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Remark[section]

# Energy and spectral efficiency optimization methods in 5G massive mimo systems

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***Original Research Article***

## Abstract

Radio technologies are driven by a never-ending quest for capacity and thus high throughput, associated with a constant need to improve the quality of the transmission. Thus, the massive MIMO (Multiple Input Multiple Output) technology has been found to be a powerful system to optimize the trade-off between energy and spectral efficiency in fifth generation (5G) cellular networks. In this work, i analyse the trade-off between energy efficiency and spectral efficiency for massive MIMO in wireless communication networks in relation to the power allocation problem. Also i normalize the energy and spectral efficiency to make them comparable and convert the MOO problem into a single object optimization (SOO) problem by the weighted product scalarization method. Finally, i show that the objective function of this SOO problem, i.e. the energy and spectral efficiency trade-off metric that we characterise, is quasi-concave with the emission power and we obtain a unique globally optimal solution. The numerical results attest to the performance of the proposed method.

*Keywords: Optimization; Pareto; Trade-off; MIMO; Spectral efficiency ; Energy efficiency; scalarization method*

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# 1 Introduction

For some time now, Multiple Input Multiple Output (MIMO) multi-antenna techniques have been prominent in scientific research. Massive multiple-input multiple-output systems are being realised mainly for 5G wireless cellular networks to meet a need for the explosive growth of broadband applications using base stations equipped with a large number of antenna elements to provide better service to active users . It is necessary to mention that the advantages of massive MIMO come from the fact that the number of antennas at the BS is much higher than the total number of antennas of the users of interest. Also the commissioning of a large number of antennas in massive MIMO systems improves or increases the power gain, which translates into a decrease in transmit power and favours the increase of both EE and SE . Another important thing to note is that although the power consumption of UEs has increased considerably, battery capacities still remain limited, which leads to the search for EE evolution in 5G massive MIMO systems[2]. A massive antenna array considered at the base station level has the capacity to provide a more excellent connection to several single active users in the same time-frequency resource.

Maximising EE is equivalent to minimising SE, so they cannot be optimised simultaneously, a trade-off must be found. The energy and spectral efficiency are almost concave for most of the time due to the large number of antennas, the cost of the hardware and the noise amplifier, which leads to a reduction of the radio frequency (RF) chains [7]-[15]. A large number of antennas at the precoder and linear decoder are significant for choosing the optimal antenna to reduce noise with uncorrelated interference as much as possible. In addition, the use of more antennas leads to a higher power consumption of the circuit. In terms of energy efficiency, the significant increase in base stations in cellular networks shows that the total energy consumed in the whole network is about $60 - 80\%[19] - [20]$. The massive MIMO system aims at reducing the transmit power created by high power gain and provides higher EE. In addition, to improve spectral efficiency it is necessary to have a proper distribution of users in each cell and spectrum allocation. Among the known works, the author of [23] established the optimization of EE and SE trade-off associated with the number of users, number of antennas, transmitted power and consideration of transport capacity to improve the EE-SE performance. By ensuring a good rate equity, the lower level power with the combination of users improves the EE and SE performance. At the same time, the author of [24] studied EE and SE based on heterogeneous K-tier networks by offloading the data traffic onto the MIMO system of massive small cells. Another author in [25] studied the maximisation of EE-SE in a massive downlink MIMO system, based on a mutlti-objective optimisation approach. In this paper i analyse the trade-off between energy efficiency and spectral efficiency for massive MIMO in wireless communication networks in relation to the power allocation problem. Also i normalize the energy and spectral efficiency to make them comparable and convert the MOO problem into a single object optimization (SOO) problem by the weighted product scalarization method. Finally, i show that the objective function of this SOO problem, i.e. the energy and spectral efficiency trade-off metric that we characterise, is quasi-concave with the emission power and we obtain a unique globally optimal solution.

# 2 System Model

In this section with a massive MIMO system and multiple uplink users, i consider that each cell contains one base station,and hence each base station has multiple transmit antennas $M$ and $K$ unique users. I also assume that $M \gg K$.

The received signal at base station (BS) is thus given by[2]:

$$y_i = \sum_{j=1}^{N} H_{i,j} x_j w_j + \sum_{k=1,k\neq i}^{K} H_{i,k} x_k w_k + n_i, i = 1, ..., N_R \tag{2.1}$$

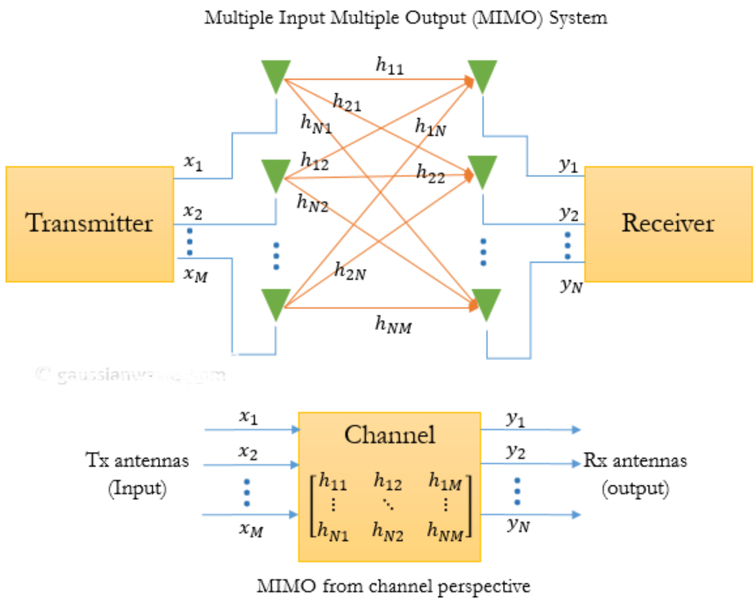


Figure 1: Model and characterize MIMO channel

where $N$ represents the number of signal sources, $w_j$ is the linear pre-coder,$n_i$ is the mean power of the noise with variance of $\mathbb{E}(|n_i|^2) = N_0, x_j \sim \mathcal{N}(0, \mathbf{I_K})$ represents data vector transmission, $H_{i,j}$ represents the propagation channel matrix of the desired signal, $H_{i,k}$ is the propagation channel matrix of each interference signal. It is fair to say that the pre-coders of neighbouring cells will cause inter-cell interference. In addition the signal and interference are generated by the same base station system which justifies that the coefficients of the signal and interference channels can be evaluated.

The signal-to-interference-plus-noise ratio (SINR) is one of the very important metrics as it is widely used in theoretical studies specifically in channel capacity, and it allows an upper limit to be set on the information-carrying capacity. It is used to set an upper limit on the information carrying capacity of a given communication system [18], [19]:

$$\gamma_k = \frac{P\frac{M}{K}|H_{i,j}w_i|^2}{P\frac{M}{K}\sum_{k=1,k\neq j}^{N}|H_{i,k}w_k|^2 + N_0} \tag{2.2}$$

## 2.1 Spectral Efficiency and Energy Efficiency

efficiency (SE), defined as the throughput per unit of bandwidth, is one of the most important metrics for wireless network design. With the increasing attention on energy saving and the development of green radio techniques [20], [21], energy efficiency (EE), defined as the number of bits that can be transmitted per unit of energy consumption [18], is also an important metric for wireless communications.

From the derivative of the SINR, the higher SE in the upper bound per unit of bandwidth for a massive MIMO system can be represented according to the number of users and available antennas as The spectral efficiency is defined by [22]:

$$\eta_k^{SE} = K\mathbb{E}\left[\log_2\left(1 + \frac{P\frac{M}{K}|H_{i,j}w_i|^2}{\underbrace{P\frac{M}{K}\sum_{k=1,k\neq j}^{N}|H_{i,k}w_k|^2 + N_0}_{I}}\right)\right] \tag{2.3}$$

where i have B which is the bandwidth. As a result, i obtain the achievable SE by reducing the interference to the transmission signal from a large number of antennas and users, so that [22]:

$$\eta_k^{SE} = K\log_2\left(1 + \frac{PM}{(I+N_0)K}\right) \tag{2.4}$$

where we define the signal I which represents the accuracy of the interference between the users. A massive MIMO system can improve the EESE trade-off by maximizing SE and reducing the transmit power as the number of transmit antennas increases. By applying the maximal SE for users, i present the optimal number of transmit antennas and total transmit power consumption for a Pareto-optimal system with a maximized EE, which can be expressed as

$$\eta_k^{EE} = \frac{BK\log_2\left(1 + \frac{PM}{(I+N_0)K}\right)}{\mu P + MP_c} \tag{2.5}$$

## 2.2 The EE-SE Trade-off Relationship

In this paper our objective is to optimize the EE-SE trade-off in a massive MIMO system through uplink power allocation where a maximum SE corresponds to a maximum transmit power and to increase the number of available antennas, so that EE aims to reduce the corresponding high power consumption. In the same section, i study the trade-off between EE and SE based on a multi-objective optimisation problem aiming at minimising the total transmission power as follows :

$$\textbf{P(1)} \quad \max_{P}\{\eta_k^{SE}(P), \eta_k^{EE}(P)\} \quad s.t. \quad P \in \mathcal{P}$$

Where:
$\mathcal{P} = \{P|0 \leq P \leq P_{max}\}$ is the transmit power constraint with $P_{max}$ denoting the maximal transmit power.
To maximize the efficiency energy, we consider the function $\eta_k^{EE}$. We have since these two metrics are contradictory: to maximize one conducted to reduce the second. It is necessary to achieve a multi-objectives optimization that will drive to a compromise between the two therefore.
Let's note $\eta_k^{EE}(P)$ and $\eta_k^{SE}(P)$ the two functions objectives: In a multi-objectives optimization (Multiple Objectifies Optimization, MOO), the notion of optimum of Pareto is important [6]:

**Dfinition 2.1.** *A point $x \in \Omega$ is said Pareto-Optimal if and only if none does not exist other point $x_1 \in \Omega$, such that $\eta_k^{EE}(x_1) \geq \eta_k^{EE}(x)$ and $\eta_k^{SE}(x_1) \geq \eta_k^{SE}(x)$.[6]*

**Dfinition 2.2.** *A set of points is said Pareto-Optimal if all points of that are together Pareto-Optimal.* [6]
*In other words, a point is Pareto optimal if there is no other point that can improve both $\eta_k^{SE}$ and $\eta_k^{EE}$ simultaneously.*
*To solve an MOO, the Pareto optimal set needs to be characterized, in which we can find the globally optimal solution*[6].

**Lemma 1**
$\eta_k^{SE}$ is strictly increasing and concave with $P$.
**Proof**

$$\frac{\partial \eta_k^{SE}}{\partial P} = \frac{\partial \left( K \log_2 \left( 1 + \frac{PM}{(I+N_0)K} \right) \right)}{\partial P}$$

$$\frac{\partial \eta_k^{SE}}{\partial P} = \frac{\frac{M}{(I+N_0)K}}{\left(1 + \frac{PM}{(I+N_0)K}\right) \ln 2} = \frac{MK}{((I+N_0)K + PM) \ln 2} > 0$$

from where $\eta_k^{SE}$ is strictly increasing.

$$\frac{\partial^2 \eta_k^{SE}}{\partial P^2} = -\frac{M^2 K}{((I+N_0)K + PM)^2 \ln 2} < 0$$

from where $\eta_k^{SE}$ is concave.
**Lemma 2**
There exists one and only one point $P \in [0, +\infty[$ that maximizes $\eta_k^{EE}(P)$. $\eta_k^{EE}(P)$ is strictly increasing and concave at $P \in [0, P^*[$
while strictly decreasing and neither concave nor convex at
$P \in [P^*, +\infty[$.

**Proof**
we get the first derivative of $\eta_k^{EE}$ as follows:

$$\frac{\partial \eta_k^{EE}}{\partial P} = \frac{\theta(P)}{\ln 2((I+N_0)K + PM)[\mu P + MP_c]^2} \tag{2.6}$$

where

$$\theta(P) = MK(\mu P + MP_c) - ((I+N_0)K + PM)\mu \ln 2K \log_2 \left(1 + \frac{PM}{(I+N_0)K}\right) \tag{2.7}$$

So the sign of $\frac{\partial \eta_k^{EE}}{\partial P}$ depends on the one of $\theta(P)$. Besides

$$\frac{\partial \theta}{\partial P} = -M\mu K \ln 2 \log_2 \left(1 + \frac{PM}{(I+N_0)K}\right) < 0.$$

$\theta(P)$ is strictly decreasing on $[0, +\infty[$ and $\theta(0) = M^2 K P_c > 0$ and

$$\lim_{P \to +\infty} \theta(P) = -\infty$$

. So it exists $P^*$ such as $\theta(P^*) = 0$ it implies that $\frac{\partial \eta_k^{EE}}{\partial P}(P^*) = 0$
$\forall P \in [0, P^*[, \theta(P) > 0 \Longleftrightarrow \frac{\partial \eta_k^{EE}}{\partial P} > 0$ what proves that $\eta_k^{EE}$ is strictly increasing $P$ for $P \in [0, P^*[$
and $\forall P \in [P^*, +\infty[$,

$$\theta(P) < 0 \Longleftrightarrow \frac{\partial \eta_k^{EE}}{\partial P} < 0.$$

What proves that $\eta_k^{EE}$ is strictly decreasing with $P$ for $P \in [P^*, +\infty[$.
Therefore, i define the Pareto optimal set of P(1), by $\mathcal{P}^{POS}$, which is determined by the following proposition.

**Proposition 2.1.**

$$\mathcal{P}^{POS} = \begin{cases} P|P^* \leq P \leq P_{max} & \text{if } P^* < P_{max} \\ P|P = P_{max} & \text{if } P^* \geq P_{max} \end{cases} \tag{2.8}$$

**Proof**

If $P^* \geq P_{max}, \eta_k^{EE}$ and $\eta_k^{SE}$ are strictly decreasing with $P$ on $[0, P_{max}[$. Then $\forall P \in [0, P_{max}[$, we have $\eta_k^{SE}(P_{max}) > \eta_k^{SE}(P)$ and $\eta_k^{EE}(P_{max}) > \eta_k^{EE}(P)$, from where the result

$$\mathcal{P}^{POS} = \{P|P = P_{max}\}$$

If $P^* < P_{max}, \eta_k^{EE}(P)$ is increasing with $P$ on $[0, P^*[$ and is decreasing with $P$ on $[P^*, P_{max}[$. Otherwise $\forall P \in [0, P^*[, \eta_k^{SE}(P^*) > \eta_k^{SE}(P)$ and $\eta_k^{EE}(P^*) > \eta_k^{EE}(P)$, what means that $[O, P^*[ \subsetneq \mathcal{P}^{POS}$.
However $\forall P \in [P^*, P_{max}]$ no point exists $P^{'} \in [P^*, P_{max}]$ verifying that $\eta_k^{SE}(P^{'}) > \eta_k^{SE}(P)$ et $\eta_k^{EE}(P^{'}) > \eta_k^{EE}(P)$ from where $\mathcal{P}^{POS} = \{P|P^* \leq P \leq P_{max}\}$.

I note that according to Proposition 1, in the case where
$P^* \geq P_{max}$, $\mathcal{P}^{POS}$ has a single point, this point is $P_{max}$, which means that the globally optimal solution obtained for **P(1)** is unique. This means that in the following i will study the case of
$P^* < P_{max}$, where the globally optimal solution we have to find for **P(1)** must be Pareto optimal, so i will equivalently transform the MOO problem **P(1)** into **P(2)**.

**P(2)** $\max\limits_{P}\{\eta_k^{SE}(P), \eta_k^{EE}(P)\} \quad s.t. \quad P \in \mathcal{P}^{POS}$

Where $\mathcal{P}^{POS} = \{P|P^* \leq P \leq P_{max}\}$

I have shown that all points of the $\mathcal{P}^{POS}$ are Pareto optimal. Next i will look for a unique global solution. For this purpose we will apply the scalarization method. This allows us to transform the **P(2)** MOO problem into a single object optimisation (SOO) problem **P(3)**. Also i will make the energy efficiency and spectral functions comparable in order to apply the scalarization methods [9]. I set up a process of normalization of the functions so that EE and SE are comparable to be associated with a utility function. I define the functions normalized following:

$$\begin{cases} \eta_k^{SE}(norm)(P) = \dfrac{\eta_k^{SE}(P)}{\eta_k^{SE}(max)} \\ \eta_k^{EE}(norm)(P) = \dfrac{\eta_k^{EE}(P)}{\eta_k^{EE}(max)} \end{cases} \tag{2.9}$$

With $\eta_k^{SE}(max) = \eta_k^{SE}(P_{max})$ and $\eta_k^{EE}(max) = \eta_k^{EE}(P^*)$

**P(3)** $\max\limits_{P}\{\eta_k^{SE}(norm)(P), \eta_k^{EE}(norm)(P)\} \quad s.t. \quad P \in \mathcal{P}^{POS}$

I obtain the utility function U, with $w \in [0, 1]$ as the following expression:

$$U(P) = [\eta_k^{SE}(norm)(P)]^{\omega} \times [\eta_k^{EE}(norm)(P)]^{1-\omega} \tag{2.10}$$

I formulate a new problem **P(4)** with the function $U$,

**P(4)** $\max\limits_{P} U(P) \quad s.t. \quad P \in \mathcal{P}^{POS}$

Solving problem **P(1)** is the same as solving problem **P(4)**.
To solve the problem **P(4)**, i consider the following lemma:
**Lemma 3** $U(P)$ is strictly quasi-concave in $\mathcal{P}^{POS}$

**Proof**

$$T(P) = \ln U(P)$$
$$T(P) = \omega \ln \eta_k^{SE}(norm)(P) + (1-\omega)\ln \eta_k^{EE}(norm)(P)$$
$$\text{T(P)}{=}\ln \eta_k^{SE}(P) - (1-\omega)\ln[\mu P + MP_c] - \varphi.$$

Where $\varphi = \omega \ln \eta_k^{SE}(P_{max}) + (1-\omega)\ln \eta_k^{EE}(P^*)$ is a constant.
The first derivative of $T(P)$ is :

$$\frac{\partial T}{\partial P} = \frac{\frac{\partial \eta_k^{SE}}{\partial P}}{\eta_k^{SE}(P)} - \frac{(1-\omega)\mu}{\mu P + MP_c}$$
$$\frac{\partial T}{\partial P} = \frac{M(\mu P + MP_c) - (1-\omega)\mu((I+N_0)K + PM)\ln 2 \log_2\left(1 + \frac{PM}{(I+N_0)K}\right)}{(\mu P + MP_c)((I+N_0)K + PM)\mu \ln 2 \log_2\left(1 + \frac{PM}{(I+N_0)K}\right)}$$
$$\frac{\partial T}{\partial P} = \frac{\beta(P) - (1-\omega)\mu}{\mu P + MP_c}.$$

. Where $\beta(P) = \dfrac{M(\mu P + MP_c)}{((I+N_0)K + PM)\ln 2 \log_2\left(1 + \frac{PM}{(I+N_0)K}\right)}$.

By continuation $\dfrac{\partial T}{\partial P} = 0$ imply that

$$\beta(P) = (1-\omega)\mu. \qquad (2.11)$$

Otherwise $\beta(P) = \varphi(P) + \chi(P)$

With $\varphi(P) = \dfrac{\mu MP}{((I+N_0)K + PM)\ln 2 \log_2\left(1 + \frac{PM}{(I+N_0)K}\right)}$

and $\chi(P) = \dfrac{M^2 P_c}{((I+N_0)K + PM)\ln 2 \log_2\left(1 + \frac{PM}{(I+N_0)K}\right)}$.

In the following we assume $\xi = ((I+N_0)K + PM)\ln 2 \log_2\left(1 + \frac{PM}{(I+N_0)K}\right)$. The derivative of the first term is:

$$\frac{\partial \varphi}{\partial P} = \frac{\mu M \xi - \mu MP\left(M \log_2\left(1 + \frac{PM}{(I+N_0)K}\right)\ln 2 + M\right)}{\xi^2} < 0$$

Also the derivative of the second term:

$$\frac{\partial \chi}{\partial P} = -\frac{M^2 P_c\left[M \ln 2 \log_2\left(1 + \frac{PM}{(I+N_0)K}\right) + M\right]}{\xi^2} < 0$$

So the function $\beta$ is strictly decreasing with $P$.
We have

$$\theta(P^*) = MK(\mu P^* + MP_c) - ((I+N_0)K + P^*M)\mu \ln 2K \log_2\left(1 + \frac{P^*M}{(I+N_0)K}\right) = 0$$

what is equivalent to

$$\frac{M(\mu P^* + MP_c)}{((I+N_0)K + P^*M)\ln 2 \log_2\left(1 + \frac{P^*M}{(I+N_0)K}\right)} = \mu$$

From where $\beta(P^*) = \mu$
By continuation for $P \in \mathcal{P}^{POS}$, we have

$$0 < \beta(P_{max}) \leq \beta(P) \leq \beta(P^*) = \mu$$

**Case** $1 : \omega > 1 - \dfrac{\beta(P_{max})}{\mu}$
In this case, we have $(1-\omega)\mu < \beta(P_{max}) \leq \beta(P)$. Then the equation $(2.13)$ has no solution and $\dfrac{\partial T}{\partial P} > 0$. Then, $T(P)$ is strictly increasing in $\mathcal{P}^{POS}$.
**Case** $2 : \omega \leq 1 - \dfrac{\beta(P_{max})}{\mu}$
In this case, we have $\beta(P_{max}) \leq (1-\omega)\mu \leq \mu$.Then the equation $(2.13)$ admits an unique solution in $P^{**}$ because $\beta(P)$ is strictly decreasing . $\forall P \in [P^*, P^{**}[, (1-\omega)\mu < \beta(P)$ we have $\dfrac{\partial T}{\partial P} > 0$. And $\forall P \in ]P^{**}, P_{max}], (1-\omega)(1+\alpha) > \beta(P)$ we have $\dfrac{\partial T}{\partial P} > 0$
Then, $T(P)$ is strictly increasing and is strictly decreasing in $\mathcal{P}^{POS}$, which means that $T(P)$ is strictly quasi-concave in $\mathcal{P}^{POS}$.
By continuation $U(P) = \exp T(P)$ is increasing with $T(P)$.
$U(P)$ is strictly quasi-concave in $\mathcal{P}^{POS}$.
Moreover, with the proof of Lemma $3$, we also have the optimal solution of P$(4)$ , as described by the following

**Proposition 2.2.** *The optimal solution for* $P(4)$ *is*

$$\mathcal{P}^{opt} = \begin{cases} P^* & \text{if } \omega = 0 \\ P_{max} & \text{if } \omega > 1 - \dfrac{\beta(P_{max})}{\mu} \\ P^{**} & \text{if } \omega \leq 1 - \dfrac{\beta(P_{max})}{\mu} \end{cases} \tag{2.12}$$

**Proof**
The proof of this proposition justifies it self by the cases exposed higher.

# 3 Discussion

for $\omega = 1$, we consider the function $\eta_k^{SE}(P)$.
Then $\omega > 1 - \dfrac{\beta(P_{max})}{\mu}$ and the optimal power is $\mathcal{P}^{opt} = P_{max}$

for $\omega = 0$, i consider the function $\eta_k^{SE}(P)$.
Then $\omega \leq 1 - \dfrac{\beta(P_{max})}{\mu}$ and the optimal power is $\mathcal{P}^{opt} = P^{**}$
Besides $\beta(P^{**}) = (1-\omega)\mu = \beta(P^*)$
As $\beta$ is a strictly decreasing function with $P$ then $P^* = P^{**}$.
By continuation $\mathcal{P}^{opt} = P^*$.

For $0 < \omega < 1$ i consider the two functions $\eta_k^{SE}$ and $\eta_k^{EE}$ and i get the optimal power following values of $\omega$.

**Proprit 3.1.** *When* $\omega = 0$ *the optimal power is* $\mathcal{P}^{opt} = P^*$.
*For* $0 \leq \omega \leq 1 - \dfrac{\beta(P_{max})}{\mu}, \mathcal{P}^{opt}$ *is strictly increasing with* $\omega$. *For* $\omega > 1 - \dfrac{\beta(P_{max})}{\mu}$ *we have* $\mathcal{P}^{opt} = P_{max}$.

**Proof**
$1-\dfrac{\beta(P)}{\mu}$ is a strictly increasing function with $P$ from where $\mathcal{P}^{opt}$ is a strictly increasing function with $P$.

**Proprit 3.2.** *The function $\eta_k^{SE}(norm)(P^{opt})$ is increasing with $\omega$ while the function $\eta_k^{EE}(norm)(P^{opt})$ is decreasing with $\omega$. In more one exists and unique $\omega^0 \in [0, 1-\dfrac{\beta(P_{max})}{\mu}]$ such as* $\eta_k^{SE}(norm)(P^{opt}) = \eta_k^{EE}(norm)(P^{opt}) = U(P_{opt})$

**Proof**
I have $P^{opt}$ is non-decreasing with $\omega$ and

$$\eta_k^{SE}(norm)(P^{opt}) = \frac{\eta_k^{SE}(P^{opt})}{\eta_k^{SE}(max)}$$

is non-decreasing with $\omega$. In the same way i get that $\eta_k^{EE}(norm)(P^{opt})$ is non-increasing with $\omega$. Also the functions $\eta_k^{SE}(norm)(P^{opt})$ and $\eta_k^{EE}(norm)(P^{opt})$ are respectively strictly increasing and decreasing with $\omega$ and on the interval $[0; 1-\dfrac{\beta(P_{max})}{\mu}]$ and are constant for $\omega \in ]1-\dfrac{\beta(P_{max})}{\mu}, 1]$ since $P^{opt} = P^{max}$ for $\omega \in ]1-\dfrac{\beta(P_{max})}{\mu}, 1]$. Besides when $\omega = 0$
we have $P^{opt} = P^*$ then $\eta_k^{EE}(norm)(P^{opt}) = 1 > \eta_k^{SE}(norm)(P^{opt})$. When $\omega \in ]1-\dfrac{\beta(P_{max})}{\mu}, 1]$, i have $P^{opt} = P_{max}$ then

$$\eta_k^{SE}(norm)(P^{opt}) = 1 > \eta_k^{EE}(norm)(P^{opt}).$$

Then only one and one exists
$\omega^0 \in ]0; 1-\dfrac{\beta(P_{max})}{\mu}]$ such as
$\eta_k^{SE}(norm)(P^{opt}) = \eta_k^{EE}(norm)(P^{opt})$.
So

$$U(P^{opt}) = \eta_k^{SE}(norm)(P^{opt}) \times \eta_k^{SE}(norm)(P^{opt}) = \eta_k^{SE}(norm)(P^{opt}) = \eta_k^{EE}(norm)(P^{opt})$$

# 4 Numerical Result of the Simulation

The numerical results presented in this section are obtained using MATLAB based on Monte Carlo simulations .We note that the increasing number of base stations leads to an increase in power consumption, which proves that the $\eta_k^{EE}(P)$ first increases and then decreases. This is the reason why the trade-off presented in EE-SE has a quasi-concave relationship with the transmission power.

The optimal values obtained for EE and SE are realised on the number of users distributed with a fixed transmit power. The trade-off between EE and SE grows simultaneously with a first minimum SE associated with the number of user antennas with lower transmit power. A number of the employed active users $K = 4$, the EE increases with the SE and gives the value $EE = 3.5Mbit/J$, at $SE = 25bit/s/Hz$. Meanwhile, when the optimal transmit power $P = 20dBm$, at a distributed number of active users increases to $K = 8$, the $EE = 5Mbit/J$ with $SE = 40bit/s/Hz$ and become small values due to a large transmit power. After this value, the EE starts to decrease with SE due to the complex processing complexity and high operation cost for radio frequency chains at the employed antenna in a massive MIMO system.
In Fig.$2$ i see that the optimum power increases with the parameter $\omega$.

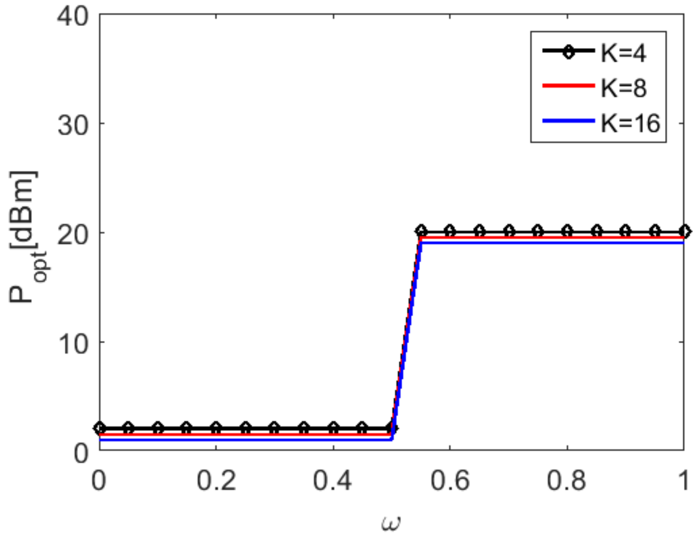


Figure 2: Evolution of the optimal power transmitted

In Figure $3$ i illustrate the trade-off between EE and SE. Also i note that the transmission power and the number of available antennas give the possibility to optimise the EE-SE trade-off by admitting a multi-objective optimisation problem.

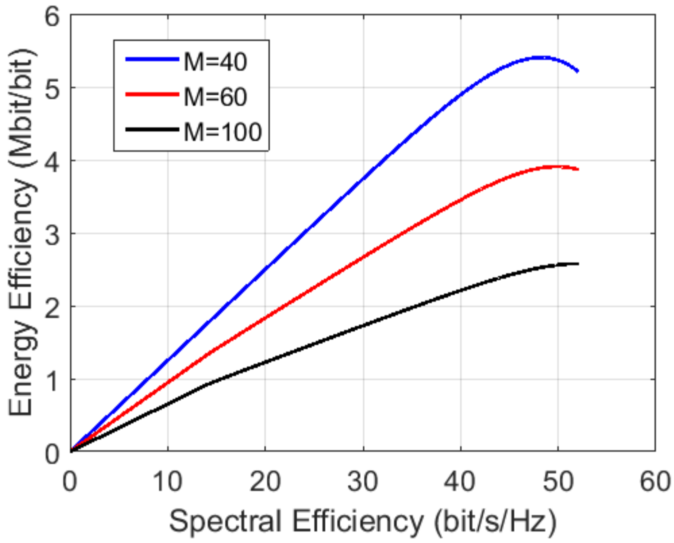


Figure 3: The trade-off between EE and SE

# 5 Conclusion

In this research work, I investigated the trade-off between EE and SE at the massive MIMO system level as a function of the first derivative in multi-objective problems for the transmitted power in a massive MIMO downlink system. The scalar method and making the system Pareto-optimal solved the optimisation problem and satisfied the maximum EE values.

Also, the trade-off between EE and SE can be obtained based on the optimal allocation of transmit power in each cell and the number of antenna elements when the value of SE is high. Therefore, the Pareto-optimal optimisation problem is flexible in that it allows the EE-SE trade-off to be manipulated according to the situation. Therefore, the consideration of dominance criteria in the sense of The Pareto-optimal criteria for solving the multi-objective optimisation problem favoured the reduction of energy consumption on the one hand and fuel consumption on the other.

The simulation results show that the EE and SE optimisation is achieved by minimising the total energy consumption with a proportional allocation of the transmission power when the number of transmitted antennas $M$ is constant.

Our research study can present investigative clues for researchers or companies to find the optimal power and number of antennas available in a massive MIMO downlink system at the base station in order to find the specific EE and SE required. More importantly, the study of the optimisation of the trade-off between EE and SE in a MIMO system, favours the operation cost of the number of antennas in use and the transmit power.

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