Original Research Article

MODELLING NEONATAL MORTALITY RATE IN NIGERIA USING A CONTINUOUS POISSON-LINDLEY DISTRIBUTION

Abstract:
Nigeria's effort to reduce under-five mortality has been biased in favour of childhood mortality to the neglect of neonates and as such the literature is short of adequate information on the determinants of neonatal mortality, whereas studies have shown that about half of infant deaths occur in the neonatal period. Knowledge of the determinants of neonatal mortality is essential for the design of intervention programmes that will enhance neonatal survival. Therefore, this study was conducted to investigate the trends in neonatal mortality in Nigeria. It also proposed a Poisson based continuous probability distribution called Poisson-Lindley distribution to neonatal mortality rate in Nigeria. Some properties of the new model and other relevant measures were obtained. The unknown parameters of the model were also estimated using the method of maximum likelihood. The fitness of the proposed model to the neonatal mortality rate was considered using a dataset on neonatal mortality rate from 1967 to 2019.

Keywords: Neonatal Mortality Rate, Poisson distribution, Lindley distribution, Poisson-Lindley distribution, statistical properties, parameter estimation, applications.

1 Introduction

Infant Mortality which is the probability of a child dying between birth and the first birthday is one of the most useful indicators for assessing the general level of health and development of a society. It gives an overview of the functionality of a country's healthcare system, socioeconomic situation, and the state of maternal and child health. Globally, 85 and 29% of deaths among children occurred in the first 5 years of life and during infancy respectively. More than 50 % of these deaths occurred in Sub-Saharan Africa (SSA) with Infant Mortality Rate (IMR) of 62 deaths per 1000 in 2018 [1]. Improvement in child health, survival and life expectancy has been a concerted and continuous global effort as indicated in the Sustainable Development Goals–3 (SDGs 3) which aims to end preventable deaths among children under-5 years of age and targets reduction of under-five mortality to as low as 25 per 1000 live births by 2030 [2]. However, feasibility of the realization of this target is doubtful due to slow pace of mortality reduction in Sub-Saharan Africa as many countries in this world sub-region may likely fall short of the SDG target [1]. Infant mortality rate is the number of children that die under one year of age in a given

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year per 1,000 live births and the neonatal mortality rate is the number of children that die under 28 days of age in a given year, per 1,000 live births.

Nigeria with a population of over 200 million is one of the five countries that accounted for half of global burden of infant mortality occupying a second position after India [1, 3]. In Nigeria, previous studies have estimated a decline in IMR from 125 in 1990 to 67 in 2018 [3, 4]. Despite this achievement in IMR reduction over the years, the current level is higher than the IMR estimates for other countries in Sub-Saharan Africa like South Africa (28/1000), Kenya (31/1000) and Ghana (35/1000) which are already close to achieving the SDGs - target 3. Survival of infants in Nigeria is challenged by the prevailing poor health service delivery and malnutrition as a result of poverty which ravages the nation. Some preventable health/environmental related conditions (infectious diseases, chronic health conditions of the mother, obstetric and non-obstetric complications, lack of immunization, and other prevalent childhood diseases), socio-demographic characteristics (place of residence, region, religion, marital status and education level) and biological factors associated with mothers have been found in the literature as additional sources of threat to the survival chances of infants in Nigeria [1, 5–7]. It is important to note that risk of adverse pregnancy outcome like infant mortality are unevenly distributed among women population owing to the variation in their biologic features and demographic composition [8].

The Lindley is a probability distribution that was investigated in context of fiducial statistic as an alternative to Bayesian theory by [9]. Its fundamental properties with application to waiting time data were discussed by [10]. Afterwards, many researchers have studied this distribution, for instance, [11] applied the distribution to competing risks lifetime data. [12] estimated the parameter of the distribution with progressive Type-II censoring scheme and showed that it may be a better lifetime model than exponential, lognormal and gamma distributions in some real life situations. Singh & Gupta [13] used the distribution under load sharing system models. Al-Mutairi et al. [14] developed an inferential procedure of the stress-strength parameter when both stress and strength variables follow Lindley distribution and discovered that the distribution is useful when the data has an increasing failure rate. All these make the use of Lindley distribution in lifetime data analysis more frequent than the exponential distribution. Despite the important properties and various applications of the Lindley distribution in many disciplines, its applicability may be restricted to non-monotone hazard rate data according to [15]. To solve the above mentioned problem therefore, several extensions of the Lindley distribution have been proposed in literature and some of the recent generalizations are; the transmuted Lindley distribution by [16], the exponentiated Power Lindley distribution by [17], Generalized Lindley distribution by [18], Transmuted Generalized Lindley distribution by [19], Extended Power Lindley distribution by [20], Transmuted Two-Parameter Lindley distribution by [21] and A Three Parameter Lindley distribution by [22]. Akmakyapan & Kadlar [23]proposed two parameter generalizations of the Lindley distribution, called the power Lindley distribution which was generated using the power transformations to the Lindley distribution. Ashour & Eltehiwy [24] studied the transmuted Lindley-geometric

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distribution. The beta-Lindley distribution was also introduced by [25]. Ashour & Eltehiwy [24] studied the statistical and mathematical properties of Kumaraswamy Quasi-Lindley distribution and Kumaraswamy Lindley distribution was proposed and discussed by [27].

The cumulative distribution function (c.d.f) and probability density function (pdf) of the Lindley distribution (LinD) are defined as:

$$G(x) = 1 - \left(1 + \frac{\theta x}{\theta + 1}\right) e^{-\theta x} \tag{1}$$

and

$$g(x) = \frac{\theta^2}{\theta + 1}(1 + x) e^{-\theta x} \tag{2}$$

respectively, for $x > 0$ and $\theta > 0$ where θ is the scale parameter of LinD.

In recent times, a number of authors have developed efficient families of probability distributions and it has been proven that they produce more flexible probability models. These proposed families among others include the quadratic rank transmutation map proposed by [28], the Weibull-X family of distribution by [29], the Weibull-G family of distributions by [30], the Gamma-X family by [31], a Lomax-G family by [32], a new Weibull-G family by [33], a Lindley-G family by [34], a Poisson-X family by [35], a Gompertz-G family by [36], an odd Lindley-G family by [37] and an odd Lomax generator of distributions by [38].
Following the above listed families and the related extended probability distributions, this study will propose another extension of the Lindley distribution by using the Poisson-X family proposed by [35], this proposed distribution is called “the Poisson-Lindley distribution (PoisLinD)”.

The rest of this paper is organized in sections as follows: the newly proposed distribution is defined with its plots in section 2. Section 3 presents statistical properties of the new distribution. Section 4 looks at the estimation of parameters using maximum likelihood estimation. An application of the Poisson-Lindley distribution and other related distributions to neonatal mortality rate data is presented in section 5 and the final summary and conclusion is provided in section 6.

2. A Poisson-Lindley Distribution (PoisLinD)

According to [35], the cdf and pdf of a Poisson-X family of distributions are respectively given by;

$$F(x) = \left(1 - e^{-1}\right)^{-1}\left(1 - e^{-[G(x)]^{\alpha}}\right) \tag{3}$$

and

$$f(x) = \alpha\left(1 - e^{-1}\right)^{-1} g(x)\left[G(x)\right]^{\alpha - 1} e^{-[G(x)]^{\alpha}} \tag{4}$$

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where; $x > 0$, and α is the extra shape parameter, $G(x)$ and $g(x)$ are the cdf and pdf of any continuous distribution to be modified respectively.

Equations (1) and (2) are substituted into equation (3) and (4) respectively. On simplification, the cdf and pdf of the PoisLinD are obtained as equations (5) and (6) respectively, as follow

$$F(x)=\left(1-e^{-1}\right)^{-1}\left(1-e^{-\left[1-\left(1+\frac{\theta x}{\theta+1}\right)e^{-\theta x}\right]^{\alpha}}\right) \tag{5}$$

and

$$f(x)=\frac{\alpha\theta^{2}}{\theta+1}(1+x)e^{-\theta x}\left(1-e^{-1}\right)^{-1}\left[1-\left(1+\frac{\theta x}{\theta+1}\right)e^{-\theta x}\right]^{\alpha-1}e^{-\left[1-\left(1+\frac{\theta x}{\theta+1}\right)e^{-\theta x}\right]^{\alpha}} \tag{6}$$

where $x>0,\theta>0,\alpha>0,$ α and θ are the shape and scale parameters of the PoisLinD respectively.

Plots of the pdf and cdf of the PoisLinD using some parameter values are presented in **figure 1** as follows.

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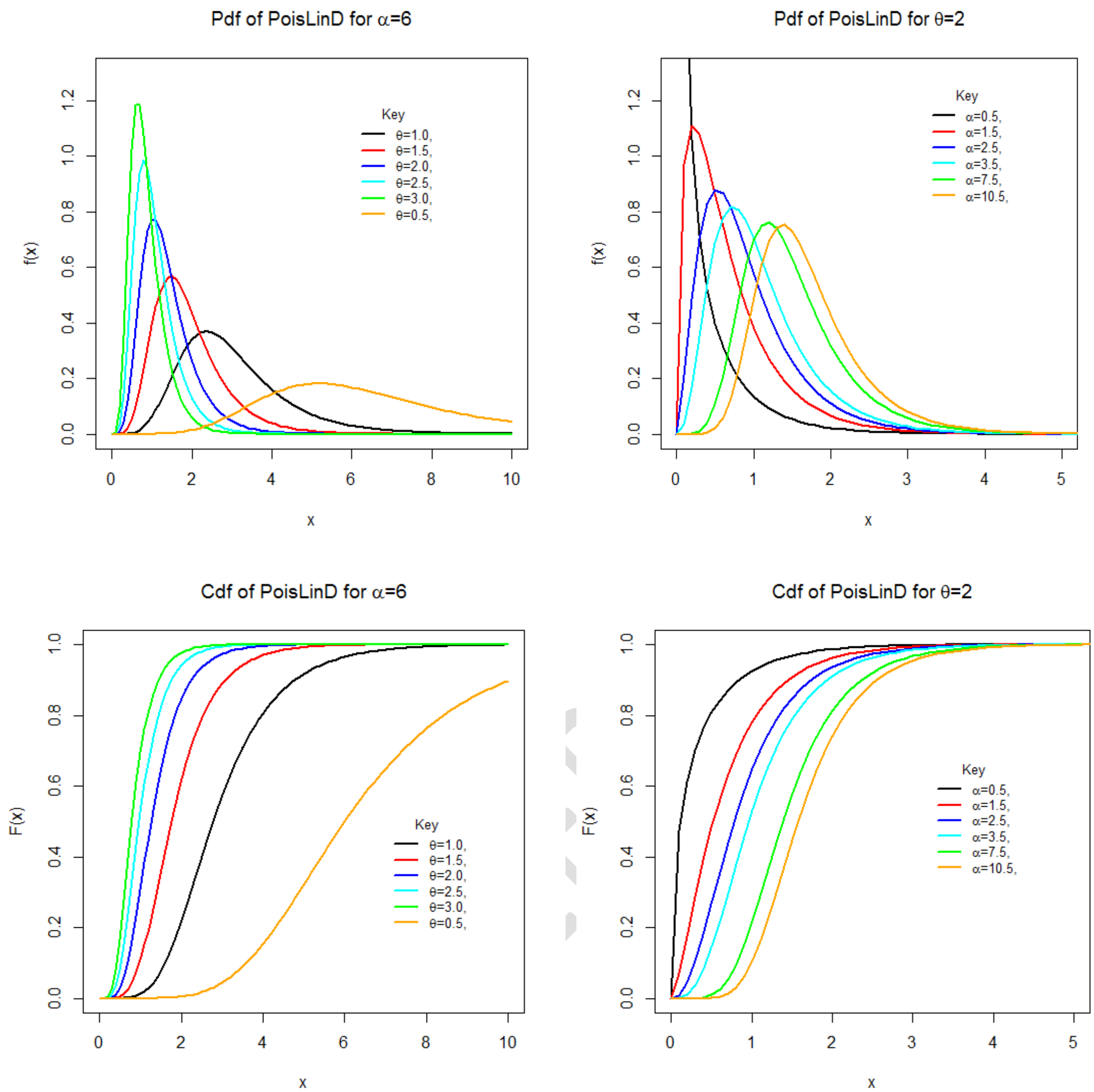

Figure. **1**: PDF and CDF of the PoisLinD for different values of the parameters.

Looking at Figure 1 above, it can be seen that the pdf of PoisLinD distribution is positively skewed and takes various shapes in relation to the parameter values. Also, the plot of the cdf shows that the cdf tends to the value of one when x approaches infinity and equals zero when x tends to zero as normally expected.

3. Reliability analysis of the PoisLinD.

The Survival function describes the likelihood that a system or an individual will not fail after a given time. Mathematically, the survival function is given by:

$$S(x)=1-F(x) \tag{11}$$

Applying the cdf of the PoisLinD in (11), the survival function for the PoisLinD is obtained as:

$$S(x)=1-\left(1-e^{-1}\right)^{-1}\left(1-e^{-\left[1-\left(1+\frac{\theta x}{\theta+1}\right)e^{-\theta x}\right]^{\alpha}}\right) \tag{12}$$

Hazard function is a function that describes the chances that a product or component will breakdown over an interval of time. It is mathematically defined as:

$$h(x)=\frac{f(x)}{S(x)}=\frac{f(x)}{1-F(x)} \tag{13}$$

Therefore, our definition of the hazard rate of the PoisLinD is given by

$$h(x)=\frac{\alpha\theta^{2}(1+x)e^{-\theta x}\left(1-e^{-1}\right)^{-1}\left[1-\left(1+\frac{\theta x}{\theta+1}\right)e^{-\theta x}\right]^{\alpha-1}e^{-\left[1-\left(1+\frac{\theta x}{\theta+1}\right)e^{-\theta x}\right]^{\alpha}}}{(\theta+1)\left\{1-\left(1-e^{-1}\right)^{-1}\left(1-e^{-\left[1-\left(1+\frac{\theta x}{\theta+1}\right)e^{-\theta x}\right]^{\alpha}}\right)\right\}} \tag{14}$$

where $\alpha,\theta>0$.

The figure below presents a plot of both the survival function (SF) and hazard function (HF) of PoisLinD based on arbitrary parameter values as follows:

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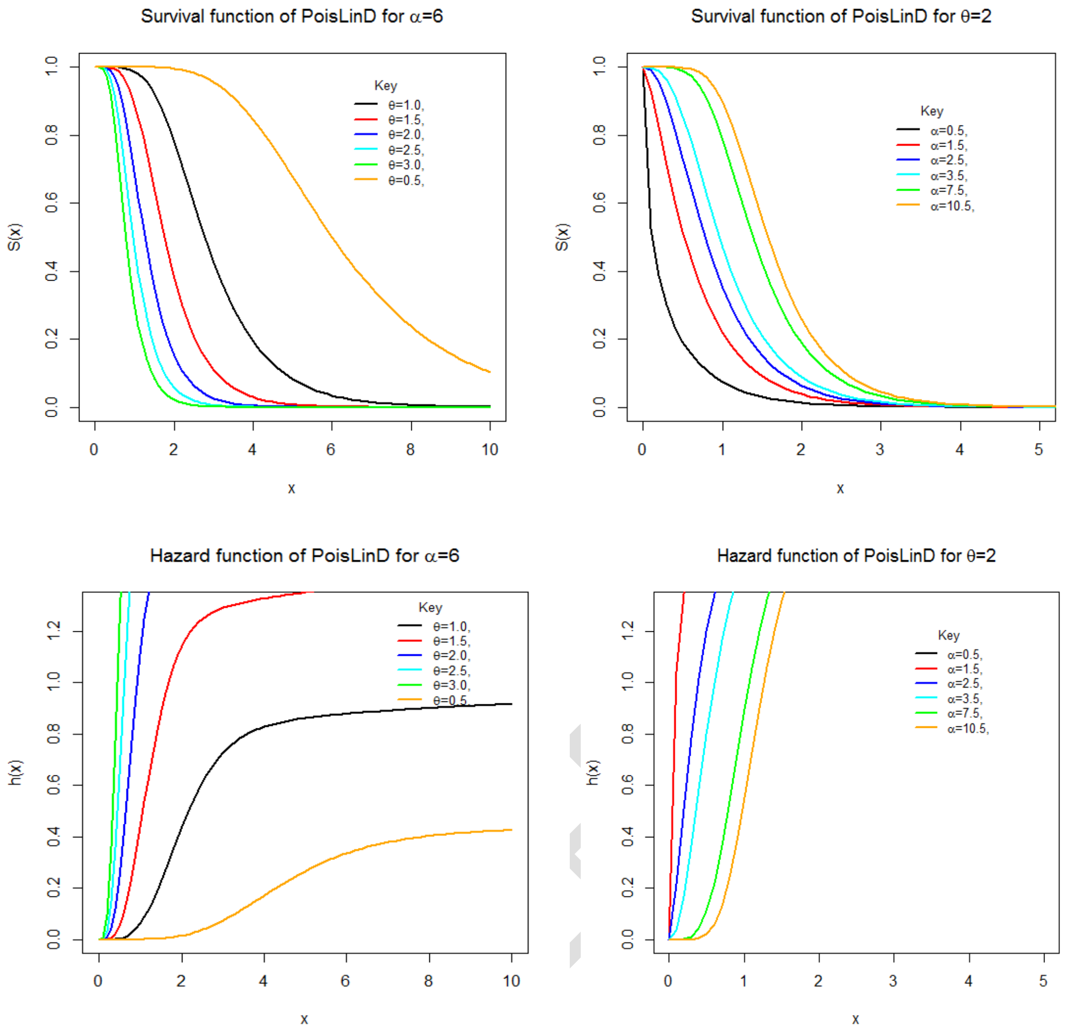

Figure 2: SF and HF of PoisLinD for Selected Values of the Parameters.

The plots in figure **2** show that the chances of survival are very certain at the beginning or early age and begin to reduce as the time advances and tend to zero at infinity. The plots also revealed that the proposed distribution has an increasing failure rate which implies that the probability of failure for any random variable following a PoisLinD increases as time increases, that is, probability of failure or death increases as the process or event progresses.

4. Estimation of unknown Parameters of the PoisLinD

In this section, the estimation of the parameters of the PoisLinD is done by using the method of maximum likelihood estimation (MLE). Let $X_1, X_2, \ldots\ldots, X_n$ be a sample of size 'n' independently and identically distributed random variables from the PoisLinD with unknown parameters α and θ defined previously.

The likelihood function of the PoisLinD using the pdf in equation (6) is given by;

$$L(\underline{X}/\alpha,\theta)=\left(\frac{\alpha\theta^2}{\theta+1}\right)^n\left(1-e^{-1}\right)^{-n}e^{-\sum_{i=1}^{n}\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right]^{\alpha}}\prod_{i=1}^{n}\left\{(1+x_i)e^{-\theta\sum_{i=1}x_i}\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right]^{\alpha-1}\right\} \quad (15)$$

Let the natural logarithm of the likelihood function be, $l=\log L(\underline{X}\mid\alpha,\theta)$, therefore, taking the natural logarithm of the function above gives:

$$l=n\log\alpha+2n\log\theta-n\log(\theta+1)-n\log\left(1-e^{-1}\right)-\sum_{i=1}^{n}\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right]^{\alpha}+\sum_{i=1}^{n}\log(1+x_i)-\theta\sum_{i=1}^{n}x_i \quad (16)$$
$$+(\alpha-1)\sum_{i=1}^{n}\log\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right]$$

Differentiating l partially with respect to α and θ respectively gives the following results:

$$\frac{\partial l}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right]^{\alpha}\ln\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right]+\sum_{i=1}^{n}\log\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right] \quad (17)$$

$$\frac{\partial l}{\partial\theta}=\frac{2n}{\theta}-\frac{n}{\theta+1}-\sum_{i=1}^{n}x_i-\sum_{i=1}^{n}\left\{\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right]^{\alpha-1}x_ie^{-\theta x_i}\left(1-\frac{1-\theta x_i(\theta+1)}{(\theta+1)^2}\right)\right\}+(\alpha-1)\sum_{i=1}^{n}\left\{\frac{x_ie^{-\theta x_i}\left(1-\frac{1-\theta x_i(\theta+1)}{(\theta+1)^2}\right)}{\left[1-\left(1+\frac{\theta x_i}{\theta+1}\right)e^{-\theta x_i}\right]}\right\} \quad (18)$$

When (17) and (18) are equated to zero (0) and solved, the solution of the non-linear system of equations above will give the maximum likelihood estimates $\hat{\alpha}$ and $\hat{\theta}$ of parameters α and θ respectively. However, these solutions cannot be obtained manually except numerically with the aid of suitable statistical software such as *R* software as used in this study.

5. APPLICATIONS

This section presents a dataset on the rate of infant mortality in Nigeria from the year 1964 to the year 2019. The descriptive statistics and graphs of the dataset are also presented.

This data can be obtained from www.data.unicef.org.

The following table and figures present a good exploration of the dataset with some explanations.

Table 1: Summary statistics of the Neonatal mortality rate from 1967 to 2019.

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis

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Values	53	35.85	39.86	49.53	53.02	48.83	69.30	87.05279	0.45574	-0.58921

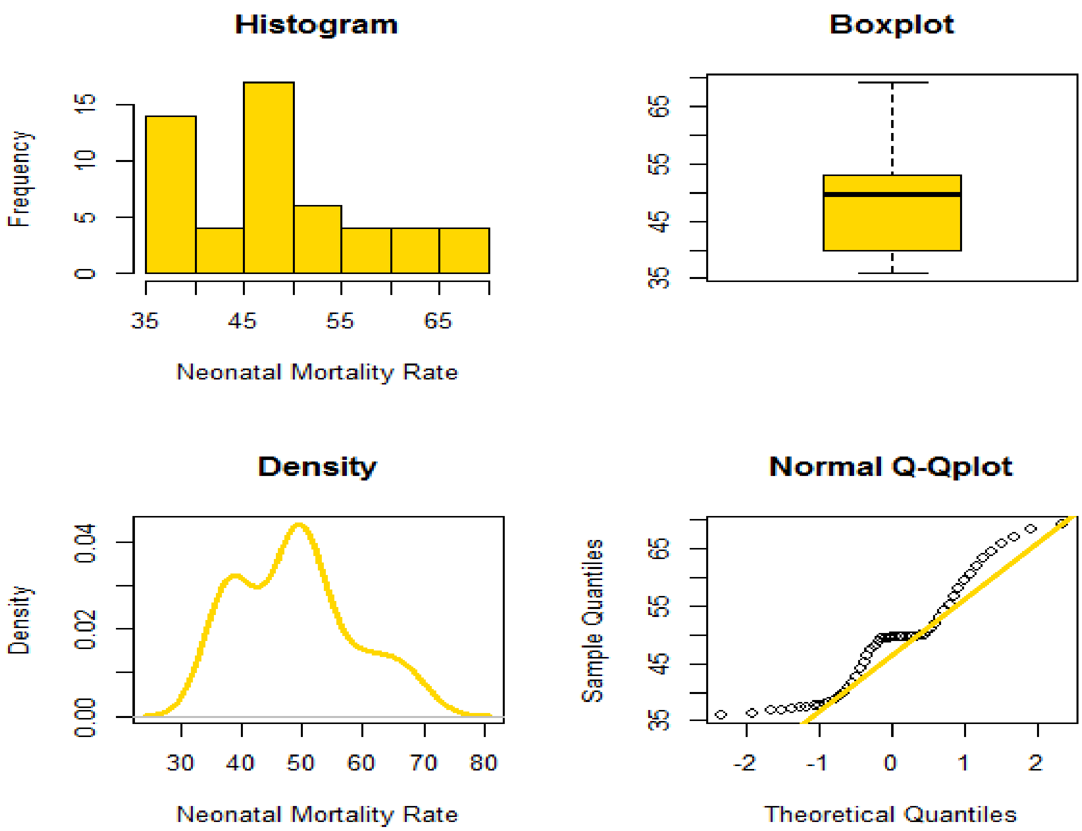

Figure 3: A graphical summary of the dataset

A summary of the dataset in Table 1 and Figure 3 has shown that neonatal mortality rate in Nigeria is bimodal and positively skewed.

Also, the trend in the rate of neonatal mortality in Nigeria from 1967 to 2019 using a line plot is shown in Figure 4 below.

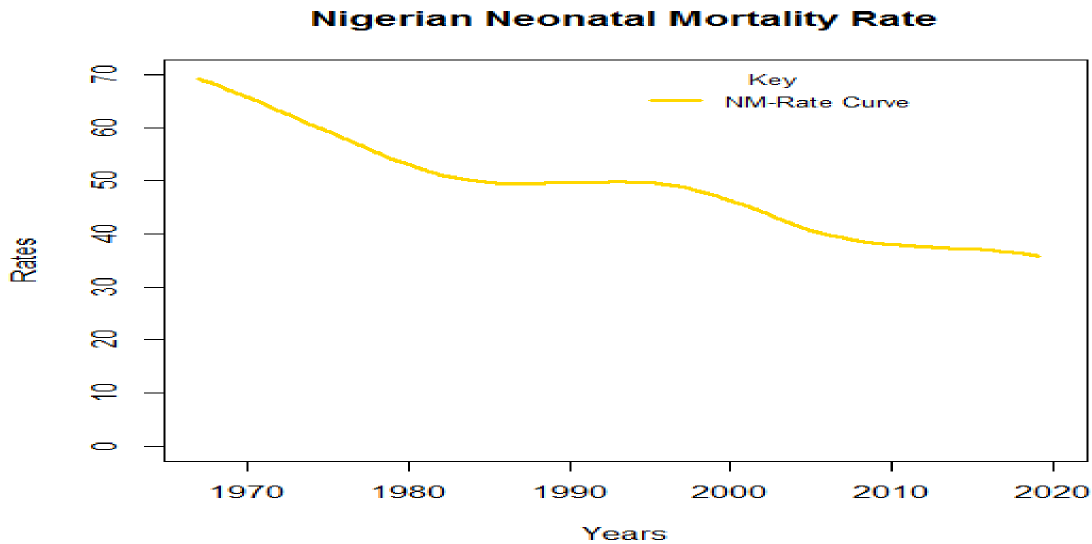

Figure 4: A Line plot of Neonatal Mortality Rate in Nigeria from 1967 to 2019

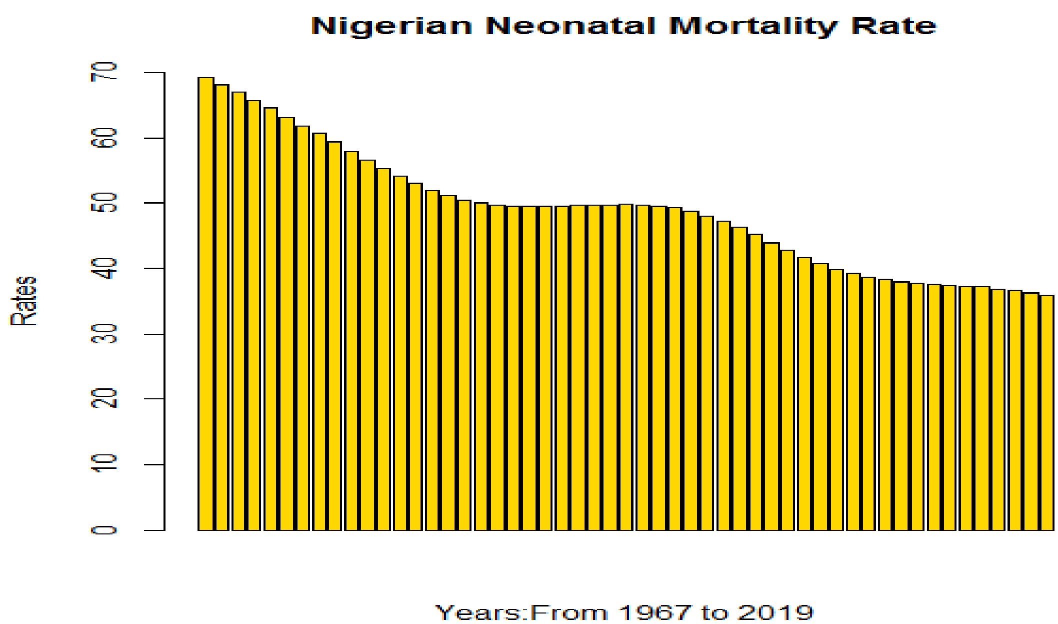

Figure 5: Bar chart of Neonatal Mortality Rate from 1967 to 2019

From Figure 4 and 5, the plots reveal the trend in the rate of neonatal mortality which show that neonatal mortality rate has been a very major problem in the past and even today because Nigeria has not been able to reduce its death rate to a minimum level as indicated by the line plot and the bar chart.

This clearly calls for more efforts from both private and government agencies to bring the curve to a bearable minimum level. Consequently, this article fits the Poisson-Lindley distribution (PoisLinD) to the above dataset in comparison with other existing probability Lindley related distributions.

To choose the best model for the data, the following model selection criteria were used: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan Quin Information Criterion (HQIC), Anderson-Darling (A*), Cramèr-Von Mises (W*) and Kolmogorov-smirnov (K-S) statistics. Adequate information on these criteria or statistics (A*, W* and K-S) can be found in [39].

Note: the probability model or distribution with the lowest values of these criteria is considered to be the best model that fit the dataset. Also, all the required computations are performed using the R package "AdequacyModel" which is freely available from http://cran.r-project.org/web/packages/AdequacyModel/AdequacyModel.pdf. The results from this R package and the commands are shown in tables 2-4.

Tables 2 lists the Maximum Likelihood Estimates of the model parameters, Table 3 presents the statistics AIC, CAIC, BIC and HQIC while A*, W* and K-S for the fitted models are given in Table 4.

Table 2: Maximum Likelihood Parameter Estimates for the dataset

Distributions	Parameter Estimates		
PoisLinD	$\hat{\theta}$=0.06147592	$\hat{\alpha}$=4.68515022	-
TraTPLinD	$\hat{\theta}$=0.04949141	$\hat{\alpha}$=4.08919046	$\hat{\lambda}$=-0.99640859
LomLinD	$\hat{\theta}$=0.03942008	$\hat{\alpha}$=4.88556302	$\hat{\beta}$=6.97045762
LinD	$\hat{\theta}$=0.04016662	-	-
TraLinD	$\hat{\theta}$=0.05063256	-	$\hat{\lambda}$=-0.33400161

Table 3: The statistics ℓ, AIC, CAIC, BIC and HQIC for the dataset

Distribution	$\hat{\ell}$	AIC	CAIC	BIC	HQIC	Ranks
PoisLinD	210.9511	425.9022	426.1422	429.8428	427.4176	1st
TraTPLinD	225.8028	457.6057	458.0955	463.5165	459.8787	2nd
LomLinD	249.6315	505.2629	505.7527	511.1738	507.536	3rd
LinD	240.5366	483.0731	483.1515	485.0434	483.8308	4th
TraLinD	236.9102	477.8204	478.0604	481.761	479.3357	5th

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Table 4: The A^*, W^*, K-S statistic and P-values based on the dataset.

Distribution	A^*	W^*	K-S	P-Value (K-S)	Ranks
PoisLinD	1.112935	0.1799556	0.2111	0.01499	1st
TraTPLinD	1.096892	0.1742217	0.28506	0.000264	2nd
LomLinD	1.097486	0.1749093	0.39879	4.067e-08	3rd
LinD	1.098607	0.1736916	0.43507	1.218e-09	4th
TraLinD	1.101905	0.1736544	0.47346	1.983e-11	5th

Figure 6 presents a histogram and estimated densities and cdfs of the fitted models to the Neonatal mortality rate data.

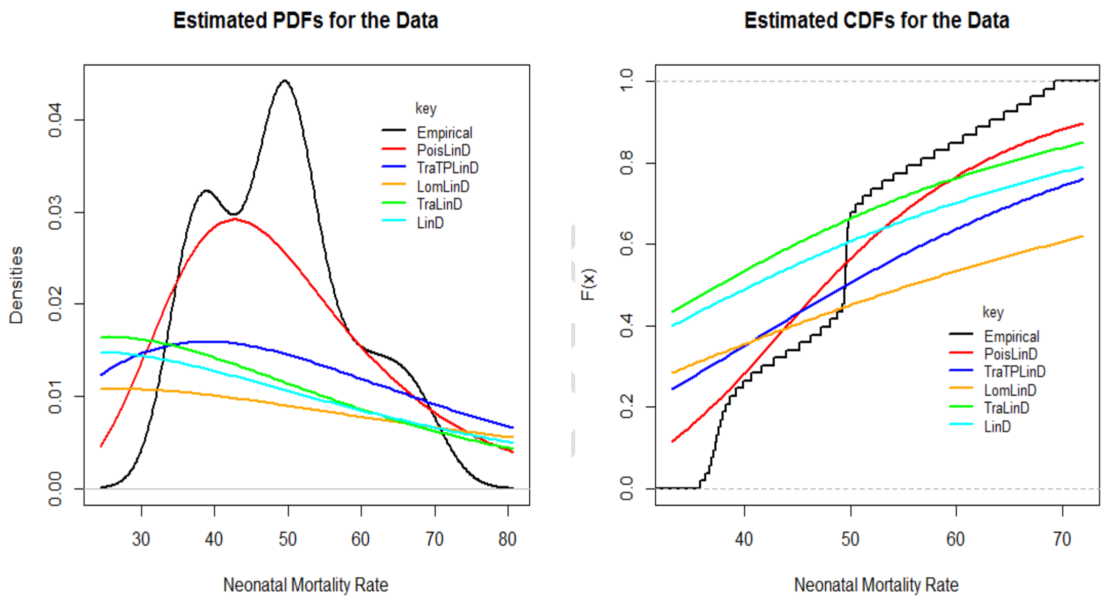

Figure 6: Estimated densities and cdfs of the fitted distributions to the Neonatal mortality rate data.

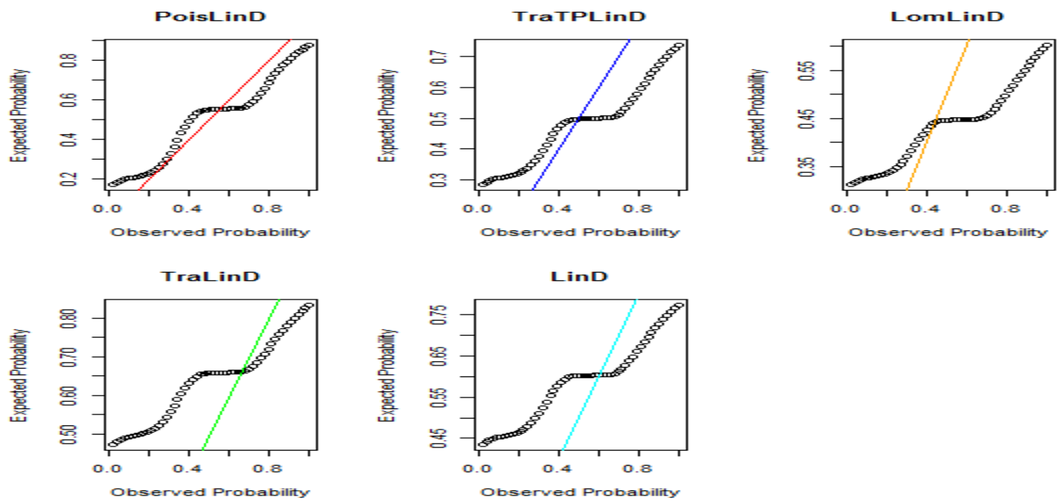

Figure 7: Probability plots for the fitted distributions based on the Neonatal mortality rate data.

Interpreting the results from the tables above, it is shown that table 2 presents the parameter estimates and table 3 lists the values of AIC, CAIC, BIC and HQIC for the fitted distributions using a dataset on neonatal mortality rate in Nigeria which is skewed to the right. The values of AIC, CAIC, BIC and HQIC in table 3 are smaller for the PoisLinD compared to those of the other four distributions and this result indicates that the Poisson-Lindley distribution (PoisLinD) is better than the other fitted distributions. This entire result was confirmed using the other statistics in table 4 and the plots in figure 6 and figure 7 respectively. Conclusively, it has shown that adding a parameter(s) to a probability distribution generate a distribution with greater flexibility in modeling real life data as reported by many other authors in previous studies.

6 Summary and Conclusion

This study considered a Poisson-X family to define and study a Poisson-Lindley distribution leading to a new distribution called "Poisson-Lindley distribution". The article derived and studied some properties of the proposed distribution with graphical analysis and discussion on its usefulness and applications. The method of maximum likelihood was used to estimate the unknown model parameters. As presented in the previous section, the article applied the proposed model to neonatal mortality rate and found that its fitness to the data was better as compared to the transmuted two-parameter Lindley distribution (TraTPLinD), Lomax-Lindley distribution (LomLinD), Transmuted Lindley distribution (TraLinD) and the Lindley distribution (LinD) based on the neonatal mortality dataset used in this study.

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