Original Research Article

An Inventory Analysis for Deteriorating Items Incorporating The Effect of Preservation Technology Investment When Demand Is Time Dependent

ABSTRACT

The goal of this work is to characterize preservation technique for deteriorating items in order to lessen the rate of deterioration. This study looks at a degrading inventory model with time-dependent demand that is a quadratic function of time. The rate of degradation in this model is determined by deterministic deterioration, which is a quadratic function of time. The main objective is to determine the effect of preservation technology for deteriorating items where the demand function is time variable and analyze the significance of preservation technology. The major goal of this article is to look at the inventory model's minimum cost per unit time. With the help of a numerical example, the results are validated. The ideal solution is subjected to a sensitivity analysis with regard to several parameters. Finally, a graph depicts the behavior of the relationship between parameters and total inventory cost.

Key Words: Inventory, Determistic Inventory Model, Preservation Technology Investment, Deterioration items, Sensitivity analysis.

1. INTRODUCTION

A store of items kept for future production or sales is referred to as an inventory. All materials, components, supplies, tools, in-process or finished commodities that are documented in an organization's books and held in its stocks, warehouse, or factory for a period of time are referred to as inventory in its broadest sense. Because inventory is a controlled component, there are numerous strategies to reduce inventory costs in terms of both investment and quality. Inventory management is one of the most challenging planning and control concerns that managers confront today, especially in manufacturing. The goal of inventory management is to keep stock levels as low as possible while ensuring that supplies are available for ongoing activities. When making inventory decisions, managers must strike a balance between numerous cost components, including the costs of delivering commodities, inventory-holding fees, and charges associated with insufficient inventories.

The deterioration of items and demand is an important aspect of the research in inventory systems. In an inventory system, the effect of deterioration is critical. Deterioration is described as decay or damage to an item to the point where it can no longer be used for its intended purpose. Deteriorating goods can be classified in two ways, according to Wee [1]. The first group includes goods such as meat, vegetables, fruit, and flowers that have rotted, been damaged, and have evaporated or expired over time. The second category, on the other hand, refers to objects that lose part or all of their value over time as a result of new technology, such as computer chips, mobile phones, and fashion items. Whitin [2] was the first to notice the inventory system for deteriorating things when he researched fashion items that were decaying at the conclusion of the scarcity era. Then Ghare and Schrader [3] expanded on their finding that the consumption of degrading items followed an exponential function of time. In an inventory system, demand is a critical factor. Demand is defined as the rate at which customers wish to purchase a thing. According to economic theory, demand is made up of two factors: taste and ability. Most scholars these days are focusing their efforts on a time-dependent demand function. In general, consistent demand rates should not be considered for products such as attractive clothing, computer equipment, and so on. Donaldson [4] was the first to investigate the inventory replenishment method for a demand function with a linear trend. Singh and Pattanayak [5] proposed an EOQ model for degrading products that degrades at a variable deterioration rate and has a linear demand function with partial backlog. A deteriorating inventory model with time variable demand and holding cost was interpreted by V. K. Mishra and L. S. Singh [6]. S. Khanra, S. K. Ghosh, and S. K. Chaudhuri [7] built an EOQ model for degrading items with time dependent quadratic demand and allowable payment delay.

Preservation technology is a technique for slowing down the deterioration of deteriorating objects. Because of rapid social movements and the fact that a preservation system can significantly reduce the effect of product deterioration, assisting the retailer in decreasing economic losses, preservation technique consideration is of crucial importance. The majority of food items, such as bread, fruits, and vegetables, start to decay as soon as they are produced. Hsu et al. [8] proposed a deteriorating inventory model with constant demand rate and exponential decay in which the retailer is allowed to spend in preservation technologies to lower the deterioration rate in order to accord with the practical inventory condition. However, the preservation technology cost is assumed to be a fixed cost per inventory cycle and this seems to be unrealistic. If new equipment's, such as refrigeration units, are acquired, capital costs will occur .Dye and Hsieh [9] then presented an extended model of Hsu et al. [8] by assuming the preservation technology cost is a function of the length of replenishment cycle. They improved on Hsu et al [8].model by including time-varying degradation and reciprocal time-dependent partial backlogging rates. T.P. Hsieh and C.Y. Dye [10] proposed a production-inventory model that takes into account the influence of preservation technology investment when demand varies over time. Mishra V. K. [11] attempt to create an inventory model for deteriorating products that takes into account the fact that preservation technology (PT) can greatly reduce the rate of deterioration. Pervin M, Roy S. K. & Weber G. W [12] formulate and solve an Economic Production Quantity inventory model with deteriorating items.

This research focuses on how a retailer's non-instantaneously deteriorating items affect preservation technology investment and inventory decisions. Besides, it analyzes sensitivity analysis to investigate how they are affected by cost factors. Basically, quadratic function of time express the steady growth or decline of the demand in the best way. Also this research considers time dependent deterioration rate which cannot be constant in real situation. Due to various causes deterioration rate can be changed and shortages are not allowed in the model of the present research. The model is solved to optimize the total cost which is to be minimized. A numerical example is analyzed to check the validity of the model and the sensitivity of different parameters is observed graphically.

2. ASSUMPTIONS

The following assumptions are explored in this paper in order to create the suggested mathematical model of the inventory system:

I. The inventory system involves single type of items.

II. Replenishment rate is infinite, i.e. Replenishment rate is instantaneously.

III. The demand rate of the item is considered by a quadratic and continuous function of time. $D(t)$ is the time dependent demand function which is defined by $D(t) = a + bt + ct^2$, where $a, b, c > 0$. Here a is the initial rate of demand, b is the rate at which the demand rate increases and c is the rate at which the change in the demand rate itself increases.

IV. The deterioration rate is a variable rate of deterioration on on-hand inventory per unit period, and the degraded products are not repaired or replenished within the cycle. Variable rate of deterioration of an item where $\theta(t) = \theta t^2; 0 < \theta < 1$.

V. $\tau\rho$ = The rate of deterioration as a result, which is taken into account by $\tau_\rho = (t) - m = \theta t^2 - m$

VI. m = The reduced deterioration rate

VII. Shortages are not allowed.

VIII. There is no provision for repair or replacement of deteriorated units.

IX. Time horizon is infinite.

3. Notations

T The length of replenishment cycle in traditional system (per year)

$I(t)$ The inventory level at time t $(0 < t < T)$

A The ordering cost of inventory per order ($TK/order$)

h The unit holding cost per unit time

p The purchase cost per unit of item

$D(t)$ Demand rate of time which is defined by

$D(t) = a + bt + ct^2, a > 0, b \neq 0$ & $c \neq 0$. Here a is the initial rate of demand, b is the rate with which the demand rate increases and the rate of change in the demand rate itself changes at a rate c.

$\theta(t)$ The variable rate of deterioration of an item where $\theta(t) = \theta t^2; 0 < \theta < 1$

m The reduced deterioration rate

$\tau\rho$ The resultant deterioration rate

IM The maximum inventory level during $[0, T]$

TC The average cost per unit time.

PT Preservation technology

4. MATHEMATICAL FORMULATION

Ghare and Schrader [3], indicates that the consumption of decaying goods was nearly tied to a negative exponential function of time. Based on the criteria, they proposed and introduced an inventory model as stated below

$$\frac{dI(t)}{dt} + \theta I(t) = -f(t)$$

In that model, θ denotes the rate at which deteriorating things deteriorate, $I(t)$ denotes the inventory level at the time, and $f(t)$ is the demand rate with regard to time.The inventory level $I(t)$ at time t generally decreases from initial inventory to meet markets demand and products deterioration and reaches to zero at T. Hence, the variation of inventory with respect to time can be described by the governing differential equation:

$$\frac{dI(t)}{dt} + \tau_\rho I(t) = -(a + bt + ct^2)$$

$$\Rightarrow \frac{dI(t)}{dt} + (\theta(t) - m)I(t) = -(a + bt + ct^2)$$

$$\Rightarrow \frac{dI(t)}{dt} + (\theta t^2 - m)I(t) = -(a + bt + ct^2) \qquad \text{... ... ... (1)}$$

With conditions $0 \le t \le T$, $(0) = IM$, $(t) = 0$ $if\ t =$ ... ... (2)

and resultant deterioration rate $\tau_\rho = \theta(t) - m$

5. ANALYTIC SOLUTION OF THE MODEL

The inventory depletes during the period $[0, T]$ due to the deterioration and demand. The inventory level at any time during $[0, T]$ is described by equation (1) which is a first order and first degree ordinary differential equation.

Integrating factor I.F = $e^{\int (\theta t^2 - m)\, dt} = e^{\frac{\theta t}{3} - mt}$

Therefore the general solution is obtained by

$$I(t)e^{\frac{\theta t^3}{3}-mt} = -\int (a+bt+ct^2)e^{\frac{\theta t^3}{3}-mt}dt$$

$$= -\int (a+bt+ct^2)(1+\frac{\theta t^3}{3}-mt)dt$$

[Neglecting higher degree of theta]

$$= \ -a\left(t+\frac{\theta t^4}{12}-\frac{mt^2}{2}\right)-b\left(\frac{t^2}{2}+\frac{\theta t^5}{15}-\frac{mt^3}{3}\right)-c\left(\frac{t^3}{3}+\frac{\theta t^6}{18}-\frac{mt^4}{4}\right)+W$$ … … … (3)

At the final stage, t = 0 if t = T in equation (3.2), we have

$$W = a\left(t+\frac{\theta t^4}{12}-\frac{mt^2}{2}\right)+b\left(\frac{t^2}{2}+\frac{\theta t^5}{15}-\frac{mt^3}{3}\right)+c\left(\frac{t^3}{3}+\frac{\theta t^6}{18}-\frac{mt^4}{4}\right)$$

Therefore,

$$I(t) = \ a\left[(T-t)+\frac{\theta}{12}(T^4-t^4)-\frac{m}{2}(T^2-t^2)\right]e^{-(\frac{\theta t}{3}-mt)}$$

$$+b\left[\frac{1}{2}(T^2-t^2)+\frac{\theta}{15}(T^5-t^5)-\frac{m}{3}(T^3-t^3)\right]e^{-(\frac{\theta t^3}{3}-mt)}$$

$$+c[\frac{1}{3}(T^3-t^3)+\frac{\theta}{18}(T^6-t^6)-\frac{m}{4}(T^4-t^4)]e^{-(\frac{\theta t^3}{3}-mt)}$$

$$= \ a\left[(T-t)+\frac{\theta}{12}(T^4-t^4)-\frac{m}{2}(T^2-t^2)\right](1-\frac{\theta t^3}{3}+mt)$$

$$+b\left[\frac{1}{2}(T^2-t^2)+\frac{\theta}{15}(T^5-t^5)-\frac{m}{3}(T^3-t^3)\right](1-\frac{\theta t^3}{3}+mt)$$

$$+c[\frac{1}{3}(T^3-t^3)+\frac{\theta}{18}(T^6-t^6)-\frac{m}{4}(T^4-t^4)](1-\frac{\theta t^3}{3}+mt)$$

$$= a\{\left[(T-t)+\frac{\theta}{12}(T^4-t^4)-\frac{m}{2}(T^2-t^2)\right]$$

$$-\frac{\theta}{3}\left[(Tt^3-t^4)+\frac{\theta}{12}(T^4t^3-t^7)-\frac{m}{2}(T^2t^3-t^5)\right]$$

$$+[m\left\{(tT-t^2)+\frac{\theta}{12}(T^4t-t^5)-\frac{m}{2}(T^2t-t^3)\right]\}$$

$$+b\{\left[\frac{1}{2}(T^2-t^2)+\frac{\theta}{15}(T^5-t^5)-\frac{m}{3}(T^3-t^3)\right]$$

$$-\frac{\theta}{3}\left[\frac{1}{2}(T^2t^3-t^5)+\frac{\theta}{15}(T^5t^3-t^8)-\frac{m}{3}(T^3t^3-t^6)\right]$$

$$+[m\left\{\frac{1}{2}(T^2t-t^3)+\frac{\theta}{15}(T^5t-t^6)-\frac{m}{3}(T^3t-t^4)\right]\}$$

$$+c\{[\tfrac{1}{3}(T^3-t^3)+\tfrac{\theta}{18}(T^6-t^6)-\tfrac{m}{4}(T^4-t^4)]-\tfrac{\theta}{3}\left[\tfrac{1}{3}(T^3t^3-t^6)+\tfrac{\theta}{18}(T^6t^3-t^9)-\right.$$

$$\left.\tfrac{m}{4}(T^4t^3-t^7)\right]+[m\left\{\tfrac{1}{3}(T^3t-t^4)+\tfrac{\theta}{18}(T^6t-t^7)-\tfrac{m}{4}(T^4t-t^5)\right]\} \quad \dots\dots\dots (4)$$

We have, Inventory holding cost per cycle,

IHC= $h\int_0^T I(t)\,dt$

$$= h\bigg[\bigg\{a\bigg[\bigg(T^2 - \frac{T^2}{2}\bigg) + \frac{\theta}{12}\bigg(T^5 - \frac{T^5}{5}\bigg) - \frac{m}{2}\bigg(T^3 - \frac{T^3}{2}\bigg)\bigg]$$

$$- \frac{\theta}{3}\bigg[\bigg(\frac{T^5}{4} - \frac{T^5}{5}\bigg) + \frac{\theta}{12}\bigg(\frac{T^8}{4} - \frac{T^8}{8}\bigg) - \frac{m}{2}\bigg(\frac{T^6}{4} - \frac{T^6}{6}\bigg)$$

$$+ m\bigg\{\bigg(\frac{T^3}{2} - \frac{T^3}{3}\bigg) + \frac{\theta}{12}\bigg(\frac{T^6}{2} - \frac{T^6}{6}\bigg) - \frac{m}{2}\bigg(\frac{T^4}{2} - \frac{T^4}{4}\bigg)\bigg\}\bigg]$$

$$+ b\{\bigg[\frac{1}{2}\bigg(T^3 - \frac{T^3}{2}\bigg) + \frac{\theta}{15}\bigg(T^6 - \frac{T^6}{6}\bigg) - \frac{m}{3}\bigg(T^4 - \frac{T^4}{4}\bigg)] - \frac{\theta}{3}[\frac{1}{2}\bigg(\frac{T^6}{4} - \frac{T^6}{6}\bigg)$$

$$+ \frac{\theta}{15}\bigg(\frac{T^9}{4} - \frac{T^9}{9}\bigg) - \frac{m}{3}\bigg(\frac{T^7}{4} - \frac{T^7}{7}\bigg)] + m[\frac{1}{2}\bigg(\frac{T^4}{2} - \frac{T^4}{4}\bigg) + \frac{\theta}{15}\bigg(\frac{T^7}{2} - \frac{T^7}{7}\bigg)$$

$$- \frac{m}{3}\bigg(\frac{T^5}{2} - \frac{T^5}{5}\bigg)]\} + c\{[\frac{1}{3}\bigg(T^4 - \frac{T^4}{2}\bigg) + \frac{\theta}{18}\bigg(T^7 - \frac{T^7}{7}\bigg) - \frac{m}{4}\bigg(T^5 - \frac{T^5}{5}\bigg)]$$

$$- \frac{\theta}{3}[\frac{1}{3}\bigg(\frac{T^7}{4} - \frac{T^7}{7}\bigg) + \frac{\theta}{18}\bigg(\frac{T^{10}}{4} - \frac{T^{10}}{10}\bigg) - \frac{m}{4}\bigg(\frac{T^8}{4} - \frac{T^8}{8}\bigg)] + m[\frac{1}{3}\bigg(\frac{T^5}{2} - \frac{T^5}{5}\bigg)$$

$$+ \frac{\theta}{18}\bigg(\frac{T^8}{2} - \frac{T^8}{8}\bigg) - \frac{m}{4}\bigg(\frac{T^6}{2} - \frac{T^6}{6}\bigg)]\bigg]$$

$$= h\{\bigg[\bigg\{a\bigg(\frac{T^2}{2} + \frac{\theta T^5}{15} - \frac{mT^3}{3}\bigg) - \frac{a\theta}{3}\bigg(\frac{T^5}{20} + \frac{\theta T^8}{96} - \frac{mT^6}{24}\bigg) + ma\bigg(\frac{T^3}{6} + \frac{\theta T^6}{36} - \frac{mT^4}{8}\bigg)\bigg\}\bigg]$$

$$+ \bigg[\bigg\{b\bigg(\frac{T^3}{3} + \frac{\theta T^6}{18} - \frac{mT^4}{4}\bigg) - \frac{b\theta}{3}\bigg(\frac{T^6}{24} + \frac{\theta T^9}{108} - \frac{mT^7}{28}\bigg)$$

$$+ mb\bigg(\frac{T^4}{8} + \frac{\theta T^7}{42} - \frac{mT^5}{10}\bigg)\bigg\}\bigg]$$

$$+ \bigg[\bigg\{c\bigg(\frac{T^4}{4} + \frac{\theta T^7}{21} - \frac{mT^5}{5}\bigg) - \frac{c\theta}{3}\bigg(\frac{T^7}{28} + \frac{\theta T^{10}}{120} - \frac{mT^8}{32}\bigg)$$

$$+ mc\bigg(\frac{T^5}{10} + \frac{\theta T^8}{48} - \frac{mT^6}{12}\bigg)\bigg\}\bigg]\}$$

... . (5)

At the initial stage of inventory, $t = 0, I(0) = IM$

$$IM = a\{\left[(T-t) + \frac{\theta}{12}(T^4 - t^4) - \frac{m}{2}(T^2 - t^2)\right] - \frac{\theta}{3}\left[(Tt^3 - t^4) + \frac{\theta}{12}(T^4t^3 - t^7) - \frac{m}{2}(T^2t^3 - t^5)\right] + [m\left\{(tT - t^2) + \frac{\theta}{12}(T^4t - t^5) - \frac{m}{2}(T^2t - t^3)\right]\}$$

$$+b\{\left[\frac{1}{2}(T^2 - t^2) + \frac{\theta}{15}(T^5 - t^5) - \frac{m}{3}(T^3 - t^3)\right] - \frac{\theta}{3}\left[\frac{1}{2}(T^2t^3 - t^5) + \frac{\theta}{15}(T^5t^3 - t^8) - \frac{m}{3}(T^3t^3 - t^6)\right] + [m\left\{\frac{1}{2}(T^2t - t^3) + \frac{\theta}{15}(T^5t - t^6) - \frac{m}{3}(T^3t - t^4)\right]\}$$

$$+c\{[\frac{1}{3}(T^3 - t^3) + \frac{\theta}{18}(T^6 - t^6) - \frac{m}{4}(T^4 - t^4)] - \frac{\theta}{3}\left[\frac{1}{3}(T^3t^3 - t^6) + \frac{\theta}{18}(T^6t^3 - t^9) - \frac{m}{4}(T^4t^3 - t^7)\right] + [m\left\{\frac{1}{3}(T^3t - t^4) + \frac{\theta}{18}(T^6t - t^7) - \frac{m}{4}(T^4t - t^5)\right]\}$$

$$= a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right) + c(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 - \frac{m}{4}T^4)$$

We have, ordering size, $Q = IM + IB$

$$= a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right) + c(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 - \frac{m}{4}T^4)$$

We have, Purchase cost $P_c = p \times Q$

$$= p[a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right) + c\left(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 - \frac{m}{4}T^4\right)]$$

Therefore, total cost (TC) per unit time is considered by

$$TC = \frac{Ordering\ cost + Holding\ cost + Purchase\ cost + Preservation\ cost}{T}$$

$$= \frac{1}{T}[A + h\{\left[\left\{a\left(\frac{T^2}{2} + \frac{\theta T^5}{15} - \frac{mT^3}{3}\right) - \frac{a\theta}{3}\left(\frac{T^5}{20} + \frac{\theta T^8}{96} - \frac{mT^6}{24}\right) + ma\left(\frac{T^3}{6} + \frac{\theta T^6}{36} - \frac{mT^4}{8}\right)\right\}\right]$$
$$+ \left[\left\{b\left(\frac{T^3}{3} + \frac{\theta T^6}{18} - \frac{mT^4}{4}\right) - \frac{b\theta}{3}\left(\frac{T^6}{24} + \frac{\theta T^9}{108} - \frac{mT^7}{28}\right) + mb\left(\frac{T^4}{8} + \frac{\theta T^7}{42} - \frac{mT^5}{10}\right)\right\}\right]$$
$$+ \left[\left\{c\left(\frac{T^4}{4} + \frac{\theta T^7}{21} - \frac{mT^5}{5}\right) - \frac{c\theta}{3}\left(\frac{T^7}{28} + \frac{\theta T^{10}}{120} - \frac{mT^8}{32}\right) + mc\left(\frac{T^5}{10} + \frac{\theta T^8}{48} - \frac{mT^6}{12}\right)\right\}\right]\}$$
$$+p\left[a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right) + c\left(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 - \frac{m}{4}T^4\right)\right] + \xi]$$

$$= \frac{1}{T}[A + h\{\left[\left\{\begin{array}{c} a\left(\frac{T^2}{2} + \frac{\theta T^5}{15} - \frac{mT^3}{3}\right) - \frac{a\theta}{3}\left(\frac{T^5}{20} + \frac{\theta T^8}{96} - \frac{mT^6}{24}\right) + \\ \left(\frac{ma}{2} + b\right)\left(\frac{T^3}{3} + \frac{\theta T^6}{12} - \frac{mT^4}{4}\right) - \frac{b\theta}{3}\left(\frac{T^6}{24} + \frac{\theta T^9}{108} - \frac{mT^7}{28}\right) + \left(\frac{mb}{2} + c\right) \\ \left(\frac{T^4}{4} + \frac{\theta T^7}{21} - \frac{mT^5}{5}\right) - \frac{c\theta}{3}\left(\frac{T^7}{28} + \frac{\theta T^{10}}{120} - \frac{mT^8}{32}\right) \end{array}\right\}\right]\}$$
$$+p\left[a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right) + c\left(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 - \frac{m}{4}T^4\right)\right] + \xi]$$

… … … (6)

To get the optimal solution we need to solve the following equations

$$\frac{dTC}{dT} = 0 \; and \; \frac{d^2TC}{dT} = 0$$

Therefore

$$\frac{dTC}{dT} = \frac{1}{T}\left[h\left\{\left[a + \left(\frac{ma}{2} + b\right)T + \left(\frac{mb}{2} + c\right)T^2\right]\left(T + \frac{\theta}{12}T^4 - mT^2\right) - \frac{\theta}{3}\left(\frac{T^4}{4} + \frac{\theta T^7}{12} - \frac{mT^5}{4}\right)(a + bT + cT^2)\right\} + p\left\{T(b - ma) + (c - mb)T^2 + \left(\frac{a\theta}{3} - cm\right)T^3 + \frac{\theta}{3}(bT^4 + cT^5)\right\}\right]$$

$$-\frac{1}{T^2}\Bigg[A + h\left\{\left[\left\{\begin{array}{c} a\left(\frac{T^2}{2} + \frac{\theta T^5}{15} - \frac{mT^3}{3}\right) - \frac{a\theta}{3}\left(\frac{T^5}{20} + \frac{\theta T^8}{96} - \frac{mT^6}{24}\right) + \\ \left(\frac{ma}{2} + b\right)\left(\frac{T^3}{3} + \frac{\theta T^6}{12} - \frac{mT^4}{4}\right) - \frac{b\theta}{3}\left(\frac{T^6}{24} + \frac{\theta T^9}{108} - \frac{mT^7}{28}\right) + \left(\frac{mb}{2} + c\right) \\ \left(\frac{T^4}{4} + \frac{\theta T^7}{21} - \frac{mT^5}{5}\right) - \frac{c\theta}{3}\left(\frac{T^7}{28} + \frac{\theta T^{10}}{120} - \frac{mT^8}{32}\right) \end{array}\right\}\right]\right\}$$

$$+p\left[a\left(T + \frac{\theta}{12}T^4 - \frac{m}{2}T^2\right) + b\left(\frac{1}{2}T^2 + \frac{\theta}{15}T^5 - \frac{m}{3}T^3\right) + c\left(\frac{1}{3}T^3 + \frac{\theta}{18}T^6 - \frac{m}{4}T^4\right)\right] + \xi\Bigg] \quad \ldots\ldots\ldots (7)$$

$$\frac{d^2TC}{dt^2} = \frac{1}{T}\Big[h\Big\{\Big[\Big(\frac{ma}{2}+b\Big)+(mb+2c)T\Big]\Big(T+\frac{\theta}{12}T^4-mT^2\Big)+$$

$$\Big[a+\Big(\frac{ma}{2}+b\Big)T+\Big(\frac{mb}{2}+c\Big)T^2\Big(1+\frac{4\theta}{12}T^3-2mT\Big)\Big]-\frac{\theta}{3}\Big(\frac{T^4}{4}+\frac{\theta T^7}{12}-\frac{mT^5}{4}\Big)(b+2cT)-$$

$$\frac{\theta}{3}\Big(T^3+\frac{7\theta T^6}{12}-\frac{5mT^4}{4}\Big)(a+bT+cT^2)\Big\}+p\Big\{(b-ma)+2T(c-mb)+$$

$$(a\theta-3cm)T^2+\frac{\theta}{3}(4bT^3+5cT^4)\Big\}\Big]-\frac{1}{T^2}\Big[h\Big\{\Big[a+\Big(\frac{ma}{2}+b\Big)T+\Big(\frac{mb}{2}+c\Big)T^2\Big]\Big(T+$$

$$\frac{\theta}{12}T^4-mT^2\Big)-\frac{\theta}{3}\Big(\frac{T^4}{4}+\frac{\theta T^7}{12}-\frac{mT^5}{4}\Big)(a+bT+cT^2)\Big\}+p\Big\{T(b-ma)+(c-mb)T^2+$$

$$\Big(\frac{a\theta}{3}-cm\Big)T^3+\frac{\theta}{3}(bT^4+cT^5)\Big\}\Big]-\frac{1}{T^2}\Big[\Big[h\Big\{\Big[a+\Big(\frac{ma}{2}+b\Big)T+\Big(\frac{mb}{2}+c\Big)T^2\Big]\Big(T+$$

$$\frac{\theta}{12}T^4-mT^2\Big)-\frac{\theta}{3}\Big(\frac{T^4}{4}+\frac{\theta T^7}{12}-\frac{mT^5}{4}\Big)(a+bT+cT^2)\Big\}+p\Big\{T(b-ma)+(c-mb)T^2+$$

$$\Big(\frac{a\theta}{3}-cm\Big)T^3+\frac{\theta}{3}(bT^4+cT^5)\Big\}\Big]\Big]+\frac{1}{T^3}\Big[A+$$

$$h\Big\{\left[\left\{\begin{array}{c} a\Big(\frac{T^2}{2}+\frac{\theta T^5}{15}-\frac{mT^3}{3}\Big)-\frac{a\theta}{3}\Big(\frac{T^5}{20}+\frac{\theta T^8}{96}-\frac{mT^6}{24}\Big)+ \\ \Big(\frac{ma}{2}+b\Big)\Big(\frac{T^3}{3}+\frac{\theta T^6}{12}-\frac{mT^4}{4}\Big)-\frac{b\theta}{3}\Big(\frac{T^6}{24}+\frac{\theta T^9}{108}-\frac{mT^7}{28}\Big)+\Big(\frac{mb}{2}+c\Big) \\ \Big(\frac{T^4}{4}+\frac{\theta T^7}{21}-\frac{mT^5}{5}\Big)-\frac{c\theta}{3}\Big(\frac{T^7}{28}+\frac{\theta T^{10}}{120}-\frac{mT^8}{32}\Big) \end{array}\right\}\right]\Big\}$$

$$+p\Big[a\Big(T+\frac{\theta}{12}T^4-\frac{m}{2}T^2\Big)+b\Big(\frac{1}{2}T^2+\frac{\theta}{15}T^5-\frac{m}{3}T^3\Big)$$

$$+c\Big(\frac{1}{3}T^3+\frac{\theta}{18}T^6-\frac{m}{4}T^4\Big)\Big]+\xi\Big]$$

6. RESULT DISCUSSION AND COMPUTATIONAL ANALYSIS:

In this section, a numerical example is considered to illustrate this maintenance model and used MATLAB to obtain the result. Parameter numerical example has been considered to check the validity inventory model in proper units:

$$A = 10\ tk\ per\ order, \quad a = 8, \quad b = 6, \quad c = 3, \quad h = 1\ tk\ per\ kg, \quad p = 1,$$
$$\theta = 0.85, m = 0.00001, \xi = 2\ tk\ per\ kg$$

When the equation is solved, the best value of T = 1.55 is achieved, and the minimal average cost is calculated as TC = 47.1367.

7. SENSITIVITY ANALYSIS:

Sensitivity analysis indicates the amount to which changes in the model's input parameter values affect the model's optimal answer. Here, the sensitivity analysis for total cost per unit time TC is carried out with respect to the changes in the values of the parameters of quadratic function, unit holding cost per unit time h, ordering cost A, Holding cost h, purchase cost p, deteriorating cost, preservation cost, parameters a, b & c. The sensitivity analysis is performed by considering variation in each one of the above parameters by 5% change in stipulated standard value, keeping all other remaining parameters as fixed.

8. Sensitivity of different parameters with total cost per unit time

Table 1: Sensitivity of ordering cost A

The following table shows that the change of total cost per unit time due to the change of ordering cost A and analyze the sensitivity of the parameter.

index	Parameter Value	TC
1	7.736	45.1663
2	8.144	45.4295
3	8.573	45.7063
4	9.025	45.9979
5	9.5	46.3044
6	**10.0000**	**47.1367**
7	10.5000	47.4593
8	11.0250	47.7980
9	11.5760	48.1535
10	12.1540	48.5264
11	12.7610	48.9180

Table 2: Sensitivity of constant parameter a

The following table shows that the change of total cost per unit time due to the change of constant cost a and analyze the sensitivity of the parameter.

index	Parameter Value	TC
1	6.17	42.6147
2	6.50	43.3292
3	6.85	44.0987

4	7.22	44.9121
5	7.60	45.7475
6	**8.00**	**47.1367**
7	8.40	48.0287
8	8.82	48.9654
9	9.26	49.9466
10	9.72	50.9725
11	10.20	52.0426

Table 3: Sensitivity of constant parameter b

The following table shows that the change of total cost per unit time due to the change of constant cost b and analyze the sensitivity of the parameter.

index	Parameter Value	TC
1	4.62	43.4128
2	4.87	43.9951
3	5.13	44.6006
4	5.4	45.2528
5	5.7	45.9282
6	**6.00**	**47.1367**
7	6.30	47.8437
8	6.61	48.5743
9	6.94	49.3519
10	7.28	50.1532
11	7.64	53.2214

Table 4: Sensitivity of constant parameter c

The following table shows that the change of total cost per unit time due to the change of constant cost c and analyze the sensitivity of the parameter.

index	Parameter Value	TC
1	2.19	44.6498
2	2.37	45.0892
3	2.56	45.5652
4	2.70	45.8947
5	2.85	46.2608
6	**3.00**	**47.1367**
7	3.15	47.5074
8	3.30	47.8781
9	3.46	48.2736
10	3.63	48.6937
11	3.81	48.1500

Table 5: Sensitivity of holding cost h

The following table shows that the change of total cost per unit time due to the change of holding cost h and analyze the sensitivity of the parameter.

index	Parameter Value	TC
1	0.70	41.9582
2	0.75	41.8760
3	0.8	42.8262
4	0.85	43.7764
5	0.90	44.7266
6	0.95	45.6768
7	**1.0000**	**47.1367**
8	1.0500	48.0869
9	1.1000	49.0371

10	1.1500	49.9872
11	1.2000	50.9374

Table 6: Sensitivity of deterioration cost

The following table shows that the change of total cost per unit time due to the change of deterioration cost and analyze the sensitivity of the parameter.

index	Parameter Value	TC
1	0.6400	44.2753
2	0.6800	44.7344
3	0.7200	45.1882
4	0.7600	45.6368
5	0.8000	46.0802
6	**0.8500**	**47.1367**
7	0.8900	47.5745
8	0.9300	48.0070
9	0.9700	48.4343
10	1.0100	48.8562
11	1.0600	49.3763

Table 7: Sensitivity of reduced deterioration rate

The following table shows that the change of total cost per unit time due to the change of reduced deterioration cost m and analyze the sensitivity of the parameter.

index	Parameter Value	TC
1	0.000075	46.6273
2	0.000080	46.6273
3	0.000085	46.6272
4	0.000090	46.6271

5	0.000095	46.6270
6	**0.000100**	**47.1367**
7	0.000105	47.1366
8	0.000110	47.1365
9	0.000115	47.1365
10	0.000120	47.1364
11	0.000125	47.1363

Table 8: Sensitivity of preservation technology cost

The following table shows that the change of total cost per unit time due to the change of preservation technology cost and analyze the sensitivity of the parameter.

index	Parameter Value	TC
1	1.5	46.3044
2	1.6	46.4089
3	1.7	46.4634
4	1.8	46.5579
5	1.9	46.7624
6	**2.0000**	**47.1367**
7	2.1000	47.2412
8	2.2000	47.2657
9	2.3000	47.3303
10	2.4000	47.3948
11	2.5000	47.4593

9. Graphical presentation for the effects of parameters on total cost of per unit Time:

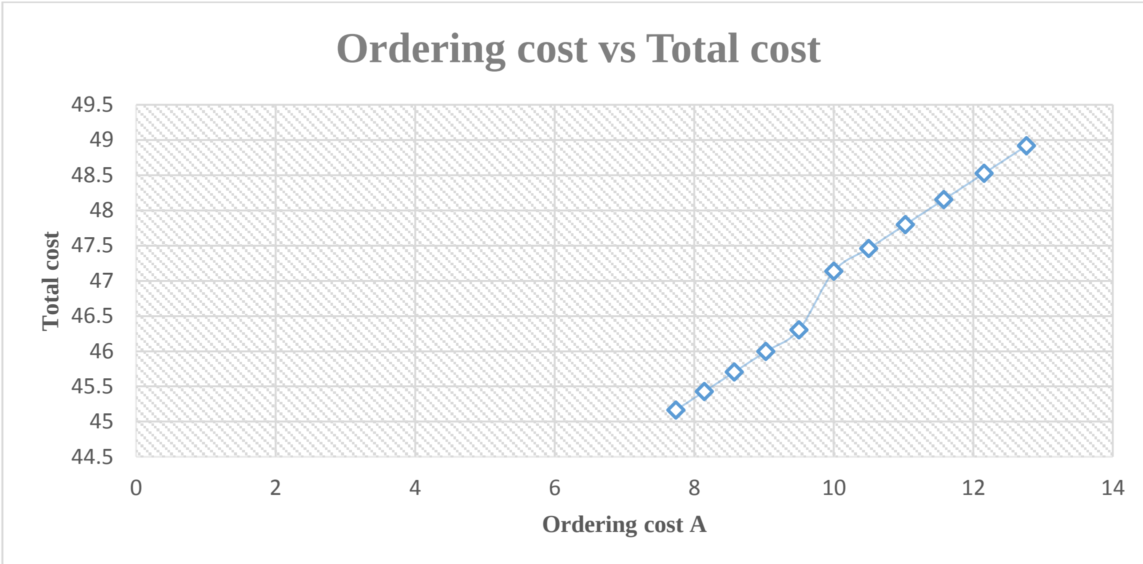

Figure 1: Sensitivity of ordering cost on total cost

When the values of ordering cost A increase, the total cost per unit time increases, and when the values of ordering cost A decrease, the total cost per unit time decreases, as illustrated in Figure 1. As a result, the cost of ordering has a substantial impact on overall cost.

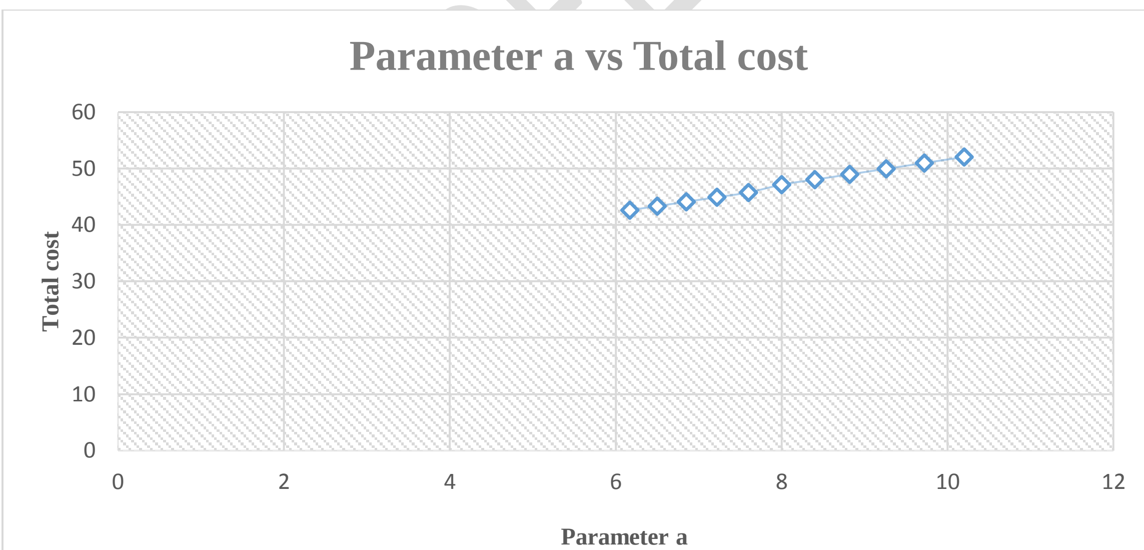

Figure 2: Sensitivity of parameter a on total cost

When the values of parameter a increase, the total cost per unit time increases, and when the values of ordering cost a decrease, the total cost per unit time decreases, as illustrated in Figure 2. It can also be deduced that the sensitivity of parameter a for each change is really close to one another.

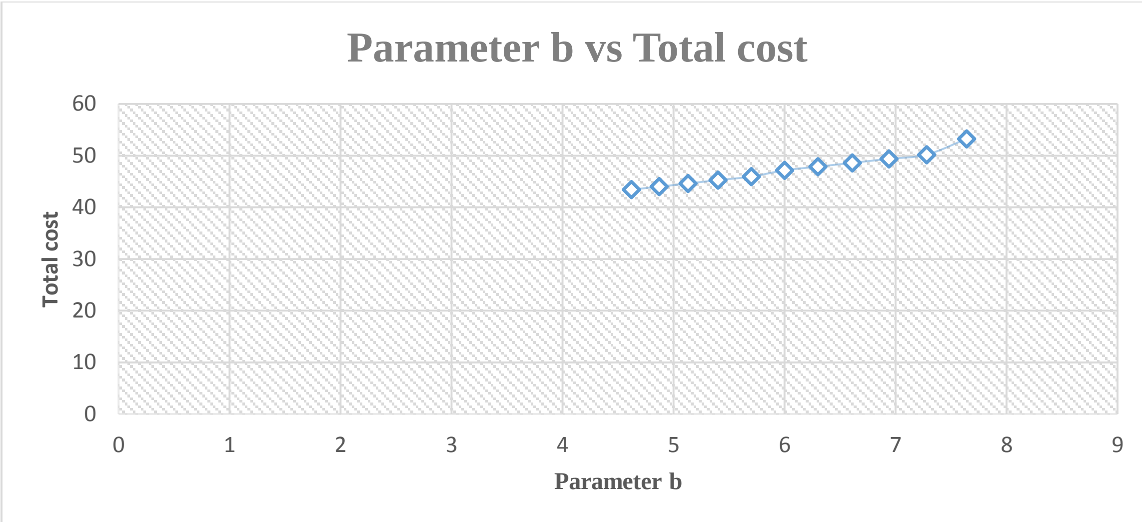

Figure 3: Sensitivity of parameter b on total cost

When the values of parameter b increase, the total cost per unit time increases, and when the values of parameter b decrease, the total cost per unit time decreases, as illustrated in Figure 3. It can also be deduced that the sensitivity of parameter b for each change is really close to one another.

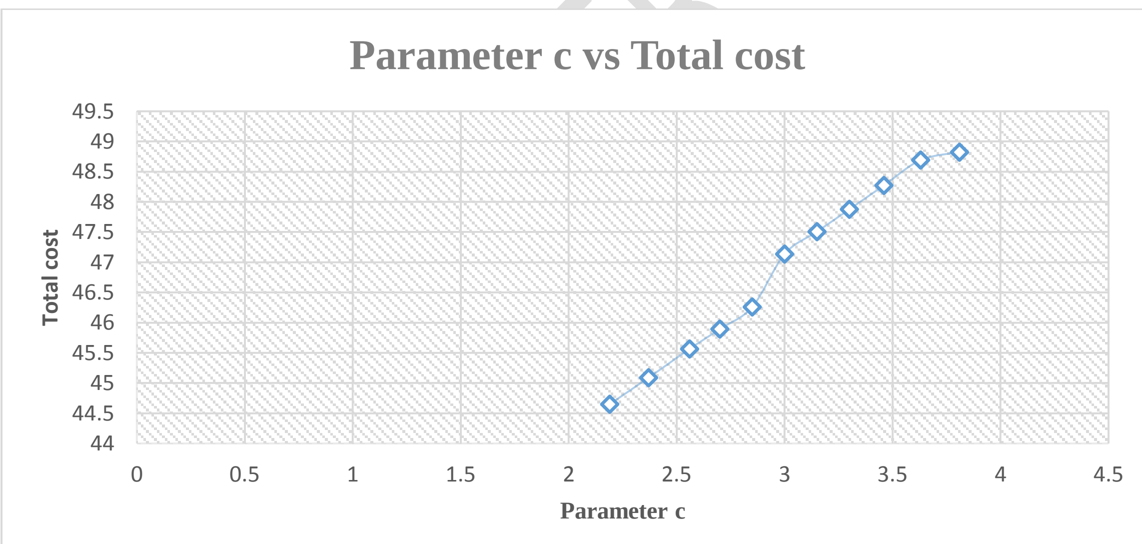

Figure 4: Sensitivity of parameter *c* on total cost

When the values of parameter b increase, the total cost per unit time increases, and when the values of parameter b decrease, the total cost per unit time decreases, as illustrated in Figure 4. It can be deduced that parameter c's sensitivity to each change is noticeable.

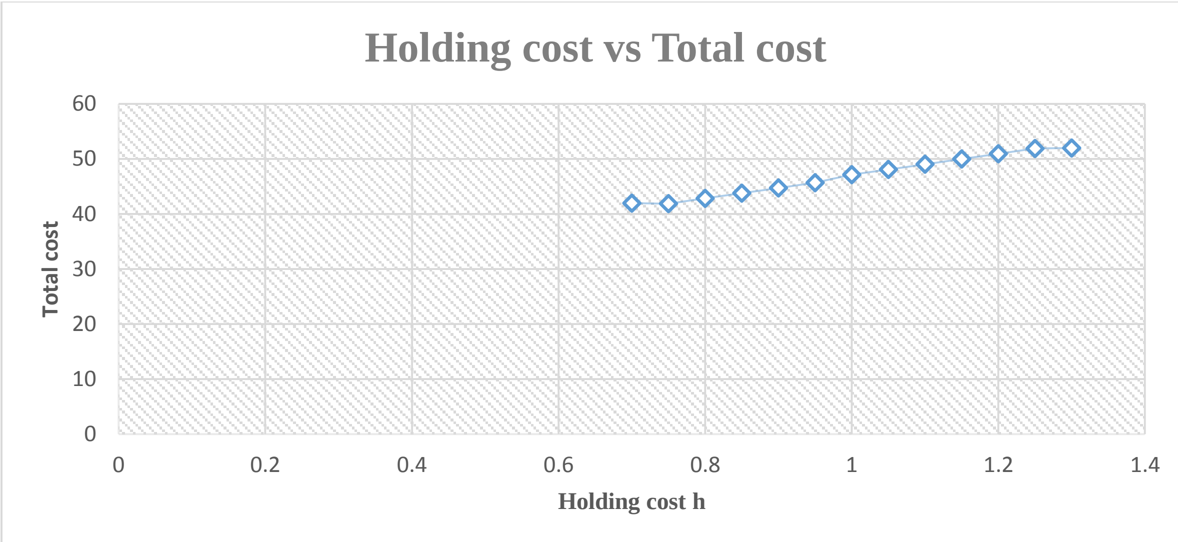

Figure 5: Sensitivity of holding cost on total cost

When the values of holding cost H increase, the total cost per unit time increases, and when the values of holding cost H decrease, the total cost per unit time decreases, as illustrated in Figure 5. It can be concluded that the sensitivity of holding cost h for each adjustment is relatively similar.

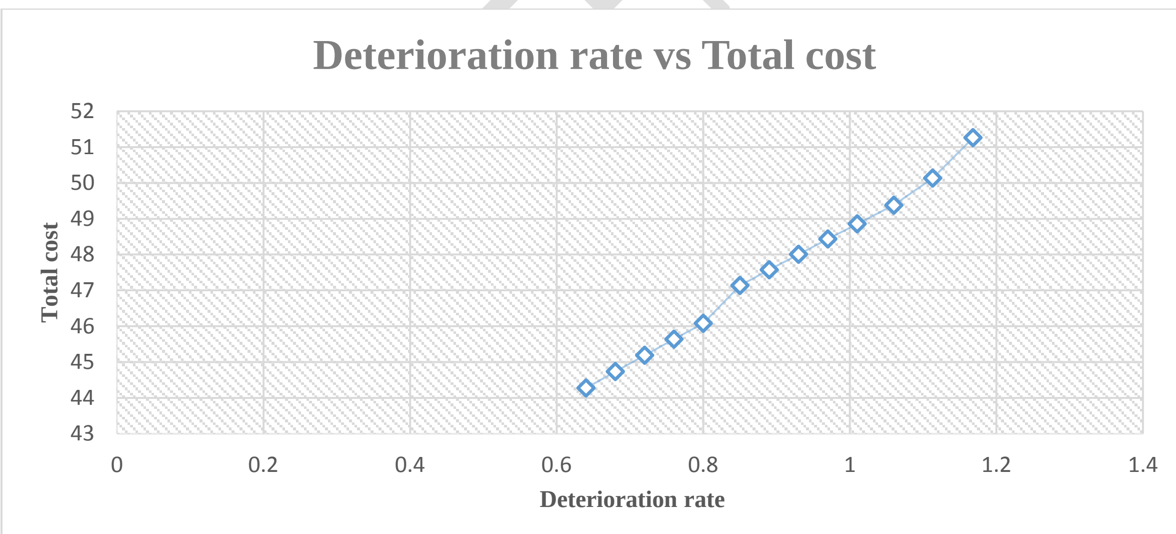

Figure 6: Sensitivity of deterioration rate on total cost

When the values of holding cost H increase, the total cost per unit time increases, and when the values of holding cost H decrease, the total cost per unit time decreases, as illustrated in Figure 6. To ensure maximum profit, the deterioration rate should be reduced.

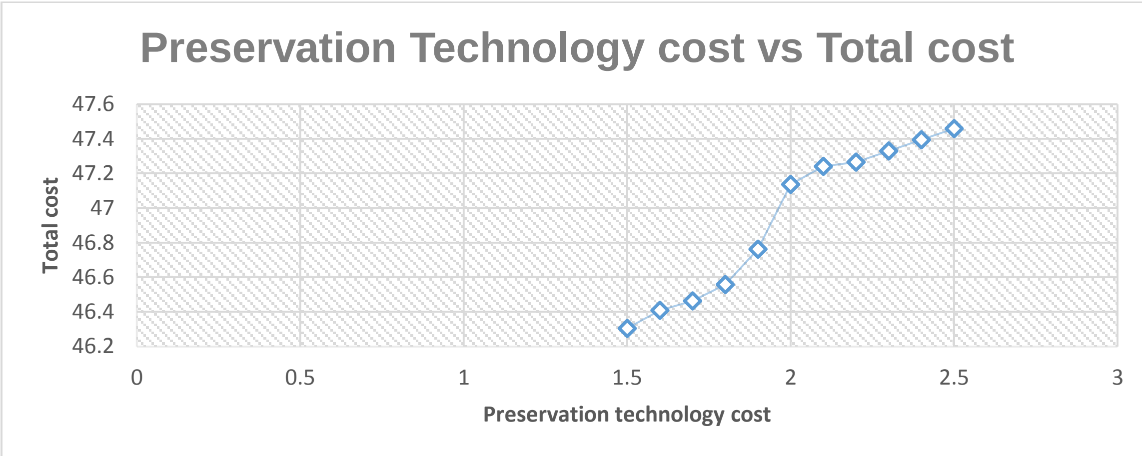

Figure 7: Sensitivity of preservation technology cost on total cost

When values of the model's preservation technology cost grow, the total cost per increases, and when the values of the model's preservation technology cost decrease, the total cost decreases, as seen in Figure 7. It was also discovered that the preservation technology cost is extremely sensitive to modifications. Hence preservation technology cost has a noticeable effects on total cost. To minimize total cost, the preservation cost is used.

10. CONCLUSION

The inventory model was upheld with a deterioration rate and no shortages. This model can be used in any industry to estimate the impact of cost per unit time when different parameters are changed. It is obvious that

- When the values of the quadratic demand function's parameters rise, the overall cost per unit time rises as well. When the values of the quadratic demand function's parameters decrease, the overall cost per unit time decreases.
- When the values of ordering cost, holding cost and purchase cost per unit time increase then the total cost per unit time increase. On the other hand, when the values of ordering cost, holding cost and purchase cost per unit time decrease then the total cost per unit time decrease.
- When the rate of degradation rises, the total cost per unit time rises rapidly, and when the rate of deterioration falls, the total cost per unit time falls rapidly.
- When the preservation technology increases, the total cost per unit time decreases and vise-versa.

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