Original Research Article

Effects of the hydro anisotropy and the magnetic field on the dynamic thermo-bi-diffusive flow in a horizontal cavity confining a porous medium saturated by a binary fluid.

Abstract

We analyze analytically the effects of anisotropy in permeability and that of a transverse magnetic field on thermal convection in a porous medium saturated with a binary fluid and confined in a horizontal cavity. The porous medium, of great extension, is subjected to various conditions at the thermal and solutal boundaries. The axes of the permeability tensor are oriented obliquely with respect to the gravitational field. Based on a scale analysis, the velocity, temperature, and heat and mass transfer rate fields were determined. These results were validated by the study of borderline cases which are: pure porous media and pure fluid media discussed in the literature. It emerges from this study that the anisotropy parameters influence the convective flow. The application of a transverse magnetic field significantly reduces the speed of the flow and thereby affects the temperature field and the rate of heat and mass transfer.

Keywords: Porous medium, convection, anisotropy, magnetic field.

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Nomenclatures

a, b, c : Constants

$\boldsymbol{A}$: Aspect ratio of the cavity (L'/H')

$\boldsymbol{B}$: Magnetic field

$\boldsymbol{g}$: Gravitational acceleration

$\boldsymbol{H'}$: Height of the cavity

$\boldsymbol{j'}$: Current density

$\bar{\bar{\boldsymbol{K}}}'$: Flow permeability tensor

$\boldsymbol{K_1}, \boldsymbol{K_2}$: Flow permeability along the principal x and y axes, respectively

$\boldsymbol{K}^*$: Permeability ratio

$\boldsymbol{q'}$: constant heat flow (per unit of surface), w.m-2

$\boldsymbol{L'}$: Thickness of the cavity

$\boldsymbol{L_x}$: length of the central region of the cavity

$\boldsymbol{Le}$: Lewis number, α_p/D

$\boldsymbol{Sh}$: Sherwood number

$\boldsymbol{Nu}$: Nusselt number

$\boldsymbol{P'}$: Pressure

$\boldsymbol{Ra}$: Rayleigh number

$\boldsymbol{N}$: Ratio volume force, (βc△C*/ βT△T*)

$\boldsymbol{T'}$: Dimensional fluid temperature

$\boldsymbol{C'}$: Dimensional fluid concentration

$\boldsymbol{T}$: Dimensionless fluid temperature

$\boldsymbol{C}$: Dimensionless fluid concentration

$\Delta \boldsymbol{T}^* = $ q'H'/kp : Temperature difference scale

$\Delta \boldsymbol{C}^* = $ j'H'/Dp : Temperature difference scale

$\boldsymbol{t}$: Dimensionless time, (t'αp /H'2)

$\boldsymbol{u'}, \boldsymbol{v'}$: Velocities components in x and y directions

$\boldsymbol{u}, \boldsymbol{v}$: Dimensionless velocities components in x and y directions

$\vec{\boldsymbol{V}}'$: Velocity of the fluid in porous medium

$\boldsymbol{x'}, \boldsymbol{y'}$: Dimensional Cartesian coordinates

$\boldsymbol{x}, \boldsymbol{y}$: Dimensionless Cartesian coordinates

$\boldsymbol{\beta_T}$: Thermal expansion's coefficient

$\boldsymbol{\alpha_p}$: thermal diffusivity kp/(ρC)f

$\boldsymbol{\varepsilon}$: Dimensionless porosity, ε'/σ

$\boldsymbol{\sigma}$: Heat capacity ratio (ρC)p/(ρC)f

$\boldsymbol{\beta_C}$: solutal expansion coefficient, kg.mol.L-1

$(\boldsymbol{\varrho C_p})_f$: Heat capacity of the fluid

$\boldsymbol{\varrho}$: Density of the fluid

$\boldsymbol{\Psi}$: dimensional stream function

$\boldsymbol{\psi}$: Dimensionless stream function (Ψ'/αp)

$\boldsymbol{\vartheta}$: kinematic viscosity of the fluid, $m^2.s^{-1}$

1. Introduction

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The effects of hydrodynamic anisotropy and transverse magnetic field on thermal convection in a porous medium are investigated analytically for fully developed flow regime. The porous medium saturated with a binary fluid, subjected to various conditions at the thermal and solutal boundaries, is anisotropic in permeability whose principal axes are oriented in a direction which is oblique to the gravity vector. On the basis of the generalized Darcy's law and within the boundary layer approximations, solutions have been obtained for the flow field the heat and mass transfer. It was found that the anisotropic permeability ratio, the orientation angle of the princi pal axes of permeability and the magnetic field affected significantly the flow regime, the heat and mass transfer.

AMAHMID et ***al*** [6] studied analytically and numerically the natural convection in a porous layer of Brinkmann doubly diffusive in a confined anisotropic porous medium taking into account the particular situation where the thermal and solutal volume forces are opposite and of the same intensity. From their studies, it appears that the increase in Da induces a decrease in the flow intensity and heat and mass transfers; likewise as R_T increases, the intensity of the flow increases monotonously; however Nusselt and Sherwood numbers tend asymptotically towards the same value independent of the of Lewis's number Le, which decreases with Da.

On the basis of Darcy flow model and the Boussinesq approximation, ***ATTIA*** et ***al*** [7] studied the Soret and Dufour effects on thermosolutal convection in a rectangular cavity filled with a porous medium saturated by a binary fluid. The horizontal walls of the cavity are subjected to uniform heat fluxes q" and species j", while the vertical walls are considered adiabatic and impermeable. As a result, Soret and Dufour effects drastically affect the stability of convection, which affects the heat and mass transfer rates. Through the Darcy flow model, ***KALLA*** [8] studied thermosolutal convection within a porous cavity saturated with a binary fluid confined in a rectangular cavity and heated from below. As a result, the convective flow induced by thermal forces, highly depends on the thermal Rayleigh number R_T. ***BENISSAAD*** and ***OUAZAA*** [9] used Darcy's model to investigate analytically and numerically natural bi-diffusive convection in a porous medium saturated with a binary fluid and confined in a rectangular cavity. The horizontal walls of the cavity are heated with a uniform flux while the vertical walls are considered adiabatic and impermeable. These authors have shown that the control parameters significantly influence the flow, heat and mass transfer. Through Brinkmann's model, ***AKOWANOU*** and ***DEGAN*** [10] studied convective transfer in a rectangular cavity, filled with a porous medium saturated with an incompressible electrically conductive fluid and subjected to a transverse magnetic field. The side walls of the cavity are

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subjected to differential heating. It appears that the convective flow is greatly influenced by the anisotropic parameters of the porous layer and by the effect of the applied transverse magnetic field. Likewise, the rate of heat transfer in the porous medium increases when the permeability in the horizontal direction is higher than that prevailing in the vertical direction ***OUAZAA*** [11] through Darcy's mathematical model studied thermosolutal convection in porous media confined in a rectangular cavity and saturated with a binary fluid. The horizontal walls of the cavity are subjected to uniform heat fluxes q' and species j' while the vertical walls are considered adiabatic and impermeable. It appears that the flow intensity and the mass and heat transfer rates are significantly influenced by the thermal Rayleigh number. ***BENISSAAD*** et ***al*** [12] studied analytically the thermosolutal natural convection in anisotropic porous medium confined in a rectangular enclosure with horizontal walls supposed to be impermeable and adiabatic using the mathematical model Darcy-Brinkman-Forcheimer. Constant and uniform temperature and concentration gradients are imposed on the vertical walls. From their study, it appears that the flow increases with an increase in the Darcy number which increases with the Rayleigh number. The tilt angle of the permeability axes greatly influences heat and mass transfer rates. Using the Darcy-Brinkmann model, ***HADIDI*** [13] studied two-dimensional thermosolutal convection in a porous cavity arranged vertically. The side walls are subjected to uniform temperature and concentration conditions while the horizontal walls are adiabatic and impermeable. The results of this investigation show that the variation in the permeability of the two layers has a very appreciable effect on the flow structure and the transfers. Through the mathematical formulation of Darcy-Brinkman-Forchheimer, ***SAFI*** [14] proceeded to the study of bi-diffusive convection in an anisotropic porous medium with for geometry used, an anisotropic porous medium saturated by a binary fluid, supposed incompressible, confined in a rectangular enclosure and arranged horizontally. The vertical walls of the cavity are subjected to constant temperatures and concentrations (Dirichlet-type boundary conditions), while the horizontal walls are kept impermeable and adiabatic. It was concluded that thermal anisotropy significantly affects heat and mass transfers as well as heat and mass transfers increase with Rayleigh number.

In order to describe the heat transfer and contaminants diffusion phenomenon through the soil, the latter is, considered as a porous medium saturated by a binary fluid since the underground water is a mixture of liquid and pollutants.

Because of the preferential orientation or asymmetric geometry of the grain or fibers encountered in porous matrix in many industrial applications and nature, as consequence, porous media are generally anisotropic in permeability. Also, analyzing the flow field, we take

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into account the magnetic earth's field which is assumed to be constant and oriented in a direction which is parallel to gravity.

The present work is devoted to the study of thermosolutal convection in a rectagular cavity filled with porous medium and saturated by a binary fluid. The porous medium is anisotropic in permeability with its principal axes oriented in a direction that is oblique to the gravity vector. Assuming the usual approximations used in the classical boundary layer problem, a fully developed flow regime solution for the problem is applied. Consequently, closed-form expressions of flow field, temperature, heat and mass transfer rate are obtained in terms of the Rayleigh number, Hartman number and the anisotropic parameters. The influence of these parameters on the convective flow and the heat and mass transfer will be investigated, since the physical problem is of significant importance to many engineering-related applications.

2. Mathematical formulation

The physical model considered in Figure 1 is that of a rectangular enclosure with flat walls. The soil, confined in the enclosure, constitutes a porous medium which is saturated with a polluted water mixture assumed to a binary fluid. The porous medium which is anisotropic in permeability whose principal axes are oriented in a direction that is non-coincident with the gravity. This medium subjected to a uniform and transverse magnetic field is heated from below by a constant flux. The anisotropy permeability of the porous medium is characterized by the anisotropy ratio $K^* = K_1/K_2$ and the inclination angle θ, defined as the angle between the vertical direction and the principal axis with the permeability K_1

From the onset of the thermal heating, the medium is registered by a thermosolutal convection phenomenon.

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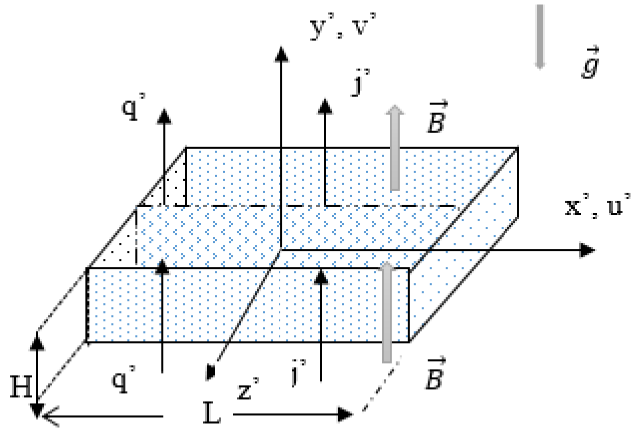

Figure 1: Physical model and coordinates system.

The regime considered here is the steady state with flow developed in the porous channels. The equations of continuity, motion, energy and concentration are written **Akowanou et all** [10] :

$$\nabla \vec{V}' = 0 \tag{1}$$

$$V' = \frac{\bar{\bar{K}}}{\mu}\left[\nabla \vec{P'} + \rho' \vec{g} + \vec{J} \wedge \vec{B}\right] \tag{2}$$

$$\nabla \vec{J}' = 0 \,; \vec{J} = \gamma\left(-\nabla \emptyset + V' \vec{B}\right) \tag{3}$$

$$\left(\rho C_p\right)_m \frac{\partial T'}{\partial t} + \left(\rho C_p\right)_f \nabla . \left(\vec{V}' T'\right) = k \nabla^2 T' \tag{4}$$

where $\bar{\bar{K}}$ represents the permeability tensor of the porous medium in the axis system shown in Figure (1). It is a tensor of order 2 which is written according to the coordinate axis system :

$$\bar{\bar{K}} = \begin{bmatrix} K_1 \cos\theta^2 + K_2 \sin\theta^2 & (K_2 - K_1)\sin\theta\cos\theta \\ (K_2 + K_1)\sin\theta\cos\theta & K_2\cos\theta^2 + K_1\sin\theta^2 \end{bmatrix} \tag{6}$$

The porous medium being electrically isolated, then the electric field is zero everywhere. On this basis we have :

$$\nabla \emptyset = 0 \tag{7}$$

Likewise

$$\vec{J}' = \gamma\left(\vec{V}' \wedge \vec{B}\right) \tag{8}$$

Using the coordinates (u', v') of the filtration rate V' defined in the plane (Ox', Oy') illustrated in Figure 1, the equations (1), (2), (3), (4) and (5) describing the phenomenon of convection in an anisotropic porous medium in permeability, formulated as primitive variables are written:

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$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (9)$$

$$\left.\begin{aligned} au' - cv' &= \frac{K_1}{\mu}\left(-\frac{\partial P'}{\partial x'} - \gamma u' B^2\right) \\ -cv' + bv' &= \frac{K_1}{\mu}\left(-\frac{\partial P'}{\partial y'} - \rho\vec{g}\right) \end{aligned}\right\} \quad (10)$$

$$\sigma\frac{\partial T'}{\partial t'} + u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \alpha\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) \quad (11)$$

$$\varepsilon\frac{\partial T'}{\partial t'} + u'\frac{\partial C'}{\partial x'} + v'\frac{\partial C'}{\partial y'} = D\left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2}\right) \quad (12)$$

With :

$$\left.\begin{aligned} a &= cos^2\theta + K^* sin^2\theta \\ b &= sin^2\theta + K^* cos^2\theta \\ c &= (1 - K^*) sin\theta cos\theta \\ K^* &= K_1/K_2 , k^* = k_{x'}/k_{y'} \end{aligned}\right\} \quad (13)$$

$$\sigma = \frac{(\rho C_p)_m}{(\rho C_p)_f} \;;\; \rho = \rho_0[1 - \beta_T(T' - T_0) - \beta_C(C' - C_0) \quad (14)$$

The dimensionless variables not only have a simplifying advantage of the equations but also they allow a better physical interpretation of the phenomenon studied.

The normalization scale factors used for the quantities of interest are :

$$\left.\begin{aligned} (x, y) &= \frac{(x', y')}{H'} & C &= \frac{C' - C_0}{\Delta C'} \\ (u, v) &= \frac{(u', v')H'}{\alpha_f} & \Delta C' &= \frac{J'H'}{D} \\ \Psi &= \frac{\Psi'}{\alpha_f} & \Delta T' &= q'H'/k \\ T &= \frac{T' - T_0}{\Delta T'} \end{aligned}\right\} \quad (15)$$

By introducing these dimensionless quantities into the conservation equations for mass (9), motion (10), energy (11) and concentration (12), we obtain respectively:

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (16)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \quad (18)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{1}{Le}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \quad (19)$$

The feeding of the equation system reveals the control parameters of the problem : $\mathbf{Ha}=\mathbf{B}(K_1\gamma/\mu)^{1/2}$ is the Hartmann number, $Ra_H = K_1 g\beta H\Delta T/(\alpha\mu)$ is the Rayleigh number of the cavity based on the height , $N=\beta_C\Delta C'/\beta_T\Delta T'$ is the ratio of the thermal and solutal volume forces , $Le=\alpha_f/D$ is the Lewis number which represents the ratio of the thermal diffusivity to the mass diffusivity of the saturated porous medium and the constants α ,β , γ

Based on the parallel flow approximation and assuming that $u=\frac{\partial\Psi}{\partial y}$ and $v=-\frac{\partial\Psi}{\partial y}$

$$(a+Ha^2)\frac{\partial^2\Psi}{\partial y^2}+b\frac{\partial^2\Psi}{\partial x^2}+2c\frac{\partial^2\Psi}{\partial x\partial y}=-Ra_H\left(\frac{\partial T}{\partial x}+N\frac{\partial C}{\partial y}\right) \quad (20)$$

$$\nabla^2 T=\frac{\partial\Psi}{\partial y}*\frac{\partial T}{\partial x}-\frac{\partial\Psi}{\partial x}*\frac{\partial T}{\partial y} \quad (21)$$

$$\nabla^2 C=Le\left(\frac{\partial\Psi}{\partial y}*\frac{\partial C}{\partial x}-\frac{\partial\Psi}{\partial x}*\frac{\partial C}{\partial y}\right) \quad (22)$$

a°) Hydrodynamic boundary conditions

$$\left.\begin{array}{c} *\,on\ vertical\ walls \\ x=\mp\frac{L}{2},\quad u=0\ ,\qquad \Psi=0 \\ *\,on\ horizontal\ walls \\ x=\mp\frac{H}{2},\quad v=0\ ,\qquad \Psi=0 \end{array}\right\} \quad (23)$$

b°) Thermal conditions

$$\left.\begin{array}{c} *\,on\ vertical\ walls \\ x=\mp\frac{L}{2},\quad u=0\ ,\qquad \Psi=0 \\ *\,on\ horizontal\ walls \\ x=\mp\frac{H}{2},\quad v=0\ ,\qquad \Psi=0 \end{array}\right\} \quad (24)$$

c°) The thermal and mass boundary conditions

When fluxes are applied to vertical and horizontal walls,

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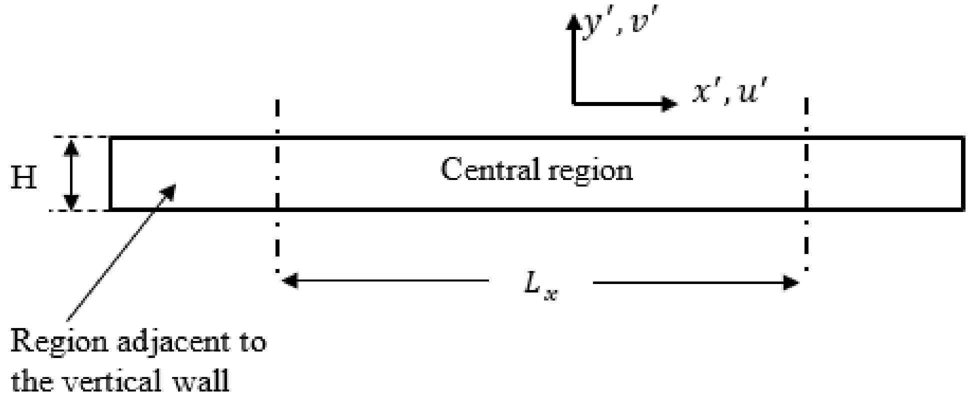
$$\left.\begin{array}{c} \boldsymbol{for\ x = \mp \frac{A}{2},\qquad \frac{\partial T}{\partial x} = \frac{\partial C}{\partial x} = 0} \\ \boldsymbol{for\ y = \mp \frac{A}{2},\qquad \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = -1} \\ \boldsymbol{With\ A = \frac{H}{L}\ represents\ the\ aspect\ ratio\ of\ the\ cavity} \end{array}\right\} \quad (25)$$

3. Method of solution

In this part, let us consider the flow motion in the central region of the cavity :



L_x and H are the characteristic scales of the variables x and y in the central region of the cavity :

$$y \sim H \quad ; \quad x \sim L_x$$

According to the analysis on the basis of equation (21) we can write

$$\frac{\boldsymbol{u}}{\boldsymbol{v}} \sim \frac{L_x}{\boldsymbol{H}} \text{ so } \frac{\boldsymbol{u}}{L_x} \sim \frac{\boldsymbol{v}}{H} \text{ and since } \mathrm{A} = \frac{\boldsymbol{H}}{\boldsymbol{L}} \ll 1 \text{ and } \mathrm{L} > L_x \ so\ \frac{\boldsymbol{L}}{H} \gg \frac{L_x}{H} \text{ where } \frac{\boldsymbol{u}}{\boldsymbol{v}} \gg 1 \quad (26)$$

From the above, we infer that the flow in the central region of the cavity is developed in the horizontal Ox region as has been discussed in detail in the past by **Cormack et al**. [2], **Vasseur et al.** [3]. So we have : u = u(y) ; v = 0 ; T = T(y), C= C(y) et $\Psi = \Psi_0$ (y)

3.1. Expression of current function, flow velocity, and temperature and concentration profiles

$$(a + Ha^2)\frac{d^2\Psi}{dy^2} = -Ra_H\left(\frac{dT}{dx} + N\frac{dC}{dx}\right) \quad (27)$$

$$\frac{d\Psi}{dy} * \frac{dT}{dx} = \frac{d^2T}{dy^2} \quad (28)$$

$$\frac{d\Psi}{dy} * \frac{dC}{dx} = \frac{1}{Le}\frac{d^2C}{dy^2} \quad (29)$$

The temperature and concentration profiles are then given by the sum of a term defining a linear longitudinal variation and another term giving the transverse distribution:

The temperature and concentration profiles are defined as follows

$$T(x, y) = C_T x + \theta_T(y) \quad (30)$$

and

$$C(x, y) = C_S x + \theta_S(y) \quad (31)$$

where C_T and C_s are coefficients expressing the temperature and concentration gradients; θ_T and θ_S are dimensionless temperatures and concentrations. So we have

$$(a + Ha^2)\frac{d^2\Psi}{dy^2} = -Ra_H(C_T + NC_S) \quad (32)$$

$$\frac{d^2\theta_S}{dy^2} = LeC_S\frac{d\Psi}{dy} \quad (34)$$

By referring to the hydrodynamic boundary conditions we obtain the expression of the current function : :

$$\Psi = \Psi_0(1 - 4y^2) \quad (35)$$

with

$$\Psi_0 = \frac{Ra_H}{8}\frac{C_T + NC_S}{(a + Ha^2)} \quad (36)$$

In addition, the flow velocity of the following fluid is written therein :

$$u = -8\Psi_0 y \quad (37)$$

As for the temperature and concentration profiles, we have:

$$T(x, y) = C_T x + \frac{C_{T.}\Psi_0}{3}(3y - 4y^3) - y \quad (38)$$

And

$$C(x, y) = C_S x + \frac{Le.C_S.\Psi_0}{3}(3y - 4y^3) - y \quad (39)$$

Temperature and concentration gradients C_T et C_S write

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$$C_T = \frac{4\beta\Psi_0}{3(2\beta + \Psi_0^2)} \; and \; C_S = \frac{4\beta Le\Psi_0}{3(2\beta + Le^2\Psi_0^2)} \text{ with } \beta = \frac{15}{16} \tag{40}$$

By combining the equation of the current function at the center of gravity with the terms of the temperature and concentration gradients we have

$$\Psi_0 = \frac{Ra_H}{8(a + Ha^2)}\left[\frac{4\beta\Psi_0}{3(2\beta + {\Psi_0}^2)} + N\frac{4\beta Le\Psi_0}{3(2\beta + Le^2{\Psi_0}^2)}\right] \tag{41}$$

From (40) we get a fifth order equation in terms of the stream function Ψ_0,

$$\Psi_0(Le^4{\Psi_0}^4 - 2\beta Le^2{\Psi_0}^2P_1 - \beta^2P_2 = 0 \tag{42}$$

$$\left.\begin{aligned} P_1 &= \frac{1}{(a + Ha^2)}\left[\frac{Ra_H}{12}Le(N + Le) - (a + Ha^2)(1 + Le^2)\right] \\ P_2 &= \frac{1}{(a + Ha^2)}\left[\frac{Ra_H}{3}Le^2(1 + NLe) - 4Le^2(a + Ha^2)\right] \\ &\frac{6(a + Ha^2)}{Le^2} \neq 0 \end{aligned}\right\} \tag{43}$$

Equation (42) indicates a possibility of five solutions, one of which is zero, the zero solution corresponds to the state of the fluid at rest

$$\Psi_0 = 0 \tag{44}$$

The other four roots of Equation (42) are written :

$$\Psi_0 = \mp\frac{\sqrt{\beta}}{Le}\left[P_1 \mp \sqrt{P_1^2 + P_2}\right]^{\frac{1}{2}} \tag{45}$$

The signs ($\pm$) on the outside of the hook indicate the counterclockwise or clockwise direction of the convective flow while the signs ($\pm$) inside the hook indicate the two possible convective solutions of our flow. As shown by **Mamou** [5] the sign (+) in our solution indicates that the flow is stable and the sign (-) corresponds to an unstable solution

3.2. Heat and Mass transfer rates

The thermal and mass transfers are expressed respectively by the number of Nusselt and Sherwood

- Nusselt number

$$N_u = \frac{1}{\Delta T} \text{ with } \Delta T = T\left(0, -\frac{1}{2}\right) - T\left(0 - \frac{1}{2}\right)T_0 \quad x = 0 \tag{46}$$

One obtains

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$$N_u = \frac{6(\Psi_0^2 + 2\beta\ )}{\Psi_0^2 + 12\beta} \tag{47}$$

- **Sherwood number**

$$Sh = \frac{1}{\Delta C}\text{with} \quad \Delta C = C\left(0, -\frac{1}{2}\right) - C\left(0 - \frac{1}{2}\right)T_0 \quad x = 0 \tag{48}$$

One obtains

$$\mathrm{Sh} = \frac{6(\ \Psi_0^2 + 2\beta\ )}{\Psi_0^2\ \mathrm{Le}^2 + 12\beta} \tag{49}$$

4. Results and discussion

Buoyancy-induced convection in a fluid saturated porous medium is of considerable interest, owing to several geophysical and engineering applications. Prominent among these are insulation techniques, flows in soils aquifers, petroleum extraction, storage of agricultural products, underground diffusion of contarninants and porous material regenerative heat exchanger. Anisotropy, which is generally a consequence of a preferential orientation or asymmetric geometry of the grain or fibers, is in fact encountered in alI those applications in industry and nature.

In all those subjects, the choice of a simple rectangular geometry is due to the cornplexity of the physical phenomenon and the inclusion of more physical realism in the matrix properties of the medium for the accurate modeling of that latter.

In order to describe the heat transfer and contaminants diffusion phenomenon through the soil, the latter is, considered as a porous medium saturated by a binary fluid since the underground water is a mixture of liquid and pollutants.

Because of the preferential orientation or asymmetric geometry of the grain or fibers encountered in porous matrix in many industrial applications and nature, in fact, as consequence porous media are generally anisotropic in permeability. Also, analyzing the flow field, we take into account the magnetic earth's field which is assumed constant and oriented in a direction which is parallel to gravity.

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The influence of these parameters and of the magnetic field is analyzed by solving analytically the system of nonlinear ordinary differential equations thanks to the "MATHALAB version 15.0" calculation software.

Figure 2.1 shows the behavior of the binary fluid in flow theme under different values of Ha.

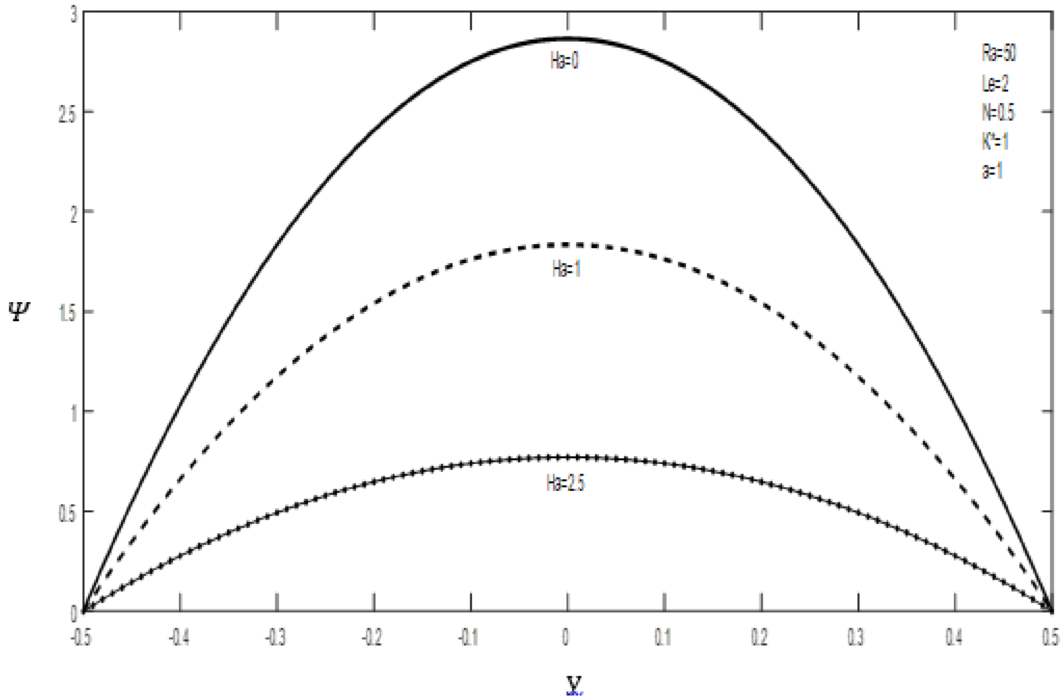

Figure.2.1. Variation of the stream function (Ψ) as a function of y for different value of Ha

The influence of Hartman number Ha on the stream function Ψ , is presented in Figure.2.1 when Ra = 50; Le = 2; N = 0.5; θ = 45°, $K^* = 1$.

Curves plotted in Figure.2.1 reveal a gradual attenuation of the convective flow as the application of the magnetic field becomes significant. We conclude that the magnetic field considerably reduces the intensity of the flow. The stream function increases to a maximum on the cavity centerline, the position of which depends upon the value of Ha and drops back to zero at the opposite wall. The maximum value reached when Ha = 0 is Ψ0 = 2.8648. A similar result has been reported by **Mamou** [5], **Ouazaa** [11] and **Kalla** [8] while studying the natural thermosolutal convection in an a rectangular enclosure.

Figure 2.2 shows the influence of the anisotropy ratio on the flow intensity. We find that the more the anisotropy ratio increases, the less the flow intensity decreases. We can then conclude that the permeability anisotropy significantly affects the flow of the fluid.

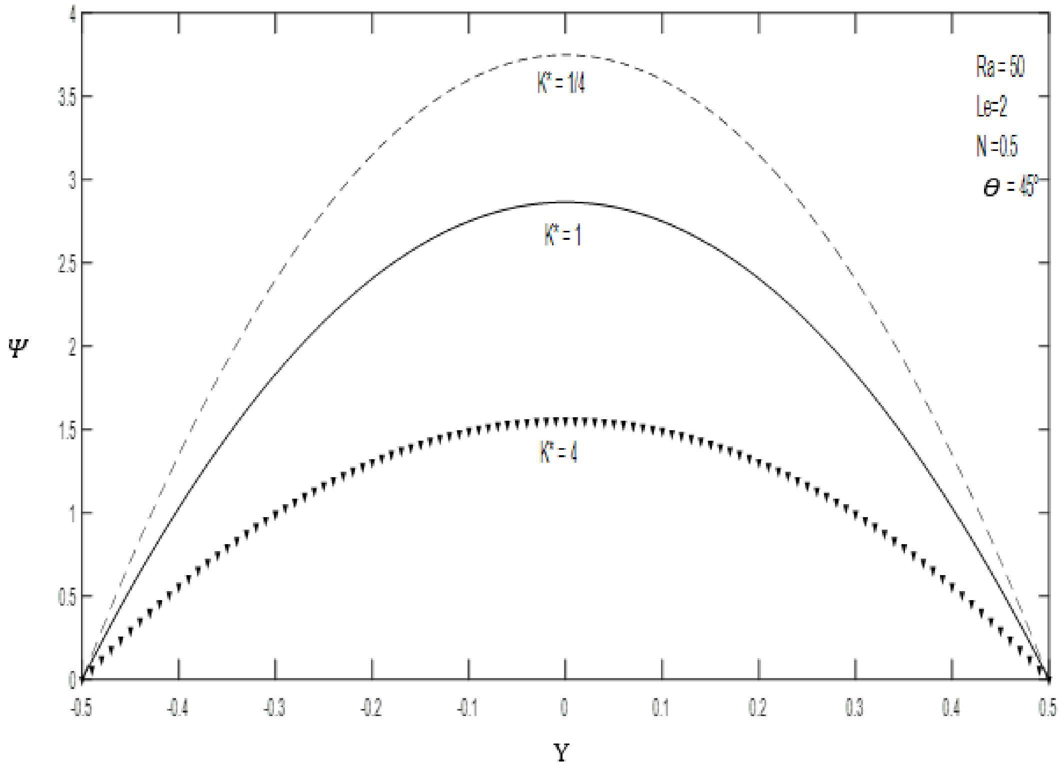

Figure 2.2 : Influence of permeability anisotropy on binary fluid flow

Figure 2.2 shows the influence of the permeability ratio K^* on the flow intensity when Ra = 50; Le = 2; N = 0.5; θ = 45°, $Ha = 0$

Figure.2.2 indicates that the velocity which is zero at the wall, because according to the hydrodynamic boundary conditions imposed**,** increases to a maximum on the channel centerline, the position of which depends upon the value of K^* and drops back to zero at the opposite wall, because of the reason explained earlier. Fig. 2(a) shows that the intensity of the convective flow is promoted with respect to that of an isotropic porous medium corresponding to $K^* = 1$, when the permeability ratio K^* is made smaller than one (i.e., $K^* = 1/4$). This is expected, because for a given value K_1, a value of K^*smaller than unity, corresponds to an increase of the permeability K_2 in the horizontal direction, thus promoting the convective circulation within the cavity. Naturally, the reverse trend is achieved when $K^*$is made large than unity (i.e., $K^* = 4$).

Figure 2.3.a illustrates the temperature and concentration profiles in the median plane (x = 0) of the cavity for Ra = 50 ; N = 2; Le = 2; Ha = 0; 1; 2.5. It can be seen that the heat and mass transfer is zero at the center of the cavity and it is also symmetrical. The first curve (T (Ha = 0), C (Ha = 0)) is the reference **Ouazaa** [11]. As the Hartmann number is varied, heat and

mass transfers also change. We deduce from this fact that the magnetic field favors the heat and mass transfer.

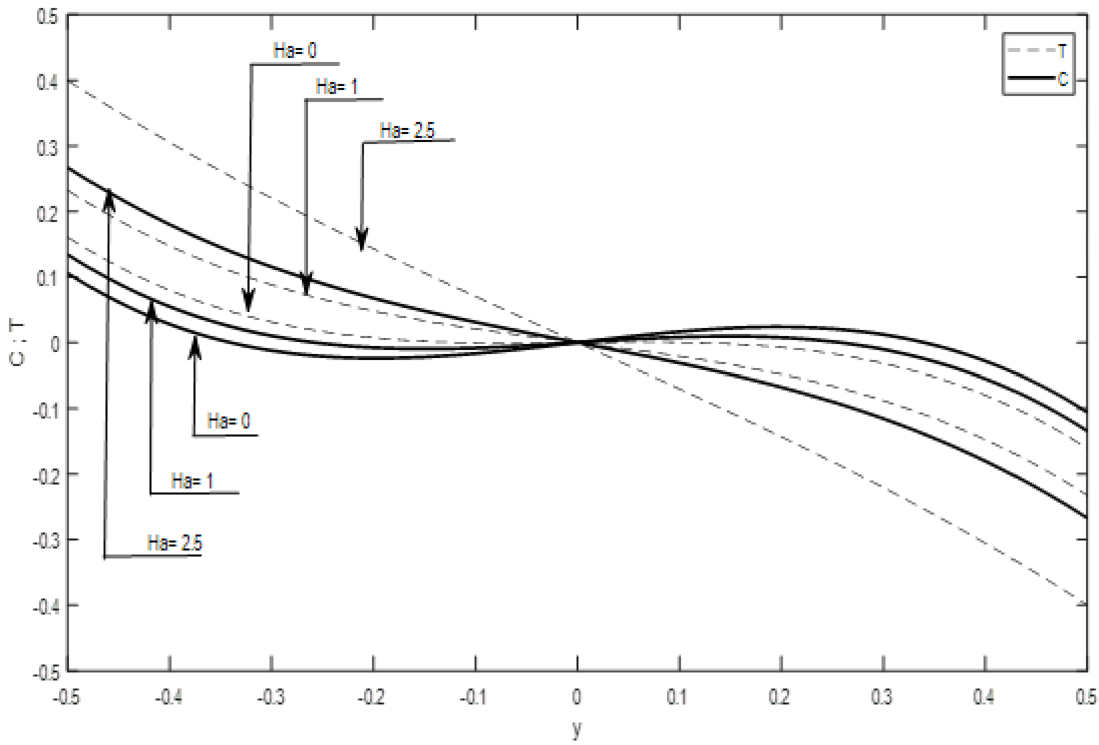

Figure2.3.a : Temperature and concentration profiles at x = 0 for different values of Ha

Figure 2.3.b shows us the influence of the Rayleigh number on the flow for different values of volume forces. The cavity being subjected to a constant and differential heat flow on the horizontal face we therefore have:

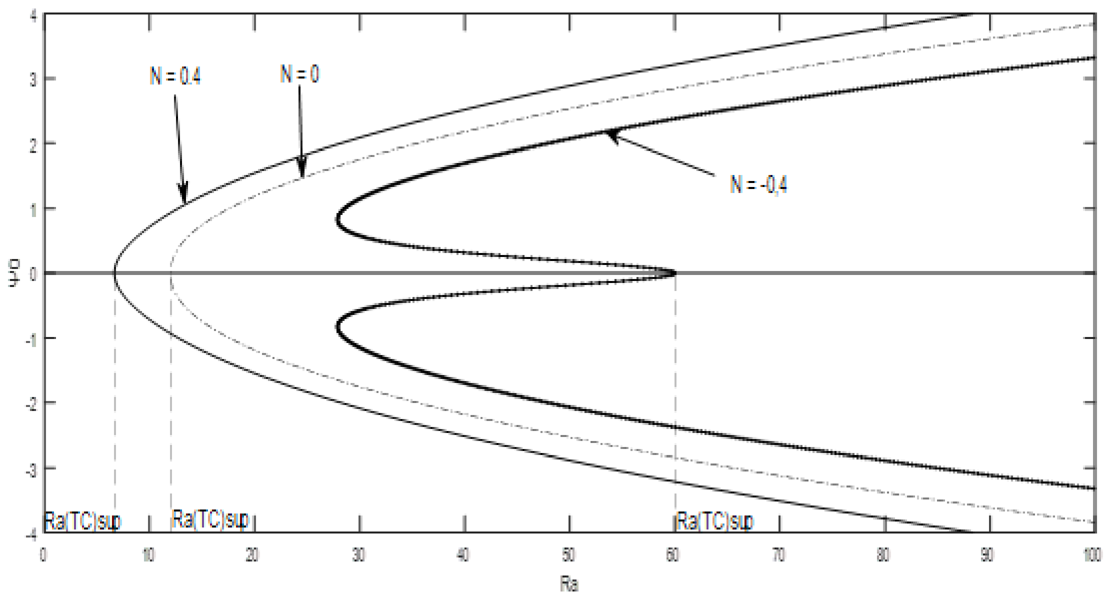

Figure2.3. b) : Bifurcation diagram Ψ_0 (Ra) for Le = 2 ; N = 0 ; N = 0.4 and N = -0.4

* when N = 0, the solutal volume force is zero, which means that the flow within this cavity is controlled by the flow of heat. Thus we notice a fork-type bifurcation which occurred for a critical Rayleigh number 12, a gold value already predicted by **Nield** [1] based on the theory of linear stability. This is the value from which the convection is started. For Ra <12 we are in the case of pure conduction and for Ra > 12 the intensity of the flow increases with possibilities of two opposite convective motions.

* When N = -0.4, the thermal and solutal volume forces act contrary, which means that the convection is opposite.

Figure 2.3.c shows the influence of the Rayleigh number on the heat transfer rate for different values of the volume ratio

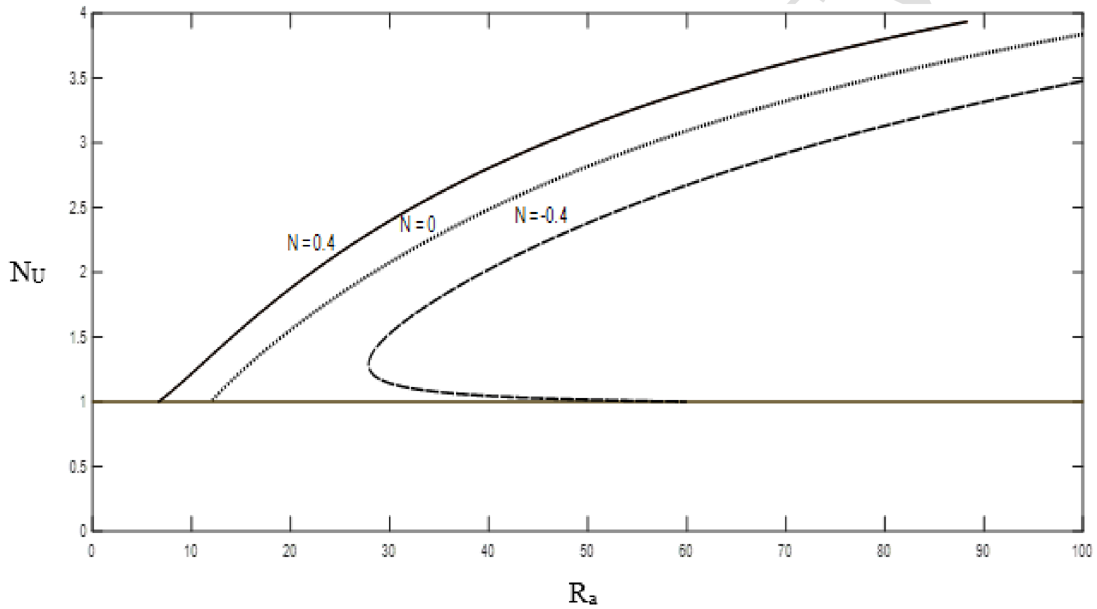

Figure 2.3.c) : Influence of the Rayleigh number on Nu for different aspect ratio value with A= 4, Le = 2

Here we examine the influence of Ra on the flow according to whether the volume forces are purely thermal (N = 0); cooperating (N> 0) or opposing (N <0). As well Ra^{sup}_{TC} (N=0.4) = 6.7 ; Ra^{sup}_{TC} (N=0) = 12 et Ra^{sup}_{TC} (N=-0.4) = 29.5 represents the supercritical Rayleigh number, value of the Rayleigh number for constant Nu. We find that for all volume forces whose Rayleigh number is less than the supercritical Rayleigh ($Ra< Ra^{sup}_{TC}$) the heat transfer rate is equal to 1 (Nu = 1), which means that we are in the presence of pure conduction. On the other hand, for Ra greater than supercritical Ra, the transfer rate increases until it reaches an asymptote value that can be deduced from equation (47)

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Figure 2.4 shows the variation of the Nusselt number as a function of the Rayleigh number for different Hartmann number.

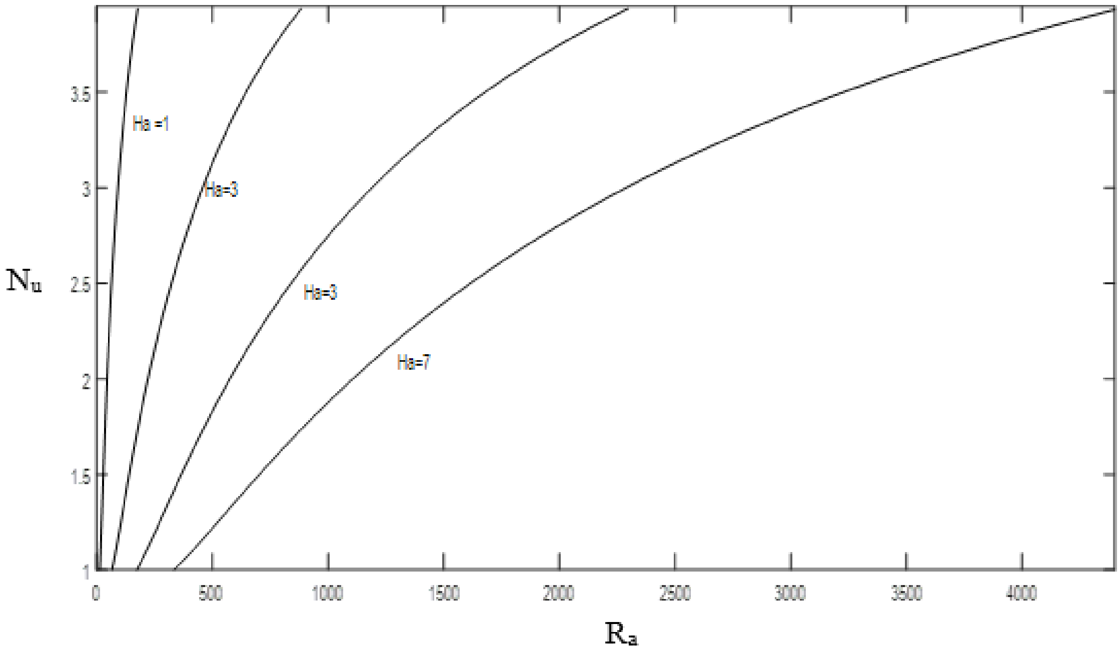

Figure 2.4 : Influence of the Rayleigh number on Nu for different value of Ha

For a fixed number of Ra, we notice that when Ha increases, the heat transfer rate decreases. It is concluded that the application of a relatively large magnetic field value reduces the heat transfer rate .

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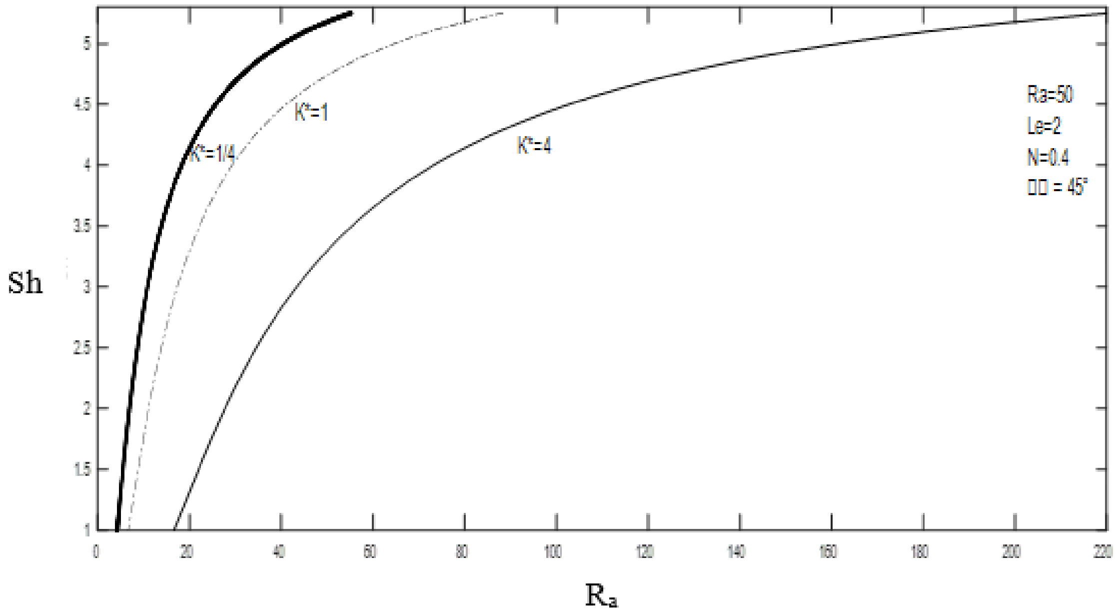

Figure 2.5 : : Influence of the Rayleigh number on Sherwood number for different values of K^* when $Le = 2$, $\theta = 45°$, $N = 0.4$

Figure 2.5 illustrates the evolution of Sherwood number versus Ra for $Le = 2$, $\theta = 45°$, $N = 0.4$ and various values of K^*.

From Figure 2.5, it is clear that for a fixed Rayleigh number Ra, when K^* is made larger, the mass transfer rate decreases as K* is made larger than unity. This follows from the fact that an increase in K^* corresponds to a decrease in the permeability K_2 when other parameters are held contant (Ra, i.e., K_1). Thus we deduce that anisotropy greatly influences the mass transfer rate.

Figures 2.6.a) and 2.6.b) show us the behavior of the heat transfer rate and the current function at the center in the center of the cavity in the presence of tilt angle under different various Rayleigh numbers and ratio of anisotropy

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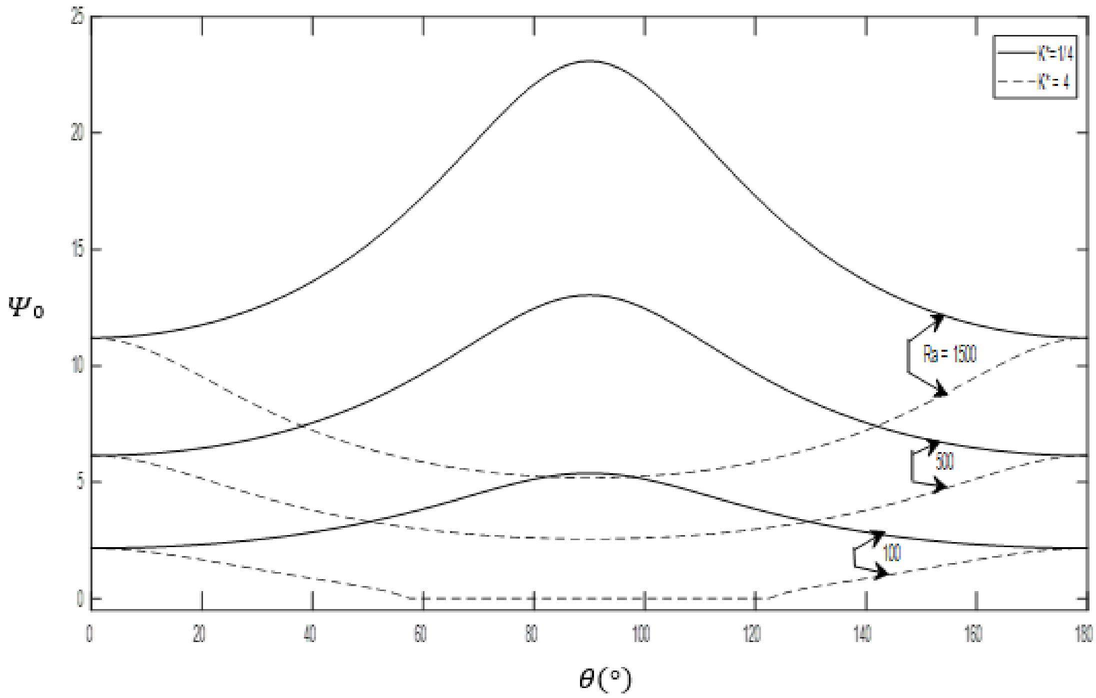

Figure 2.6.a : Effects of the angle of inclination $\boldsymbol{\theta}$ and of the Rayleigh number for K * = 4 and K * = 1/4 on the current function at the center of the cavity

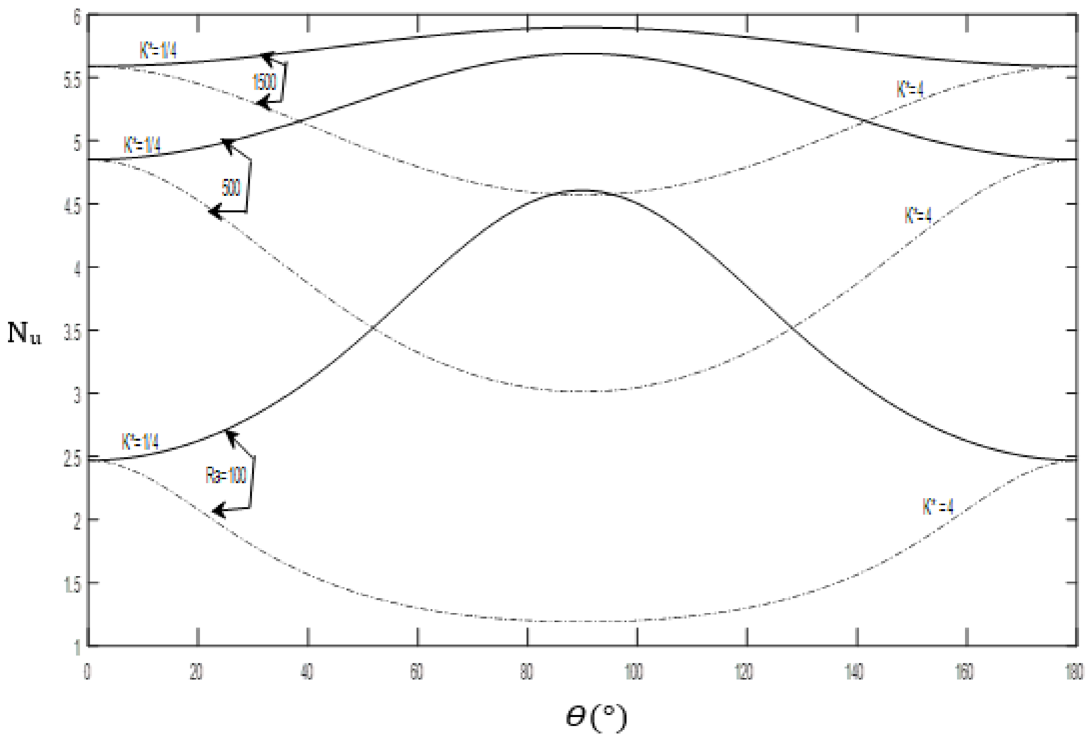

Figure 2.6.b) : Effects of the angle of inclination $\boldsymbol{\theta}$ and of the Rayleigh number for K * = 4 and K * = 1/4 on the Nusselt number

From the analysis of figure 2.6.a) and 2.6.b), we notice that the heat transfer rate is maximum for $\boldsymbol{\theta}$ = 90 ° and is minimum for $\boldsymbol{\theta}$ = 180 ° for K * = 1/4 which means that the permeability is maximum in the vertical direction and minimum in the horizontal direction. The opposite effect is observed for K * = 4, where the intensity of the unicellular convective motion and the resulting heat transfer are minimum at $\boldsymbol{\theta}$ = 90° and maximum at $\boldsymbol{\theta}$ = 0° and 180 °. The fact that for K *> 1 (K * <1) Nu is maximal (minimal) at $\boldsymbol{\theta}$ = 0 ° and 180 ° and minimal (maximal) at $\boldsymbol{\theta}$ = 90 .From Similar results have been reported in the past by **Zhang** [4] when this author studied natural convection in a laterally heated rectangular cavity . In all these studies, it is observed that a maximum (minimum) of heat transfer by natural convection is obtained when the orientation of the main axis of the anisotropic porous medium with the highest permeability is parallel (perpendicular) to the gravitational field. We then conclude that the rate of heat and the intensity of the flow all depend on the angle of orientation of the porous medium.

Moreover, considering the same conditions (Ra = 50; Le = 2; N = 0.5; θ = 45 ° and Ha = 0) **Mamou** [5] found Nu = 4.008 and Sh = 4.519; **Kalla** [8] found Nu = 4.062 and Sh = 4.658; **Ouazaa** [11] obtained Nu = 4.109 and Sh = 4.723 and for our present study we obtained Nu = 3.1090 and Sh = 4.7239. Thus we notice a satisfactory agreement with just small differences.

5. Conclusion

Our study focused on natural thermosolutal convection in a rectangular cavity confined by an anisotropic porous medium, saturated by a binary fluid and subjected to the effect of the transverse magnetic field. It emerges from this study that:

1) the intensity of the convective flow reflected by the stream function is greatly influenced on the one hand by the effect of the transverse magnetic field and on the other anisotropic parameters in permeability of the porous layer.

2) the increase in the anisotropic parameter of permeability ratio K* reduces and regulates the heat and mass transfer rate.

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