

1 Numerical analysis of an unsaturated capillary barrier cover system

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6 Abstract

7 Inclined multi-layered barriers can be used to protect underlying waste storage facilities. The intended barriers can
8 be used to confine the infiltration through implementation of the capillary barrier effect. In this study, the effect of
9 rainfall, evaporation, and transpiration on the hydraulic properties of inclined covers was assessed by performing a
10 series of simulations using HYDRUS-2D numerical models. The material of the intended layers included clay loam soil
11 as a seepage control layer, sandy soil as a moisture retention layer, and gravel as a capillary break layer. Based on
12 the key results of numerical analyses, Lateral diversion in the interface between the seepage control layer and
13 moisture retention layer occurred as a result of the significant slope of said layers and the low permeability of the
14 moisture retention layer. At the reduced degree of saturation, water did not move easily from the seepage control
15 layer to the moisture retention layer as well as from the moisture retention layer to the capillary break layer due to
16 the low hydraulic conductivity. The negative pressure head in the seepage control layer had minimal effect on the
17 water content in the moisture retention layer. Hence, the performance of this protective earthen cover can, then,
18 be guaranteed due to the current climatological conditions.

19 Keywords: Capillary Cover Barriers; Retention Capacity; Hydraulic Properties; Suction; Volumetric Water Content

20 **1. Introduction**

21 In humid regions, waste can be considered a serious threat to ground water due to percolation from infrastructures
22 such as landfills, which are the current preferred solution for disposal. (Khire, M.V., Benson, C.H., and Bosscher, P.J.
23 2000).

24 A multi-layer capillary barrier cover system is designed to reduce percolation in humid, arid, and semi-arid climates.
25 (Ogorzalek, A. et al. 2008; Ng, C. et al. 2014; Zhan, L. et al. 2016; Chen, R. et al. 2019; Chen, R et al. 2021).

26 The percolation into underlying waste in these regions can be reduced through covers made up of earthen materials
27 for promotion of the capillary barrier effect (Zaradny 1993, Morris and Stormont 1997, Bussière, Aubertin, Chapuis.
28 2003, Mallants, Volckaert, Marivoet. 1999, Aubertin, M. Cifuentes, E. Apithy, S.A. Bussière, B. Molson, J, Chapuis, R.P.
29 2009). Earthen covers are designed in various forms but are comprised of multi-layered contrasting particle sizes
30 that consist of fine-grained and coarse-grained sediments (Zaslavsky and Sinai 1981a, b; Nieber and Walter 1981;
31 Stagnitti, F., Parlange, J.-Y., Steenhuis, T.S., Parlange, M.B., and Rose, C.W. 1992; Selim 1988; Steenhuis, T.S.,
32 Parlange, J.-Y., Sanford, W.E., Heilig, A., Stagnitti, F., and Walter, M.F. 1999; Bussière, Aubertin, Chapuis. 2003). The
33 water movement restriction across the interface of these layers occurs due to the contrast in unsaturated hydraulic
34 properties employed by the materials (Tang, J., Taro, U., Huang, D., Xie, J., and Tao, S. 2020, Li. N., Jiang. H., and Li.
35 X. 2020).

36 The anisotropic ratio ($K_x^{\square}/K_y^{\square}$) is described as soil hydraulic conductivity anisotropy, where K_x^{\square} and K_y^{\square} are the
37 hydraulic conductivity in the horizontal and vertical directions, respectively. The values of the anisotropic ratio for
38 clay soil can be higher than 100 (Todd, D., 1980). Laboratory and field experiments have been shown that the amount
39 of hydraulic conductivity anisotropy is related to the degree of water saturation (McCord et al., 1991). The degree
40 of anisotropy has been explained by the anisotropy factor, defined as the ratio of diagonal components of the
41 hydraulic conductivity tensor. (Assouline, S and Or, D., 2006).

42 The results in transient seepage analysis and slope stability analysis show that when the vertical hydraulic
43 conductivity (K_y^{\square}) is constant, the horizontal hydraulic conductivity (K_x^{\square}) increases (i.e., anisotropy increases) (Yeh,
44 H and Tsai, Y., 2018). Increased hydraulic conductivity anisotropy ratio will cause a slower infiltration rate in the
45 vertical direction at the toe of the slope (Yeh, H and Tsai, Y., 2018).

46 The present study intends to investigate the water balance of a multi-layer earthen cover. The purposes of current
47 research are to minimize leachate transport by infiltrating water and reduce erosion of earthen covers by water and

48 wind. The unsaturated hydraulic performance of the earthen barrier is numerically evaluated under different
49 climatological conditions spanning the past 20 years (2000-2019) in Canada (Windsor)(Fig.1).

50 For this purpose, the useful finite element code HYDRUS-2D was used to investigate the water flux that leaves the
51 barrier. The generated water flux was investigated as vertical infiltration and as lateral flux. The numerical simulation
52 was done for a multi-layer earthen barrier. Design parameters, such as layer thickness and geoenvironmental
53 parameters of materials, follow the "Landfill Gas Collection and Control Regulation" of Ontario.

54 The program used for this study – HYDRUS – is a modeling environment for the analysis of water flow and solute
55 transport in variably saturated porous media. HYDRUS uses computational finite element models for the two- and
56 three-dimensional simulation of solutes and water through said media. The model is supported by an interactive
57 graphics-based interface for data-preprocessing, generation of structured and unstructured finite element mesh,
58 and graphic presentation of the results (*Šimůnek, J., van Genuchten, M. Th. and Šejna, M. 2012*).

59 **2. Materials and Methods**

60 **2.1. Location and Area**

61 Essex County locates in the southwestern end of Southern Ontario. The only land boundary on the east is Kent
62 County. It is bounded by the Detroit River to the west, Lake St. Clair to the north, and Lake Erie to the south. The city
63 of Windsor in the northwest section of the County is 188.7 km from London and 365.8 km from Toronto.

64 **2.2. The Classification and Description of Essex County soils**

65 Soil horizons are grouped under three main parts. The first layer has been filtrated of some mineral constituents but
66 contains an accumulation of organic materials in the upper part. The second layer is usually darker in color and
67 heavier in texture than the first layer and the structural aggregates are well formed. The parent material from which
68 the soil has developed is defined as the third layer.

69 **2.3. Climate**

70 Essex County is the earliest and warmest part of the Ontario province. The lowest annual precipitation, only 28.1
71 inches; its average snowfall is only 32 inches. In the rest of the county the mean annual temperature is 8.3°C.

72 **2.4. Barrier Design**

73 Earthen barriers are usually made up of fine-grained sediment overlying coarse-textured soil. The simulated multi-
74 layer capillary barrier consists of natural materials. The selected materials for this study's modelled barrier are clay
75 loam as the seepage control layer, sand as the moisture retention layer, and gravel as the capillary break layer.
76 Additionally, the layer has a slope of 10%, which, based on "Landfill Gas Collection and Control Regulation" of
77 Ontario, the slope can be between 5% and 25%. The slope performance includes water percolation discharge and
78 lateral diversion. The purpose of the top 0.6 m of clay loam is to serve as the seepage control layer to reduce water
79 percolation, bio-intrusion, and water storage, as well as promote plant growth. The underlying layers include 0.4 m
80 of sand and 0.2 m of gravel that are used as capillary break, to prevent downward infiltration of water as well as
81 excess-water discharge. A cross-section of the designed cover is depicted in Fig.2.

82 An essential part of protective earthen cover is vegetation. In this study, evapotranspiration was indirectly accounted
83 for by using the Hargreaves equation (*Jensen, D. T., G. H. Hargreaves, B. Temesgen, R. G. Allen. 1997; Šimůnek 2015*).
84 Likewise, soil evaporation and plant transpiration were calculated using Beer's law, which divides the solar radiation
85 component of the energy budget via interception by the canopy (*Ritchie, 1972, Asadi, Ali, Vivek K. Arora, Joe R.*
86 *Melton, and Paul Bartlett. 2018*).

87
88 The numerical analysis of water flow was confined to the three-layered barrier. The numerical simulation was done
89 by a fine computational mesh, as demonstrated in the different layers. This is an essential part in the transient state
90 of water flow modelling. Also, it is to be noted that the hydraulic properties of unsaturated materials have a non-
91 linear behavior.

92 Knowledge about the hydraulic properties of the chosen barrier materials – clay loam, sand, and gravel – help to
93 understand the hydraulic behavior of the layers. The intended hydraulic properties are soil water retention curve,
94 $\theta(h)$; and hydraulic conductivity function, $K(h)$; where h is pressure head, θ is volumetric water content, and K is

95 hydraulic conductivity. The hydraulic properties of the materials were not obtained based on real measurements.
 96 The hydraulic properties of clay loam were obtained by RETC code (*van Genuchten, M.Th., Leij, F.J., and Yates, S.R.*
 97 *1991*), while the hydraulic properties of sand and gravel were selected from literature. A short review of the selected
 98 hydraulic properties is presented below (parameter values will be discussed in Table 1).

99 **2.5. Governing Flow Equation**

100 Based on the two-dimensional isothermal uniform Darcian flow in a porous medium with variably saturated
 101 conditions, which the air phase does not consider in the water flow process, Richards' equation has been modified
 102 to introduce the governing flow equation. The modified Richard's equation, then, appears as:

$$103 \quad \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x_i} \left[K \left(K_{ij}^A \frac{\partial h}{\partial x_j} + K_{iz}^A \right) \right] - S \quad (1)$$

104 where θ is the volumetric water content $[\frac{L^3}{L^{-3}}]$, h is the pressure head [L], S is a sink term for plant water uptake
 105 $[T^{-1}]$, x_i ($i=1,2$) are the spatial coordinates [L], t is time [T], K_{ij}^A is anisotropy tensor to account for the anisotropy
 106 medium K^A , and K is the unsaturated hydraulic conductivity function $[LT^{-1}]$. The unsaturated hydraulic
 107 conductivity function is presented by

$$108 \quad K(h, x, z) = K_s(x, z) K_r(h, x, z) \quad (2)$$

109 where K_r is the relative hydraulic conductivity and K_s the saturated hydraulic conductivity $[LT^{-1}]$. Applying equation
 110 (3) to planar flow in a vertical cross-section would then introduce $x_1 = x$ as the horizontal coordinate and $x_2 = z$
 111 as the vertical coordinate, the latter coordinate taken to be positive upward. (*Šimůnek, J., van Genuchten, M. Th.*
 112 *and Šejna, M. 2012*).

113 **2.6. Unsaturated soil hydraulic properties**

114 The RETC Code is a computer program to analyze or predict the unsaturated hydraulic properties of soils: Water
 115 Retention Curve, $\theta(h)$; and Hydraulic Conductivity Function, $K(h)$. The hydraulic properties of soils are the key
 116 parameters used in water flow quantity analysis in unsaturated soils (*van Genuchten, M.Th., Leij, F.J., and Yates, S.R.*
 117 *1991*). The soil-hydraulic functions used by van-Genuchten [1980] included the statistical pore-size distribution

118 model of Mualem [1976] to obtain a predictive equation for the unsaturated hydraulic conductivity function in terms
 119 of soil water retention parameters. The expressions of van-Genuchten [1980] are presented by equations (3), (4),
 120 and (5).

$$\theta_h = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{[1 + |\alpha h|^n]^m} & h < 0 \\ \theta_s & h \geq 0 \end{cases} \quad (3)$$

$$K(h) = K_s S_e^l [1 - (1 - S_e^{1/m})^m]^2 \quad (4)$$

$$\text{Where } m = 1 - 1/n \quad (5)$$

121 The above equations contain six independent parameters:

122 θ_r (residual water content), θ_s (saturated water content), α , n (constant that define the curve shape),

123 K_s (saturated hydraulic conductivity), and l (pore-connectivity parameter). The pore-connectivity parameter in the
 124 hydraulic conductivity function was estimated [Mualem, 1976] to be about 0.5 as an average for many soils.

125 As depicted in Fig.4, there are vast differences among the shape of soil hydraulic functions and the hydraulic behavior
 126 of soils.

127 The trail of soil types is as follows (Fig.2): clay loam (upper material), sand (intermediate material) and gravel (bottom
 128 material). The barrier is characterized by a slope of 10%, a width of 10 m, and a depth of 1.2 m that includes the
 129 three mentioned layers of various thickness. The barrier consists of 4350 triangular elements present in areas where
 130 the highest fluxes occurred and where the more minute elements were considered.

131 **2.7. Initial and boundary conditions**

132 The solution of the modified - Richards' equation used in this research requires knowledge of the initial water
 133 content in the flow domain. The following equation serves that purpose.

$$134 \quad \theta(x, z, t) = \theta_0(x, z) \text{ for } t = 0 \quad (6)$$

135 In (6), θ_0 is a determined function of x and z . There are three types of conditions that are described by HYDRUS to
 136 evaluate the system-independent interactions along the boundaries of the flow region. The pressure head
 137 (Dirichlet type) boundary conditions can be presented as follows:

$$138 \quad h(x, z, t) = \psi(x, z, t) \text{ for } (x, Z) \in \Gamma_D \quad (7)$$

139 The described flux (Neumann type) boundary condition is presented by

$$140 \quad -K \left[K_{ij}^A \frac{\partial h}{\partial x_j} + K_{iz}^A \right] n_i = \sigma_1(x, z, t) \text{ for } (x, Z) \in \Gamma_N \quad (8)$$

141 As well as the described gradient boundary conditions are presented by

$$142 \quad \left[K_{ij}^A \frac{\partial h}{\partial x_j} + K_{iz}^A \right] n_i = \sigma_2(x, z, t) \text{ for } (x, Z) \in \Gamma_g \quad (9)$$

143 The Dirichlet, Neumann, and gradient type boundary segments are defined by $\Gamma_D, \Gamma_N, \Gamma_g$ respectively; $\psi [L]$, $\sigma_1 [LT-$
 144 $1]$, and $\sigma_2 [-]$ are prescribed functions of x , z and t ; and n_i are the segments of the outward unit vector normal to
 145 boundary Γ_N or Γ_g (Šimůnek, J., van Genuchten, M. Th. and Šejna, M. 2012). In the HYDRUS program, the simulation
 146 of free drainage from a deep soil profile can be presented by the implementation of a gradient boundary condition
 147 in terms of a unit vertical hydraulic gradient. This circumstance is usually presented in the vadose zone studies (Sisson
 148 1987; McCord 1991; Šimůnek, J., van Genuchten, M. Th. and Šejna, M. 2012).

149 In the present numerical simulation, the atmospheric boundary conditions are assigned to the top boundary and the
 150 free drainage boundary conditions are assigned to the bottom boundary (Fig.2). In the atmospheric boundary
 151 conditions, the external conditions control the potential fluid flux that moves across the intended boundary. Also,
 152 the actual flux depends on the dominant soil moisture conditions. The numerical solution of the modified form of
 153 Richards' equation is as follows:

$$154 \quad \left| K \left[K_{ij}^A \frac{\partial h}{\partial x_j} + K_{iz}^A \right] n_i \right| \leq E \quad (10)$$

$$155 \quad h_A \leq h \leq h_S \quad (11)$$

156 where E is the maximum potential rate of infiltration or evaporation under the current atmospheric conditions, h is
 157 the pressure head at the soil surface, and h_A and h_S are the minimum and maximum pressure heads allowed under
 158 the dominant soil conditions — respectively. Based on the HYDRUS assumption, any excess water on the soil surface
 159 is immediately eliminated. (Neuman, S. P., R. A. Feddes, and E. Bresler. 1974; Šimůnek, J., van Genuchten, M. Th. and
 160 Šejna, M. 2012). Gradient type boundary conditions, exclusively as the unit gradient boundary condition or the Free
 161 Drainage boundary condition, were implemented by HYDRUS. Whenever the flow is contrary to the particular axis,
 162 the gradients in the x-direction (from right to left) and in the y-direction (from back to front) are positive (Šimůnek,
 163 J., van Genuchten, M. Th. and Šejna, M. 2012). The long-term daily climatological data – comprised of precipitation,
 164 evaporation, and transpiration — is assigned to the top boundary condition (the meteorological stations are
 165 "Windsor Riverside" and "Windsor A" which are situated in Windsor city, Ontario province, Canada), while a unit
 166 vertical gradient corresponds to simulate free drainage in the bottom boundary condition. The right and left
 167 boundaries of the domain assume zero flux boundary conditions. The initial water content for each soil type is
 168 included in Table 1 and it was considered throughout the barrier.

169 **2.8. Numerical solution strategy**

170 The initial water content at field capacity is calculated as follows:

$$S_{fc} = \frac{\theta_{fc} - \theta_r}{\theta_s - \theta_r} = n^{-0.6(2 + \log_{10} K_s)} \quad (12)$$

171 where θ_{fc} and S_{fc} are the water content and saturation at field capacity, respectively. θ_r , θ_s , n , and K_s are the soil
 172 hydraulic parameters for the van Genuchten model (Twarakavi et al., 2009). The water content at field capacity
 173 corresponds to the hydraulic conductivity of about 0.01 cm/d.

174 The Galerkin finite element method with linear basis functions was used to obtain the solution for the modified form
 175 of Richards' equation dependent on the imposed initial water content and boundary conditions. Either the Gaussian
 176 elimination or the conjugate gradient method was used to solve each iteration system of linearized algebraic
 177 equations. The iterative process was continued until a satisfactory degree of convergence was obtained, i.e., until at
 178 all nodes in the unsaturated region, the absolute value of change in water content between two consecutive

179 iterations became less than some small value determined by the imposed absolute value of water content
 180 $(0.0003 \frac{cm^3}{cm^3})$ tolerance.

181 **2.9. Water balance computations**

182 Water balance computations were performed 480 times for the three subregions of flow domain. The water balance
 183 information for each subregion is comprised of the actual volume of water, $V(cm^2)$, in the respective subregion;
 184 and the rate, $O(cm^2/day)$, of inflow or outflow to or from the subregion. V and O are given by

$$185 \quad V = \sum_e \kappa \frac{\theta_i + \theta_j + \theta_k}{3} \quad \text{for 2D (13)}$$

$$186 \quad O = \frac{V_{new} - V_{old}}{\Delta t} \quad (14)$$

187 where $\theta_i, \theta_j, \theta_k$ are calculated water contents at the corner nodes of element e , and where V_{new} and V_{old} are
 188 volumes of water in the subregion computed at the current and previous time levels, respectively. The absolute
 189 error in the mass balance is computed by

$$190 \quad \varepsilon_a^w = V_t - V_0 + S_t \int_0^t T_a dt - \int_0^t \sum_{n_r} Q_n dt \quad (15)$$

191 where V_t and V_0 are the volumes of water in the flow domain at time t and zero, respectively, as calculated with
 192 equation (12). The third term on the right side of the equation represents the cumulative root water uptake amount,
 193 while the fourth term gives the cumulative flux through nodes (n_r) located along the boundary of the flow domain
 194 or at internal source and sink nodes.

195 The accuracy of the numerical solution is evaluated in terms of the relative error, ε_r^w [%], in the water mass balance
 196 as follows:

$$197 \quad \varepsilon_r^w = \frac{|\varepsilon_a^w|}{\max[\sum_e |V_0^e - V_t^e|, S_t \int_0^t T_a dt - \int_0^t \sum_{n_r} |Q_n| dt]} \quad (16)$$

198 where V_t^e and V_0^e are the volumes of water in element e at times t and zero, respectively. Note that HYDRUS does
 199 not relate the absolute error to the volume of water in the flow domain, but instead to the maximum value of two
 200 quantities. The first quantity represents the sum of the absolute changes in water content over all elements, whereas

201 the second quantity is the sum of the absolute values of all fluxes in and out of the flow domain. This criterion is
202 much stricter than the usual criterion involving the total volume of water in the flow domain; this is because
203 cumulative boundary fluxes are often much smaller than the volume in the domain, especially at the beginning of
204 the simulation.

205 Also, the water balance was calculated throughout the flow domain, made up of the three subregions previously
206 adopted.

207 **3. Results and discussion**

208 Based on assigned initial water content and boundary conditions, flow in the multi-layer earthen cover was transient.
209 The mean pressure head in all layers is not constant (Fig.5). The water balance error at transient state, computed
210 with equation (15), reveals the water balance error to be variable between 0.028% and 0.156%.

211 The total flux that leaves the simulation domain across the bottom boundary presents several noticeable features
212 (Fig. 5). The main reasons that explain water drainage are water infiltration into the barrier, run-off from the sloping
213 soil, and a rapid decrease in flux due to water drainage from the bottom. When the flux increased and was followed
214 by a decrease, water drainage from either sand or gravel layers was considered.

215 The spatial variation of the Darcy flux across the bottom boundary is displayed in Fig. 6. The number of FE element
216 associated with each node on the cross-section is the main factor during velocity calculation. A decreasing behavior
217 was observed in an area close to the left and right boundary of the barrier. This is the result of number of FE
218 connected with each node through the bottom boundary. The number of the nodes in the bottom boundary is 101.

219 Note that primary results of the Richards solution are the pressure heads. The pressure head and the velocity have
220 direct relation.

221 Conclusively, a lateral hydraulic gradient exists between the clay loam and sand, creating a lateral flux into the sand
222 (Fig. 7). Based on the average flux across the bottom boundary (free drainage boundary condition) the highest mean
223 velocity occurs in week 867 from 1043 weeks. Hence, the highest velocity in the respective time occurs at the bottom

224 of the 3rd layer (it occurs 860 cm from the left boundary of the simulation). The lower peaks in the bottom layer are
225 observed approximately 20 cm from the left boundary and 20 cm from the right boundary in the gravel layer (Fig.8).

226 Lastly, the cumulative flux across the bottom boundary was calculated. The total cumulative flux equals the
227 summation of the fluxes of each element, times the element length (L^2T^{-1}). This was equal to $1505.5 \text{ cm}^2/\text{day}$.
228 This value equals 78.91% of the cumulative net rainfall that entered the barrier from the atmospheric boundary.
229 Therefore, 21.09% of the rainfall can be considered runoff (Fig.9).

230 The simulated mean pressure head for all barrier layers at transient state was summarized in table 2.

231 As expected, the second layer (moisture retention layer) held more water, and the mean pressure head in the third
232 layer held less, due to the high retention capacity of the sand layer and the lateral diversion phenomenon.

233 Based on Darcy fluid velocities (Fig.7), fluxes in the sand layer are parallel to the slope. In the intended times, the
234 significant amount of water that passed through the sand layer was laterally diverted. The capillary barrier effect
235 was depicted at $t=2008$ days and $t=6023$ days when a considerable volume of water was diverted along the interface
236 of the moisture retention layer and the capillary break layer. The capillary barrier effect was presented in the
237 interface when the fine-grained material overlaid the coarse-grained one. In unsaturated conditions, the pressure
238 head in the fine-grained layer was higher than the pressure head in the coarse-grained material. As a result, the
239 moisture retention layer could store water. Furthermore, water was diverted laterally due to the slope of the layer.

240 Based on the mean pressure head for various climatological conditions, the seepage control layer was -326.91 cm at
241 its lowest and -11.75 cm at its highest (Fig.10). Based on the water retention curve shape of the clay loam (Fig.3),
242 the negative pressure heads have a noticeable effect on water content and degree of saturation.

243 The profile of water content (Fig.11) along a perpendicular line at 250 cm from the left boundary of the earthen
244 cover was depicted for changeable climatological conditions. In the first layer, the infiltration developed between
245 $t=1825$ days and $t=7300$ days. The water content in the top 22.7 cm of the sand layer showed a steady-state behavior,
246 but then reduced suddenly. This event is introduced as the main reason for the lateral diversion of water. The water
247 content in the gravel layer was reduced as well, but its behavior was not linear, and the bottom of the layer was not
248 saturated.

249 In this calculation, the distribution of hydraulic properties was assumed to be homogeneous .

250 **4. Conclusions**

251 Based on the numerical analysis results at transient state conditions, the lateral drainage of water occurred at the
252 interface between the sand layer and the gravel layer. In the current transient state for variable climatological
253 conditions, 21% of water infiltration was converted to lateral drainage. Also, the mean flux that permeated the
254 bottom boundary was 206.70 cm/day (2.39×10^{-6} m/s). Hence, this mean flux had minimal effect in the variable
255 climatological conditions. The performance of this protective earthen cover can, then, be guaranteed due to the
256 current climatological conditions.

257 The results of negative pressure head in the seepage control layer showed that this layer is usually under unsaturated
258 conditions. Hence, this circumstance will cause the formation of cracks in the clay.

259 Future studies should address the investigation of multi-layered earthen covers using experimental data from
260 laboratory prototypes or field-scale studies. In such studies, numerical calculations – such as the ones presented
261 here – may be appropriate in selecting improved barrier designs.

262 **5. Data Availability Statement**

263 Some or all data, models, and code that support the findings of this study are available through the corresponding
264 author upon reasonable request.

265 **6. Author contributions**

266 All authors contributed to the study conception and design. Material preparation, data collection and analysis were
267 performed by Mohammad Ali Hagh Shenaa, Hassan Sharafi. The first draft of the manuscript was written by
268 Mohammad Ali Hagh Shenaa and Ana Colindres. All authors commented on previous versions of the manuscript. All
269 authors read and approved the final manuscript.

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Table 1

Soil water retention and hydraulic conductivity parameters and initial soil water content (θ_i) for soil types used in the water flow calculations

Parameter	Perth Clay Loam	Clinton Sand	Clinton Gravel
Material number	1	2	3
Layer number	1	2	3
Layer thickness(cm)	60	40	20
$\theta_r(cm^3/cm^3)$	0.0696	0	0.0016
$\theta_s(cm^3/cm^3)$	0.4142	0.2323	0.2378
$\alpha (cm)^{-1}$	0.0115	1.1956	0.3580
n	1.4562	2.4925	2.1014
$K_s (\frac{cm}{d})$	7.37	3196.8	43200
Source	<i>van Genuchten, M.Th., Leij, F.J., and Yates, S.R. 1991., Richards and Morwick 1949</i>	<i>Parent 2003</i>	<i>Parent 2003</i>
$\theta_i(cm^3/cm^3)$	0.2501	0.01138	0.01389
Source	<i>Twarakavi, N. K. C, M. Sakai, and J. Šimůnek. 2009; Šimůnek, J., van Genuchten, M. Th. and Šejna, M. 2012</i>		

Table 2

Summary of simulation results: hmean is mean pressure head,

		Layers		
		1	2	3
Time (day)	Property			
183	hMean	-36.35	-10.84	-59.80
913		-61.11	-27.03	-146.49
1278		-39.77	-19.81	-107.93
1643		-51.59	-20.19	-103.68
2008		-73.07	-34.07	-176.60
2373		-51.40	-19.37	-92.72
2738		-39.16	-12.67	-57.83
3833		-40.17	-12.89	-59.38
4563		-54.22	-19.00	-88.89
6023		-93.04	-39.18	-192.37
6753		-41.84	-13.79	-63.14



Fig 1. The map of Widsor

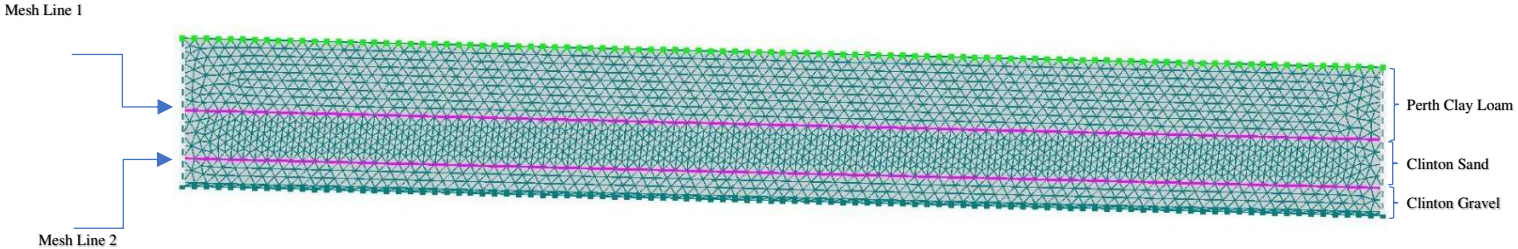


Fig 2. Distribution of soil layers within the barrier (from top to bottom): Perth Clay Loam (60 cm); Clinton Sand (40 cm); Clinton Gravel (20 cm)

An essential part of protective earthen cover is vegetation. In this present study evapotranspiration was indirectly

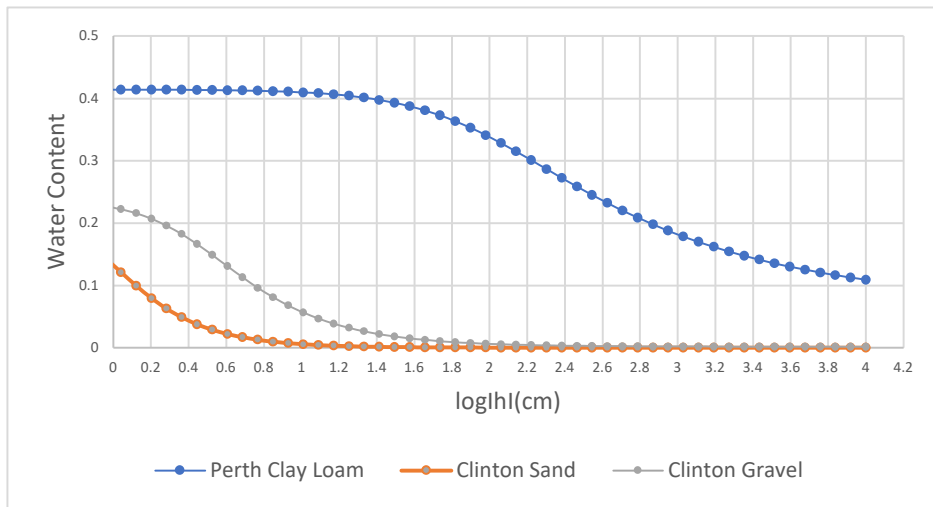


Fig 3. Water Retention Curve (WRC)

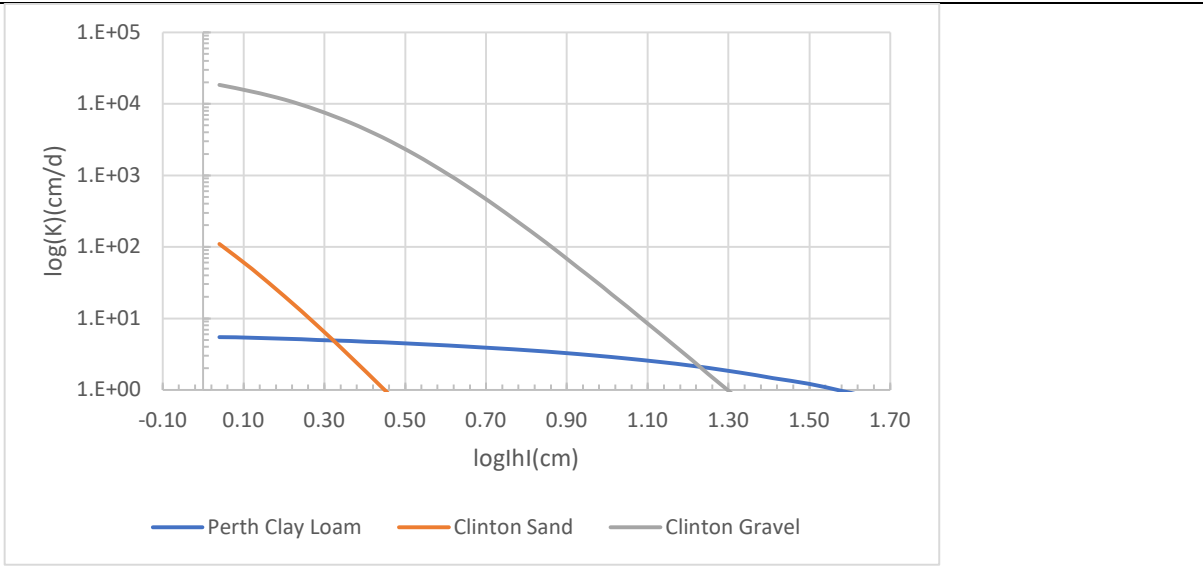


Fig.4. Shows the soil water pressure head and hydraulic conductivity function obtained from equation (3) and (4).

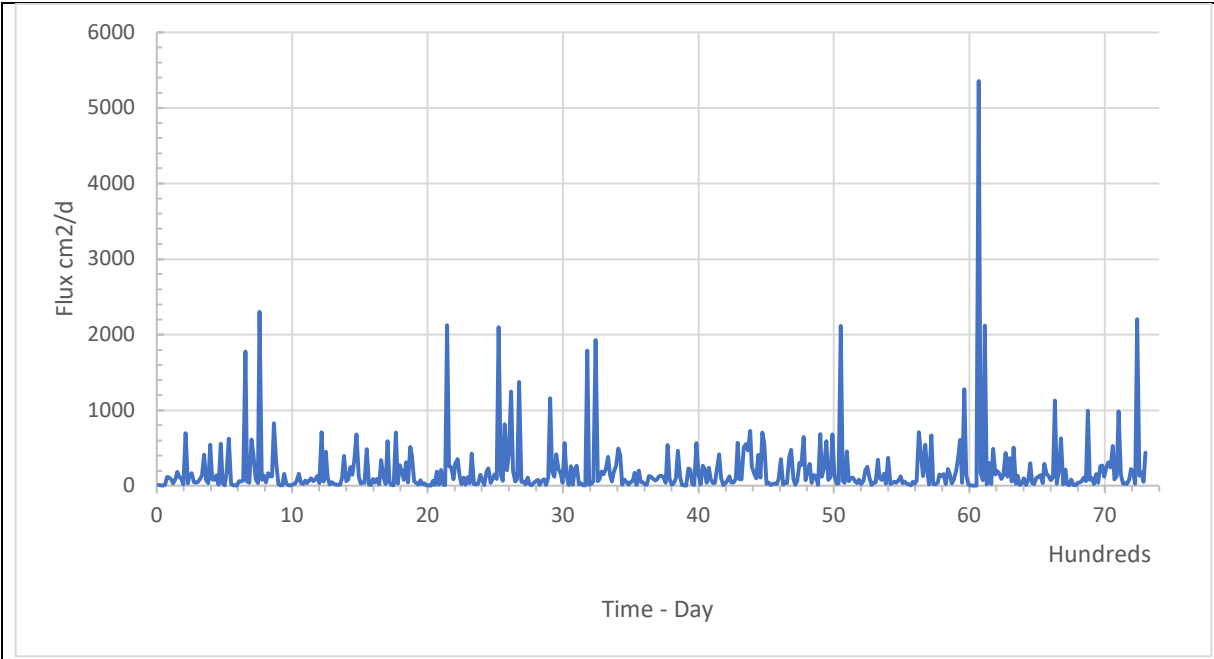


Fig 5. Total flux across bottom boundary

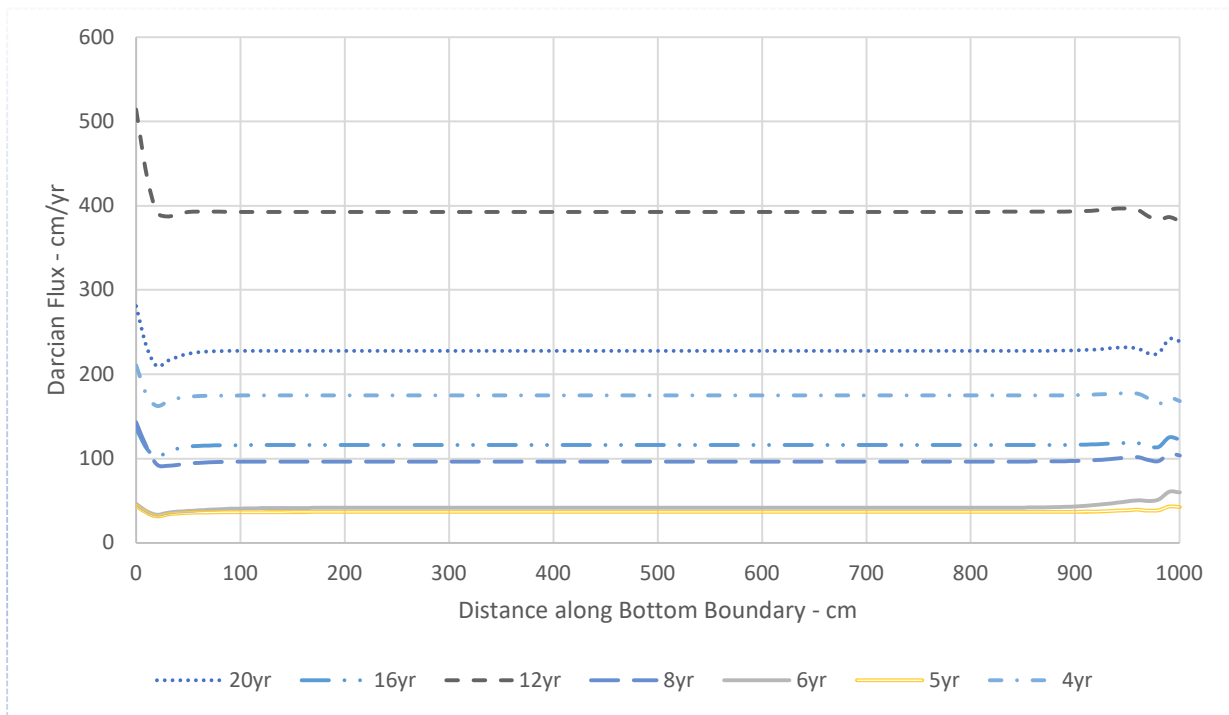


Fig 6. Variation of the Darcian flux along the bottom boundary

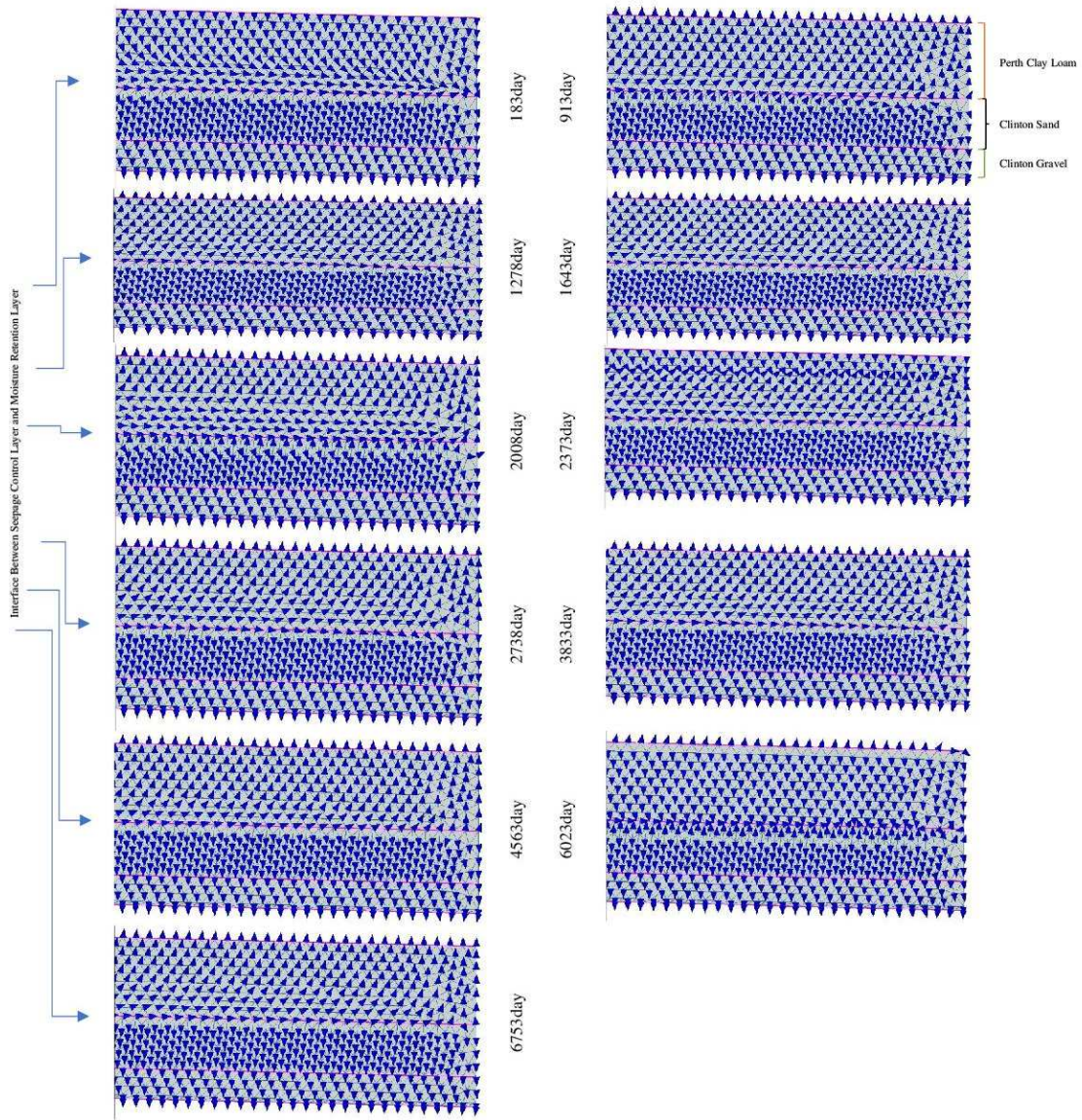


Fig 7. Darcian velocity at eleven different times for various climatological conditions.
(Lateral Drainage depicted in the interface between seepage control layer and moisture retention layer)

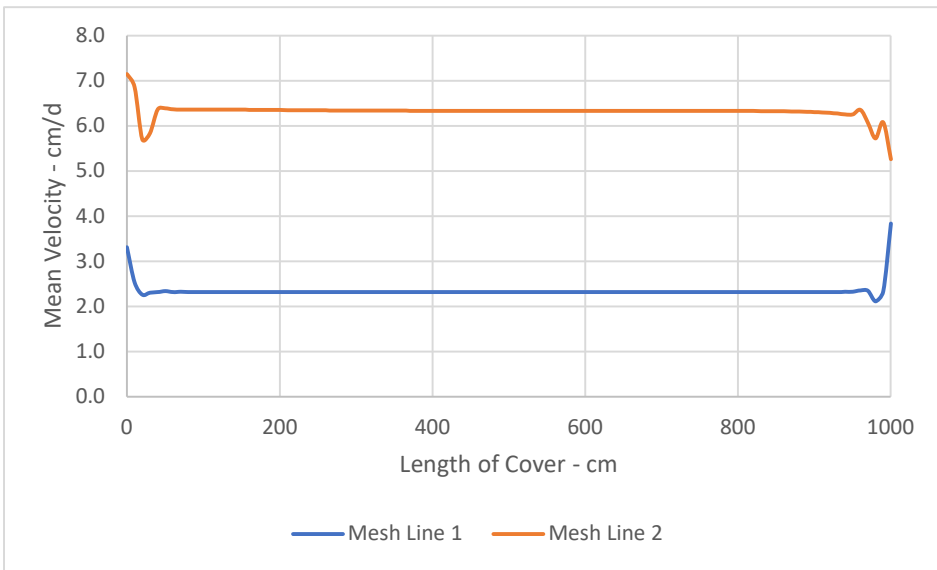


Fig 8. Flux Velocity across the bottom boundary at 867th week

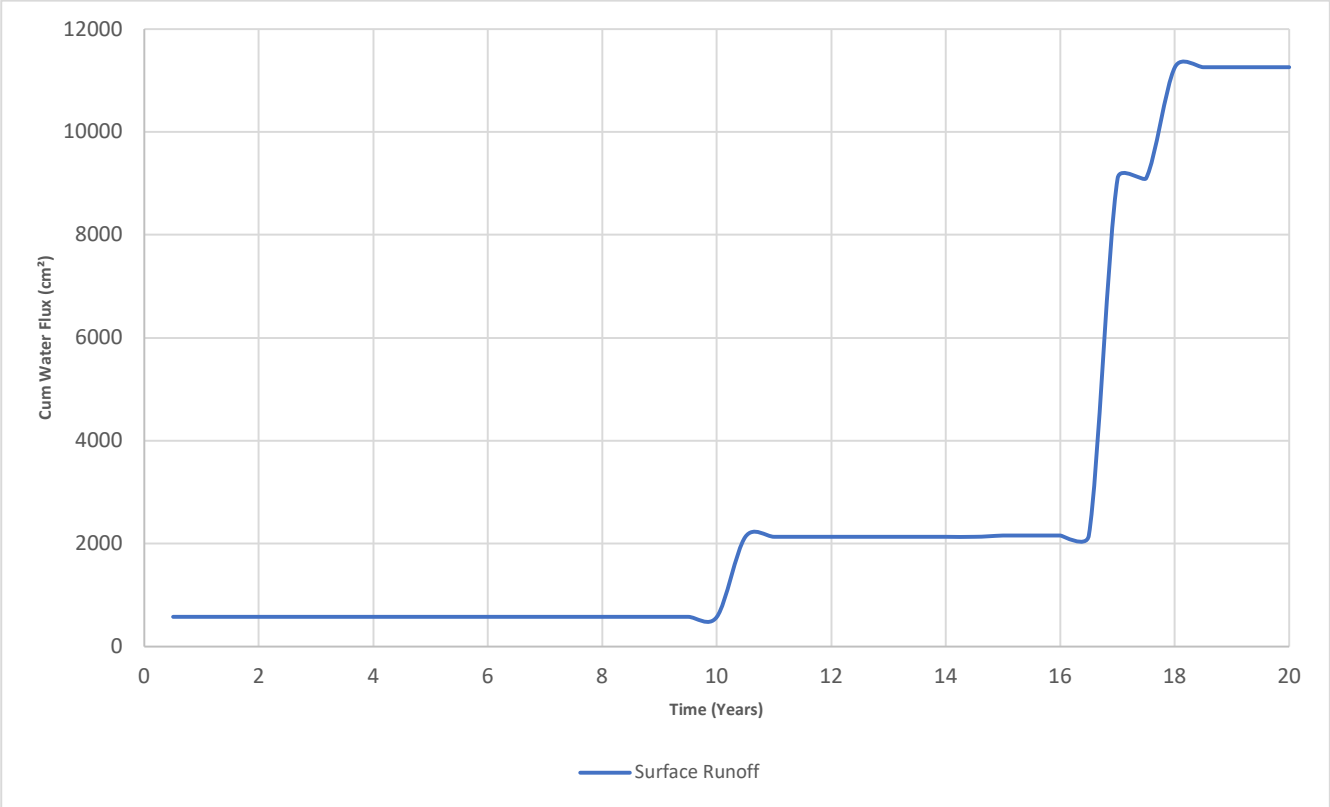


Fig 9. Cumulative Surface Runoff

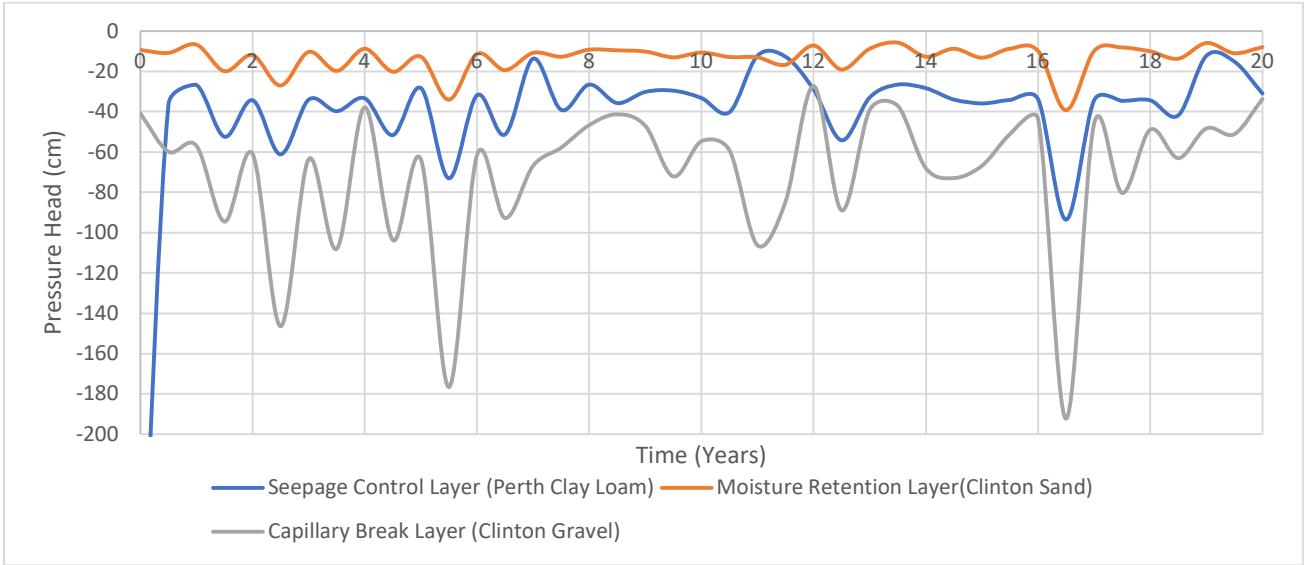


Fig. 10. Mean pressure head in layers

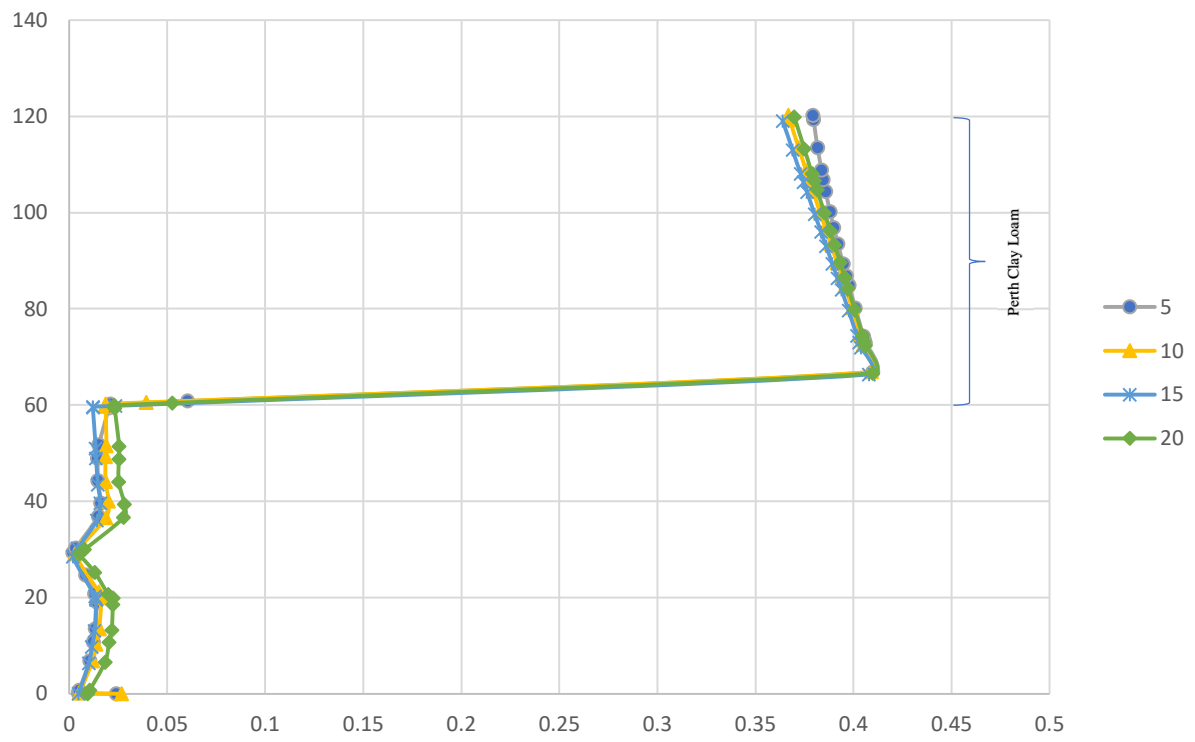


Fig 11. Soil water content versus depth at four different times along a vertical line at $x=250$ cm from the left boundary