\hat{x}^3) + \varnothing\hat{\psi}_{\hat{y}^3}(\varnothing\hat{y}^3) + \cdots + \varnothing\hat{\psi}_{\hat{w}^3}(\varnothing\hat{w}^3)\right) ;\\
&= \sin^2 \varnothing\hat{\psi}_{\hat{x}^3}(\varnothing\hat{x}^3) + \sin^2 \varnothing\hat{\psi}_{\hat{y}^3}(\varnothing\hat{y}^3) + \cdots + \sin^2 \varnothing\hat{\psi}_{\hat{w}^3}(\varnothing\hat{w}^3) .
\end{aligned}
\tag{61}
$$

The corresponding resultant absolute intrinsic curvature parameters are

$$
\begin{aligned}
(\varnothing\hat{\bar{k}}_1)^2 &= \varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1)^2 + \varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1)^2 + \cdots + \varnothing\hat{k}_{\hat{w}^1}(\varnothing\hat{w}^1)^2; \\
(\varnothing\hat{\bar{k}}_2)^2 &= \varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2)^2 + \varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2)^2 + \cdots + \varnothing\hat{k}_{\hat{w}^2}(\varnothing\hat{w}^2)^2; \\
(\varnothing\hat{\bar{k}}_3)^2 &= \varnothing\hat{k}_{\hat{x}^3}(\varnothing\hat{x}^3)^2 + \varnothing\hat{k}_{\hat{y}^3}(\varnothing\hat{y}^3)^2 + \cdots + \varnothing\hat{k}_{\hat{w}^3}(\varnothing\hat{w}^3)^2 \, .
\end{aligned} \tag{62}
$$

Although equations (60) – (63) show no ceiling on the resultant absolute intrinsic angle, $\varnothing\hat{\bar{\psi}}_q$; $q = 1, 2$ or 3, it is known that $\varnothing\hat{\bar{\psi}}_q$ has a maximum value of, $\varnothing\hat{\bar{\psi}}_q = \varnothing\hat{\pi}/2$, since then the curved absolute intrinsic metric 'dimension' $\hat{w}^q$ of the last (i.e. the Nth) absolute intrinsic metric space will lie along the vertical, parallel to the absolute intrinsic metric time 'dimension' $\varnothing\hat{x}^0$. This implies that there is a ceiling on the number of absolute intrinsic metric spaces that can be superposed. The implication of going beyond the ceiling, that is, for making, $\varnothing\hat{\bar{\psi}}_q > \varnothing\hat{\pi}/2$, will be derived elsewhere with further development.

The resultant absolute intrinsic metric tensor $\varnothing\hat{\bar{g}}_{ik}$ of the last (i.e. the Nth) absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3_{(N)}$ relative to the underlying flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ is given in terms of the resultant absolute intrinsic angles and the resultant absolute intrinsic curvature parameters in systems (62) and (63) respectively as

$$
\varnothing\hat{\bar{g}}^{(2)}_{ik} = \begin{pmatrix} \frac{1}{1-\sin^2\varnothing\hat{\bar{\psi}}_1} & 0 & 0 \\ 0 & \frac{1}{1-\sin^2\varnothing\hat{\bar{\psi}}_2} & 0 \\ 0 & 0 & \frac{1}{1-\sin^2\varnothing\hat{\bar{\psi}}_3} \end{pmatrix} , \tag{63}
$$

or

$$
\varnothing\hat{\bar{g}}^{(2)}_{ik} = \begin{pmatrix} \frac{1}{1-(\varnothing\hat{\bar{k}}_1)^2} & 0 & 0 \\ 0 & \frac{1}{1-(\varnothing\hat{\bar{k}}_2)^2} & 0 \\ 0 & 0 & \frac{1}{1-(\varnothing\hat{\bar{k}}_3)^2} \end{pmatrix} . \tag{64}
$$

The resultant absolute intrinsic Ricci tensor is likewise given in terms of the resultant absolute intrinsic angles and resultant absolute intrinsic curvature parameters as

$$
\varnothing\hat{\bar{R}}_{ik} = \begin{pmatrix} \frac{\sin^2\varnothing\hat{\bar{\psi}}_1}{1-\sin^2\varnothing\hat{\bar{\psi}}_1} & 0 & 0 \\ 0 & \frac{\sin^2\varnothing\hat{\bar{\psi}}_2}{1-\sin^2\varnothing\hat{\bar{\psi}}_2} & 0 \\ 0 & 0 & \frac{\sin^2\varnothing\hat{\bar{\psi}}_3}{1-\sin^2\varnothing\hat{\bar{\psi}}_3} \end{pmatrix} ; \tag{65}
$$

or

$$\varnothing\hat{\bar{R}}_{ik} = \begin{pmatrix} \frac{(\varnothing\hat{\bar{k}}_1)^2}{1-(\varnothing\hat{\bar{k}}_1)^2} & 0 & 0 \\ 0 & \frac{(\varnothing\hat{\bar{k}}_2)^2}{1-(\varnothing\hat{\bar{k}}_2)^2} & 0 \\ 0 & 0 & \frac{(\varnothing\hat{\bar{k}}_3)^2}{1-(\varnothing\hat{\bar{k}}_3)^2} \end{pmatrix} . \qquad (66)$$

The resultant absolute intrinsic line element on the last (i.e. the Nth) absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3_{(N)}$ relative to the the underlying flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ is the following

$$(d\varnothing\hat{\bar{s}})^2 = (\mathrm{d}\varnothing\hat{\bar{x}}^0)^2 - \frac{\big((\mathrm{d}\varnothing\hat{w}^1)^2 + (\mathrm{d}\varnothing\hat{w}^2)^2 + (\mathrm{d}\varnothing\hat{w}^3)^2\big)}{1-(\varnothing\hat{\bar{k}})^2} \qquad (67)$$

where $\varnothing\hat{\bar{k}} = \varnothing\hat{\bar{k}}_1 = \varnothing\hat{\bar{k}}_2 = \varnothing\hat{\bar{k}}_3$ is assumed.

As mentioned earlier, only the situation where $\varnothing\hat{\bar{k}}_1$, $\varnothing\hat{\bar{k}}_2$ and $\varnothing\hat{\bar{k}}_3$ are all identical to $\varnothing\hat{\bar{k}}$, as assumed in writing Eq. (68), shall be of relevance in absolute intrinsic Riemann geometry ultimately. For that situation, the two absolute intrinsic metric tensor equations (33) and (45), derived in the context of absolute intrinsic Riemann geometry earlier, are given in terms of the resultant absolute intrinsic tensors, $\varnothing\hat{\bar{g}}_{ik}$ and $\varnothing\hat{\bar{R}}_{ik}$, and resultant absolute intrinsic curvature parameter as

$$\varnothing\hat{\bar{g}}_{ik} - \varnothing\hat{\bar{R}}_{ik} = \delta_{ik} \qquad (68)$$

and

$$\varnothing\hat{\bar{R}}_{ik} - (\varnothing\hat{\bar{k}})^2\varnothing\hat{\bar{g}}_{ik} = 0 \; . \qquad (69)$$

The solution to equations (69) and (70) are equations (65) and (67) with, $(\varnothing\hat{\bar{k}})^2 = (\varnothing\hat{\bar{k}}_1)^2 = (\varnothing\hat{\bar{k}}_2)^2 = (\varnothing\hat{\bar{k}}_3)^2$, assumed.

The first step of the formulation of absolute intrinsic Riemann geometry in a situation where two or a larger number of parallel absolute intrinsic metric spaces co-exist (or are superposed) has again been accomplished in this sub-section. Let us now proceed to the second step namely, obtaining resultant absolute intrinsic coordinate projection relations, when two or a larger number of parallel absolute intrinsic metric spaces co-exist.

2.6.2 The resultant absolute intrinsic coordinate projection relations when two or a larger number of parallel absolute intrinsic metric spaces co-exist

Now let us redraw Fig. 6 while showing certain detail required for this sub-sub-section as Fig. 7. Let us consider elementary absolute intrinsic metric coordinate intervals, $d\varnothing\hat{y}^1$ and $d\varnothing\hat{y}^2$, defined about point $(\varnothing\hat{y}^1, \varnothing\hat{y}^2)$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$, to be the dimensions of a locally flat region (or frame) on $\varnothing\hat{\mathbb{M}}^2_{(2)}$. The interval $d\varnothing\hat{y}^1$ about point $\varnothing\hat{y}^1$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$ projects component $d\varnothing\hat{x}^1$ into the underlying curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^1$, which lies along the tangent to the curved $\varnothing\hat{x}^1$ at point $\varnothing\hat{x}^1$ of $\varnothing\hat{x}^1$, as shown in Fig. 7. Likewise the interval $d\varnothing\hat{y}^2$ about point $\varnothing\hat{y}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(2)}$, projects component $d\varnothing\hat{x}^2$ into the underlying curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^2$ of $\varnothing\hat{\mathbb{M}}^2$, which lies along the tangent to the curved $\varnothing\hat{x}^2$ at point $\varnothing\hat{x}^2$ of $\varnothing\hat{x}^2$, as also shown in Fig. 7. The following projection relations obtain from elementary coordinate geometry

$$\begin{aligned} d\varnothing\hat{x}^1 &= d\varnothing\hat{y}^1 \cos\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1) \; ; \\ d\varnothing\hat{x}^2 &= d\varnothing\hat{y}^2 \cos\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2) \; . \end{aligned} \tag{70}$$

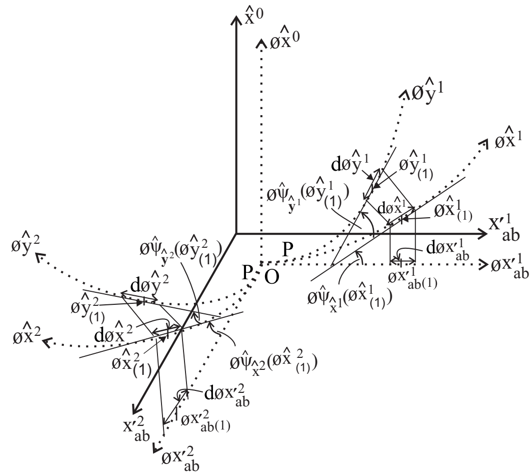

Figure 7: Obtaining resultant coordinate interval projections of two co-existing parallel absolute intrinsic metric spaces.

In turn, the component $d\varnothing\hat{x}^1$ projected about an arbitrary point $\varnothing\hat{x}^1$ of the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^1$ of $\varnothing\hat{\mathbb{M}}^2$, projects component $d\varnothing x'^1_{ab}$ about the corresponding arbitrary point $\varnothing x'^1_{ab}$ of its underlying straight line absolute proper intrinsic metric dimension $\varnothing x'^1_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$, and the component $d\varnothing\hat{x}^2$ projected about the point $\varnothing\hat{x}^2$ of the curved absolute intrinsic metric 'dimension' $\varnothing\hat{x}^2$, projects component $d\varnothing x'^2_{ab}$ about the corresponding arbitrary point $\varnothing x'^2_{ab}$ of its underlying straight line absolute proper intrinsic metric dimension $\varnothing x'^2_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$, as shown in Fig. 7.

Again the following coordinate projection relations obtain from Fig. 7 from elementary coordinate geometry

$$\begin{aligned} \mathrm{d}\varnothing x'^1_{ab} &= \mathrm{d}\varnothing\hat{x}^1 \cos\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)\ ; \\ \mathrm{d}\varnothing x'^2_{ab} &= \mathrm{d}\varnothing\hat{x}^2 \cos\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)\ . \end{aligned} \tag{71}$$

Then by combining systems (71) and (72) the following obtain.

$$\begin{aligned} \mathrm{d}\varnothing x'^1_{ab} &= \mathrm{d}\varnothing\hat{y}^1 \cos\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)\cos\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)\ ; \\ \mathrm{d}\varnothing x'^2_{ab} &= \mathrm{d}\varnothing\hat{y}^2 \cos\varnothing\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)\cos\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)\ . \end{aligned} \tag{72}$$

System (73) gives the resultant length contraction relations of the absolute intrinsic metric coordinate intervals of the absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(2)}$ with respect to observers in $\mathbb{E}'^2$. They become the following in terms of absolute intrinsic curvature parameters

$$\begin{aligned} \mathrm{d}\varnothing x'^1_{ab} &= \mathrm{d}\varnothing\hat{y}^1(1-\varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1))^{1/2}(1-\varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1))^{1/2}\ ; \\ \mathrm{d}\varnothing x'^2_{ab} &= \mathrm{d}\varnothing\hat{y}^2(1-\varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2))^{1/2}(1-\varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2))^{1/2}\ , \end{aligned} \tag{73}$$

where the definitions of the absolute intrinsic curvature parameters of system (14) have been used.

Systems (73) and (74) admit of generalization to a situation where any number N of 'two-dimensional' parallel absolute intrinsic metric spaces co-exist, in which case they become the following respectively,

$$\begin{aligned} \mathrm{d}\varnothing x'^1_{ab} &= \mathrm{d}\hat{w}^1 \cos\varnothing\hat{\psi}_{\hat{x}^1}(\varnothing\hat{x}^1)\cos\varnothing\hat{\psi}_{\hat{y}^1}(\varnothing\hat{y}^1)\ \cdots\ \cos\varnothing\hat{\psi}_{\hat{w}^1}(\varnothing\hat{w}^1)\ ; \\ \mathrm{d}\varnothing x'^2_{ab} &= \mathrm{d}\varnothing\hat{w}^2 \cos\hat{\psi}_{\hat{x}^2}(\varnothing\hat{x}^2)\cos\varnothing\hat{\psi}_{\hat{y}^2}(\varnothing\hat{y}^2)\ \cdots\ \cos\varnothing\hat{\psi}_{\hat{w}^2}(\varnothing\hat{w}^2)\ . \end{aligned} \tag{74}$$

and

$$\begin{aligned} \mathrm{d}\varnothing x'^1_{ab} &= \mathrm{d}\hat{w}^1(1-\varnothing\hat{k}_{\hat{x}^1}(\varnothing\hat{x}^1))^{\frac{1}{2}}(1-\varnothing\hat{k}_{\hat{y}^1}(\varnothing\hat{y}^1))^{\frac{1}{2}}\ \cdots\ (1-\varnothing\hat{k}_{\hat{w}^1}(\varnothing\hat{w}^1))^{\frac{1}{2}}\ ; \\ \mathrm{d}\varnothing x'^2_{ab} &= \mathrm{d}\hat{w}^2(1-\varnothing\hat{k}_{\hat{x}^2}(\varnothing\hat{x}^2))^{\frac{1}{2}}(1-\varnothing\hat{k}_{\hat{y}^2}(\varnothing\hat{y}^2))^{\frac{1}{2}}\ (1-\varnothing\hat{k}_{\hat{w}^2}(\varnothing\hat{w}^2))^{\frac{1}{2}}\ , \end{aligned} \tag{75}$$

where the uppermost Nth curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2_{(N)}$ has absolute intrinsic 'dimensions' $\varnothing\hat{w}^1$ and $\varnothing\hat{w}^2$ with absolute intrinsic curvature parameters, $\varnothing\hat{k}_{\hat{w}^1}(\varnothing\hat{w}^1)$ and $\varnothing\hat{k}_{\hat{w}^2}(\varnothing\hat{w}^2)$, at an arbitrary point on $\varnothing\hat{\mathbb{M}}^2_{(N)}$.

Systems (72) through (76) for superposition of 'two-dimensional' parallel absolute intrinsic metric spaces admit of easy and direct extension to superposition of 'three-dimensional' parallel absolute intrinsic metric spaces, in which case, a third expression for $\mathrm{d}\varnothing x'^3_{ab}$ must be added to each of the systems.

Now the flat two-dimensional absolute proper metric 2-space $\mathbb{E}'^2_{ab}$ is the outward manifestation of $\varnothing\mathbb{E}'^2_{ab}$ in Figs. 1a&b through Fig. 7 of this paper. The absolute intrinsic coordinate projection expressions relating the absolute proper intrinsic metric coordinate intervals, $\mathrm{d}\varnothing x'^1_{ab}$ and $\mathrm{d}\varnothing x'^2_{ab}$ of $\varnothing\mathbb{E}'^2_{ab}$, to the absolute intrinsic metric coordinate intervals, $\mathrm{d}\varnothing\hat{w}^1$ and $\mathrm{d}\varnothing\hat{w}^2$ of $\varnothing\hat{\mathbb{M}}^2_{(N)}$, respectively, likewise have their outward manifestations. The outward manifestations of systems (75) and (76), obtained by simply removing the symbol $\varnothing$ are the following respectively,

$$\begin{aligned} \mathrm{d}x'^1_{ab} &= \mathrm{d}\hat{x}^1 \cos\hat{\psi}_{\hat{x}^1}(\hat{x}^1)\cos\hat{\psi}_{\hat{y}^1}(\hat{y}^1)\ \cdots\ \cos\hat{\psi}_{\hat{w}^1}(\hat{w}^1)\ ; \\ \mathrm{d}x'^2_{ab} &= \mathrm{d}\hat{x}^2 \cos\hat{\psi}_{\hat{x}^2}(\hat{x}^2)\cos\hat{\psi}_{\hat{y}^2}(\hat{y}^2)\ \cdots\ \cos\hat{\psi}_{\hat{w}^2}(\hat{w}^2)\ , \end{aligned} \tag{76}$$

and

$$\begin{aligned} \mathrm{d}x'^1_{ab} &= \mathrm{d}\hat{x}^1(1-\hat{k}_{\hat{x}^1}(\hat{x}^1))^{1/2}(1-\hat{k}_{\hat{y}^1}(\hat{y}^1))^{1/2}\ \cdots\ (1-\hat{k}_{\hat{w}^1}(\hat{w}^1))^{1/2}\ ; \\ \mathrm{d}x'^2_{ab} &= d\hat{x}^2(1-\hat{k}_{\hat{x}^2}(\hat{x}^2))^{1/2}(1-\hat{k}_{\hat{y}^2}(\hat{y}^2))^{1/2}\cdots\cdots(1-\hat{k}_{\hat{w}^2}(\hat{w}^2))^{1/2}\ . \end{aligned} \tag{77}$$

It is to be recalled that $\hat{x}^{\,i}$ are the absolute metric dimensions of a curved absolute metric 2-space $\hat{\mathbb{M}}^2$, as the outward manifestation of $\varnothing\hat{\mathbb{M}}^2$, but which does not actually exist in the absolute intrinsic Riemann geometry of Figs. 1a and 1b. Thus $\hat{x}^{\,i}$ are the outward manifestations of $\varnothing\hat{x}^{\,i}$.

In the case of a singular absolute intrinsic metric space, systems (77) and (78) simplify respectively as follows

$$dx'^1_{ab} = d\hat{x}^1\cos\hat{\psi}_{\hat{x}^1}(\hat{x}^1)\ ;\quad dx'^2_{ab} = d\hat{x}^2\cos\hat{\psi}_{\hat{x}^2}(\hat{x}^2) \tag{78}$$

and

$$dx'^1_{ab} = d\hat{x}^1(1-\hat{k}_{\hat{x}^1}(\hat{x}^1))^{\frac{1}{2}};\ dx'^2_{ab} = d\hat{x}^2(1-\hat{k}_{\hat{x}^2}(\hat{x}^2))^{\frac{1}{2}} \tag{79}$$

Systems (79) or (80) is the outward (or physical) manifestation of system (1) derived from Figs. 1a&b. System (79) or (80) expresses the evolution of the flat two-dimensional absolute proper metric space $\mathbb{E}'^2_{ab}$ from the flat 'two-dimensional' absolute metric space $\hat{\mathbb{E}}^2$ with the presence of absolute intrinsic Riemann geometry. It is to be noted however that the absolute metric coordinates, $\hat{x}^1$ and $\hat{x}^2$, of the absolute metric space $\hat{\mathbb{E}}^2$ are not curved despite systems (79) and (80). It is only the absolute intrinsic metric coordinates, $\varnothing\hat{x}^{\,1}$ and $\varnothing\hat{x}^{\,2}$, of $\varnothing\hat{\mathbb{E}}^2$ that are curved to form $\varnothing\hat{\mathbb{M}}^2$.

Once the absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1$ and $\varnothing\hat{x}^2$, of the initially flat absolute intrinsic metric space $\varnothing\hat{\mathbb{E}}^2$ underlying $\hat{\mathbb{E}}^2$ in the reference geometry of absolute intrinsic Riemann geometry of Fig. 7 of part one of this paper [1], reproduced as Fig. 4 of this article, become curved to form the absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$, and projects absolute proper intrinsic metric dimensions, $\varnothing x'^1_{ab}$ and $\varnothing x'^2_{ab}$, respectively along the horizontal in Figs. 1a&b, then the projective absolute proper intrinsic metric dimensions, $\varnothing x'^1_{ab}$ and $\varnothing x'^2_{ab}$, of $\varnothing\mathbb{E}'^2_{ab}$ along the horizontal are made manifested in the absolute proper metric space dimensions, x'^1_{ab} and x'^2_{ab}, respectively of the flat absolute proper metric space $\mathbb{E}'^2_{ab}$ along the horizontal, without any need to prescribe the curvature of the 'dimensions', $\hat{x}^1$ and $\hat{x}^2$, of the initially flat absolute metric space $\hat{\mathbb{E}}^2$ overlying the initially flat absolute intrinsic metric space $\varnothing\hat{\mathbb{E}}^2$ in Fig. 4. The flat absolute proper metric space $\mathbb{E}'^2_{ab}$ has simply evolved from the initial flat absolute space $\hat{\mathbb{E}}^2$ by virtue of the evolution of curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^2$ from the initial flat $\varnothing\hat{\mathbb{E}}^2$ in Fig. 4.

The results of the first step of the formulation of absolute intrinsic Riemann geometry for a singular 'two-dimensional' or 'three-dimensional' absolute intrinsic metric space in sub-sections 1.1 through 1.5, and for two or a larger number of co-existing parallel 'two-dimensional' or 'three-dimensional' absolute intrinsic metric spaces in sub-section 1.6, are valid with respect to 2-observers or 3-observers in the flat relative proper (or physical) metric space $\mathbb{E}'^2$ or $\mathbb{E}'^3$.

Now let us rewrite system (75) in terms of resultant absolute intrinsic angles $\varnothing\hat{\psi}_{1\mathrm{res}}$ and $\varnothing\hat{\psi}_{2\mathrm{res}}$ as follows

$$\mathrm{d}\varnothing x'^1_{ab} = \mathrm{d}\hat{w}^1 \cos\varnothing\hat{\psi}_{1\mathrm{res}}; \quad \mathrm{d}\varnothing x'^2_{ab} = \mathrm{d}\hat{w}^2 \cos\varnothing\hat{\psi}_{2\mathrm{res}}. \tag{80}$$

And let us rewrite system (76) in terms of resultant absolute intrinsic curvature parameters $\varnothing\hat{k}_{1\mathrm{res}}$ and $\hat{k}_{2\mathrm{res}}$ as

$$\mathrm{d}\varnothing x'^1_{ab} = \mathrm{d}\hat{w}^1(1-\varnothing\hat{k}^2_{1\mathrm{res}})^{1/2}\ ; \quad \mathrm{d}\varnothing x'^2_{ab} = \mathrm{d}\hat{w}^2(1-\varnothing\hat{k}^2_{2\mathrm{res}})^{1/2}\ . \tag{81}$$

Then as follows from systems (75) and (81)

$$\begin{aligned}\cos\varnothing\hat{\psi}_{1\mathrm{res}} &= \cos(\varnothing\hat{\psi}_{\hat{x}^1} + \varnothing\hat{\psi}_{\hat{y}^1} + \cdots + \varnothing\hat{\psi}_{\hat{w}^1}) = \cos\varnothing\hat{\psi}_{\hat{x}^1}\cos\varnothing\hat{\psi}_{\hat{y}^1}\cdots\cos\varnothing\hat{\psi}_{\hat{w}^1}\ ;\\ \cos\varnothing\hat{\psi}_{2\mathrm{res}} &= \cos(\varnothing\hat{\psi}_{\hat{x}^2} + \varnothing\hat{\psi}_{\hat{y}^2} + \cdots + \varnothing\hat{\psi}_{\hat{w}^2}) = \cos\varnothing\hat{\psi}_{\hat{x}^2}\cos\varnothing\hat{\psi}_{\hat{y}^2}\cdots\cos\varnothing\hat{\psi}_{\hat{w}^2}\ .\end{aligned} \tag{82}$$

And as follows from systems (78) and (82),

$$\begin{aligned}\varnothing\hat{k}^2_{1\mathrm{res}} &= 1-(1-\varnothing\hat{k}^2_{\hat{x}^1})(1-\varnothing\hat{k}^2_{\hat{y}^1})\cdots(1-\varnothing\hat{k}^2_{\hat{w}^1})\ ;\\ \varnothing\hat{k}^2_{2\mathrm{res}} &= 1-(1-\varnothing\hat{k}^2_{\hat{x}^2})(1-\varnothing\hat{k}^2_{\hat{y}^2})\cdots(1-\varnothing\hat{k}^2_{\hat{w}^2})\ .\end{aligned} \tag{83}$$

System (83) expresses the rule for the composition of the absolute intrinsic angles, $\varnothing\hat{\psi}_{\hat{x}^1}, \varnothing\hat{\psi}_{\hat{y}^1}, \ldots, \varnothing\hat{\psi}_{\hat{w}^1}$ and $\varnothing\hat{\psi}_{\hat{x}^2}, \varnothing\hat{\psi}_{\hat{y}^2}, \ldots, \varnothing\hat{\psi}_{\hat{w}^2}$, for the purpose of obtaining resultant absolute intrinsic coordinate projections (or resultant intrinsic length contraction formulae) in the context of absolute intrinsic Riemann geometry, while system (84) expresses the corresponding rule for the composition of absolute intrinsic curvature parameters, $\varnothing\hat{k}_{\hat{x}^1}$, $\varnothing\hat{k}_{\hat{y}^1}$, ..., $\varnothing\hat{k}_{\hat{w}^1}$ and $\varnothing\hat{k}_{\hat{x}^2}$, $\varnothing\hat{k}_{\hat{y}^2}$, ..., $\varnothing\hat{k}_{\hat{w}^2}$. Those rules shall be written more compactly as

$$\begin{aligned}\cos\varnothing\hat{\psi}_{i\,\mathrm{res}} &= \cos(\varnothing\hat{\psi}_{i\,1} + \varnothing\hat{\psi}_{i\,2} + \cdots + \varnothing\hat{\psi}_{i\,N})\\ &= \cos\varnothing\hat{\psi}_{i\,1}\cos\varnothing\hat{\psi}_{i\,2}\cdots\cos\varnothing\hat{\psi}_{i\,N}\ ; \quad i = 1,\,2,\,3\ .\end{aligned} \tag{84}$$

and

$$\varnothing\hat{k}^2_{i\text{res}} = 1 - (1 - \varnothing\hat{k}^2_{i1})(1 - \varnothing\hat{k}^2_{i2}) \cdots (1 - \varnothing\hat{k}^2_{iN}) \; ; \quad i = 1,\, 2,\, 3 \,. \tag{85}$$

where $i\,(= 1, 2 \text{ or } 3)$ refers to the three curved absolute intrinsic metric 'dimensions' of the absolute intrinsic metric spaces superposed, and N is the number of absolute intrinsic metric spaces superposed. One observes from Eq. (85) that if $\varnothing\hat{\psi}_{iq} = 90^{\text{o}}$; $q = 1 \,\text{or}\, 2 \,\text{or}\, 3 \,\ldots\, \text{or}\, N$, then $\cos\varnothing\hat{\psi}_{i\,q} = 0$ and $\cos\varnothing\hat{\psi}_{i\,\text{res}} = 0$. Hence $\varnothing\hat{\psi}_{i\,\text{res}} = 90^{\text{o}}$ too. Also if $\varnothing\hat{k}_{i\,q} = 1$; q=1 or 2 or 3 ... or N, which corresponds to $\varnothing\hat{\psi}_{i\,q} = 90^{\text{o}}$ from, $\varnothing\hat{k}_{i\,q} = \sin\varnothing\hat{\psi}_{i\,q}$, then $\varnothing\hat{k}_{1\,\text{res}} = 1$ too. These results show that the rules for composition of absolute intrinsic angles and absolute intrinsic curvature parameters for the purpose of obtaining resultant intrinsic coordinate projections (or resultant intrinsic length contraction formulae) in absolute intrinsic Riemann geometry, do not lead to values of resultant absolute intrinsic angles larger than 90^{o} or resultant absolute intrinsic curvature parameters larger than unity.

In other words, absolute intrinsic angle, $\varnothing\hat{\psi} = 90^{\text{o}}$, is an invariant absolute intrinsic angle and absolute intrinsic curvature parameter, $\varnothing\hat{k} = 1$, is an invariant absolute intrinsic curvature parameter in the rules for composition of absolute intrinsic angles and absolute intrinsic curvature parameters, for the purpose of obtaining resultant absolute intrinsic coordinate projection relations, or resultant intrinsic length contraction formulae, with respect to observers in the underlying relative (or physical) proper Euclidean 3-space $\mathbb{E}'^3$, when two or a larger number N of absolute intrinsic metric spaces co-exist.

Finally it is important to remark the major difference between the rule for composition of absolute intrinsic angles of system (62) (or the equivalent rule for composition of absolute intrinsic curvature parameters of system (63)), for the purpose of writing the resultant absolute intrinsic line element, the resultant absolute intrinsic metric tensor and the resultant absolute intrinsic Ricci tensor, and the counterpart rule for composition of absolute intrinsic angles of system (83) (or its equivalent rule for composition of absolute intrinsic curvature parameters of system (84)), for the purpose of writing the resultant intrinsic coordinate projection relations or resultant intrinsic length contraction formulae, in the context of absolute intrinsic Riemann geometry. These rules are valid with respect to all 3-observers in the underlying relative (or physical) proper Euclidean 3-space $\mathbb{E}'^3$, when N parallel 'three-dimensional' absolute intrinsic metric spaces are superposed.

2.6.3 Parallelism of all absolute intrinsic metric spaces in the universe

The highly ordered situation of the co-existence of parallel absolute intrinsic metric spaces has been considered so far in this sub-section. As defined previously, a pair of 'three-dimensional' absolute intrinsic metric spaces $\varnothing\hat{\mathbb{M}}^3$ with absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1, \varnothing\hat{x}^2, \varnothing\hat{x}^3$ and $\varnothing\hat{\mathbb{M}}^3_{(2)}$, with curved absolute intrinsic 'dimensions' $\varnothing\hat{y}^1, \varnothing\hat{y}^2, \varnothing\hat{y}^3$, are parallel if each curved absolute intrinsic 'dimension' $\varnothing\hat{y}^i$ of $\varnothing\hat{\mathbb{M}}^3_{(2)}$

and the corresponding curved absolute intrinsic 'dimension' $\varnothing\hat{x}^i$ of $\varnothing\hat{\mathbb{M}}^3$ lie on the same vertical $\varnothing x'^i_{ab}\varnothing\hat{x}^0-$ plane. In this situation, the intrinsic dimensions, $\varnothing x'^1_{ab}, \varnothing x'^2_{ab}$ and $\varnothing x'^3_{ab}$, of the underlying global absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ have parametric dependence on the absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1, \varnothing\hat{x}^2$ and $\varnothing\hat{x}^3$, respectively of $\varnothing\hat{\mathbb{M}}^3$, as well as the absolute intrinsic metric 'dimensions', $\varnothing\hat{y}^1, \varnothing\hat{y}^2$ and $\varnothing\hat{y}^3$ respectively, of $\varnothing\hat{\mathbb{M}}^3_{(2)}$, prior to the superposition of $\varnothing\hat{\mathbb{M}}^3$ and $\varnothing\hat{\mathbb{M}}^3_{(2)}$ as,

$$\varnothing x'^1_{ab} = g^1(\varnothing\hat{y}^1);\ \varnothing x'^2_{ab} = g^2(\varnothing\hat{y}^2);\ \varnothing x'^3_{ab} = g^3(\varnothing\hat{y}^3) \tag{87a}$$

and

$$\varnothing x'^1_{ab} = f^1(\varnothing\hat{x}^1);\ \varnothing x'^2_{ab} = f^2(\varnothing\hat{x}^2);\ \varnothing x'^3_{ab} = f^3(\varnothing\hat{x}^3)\ . \tag{87b}$$

When $\varnothing\hat{\mathbb{M}}^3_{(2)}$ and $\varnothing\hat{\mathbb{M}}^3$ coexist, the curved absolute intrinsic metric space 'dimension' $\varnothing\hat{y}^q$ lies above the curved absolute intrinsic metric space 'dimension' $\varnothing\hat{x}^q$ on the vertical $\varnothing x'^q_{ab}\varnothing\hat{x}^0-$plane, for $q = 1, 2$ and 3, as illustrated in Fig. 6 for the superposition of the pair of parallel 'two-dimensional' absolute intrinsic metric spaces in Figs. 5a and 3b prior to their superposition.

Now let us consider the chaotic situation of the co-existence of non-parallel absolute intrinsic metric spaces. In this situation, some or all of the curved absolute intrinsic metric space 'dimensions', $\varnothing\hat{y}^q$ of $\varnothing\hat{\mathbb{M}}^3_{(2)}$, do not lie above the corresponding curved absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^q$, of $\varnothing\hat{\mathbb{M}}^3$ on the vertical $\varnothing x'^q_{ab}\varnothing\hat{x}^0$-plane. In this situation, while the absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1, \varnothing\hat{x}^2$ and $\varnothing\hat{x}^3$, of $\varnothing\hat{\mathbb{M}}^3$ are parameterized in terms of an absolute proper intrinsic metric coordinate set (or 'frame') $(\varnothing\xi'^1_{ab}, \varnothing\xi'^2_{ab}, \varnothing\xi'^3_{ab})$ in the underlying global absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$, the curved absolute intrinsic metric 'dimensions', $\varnothing\hat{y}^1, \varnothing\hat{y}^2$ and $\varnothing\hat{y}^3$ of $\varnothing\hat{\mathbb{M}}^3_{(2)}$, are parameterized in terms of a different absolute proper intrinsic metric coordinate set (or 'frame') $(\varnothing\eta'^1_{ab}, \varnothing\eta'^2_{ab}, \varnothing\eta'^3_{ab})$ in the underlying global absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ in general, prior to the superposition of $\varnothing\hat{\mathbb{M}}^3$ and $\varnothing\hat{\mathbb{M}}^3_{(2)}$. In other words, the following transformations of local intrinsic metric coordinates obtain in general prior to the superposition of $\varnothing\hat{\mathbb{M}}^3$ and $\varnothing\hat{\mathbb{M}}^3_{(2)}$

$$\varnothing\eta'^1_{ab} = \mathrm{f}^1(\varnothing\hat{y}^1);\ \varnothing\eta'^2_{ab} = \mathrm{f}^2(\varnothing\hat{y}^2);\ \varnothing\eta'^3_{ab} = \mathrm{f}^3(\varnothing\hat{y}^3) \tag{88a}$$

and

$$\varnothing\xi'^1_{ab} = \mathrm{g}^1(\varnothing\hat{x}^1);\ \varnothing\xi'^2_{ab} = \mathrm{g}^2(\varnothing\hat{x}^2);\ \varnothing\xi'^3_{ab} = \mathrm{g}^3(\varnothing\hat{x}^3) \tag{88b}$$

When $\varnothing\hat{\mathbb{M}}^3_{(2)}$ and $\varnothing\hat{\mathbb{M}}^3$ coexist, or are superposed, they are both underlay by the global flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$. However the absolute intrinsic metric 'dimensions', $\varnothing\hat{y}^1, \varnothing\hat{y}^2$ and $\varnothing\hat{y}^3$, of $\varnothing\hat{\mathbb{M}}^3_{(2)}$ are curved relative to the absolute proper intrinsic metric coordinates, $\varnothing\eta'^1_{ab}, \varnothing\eta'^2_{ab}$ and $\varnothing\eta'^3_{ab}$, respectively of one frame in $\varnothing\mathbb{E}'^3_{ab}$, while the absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1, \varnothing\hat{x}^2$ and

$\varnothing\hat{x}^3$, of $\varnothing\hat{\mathbb{M}}^3$ are curved relative to the absolute proper intrinsic metric coordinates, $\varnothing\xi'^1_{ab}, \varnothing\xi'^2_{ab}$ and $\varnothing\xi'^3_{ab}$, of another frame in $\varnothing\mathbb{E}'^3$

Having described the superposition of parallel absolute intrinsic metric spaces and the superposition of non-parallel absolute intrinsic metric spaces above, it shall now be shown that non-parallel absolute intrinsic metric spaces do not exist in nature.

As deduced from the consistent arguments leading to the isolation of absolute intrinsic Riemannian metric spaces in section 4 of part one of this paper [1], all local absolute intrinsic metric coordinate sets (or local absolute intrinsic 'frames'), $(\varnothing\hat{x}^1, \varnothing\hat{x}^2, \varnothing\hat{x}^3)$, $(\varnothing\hat{y}^1, \varnothing\hat{y}^2, \varnothing\hat{y}^3)$, $(\varnothing\hat{z}'^1, \varnothing\hat{z}^2, \varnothing\hat{z}^3)$, $(\varnothing\hat{w}'^1, \varnothing\hat{w}^2, \varnothing\hat{w}^3)$, etc, at a point on a curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ are equivalent to a singular local absolute intrinsic metric coordinate set (or local absolute intrinsic 'frame') $(\varnothing\hat{x}^1, \varnothing\hat{x}^2, \varnothing\hat{x}^3)$, with respect to observers in the relative proper metric 3-space $\mathbb{E}'^3$ underlying $\varnothing\hat{\mathbb{M}}^3$.

All the projective local absolute proper intrinsic metric coordinate sets (or local absolute proper intrinsic 'frames'), $(\varnothing x'^1_{ab}, \varnothing x'^2_{ab}, \varnothing x'^3_{ab})$, $(\varnothing y'^1_{ab}, \varnothing y'^2_{ab}, \varnothing y'^3_{ab})$, $(\varnothing z'^1_{ab}, \varnothing z'^2_{ab}, \varnothing z'^3_{ab})$ $(\varnothing w'^1_{ab}, \varnothing w'^2_{ab}, \varnothing w'^3_{ab})$, etc, at the corresponding point in the underlying projective absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$, are equivalent to a singular local absolute proper intrinsic metric coordinate set (or local absolute proper intrinsic 'frame') $(\varnothing x'^1_{ab}, \varnothing x'^2_{ab}, \varnothing x'^3_{ab})$, with respect to all 3-observers in $\mathbb{E}'^3$, as a consequence.

It follows from the foregoing paragraph that the two local frames, $(\varnothing\xi'^1_{ab}, \varnothing\xi'^2_{ab}, \varnothing\xi'^3_{ab})$ and $(\varnothing\eta_{ab}{}'^1, \varnothing\eta_{ab}{}'^2, \varnothing\eta_{ab}{}'^3)$, on the flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ that lie underneath two coexisting non-parallel absolute intrinsic metric spaces, $\varnothing\hat{\mathbb{M}}^3$ and $\varnothing\hat{\mathbb{M}}^3_{(2)}$, respectively in our discussion above, are equivalent to the singular local absolute proper intrinsic metric coordinate set (or 'frame') $(\varnothing x'^1_{ab}, \varnothing x'^2_{ab}, \varnothing x'^3_{ab})$ in the flat $\varnothing\mathbb{E}'^3_{ab}$ (where $\varnothing x'^1_{ab}$, $\varnothing x'^2_{ab}$ and $\varnothing x'^3_{ab}$ are actually the absolute proper intrinsic dimensions of $\varnothing\mathbb{E}'^3_{ab}$).

It then follows that the curved absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^2, \varnothing\hat{x}^2, \varnothing\hat{x}^3$ and $\varnothing\hat{y}^2, \varnothing\hat{y}^2, \varnothing\hat{y}^3$, of the co-existing non-parallel absolute intrinsic metric spaces, $\varnothing\hat{\mathbb{M}}^3$ and $\varnothing\hat{\mathbb{M}}^3_{(2)}$, respectively are actually curved relative to the singular local absolute proper intrinsic metric coordinate set (or 'frame') $(\varnothing x'^1_{ab}, \varnothing x'^2_{ab}, \varnothing x'^3_{ab})$ of the underlying flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$, which makes them parallel. The local absolute proper intrinsic coordinate set $(\varnothing\eta'^1_{ab}, \varnothing\eta'^2_{ab}, \varnothing\eta'^3_{ab})$ in (88a) and the local absolute proper intrinsic coordinate set $(\varnothing\xi'^1_{ab}, \varnothing\xi'^2_{ab}, \varnothing\xi'^3_{ab})$ in system (88b) must be replaced by the same local absolute proper intrinsic metric coordinate set $(\varnothing x_{ab}{}'^1, \varnothing x_{ab}{}'^2, \varnothing x_{ab}{}'^3)$.

The conclusion that follows from the preceding paragraph is that all absolute intrinsic Riemannian metric spaces in the universe are parallel, all lying above the singular absolute proper intrinsic metric coordinate set (or 'frame') $(\varnothing x'^1_{ab}, \varnothing x'^2_{ab}, \varnothing x'^3_{ab})$ of the isotropic absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ that lies underneath all curved absolute intrinsic Riemannian metric spaces.

The programme of this sub-section, which is to formulate absolute intrinsic

Riemann geometry when two or a larger number of absolute intrinsic metric spaces co-exist (both the first and second steps of the formulation), has been accomplished. An interesting and dramatic aspect of absolute intrinsic Riemann geometry shall be discussed in the next and concluding section of this article.

3 Perfect isotropy and implied 'one-dimensionality' of a curved absolute intrinsic metric '3-space', its projective flat absolute proper intrinsic metric '3-space' and the flat absolute proper metric '3-space' with respect to 3-observers in the underlying relative proper metric Euclidean 3-space

3.1 Natural contraction of the curved three- (or two-) dimensional absolute intrinsic metric space to a curved 'one-dimensional' scalar isotropic absolute intrinsic metric space with respect to all Euclidean 3-observers in the relative proper metric 3-space

As deduced from the fact that both the absolute intrinsic line element and absolute intrinsic metric tensor on an absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ are invariant with change of local absolute intrinsic coordinate set at each point on $\varnothing\hat{\mathbb{M}}^3$ in section 4 of part one of this paper [1], different local absolute intrinsic coordinate sets, $(\varnothing\hat{x}^1, \varnothing\hat{x}^2, \varnothing\hat{x}^3)$, $(\varnothing\hat{y}^1, \varnothing\hat{y}^2, \varnothing\hat{y}^3)$, $(\varnothing\hat{z}^1, \varnothing\hat{z}^2, \varnothing\hat{z}^3)$, etc, which are arbitrarily orientated relative to one another at a given point P on $\varnothing\hat{\mathbb{M}}^3$, with respect to a Riemannian observer located at the point P on $\varnothing\hat{\mathbb{M}}^3$, are identical to a singular local absolute intrinsic coordinate set $(\varnothing\hat{\xi}^1, \varnothing\hat{\xi}^2, \varnothing\hat{\xi}^3)$ at the point P on $\varnothing\hat{\mathbb{M}}^3$, with respect to Euclidean observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$ underlying the curved $\varnothing\hat{\mathbb{M}}^3$.

An implication of the foregoing paragraph is that the local absolute intrinsic metric coordinates, $\varnothing\hat{x}^1$, $\varnothing\hat{y}^1$ and $\varnothing\hat{y}^1$, etc, of the different local absolute intrinsic coordinate sets (or 'frames'), $(\varnothing\hat{x}^1, \varnothing\hat{x}^2, \varnothing\hat{x}^3)$, $(\varnothing\hat{y}^1, \varnothing\hat{y}^2, \varnothing\hat{y}^3)$, $(\varnothing\hat{z}^1, \varnothing\hat{z}^2\varnothing\hat{z}^3)$, etc, which are orientated along different directions about the point P on $\varnothing\hat{\mathbb{M}}^3$, with respect to a Riemannian observer at this point, are all identical to a singular local absolute intrinsic coordinate $\varnothing\hat{\xi}^1$ at the point P on the curved $\varnothing\hat{\mathbb{M}}^3$, with respect to the Euclidean 3-observers in $\mathbb{E}'^3$ underlying $\varnothing\hat{\mathbb{M}}^3$.

It thus follows that the different absolute intrinsic angles, $\varnothing\hat{\alpha}, \varnothing\hat{\beta}, \varnothing\hat{\gamma}$, etc, at which the local absolute intrinsic coordinates, $\varnothing\hat{x}^1, \varnothing\hat{y}^1, \varnothing\hat{z}^1$, etc, of different local absolute intrinsic coordinate sets are inclined relative to one another at the point P on $\varnothing\hat{\mathbb{M}}^3$, with respect to a Riemannian observer at this point, all vanish, that is, $\varnothing\hat{\alpha} = \varnothing\hat{\beta} = \varnothing\hat{\gamma} = 0$, with respect to observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$ underlying $\varnothing\hat{\mathbb{M}}^3$. The different local absolute intrinsic coordinates, $\varnothing\hat{x}^1, \varnothing\hat{y}^1, \varnothing\hat{z}^1$, etc, are all aligned along a singular direction, thereby constituting a singular local absolute intrinsic coordinate $\varnothing\hat{\xi}^{\,1}$ at the point P on the curved $\varnothing\hat{\mathbb{M}}^3$, with respect to observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$ consequently.

The different absolute intrinsic angles, $\varnothing\hat{\delta}, \varnothing\hat{\theta}, \varnothing\hat{\varphi}$, etc, at which the local absolute intrinsic coordinates, $\varnothing\hat{x}^2, \varnothing\hat{y}^2, \varnothing\hat{z}^2$, etc, of different local absolute intrinsic coordinate sets (or 'frames') are inclined relative to one another at point P on $\varnothing\hat{\mathbb{M}}^3$, with respect to a Riemannian observer at this point all vanish, that is, $\varnothing\hat{\delta} = \varnothing\hat{\theta} = \varnothing\hat{\varphi} = 0$, with respect to Euclidean observers in $\mathbb{E}'^3$ underlying $\varnothing\hat{\mathbb{M}}^3$. Consequently the different local absolute intrinsic coordinates, $\varnothing\hat{x}^2, \varnothing\hat{y}^2, \varnothing\hat{z}^2$, etc, are all aligned along a singular direction, thereby constituting a singular local absolute intrinsic coordinate $\varnothing\hat{\xi}^{\,2}$ at the point P on $\varnothing\hat{\mathbb{M}}^3$, with respect to Euclidean observers in $\mathbb{E}'^3$. Likewise the different local absolute intrinsic coordinates, $\varnothing\hat{x}^3, \varnothing\hat{y}^3, \varnothing\hat{z}^3$, etc, are all aligned along a singular direction, thereby constituting a singular local absolute intrinsic coordinate $\varnothing\hat{\xi}^{\,3}$ at point P on $\varnothing\hat{\mathbb{M}}^3$, with respect to all Euclidean observers in $\mathbb{E}'^3$.

As found from the foregoing two paragraphs, two directions within an elementary local neighborhood about a point P of the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$, which are distinct directions separated by absolute intrinsic Euler angles, $\varnothing\hat{\alpha}, \varnothing\hat{\beta}$ and $\varnothing\hat{\gamma}$, with respect to a Riemannian observer located at point P on $\varnothing\hat{\mathbb{M}}^3$, are the same direction with respect to observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$ underlying $\varnothing\hat{\mathbb{M}}^3$. This is so, since any magnitudes of the absolute intrinsic angles, $\varnothing\hat{\alpha}$, $\varnothing\hat{\beta}$ and $\varnothing\hat{\gamma}$, on $\varnothing\hat{\mathbb{M}}^3$ are equivalent to zero magnitudes of the corresponding relative Euler angles, α', β' and γ', in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$.

It also follows as further implication of the preceding paragraph that the singular local absolute intrinsic coordinate set $(\varnothing\hat{\xi}^{\,1}, \varnothing\hat{\xi}^{\,2}, \varnothing\hat{\xi}^{\,3})$ at point P on $\varnothing\hat{\mathbb{M}}^3$, with mutually perpendicular local absolute intrinsic coordinates, $\varnothing\hat{\xi}^{\,1}$, $\varnothing\hat{\xi}^{\,2}$ and $\varnothing\hat{\xi}^{\,3}$, is impossible with respect to Euclidean observers in $\mathbb{E}'^3$.

The end of the preceding paragraph is so, because the absolute intrinsic angle, $\varnothing\hat{\phi} = \varnothing\hat{\pi}/2$, separating the local absolute intrinsic coordinates, $\varnothing\hat{\xi}^{\,1}$ and $\varnothing\hat{\xi}^{\,2}$, and the absolute intrinsic angle, $\varnothing\hat{\theta} = \varnothing\hat{\pi}/2$, separating the local absolute intrinsic coordinates, $\varnothing\hat{\xi}^{\,2}$ and $\varnothing\hat{\xi}^{\,3}$, as well as the absolute intrinsic angle, $\varnothing\hat{\varphi} = \varnothing\hat{\pi}/2$, separating the local absolute intrinsic coordinates, $\varnothing\hat{\xi}^{\,1}$ and $\varnothing\hat{\xi}^{\,3}$, all vanish with respect to Euclidean observers in $\mathbb{E}'^3$, thereby causing, $\varnothing\hat{\xi}^{\,1}$, $\varnothing\hat{\xi}^{\,2}$ and $\varnothing\hat{\xi}^{\,3}$, to be aligned along a singular direction. They thereby constitute a singular local absolute intrinsic coordinate $\varnothing\hat{\xi}_{\rm P}$ at point P on $\varnothing\hat{\mathbb{M}}^3$ with respect to all observers in $\mathbb{E}'^3$.

The result derived at point P on the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ in the preceding paragraph obtains at every other point on $\varnothing\hat{\mathbb{M}}^3$. In other words, only singular local absolute intrinsic coordinates, $\varnothing\hat{\xi}_{\rm Q}$, $\varnothing\hat{\xi}_{\rm R}$, $\varnothing\hat{\xi}_{\rm S}$, $\varnothing\hat{\xi}_{\rm T}$, etc, exist at points Q, R, S, T, etc, on $\varnothing\hat{\mathbb{M}}^3$, with respect to the Euclidean observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$ underlying $\varnothing\hat{\mathbb{M}}^3$. When the singular indefinitely short local absolute intrinsic coordinates at every point of $\varnothing\hat{\mathbb{M}}^3$ are joined together, one obtains a continuous curved 'one-dimensional' absolute intrinsic metric space (or an absolute intrinsic Riemannian metric space 'dimension'), to be denoted by $\varnothing\hat{\rho}$, with respect to Euclidean observers in $\mathbb{E}'^3$.

The singular indefinitely short local absolute intrinsic coordinates, $\varnothing\hat{\xi}_{\mathrm{Q}}$, $\varnothing\hat{\xi}_{\mathrm{Q}}$, $\varnothing\hat{\xi}_{\mathrm{R}}$, $\varnothing\hat{\xi}_{\mathrm{S}}$, $\varnothing\hat{\xi}_{\mathrm{T}}$, etc, that exist within elementary locally flat (or Euclidean) regions at points P, Q, R, S, T, etc, on $\varnothing\hat{\mathbb{M}}^3$, with respect to the Euclidean observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$, have no unique orientations (or basis) within their locally flat elementary regions. They are consequently scalar isotropic local absolute intrinsic coordinates within their locally flat regions. The continuous curved 'one-dimensional' absolute intrinsic metric space (or a 'one-dimensional' absolute intrinsic Riemannian metric space 'dimension') $\varnothing\hat{\rho}$, obtained by joining the local absolute intrinsic coordinates, is consequently a scalar isotropic curved absolute intrinsic 'dimension' (in the small), with respect to Euclidean observers $\mathbb{E}'^3$.

An important conclusion has been reached in the preceding paragraph that, the curved absolute intrinsic metric space (or absolute intrinsic Riemannian metric space), which has been considered as 'two-dimensional' $\varnothing\hat{\mathbb{M}}^2$ or 'three-dimensional' $\varnothing\hat{\mathbb{M}}^3$, with respect to Euclidean observers in the underlying relative proper metric Euclidean space, $\mathbb{E}'^2$ or $\mathbb{E}'^3$, so far in this article, is actually a 'one-dimensional' curved scalar isotropic absolute intrinsic metric space (or a curved scalar isotropic absolute intrinsic Riemannian metric space 'dimension'), with respect to all 3-observers in the underlying relative proper physical Euclidean 3-space $\mathbb{E}'^3$.

3.2 Natural contraction of the projective flat three- (or two-) dimensional absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ to a straight line 'one-dimensional' isotropic absolute proper intrinsic metric space with respect to all 3-observers in the relative proper metric Euclidean 3-space

The 'one-dimensional' scalar isotropic absolute intrinsic metric space $\varnothing\hat{\rho}$ that is curved toward the absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ along the vertical, with respect to Euclidean observers in the flat relative proper metric space $\mathbb{E}'^3$, will naturally project one-dimensional straight line scalar isotropic absolute proper intrinsic metric space, to be denoted by $\varnothing\rho'_{ab}$, underneath the relative proper metric Euclidean 3-space $\mathbb{E}'^3$, with respect to 3-observers in $\mathbb{E}'^3$.

However let us for completeness also show below that a three-dimensional flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$, considered to be projected underneath the flat relative proper metric 3-space $\mathbb{E}'^3$ by the curved 'three-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ previously in this article and its first part, naturally contracts to a one-dimensional scalar isotropic absolute proper intrinsic metric space

$\varnothing\rho'_{ab}$ with respect to 3-observers in $\mathbb{E}'^3$.

Now any magnitudes of the absolute proper intrinsic angles, $\varnothing\alpha'_{ab}, \varnothing\beta'_{ab}$ and $\varnothing\gamma'_{ab}$, on the flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ are equivalent to zero magnitude of relative proper (or physical) angles, α', β' and γ', respectively in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$, with respect to 3-observers in $\mathbb{E}'^3$ overlying $\varnothing\mathbb{E}'^3_{ab}$. These follow from the equivalences, $\varnothing\alpha'_{ab} \equiv \varnothing\hat{\alpha} = 0 \times \alpha'$, $\varnothing\beta'_{ab} \equiv \varnothing\hat{\beta} = 0 \times \beta'$ and $\varnothing\gamma'_{ab} \equiv \varnothing\hat{\gamma} = 0 \times \gamma'$. Consequently any two distinct directions, which are separated by non-zero absolute proper intrinsic angles, $\varnothing\alpha'_{ab}, \varnothing\beta'_{ab}$ and $\varnothing\gamma'_{ab}$, on the flat three-dimensional absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$, with respect to hypothetical intrinsic 3-observers in $\varnothing\mathbb{E}'^3_{ab}$, are the same direction with respect to 3-observers in the relative proper physical Euclidean 3-space $\mathbb{E}'^3$.

A consequence of the preceding paragraph is that the mutually orthogonal local absolute proper intrinsic metric coordinates, $\varnothing\xi'^1_{ab}, \varnothing\xi'^2_{ab}$ and $\varnothing\xi'^3_{ab}$, of $\varnothing\mathbb{E}'^3_{ab}$, at a point on $\varnothing\mathbb{E}'^3_{ab}$, are impossible with respect to 3-observers in $\mathbb{E}'^3$. This is so because the absolute proper intrinsic angle, $\varnothing\phi'_{ab} \equiv \varnothing\hat{\phi} = \varnothing\hat{\pi}/2$, between the local absolute proper intrinsic coordinates, $\varnothing\xi'^1_{ab}$ and $\varnothing\xi'^2_{ab}$ and $\varnothing\theta'_{ab} \equiv \varnothing\hat{\theta} = \varnothing\hat{\pi}/2$, between the local absolute proper intrinsic coordinates, $\varnothing\xi'^2_{ab}$ and $\varnothing\xi'^3_{ab}$, as well as the absolute proper intrinsic angle, $\varnothing\varphi'_{ab} \equiv \varnothing\hat{\varphi} = \varnothing\hat{\pi}/2$, between the local absolute proper intrinsic coordinates, $\varnothing\xi'^1_{ab}$ and $\varnothing\xi'^3_{ab}$, all vanish with respect to observers in $\mathbb{E}'^3$. The three local absolute proper intrinsic coordinates, $\varnothing\xi'^1_{ab}, \varnothing\xi'^2_{ab}$ and $\varnothing\xi'^3_{ab}$ of $\varnothing\mathbb{E}'^3_{ab}$, are consequently aligned along a singular direction, thereby constituting a singular absolute proper intrinsic metric space (or a singular absolute proper intrinsic metric space dimension) denoted by $\varnothing\rho'_{ab}$ above, which underlies the relative proper metric Euclidean 3-space $\mathbb{E}'^3$, with respect to all 3-observers in $\mathbb{E}'^3$.

The one-dimensional absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ has no unique orientation (or basis) in the flat three-dimensional absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ that contracts to it. Consequently it has no unique orientation (or basis) in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$ overlying $\varnothing\mathbb{E}'^3_{ab}$, with respect to 3-observers in $\mathbb{E}'^3$. Thus $\varnothing\rho'_{ab}$ is a scalar isotropic intrinsic metric space (or scalar isotropic intrinsic metric space dimension) in $\mathbb{E}'^3$, with respect to 3-observers in $\mathbb{E}'^3$. It can be considered to lie along any direction in $\mathbb{E}'^3$ by 3-observers in $\mathbb{E}'^3$.

3.3 Natural contraction of the flat three- (or two-) dimensional absolute proper metric space $\mathbb{E}'^3_{ab}$ to a straight line 'one-dimensional' isotropic absolute proper metric space with respect to all Euclidean 3-observers in the relative proper metric 3-space

The natural contractions of the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ (or $\varnothing\hat{\mathbb{M}}^2$) in Fig. 1a to a curved 'one-dimensional absolute intrinsic metric space $\varnothing\hat{\rho}$ and the natural contraction of the flat absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ (or $\varnothing\mathbb{E}'^2_{ab}$) projected along the horizontal by the curved $\varnothing\hat{\mathbb{M}}^3$ (or $\varnothing\hat{\mathbb{M}}^2$) in Fig. 1a, to a straight line 'one-dimensional' scalar isotropic absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ along the horizontal, with respect to all 3-observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$, has been shown in the preceding two sub-sections.

The outward manifestation of the projective absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$ (or $\varnothing\mathbb{E}'^2_{ab}$)namely, the flat absolute proper metric space $\mathbb{E}'^3_{ab}$ (or $\mathbb{E}'^2_{ab}$) in Fig. 1b, likewise naturally contracts to a straight line 'one-dimensional' isotropic scalar absolute proper metric space ρ'_{ab} with respect to all 3-observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$, demonstrated as follows.

Now any magnitudes of the absolute proper angles, $\alpha'_{ab}, \beta'_{ab}$ and γ'_{ab}, (which are equivalent to absolute angles, $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ respectively), on the flat absolute proper metric space $\mathbb{E}'^3_{ab}$, are equivalent to zero magnitude of relative (or physical) proper angles, α', β' and γ', respectively in the relative proper physical Euclidean 3-space $\mathbb{E}'^3$, with respect to 3-observers in $\mathbb{E}'^3$ overlying $\mathbb{E}'^3_{ab}$. These follow from the equivalences, $\alpha'_{ab} \equiv \hat{\alpha} = 0 \times \alpha'$, $\beta'_{ab} \equiv \hat{\beta} = 0 \times \beta'$, and $\gamma'_{ab} \equiv \hat{\gamma} = 0 \times \gamma'$. (An absolute angle is equivalent to zero relative angle.) Consequently any two distinct directions, which are separated by non-zero absolute proper angles, $\alpha'_{ab}, \beta'_{ab}$ and γ'_{ab}, on the flat three-dimensional absolute proper metric space $\mathbb{E}'^3_{ab}$, with respect to hypothetical 3-observers in $\mathbb{E}'^3_{ab}$, are the same direction with respect to 3-observers in the relative (or physical) proper metric Euclidean 3-space $\mathbb{E}'^3$.

A consequence of the preceding paragraph is that the mutually orthogonal local absolute proper metric coordinates, ξ'^1_{ab}, ξ'^2_{ab} and ξ'^3_{ab} of $\mathbb{E}'^3_{ab}$, are impossible with respect to 3-observers in $\mathbb{E}'^3$. This is so because the absolute proper angle, $\phi'_{ab} \equiv \hat{\phi} = \hat{\pi}/2$, between the local absolute proper coordinates, ξ'^1_{ab} and ξ'^2_{ab}, and $\varnothing\theta'_{ab} \equiv \hat{\theta} = \hat{\pi}/2$ between the local absolute proper coordinates, ξ'^2_{ab} and ξ'^3_{ab}, as well as the absolute proper angle, $\varphi'_{ab} \equiv \hat{\varphi} = \hat{\pi}/2$, between the local absolute proper coordinates, ξ'^1_{ab} and ξ'^3_{ab}, all vanish with respect to observers in $\mathbb{E}'^3$. The three local absolute proper coordinates, ξ'^1_{ab}, ξ'^2_{ab} and ξ'^3_{ab} of $\mathbb{E}'^3_{ab}$, are consequently aligned along a singular direction, thereby constituting a singular absolute proper metric space (or absolute proper metric space dimension), denoted by ρ'_{ab} above, which underlies the relative proper metric Euclidean 3-space $\mathbb{E}'^3$, with respect to all 3-observers in $\mathbb{E}'^3$.

The one-dimensional absolute proper metric space ρ'_{ab} has no unique orientation (or unique basis) in the flat three-dimensional absolute proper space $\mathbb{E}'^3_{ab}$ that contracts to it. Consequently it has no unique orientation (or basis) in the relative (or physical) proper metric Euclidean 3-space $\mathbb{E}'^3$ overlying $\mathbb{E}'^3_{ab}$, with respect to 3-observers in $\mathbb{E}'^3$. Thus ρ'_{ab} is a scalar isotropic absolute proper metric space (or scalar isotropic absolute proper metric space dimension) embedded in $\mathbb{E}'^3$, with respect to 3-observers in $\mathbb{E}'^3$. It can be considered to lie along every direction in $\mathbb{E}'^3$ by 3-observers in $\mathbb{E}'^3$.

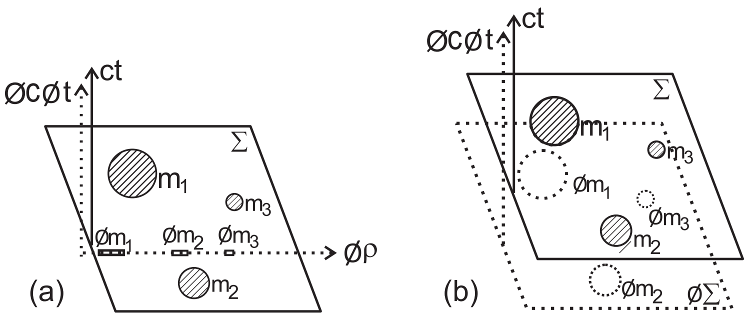

Figure 8: (a) The flat 4-dimensional spacetime and its underlying flat 2-dimensional intrinsic spacetime with the inertial masses of three objects scattered in the Euclidean 3-space and their one-dimensional intrinsic inertial masses aligned along the one-dimensional isotropic intrinsic space with respect to observers in spacetime. (b) The flat 2-dimensional intrinsic spacetime with respect to observers in spacetime in (a) is a flat four-dimensional intrinsic spacetime containing 3-dimensional intrinsic inertial masses of particles and objects in 3-dimensional intrinsic space with respect to intrinsic-mass-observers in intrinsic spacetime; Fig. 6a&b of [2].

3.4 Natural contraction of the flat three- (or two-) dimensional relative proper intrinsic metric space $\varnothing\mathbb{E}'^3$ to a straight line one-dimensional scalar isotropic relative proper intrinsic metric space with respect to all Euclidean 3-observers in the relative proper metric 3-space

The flat three dimensional relative proper intrinsic metric space $\varnothing\mathbb{E}'^3$ (or $\varnothing\mathbb{E}'^2$) underlying the curved absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ (or $\varnothing\hat{\mathbb{M}}^2$) in Fig. 1a, is also naturally contracted to a straight line one-dimensional scalar isotropic relative proper intrinsic metric space (or a scalar isotropic relative proper intrinsic space dimension), which has been denoted by $\varnothing\rho'$ since its introduction (as *ansatz*) in sub-section 4.4 of [2], and used in the articles [4–6] that follow [2].

As mentioned in [2], the three-dimensional relative proper intrinsic metric space $\varnothing\mathbb{E}'^3$ (denoted by $\varnothing\Sigma'$ in that article), with respect to intrinsic 3-observers in it, in Fig. 6a of that article, naturally contracts to a straight line one-dimensional scalar isotropic relative intrinsic metric space $\varnothing\rho'$, with respect to 3-observers in the three-dimensional relative proper Euclidean 3-space $\varnothing\mathbb{E}'^3$ (or Σ') overlying it, in Fig. 6b of that article. This fact shall be shown formally in this sub-section. Figures 6a and 6b of [2] are reproduced as Fig. 8a and 8b of this article.

Now any magnitudes of the relative proper intrinsic angles, $\varnothing\alpha', \varnothing\beta'$ and $\varnothing\gamma'$, on the flat relative proper intrinsic metric 3-space $\varnothing\mathbb{E}'^3$ are equivalent to zero magnitude of relative proper (or physical) angles α', β' and γ' respectively, in the relative proper physical Euclidean 3-space $\mathbb{E}'^3$, with respect to 3-observers in $\mathbb{E}'^3$

overlying $\varnothing\mathbb{E}'^3$. These follow from the definitions, $\varnothing\alpha' = 0 \times \alpha'$, $\varnothing\beta' = 0 \times \beta'$ and $\varnothing\gamma' = 0 \times \gamma'$. Consequently any two distinct directions, which are separated by non-zero relative proper intrinsic angles, $\varnothing\alpha', \varnothing\beta'$ and $\varnothing\gamma'$, on the flat three-dimensional relative proper intrinsic metric space $\varnothing\mathbb{E}'^3$, with respect to hypothetical intrinsic 3-observers in $\varnothing\mathbb{E}'^3$, are the same direction with respect to 3-observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$.

A consequence of the preceding paragraph is that the mutually orthogonal relative proper intrinsic metric coordinates, $\varnothing\xi'^1, \varnothing\xi'^2$ and $\varnothing\xi'^3$, of $\varnothing\mathbb{E}'^3$, at a point on $\varnothing\mathbb{E}'^3$, are impossible with respect to 3-observers in $\mathbb{E}'^3$. This is so because the relative proper intrinsic angle, $\varnothing\phi' = \varnothing\pi/2$, between the relative proper intrinsic coordinates, $\varnothing\xi'^1$ and $\varnothing\xi'^2$, and $\varnothing\theta' = \varnothing\pi/2$ between the relative proper intrinsic coordinates, $\varnothing\xi'^2$ and $\varnothing\xi'^3$, as well as the relative proper intrinsic angle, $\varnothing\varphi' = \varnothing\pi/2$, between the relative proper intrinsic coordinates, $\varnothing\xi'^1$ and $\varnothing\xi'^3$, all vanish with respect to observers in $\mathbb{E}'^3$. The three relative proper intrinsic coordinates, $\varnothing\xi'^1, \varnothing\xi'^2$ and $\varnothing\xi'^3$ of $\varnothing\mathbb{E}'^3$, are consequently aligned along a singular direction, thereby constituting a singular relative proper intrinsic metric space (or a singular relative proper intrinsic metric space dimension) $\varnothing\rho'$, which underlies the relative proper metric Euclidean 3-space $\mathbb{E}'^3$, with respect to all 3-observers in $\mathbb{E}'^3$.

The one-dimensional relative proper intrinsic metric space $\varnothing\rho'$ has no unique orientation (or basis) in the flat three-dimensional relative proper intrinsic metric space $\varnothing\mathbb{E}'^3$ that contracts to it. Consequently it has no unique orientation (or basis) in the relative (or physical) proper Euclidean 3-space $\mathbb{E}'^3$ overlying $\varnothing\mathbb{E}'^3$, with respect to 3-observers in $\mathbb{E}'^3$. Thus $\varnothing\rho'$ is a scalar isotropic intrinsic metric space (or scalar isotropic intrinsic metric space dimension) in $\mathbb{E}'^3$, with respect to 3-observers in $\mathbb{E}'^3$. It can be considered to lie along every direction in $\mathbb{E}'^3$ by 3-observers in $\mathbb{E}'^3$.

On the other hand, the relative proper (or physical) Euclidean 3-space $\mathbb{E}'^3$, in which different coordinate sets, (x'^1, x'^2, x'^3), (y'^1, y'^2, y'^3), (z'^1, z'^2, z'^3), etc, at a point on it are distinct with respect to 3-observers in it, and this is true at every point on 3-space $\mathbb{E}'^3$, is not naturally contracted with respect to 3-observers in 3-space $\mathbb{E}'^3$.

Let us give a graphical illustration of the natural contractions of the flat absolute proper and relative proper intrinsic metric 3-spaces, along with the curved absolute metric 3-space, with respect to 3-observers in the relative proper metric 3-space, described in the preceding four sub-sections. In doing this, the curved 'one-dimensional' absolute intrinsic metric space $\varnothing\hat{\rho}$, to which the curved 'three-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{M}}^3$ naturally contracts, with respect to 3-observers in the relative (or physical) proper metric Euclidean 3-space $\mathbb{E}'^3$, must be allowed to be curved onto the straight line absolute intrinsic metric time 'dimension', $\varnothing\hat{x}^0 = \varnothing\hat{c}_s\varnothing\hat{t}$, and to projects a straight line scalar isotropic absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ along the horizontal. The projective $\varnothing\rho'_{ab}$ is made manifested outwardly as 'one-dimensional' scalar isotropic absolute proper metric space ρ'_{ab} that overlies

$\varnothing\rho'_{ab}$ along the horizontal.

The contracted one-dimensional scalar isotropic relative proper intrinsic metric space $\varnothing\rho'$ likewise lies along the horizontal underneath the non-contracted flat three-dimensional relative proper metric space $\mathbb{E}'^3$ (as a hyper-surface) along the horizontal, with respect to 3-observers in $\mathbb{E}'^3$.

Naturally the straight line absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ is embedded in the straight line relative proper intrinsic metric space $\varnothing\rho'$ and the straight line absolute proper metric space ρ'_{ab} is embedded in the flat three-dimensional relative proper metric space $\mathbb{E}'^3$, in which the 3-observers are located, as illustrated in Fig. 9.

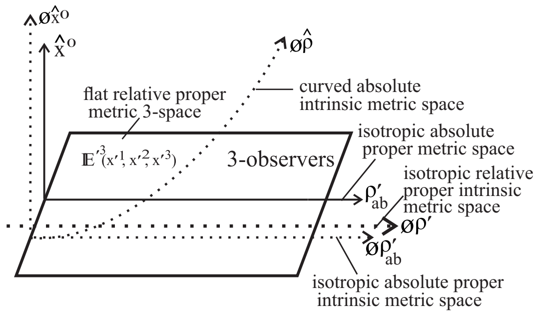

Figure 9: The curved 'three-dimensional' absolute intrinsic metric '3-space' $\varnothing\hat{\mathbb{M}}^3$, its projective flat absolute proper intrinsic metric '3-space' $\varnothing\mathbb{E}'^3_{ab}$, the flat relative proper intrinsic metric 3-space $\varnothing\mathbb{E}'^3$, as well as the flat absolute proper metric '3-space' $\mathbb{E}'^3_{ab}$, as outward manifestation of $\varnothing\mathbb{E}'^3_{ab}$, isolated in the context of absolute intrinsic Riemann geometry in [1] and illustrated in Fig. 5 of that article, reproduced as Fig. 2 of this article, contract naturally to curved isotropic scalar 'one-dimensional' absolute intrinsic metric space $\varnothing\hat{\rho}$, straight line isotropic scalar absolute proper intrinsic metric space $\varnothing\rho'_{ab}$, straight line isotropic scalar relative proper intrinsic metric space $\varnothing\rho'$ and straight line isotropic scalar absolute proper metric space ρ'_{ab} respectively, with respect to 3-observers in the flat three-dimensional relative proper metric space $\mathbb{E}'^3$.

Thus the 'three-dimensional' absolute intrinsic metric spaces $\varnothing\hat{\mathbb{M}}^3$ (which are 'three-dimensional' absolute intrinsic Riemannian metric spaces), underlay by flat three-dimensional absolute proper intrinsic metric space $\varnothing\mathbb{E}'^3_{ab}$, which have been carried along from the first part of this paper to this point, have now been found to be naturally contracted to curved 'one-dimensional' absolute intrinsic metric spaces (which are 'one-dimensional' absolute intrinsic Riemannian metric spaces) $\varnothing\hat{\mathbb{M}}^1$, underneath which lies its projective one-dimensional isotropic absolute proper intrinsic metric space (or 'dimension') $\varnothing\rho'_{ab}$, with respect to 3-observers in $\mathbb{E}'^3$.

The relative (or physical) proper metric Euclidean 3-space $\mathbb{E}'^3$ that has been known to be the outward manifestation of the 3-dimensional relative proper intrinsic metric space $\varnothing\mathbb{E}'^3$ from the first part of this paper to this point, is now the outward manifestation of the one-dimensional isotropic relative proper intrinsic metric space $\varnothing\rho'$ in Fig. 9. It may be recalled that this fact is stated as *ansatz* in sub-section

4.4 of [2], prior to formal validation of the existence of the proper intrinsic metric space $\varnothing\rho'$ underlying the proper physical Euclidean 3-space $\mathbb{E}'^3$ in nature in section 1 of [6].

The absolute intrinsic metric tensors $\varnothing\hat{g}_{ik}$ of absolute intrinsic Riemann geometry on curved 'three-di-mensional' absolute intrinsic metric spaces $\varnothing\hat{\mathbb{M}}^3$, which are 3×3 diagonal 'matrices' in section one, are actually 1×1 'matrices' or numbers $\varnothing\hat{g}_{11}$ on curved 'one-dimensional' absolute intrinsic metric spaces $\varnothing\hat{\rho}$. Likewise the absolute intrinsic Ricci tensors. The absolute intrinsic Gaussian line element written in terms of mutually orthogonal elementary intervals of absolute intrinsic metric 'dimensions', $\varnothing\hat{x}^1, \varnothing\hat{x}^2$ and $\varnothing\hat{x}^3$ of $\varnothing\hat{\mathbb{M}}^3$, as Eq. (48), is actually the following absolute intrinsic Gaussian line element in terms of elementary interval of the 'one-dimensional' curved absolute intrinsic metric space $\varnothing\hat{\rho}$, with respect to 3-observers in the underlying flat relative proper metric Euclidean 3-space $\mathbb{E}'^3$ in Fig. 9

$$\begin{aligned} d\varnothing\hat{s}^2 &= (d\varnothing\hat{x}^0)^2 - \varnothing\hat{g}_{11}(d\varnothing\hat{\rho})^2 \\ &= (d\varnothing\hat{x}^0)^2 - \frac{(d\varnothing\hat{\rho})^2}{1-(\varnothing\hat{k})^2} \, . \end{aligned} \tag{88}$$

It is to be remembered that $\varnothing\hat{\rho}$ has been formed by bundling together the curved absolute intrinsic metric space 'dimensions', $\varnothing\hat{x}^1, \varnothing\hat{x}^2$ and $\varnothing\hat{x}^3$, of $\varnothing\hat{\mathbb{M}}^3$ into 'one-dimensional' curved absolute intrinsic metric space, with respect to 3-observers in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$. In effect, $(d\varnothing\hat{x}^1)^2+(d\varnothing\hat{x}^2)^2+(d\varnothing\hat{x}^3)^2$ in Eq. (48) has simply been replaced by $(d\varnothing\hat{\rho})^2$ in Eq. (89). All absolute intrinsic Riemannian metric spaces in the universe are curved 'one-di-into 'one-dimensional' absolute intrinsic metric spaces, $\varnothing\hat{\rho}, \varnothing\hat{\rho}', \varnothing\hat{\rho}''$, etc, all of which are curved relative to the singular universal isotropic absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ (with no unique orientation in the universal proper physical Euclidean 3-space $\mathbb{E}'^3$), with respect to 3-observers in $\mathbb{E}'^3$. They are all 'parallel' absolute intrinsic metric spaces with respect to observers in $\mathbb{E}'^3$.

Illustrated in Figs. 10a&b is a situation where two absolute intrinsic metric spaces, $\varnothing\hat{\rho}$ and $\varnothing\hat{\rho}'$, co-exist (or are superposed), such that $\varnothing\hat{\rho}$ is curved relative to curved $\varnothing\hat{\rho}'$ and $\varnothing\hat{\rho}'$ is curved relative to the straight line absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ along the horizontal. For the purpose of writing the absolute intrinsic line element, the absolute intrinsic metric tensor and the absolute intrinsic Ricci tensor on the upper curved 'one-dimensional' absolute intrinsic metric space $\varnothing\hat{\rho}$, with respect to 3-observers in $\mathbb{E}'^3$, the resultant absolute intrinsic angle $\varnothing\hat{\overline{\psi}}$ of inclination of $\varnothing\hat{\rho}$ relative to $\varnothing\rho'_{ab}$ at point $\varnothing\hat{x}$ along the curved $\varnothing\hat{\rho}$ (from the origin of $\varnothing\hat{\rho}$), which corresponds to point $\varnothing\hat{x}'$ along the curved $\varnothing\hat{\rho}'$ and point $\varnothing x\_ab'$ along the underlying straight line $\varnothing\rho'_{ab}$, is given in terms of the absolute intrinsic angles, $\varnothing\hat{\psi}_2(\varnothing\hat{x})$ and $\varnothing\hat{\psi}_1(\varnothing\hat{x}')$, as follows, as derived in sub-sub-section 1.6.1 (see

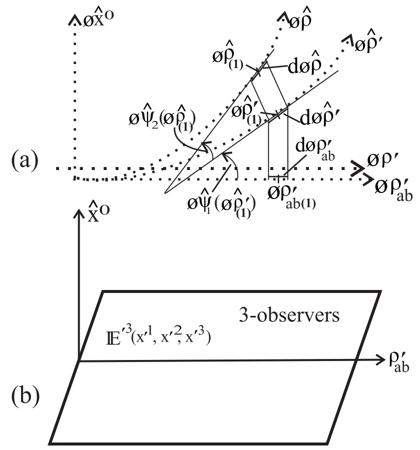

Figure 10: (**a** and **b**) Co-existing pair of 'one-dimensional' absolute intrinsic metric spaces.

Eqs. (58a) and (58b)),

$$\begin{aligned} \sin^2 \varnothing\hat{\hat{\psi}} &= \sin^2\Big(\varnothing\hat{\psi}_1(\varnothing\hat{x}') + \varnothing\hat{\psi}_2(\varnothing\hat{x})\Big) \ ; \\ &= \sin^2 \varnothing\hat{\psi}_1(\varnothing\hat{x}') + \sin^2 \varnothing\hat{\psi}_2(\varnothing\hat{x}) \ . \end{aligned} \tag{89}$$

Hence the resultant absolute intrinsic curvature parameter $\varnothing\hat{\hat{k}}$ of the upper curved absolute intrinsic metric space $\varnothing\hat{\rho}$ at point $\varnothing\hat{x}$ along $\varnothing\hat{\rho}$ in Figs. 10a&b, to appear in the component of the resultant absolute intrinsic metric tensor $\varnothing\hat{\hat{g}}_{11}$ at this point, is given in terms of the absolute intrinsic curvature parameters, $\varnothing\hat{k}_2(\varnothing\hat{x})$ and $\varnothing\hat{k}_1(\varnothing\hat{x}')$, of the curved absolute intrinsic metric spaces, $\varnothing\hat{\rho}$ and $\varnothing\hat{\rho}'$, respectively relative to $\varnothing\rho'_{ab}$ prior to their superposition as

$$\varnothing\hat{\hat{k}}^2 = \varnothing\hat{k}_1(\varnothing\hat{x}')^2 + \varnothing\hat{k}_2(\varnothing\hat{x})^2 \ . \tag{90}$$

The only component $\varnothing\hat{\hat{g}}_{11}$ of the resultant absolute intrinsic metric tensor, $\varnothing\hat{\hat{g}}_{ik}$; $i, k = 1$, at point $\varnothing\hat{x}$ on the upper curved absolute intrinsic space $\varnothing\hat{\rho}$, is then given in terms of $\varnothing\hat{\hat{k}}^2$ as

$$\varnothing\hat{\hat{g}}_{11} = \Big(1 - \varnothing\hat{\hat{k}}^2\Big)^{-1} \ = \ \Big(1 - \varnothing\hat{k}_1(\varnothing\hat{x}')^2 - \varnothing\hat{k}_2(\varnothing\hat{x})^2\Big)^{-1} \ . \tag{91}$$

The resultant absolute intrinsic line element must be written by simply replacing $\varnothing\hat{k}^2$ by $\varnothing\hat{\hat{k}}^2$ in Eq. (89) as

$$d\varnothing\hat{s}^2 = (d\varnothing\hat{x}^0)^2 - \frac{d\varnothing\hat{\rho}^2}{1 - \varnothing\hat{k}_1(\varnothing\hat{x}')^2 - \varnothing\hat{k}_2(\varnothing\hat{x})^2} \ . \tag{92}$$

This absolute intrinsic line elements shall be made complete on a curved 'two-dimensional' absolute intrinsic metric spacetime by replacing the term $(d\varnothing\hat{x}^0)^2$ with $\varnothing\hat{g}_{00}\,\varnothing\hat{c}_s{}^2 d\varnothing\hat{t}^2$ elsewhere, where the component $\varnothing\hat{g}_{00}$ of the absolute intrinsic metric tensor shall be derived.

3.5 Further on the concepts of 'absolute proper' and 'relative proper' coordinates and intrinsic coordinates

Two concepts of proper namely, "absolute proper" and "relative proper", have appeared in the absolute intrinsic Riemann geometry from the beginning of its development in the preceding article [1] up to this point in this article. The curved absolute intrinsic metric space $\varnothing\hat{\rho}$ invariantly projects the absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ along the horizontal and the relative proper intrinsic metric space $\varnothing\rho'$ appears automatically. The absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ is imperceptibly embedded in the relative proper intrinsic metric space $\varnothing\rho'$; the two thereby appearing as $\varnothing\rho'$ along the horizontal. The origin of the relative proper intrinsic metric space that appears automatically shall be explained elsewhere.

The resulting composite proper intrinsic metric space $\varnothing\rho'\ (= \varnothing\rho' \cup \varnothing\rho'_{ab})$ is then made manifested in composite proper metric Euclidean space, which is composed of three-dimensional relative proper metric Euclidean space, denoted by $\mathbb{E}'^3$, with dimensions, x'^1, x'^2 and x'^3, and the 'one-dimensional' isotropic scalar absolute proper metric space ρ'_{ab}. That is, $\mathbb{E}'^3 = \mathbb{E}'^3 \cup \rho'_{ab}$, where the absolute proper metric space ρ'_{ab} is imperceptibly embedded (as an isotropic scalar 'dimension') in the relative proper metric Euclidean 3-space $\mathbb{E}'^3$; the two thereby appearing as $\mathbb{E}'^3$. The 'one-dimensional' isotropic absolute proper metric space ρ'_{ab} is orientated along all directions in the relative proper Euclidean 3-space $\mathbb{E}'^3$, with respect to all 3-observers in $\mathbb{E}'^3$.

4 Conclusion

This article gives further support to the existence of absolute intrinsic Riemann space and absolute intrinsic Riemann geometry introduced in the first part. The metric tensor and the Ricci tensor on curved conventional metric space have counterparts pure diagonal absolute intrinsic metric tensor and absolute intrinsic Ricci tensor, derived in this article, on curved absolute intrinsic metric space. The pair of absolute intrinsic metric tensor equations derived on curved absolute intrinsic metric space are amenable to algebraic solution for the absolute intrinsic metric tensor and absolute intrinsic Ricci tensor. This and the fact that the curved 'three-dimensional' absolute intrinsic metric space naturally contracts to a curved 'one-dimensional' isotropic absolute intrinsic metric space, also derived in this article, makes absolute intrinsic Riemann geometry less rigorous to handle than the conventional Riemann geometry. The absolute intrinsic Riemann geometry of curved absolute intrinsic metric space developed in the preceding first part of this paper and this second part shall be extended to curved 'two-dimensional' absolute intrinsic Riemannian metric spacetime in the third part.

COMPETING INTERESTS

No competing interests are involved in this work.

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