Boolean Subtraction and Division with Application in Blood Transfusion

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Original Research Article

Abstract

The subject of Boolean subtraction and division first appeared in the gem *Laws of Thought* by George Boole of Great Britain. However, this subject has been exiled from the famed Boolean algebra because it was supposed that it leads to illogical expressions. A paper recently published elsewhere by the author reintroduced the subject by clarifying it and then furnishing an application of it in the design of digital circuits. The goal of this work is to demonstrate how these banished logical operations can be employed in furnishing infallible information about blood typing and cross-matching during blood transfusion.

Keywords: ABO blood group; Rhesus factor; Boolean algebra; Boolean subtraction; Boolean division; Antigenic sets; Blood transfusion; Blood compatibility equation.

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1 Introduction

One of most illustrious discoveries in the world was that of the **ABO** blood group system, a system of classification of human blood based on the inherited properties of red blood cells (erythrocytes) as determined by the presence or absence of the antigens A and B, which are carried on the surface of the red cells. Before the advent of the 20th century, there existed a hunch that the blood of all human is identical in every respect and could therefore be transfused from one individual, the donor, to another, the recipient, who was in need of blood. There were lethal consequences of such blood transfusions and these proved incomprehensible. It was at the commencement of the 20th century that the problem was unravelled by the Austrian scientist, Karl Landsteiner, who noticed that the red blood cells of some individuals were clumped by serum from other individuals. He depicted the patterns of blood clumping and substantiated that the human blood could be assorted into groups or types. This engendered great joy and marked the discovery of the first blood group system, **ABO**, and earned Landsteiner a Nobel Prize [1], [2].

In 1940 the discovery of the **Rh** system by Landsteiner and Alexander Wiener was made because they tested human red cells with antisera developed in rabbits and guinea pigs by immunization of the animals with the red cells of the rhesus monkey Macaca mulatta. Those who wish to examine the inquiries on this subject of blood grouping may consult [1], [3], [5], [7], [10], etc.

The work pertains to matters unfamiliar to the reader and enters upon subjects of special importance. Its chief goal is to show how the Boolean operations of subtraction and division, a subject which remains a dark horse for years, can be employed in furnishing infallible information about blood grouping and cross-matching during blood transfusion. We have endeavoured to treat these thoroughly, discussing both examples with that completeness which we have always found indispensable in our experience as teachers. It has been our goal to discuss all the necessary parts as completely as possible within the limits of a single paper. To understand this work, one must be willing to alter ones ideas about Boolean subtraction and division. Throughout, it is assumed that the reader can simplify algebraic expressions and solve simple equations.

The perennial problem confronting a reader of any work is that of order. In this work, I have naturally ordered the material as I myself prefer. The rest of this work is ordered as follows. Sections 2 and 3 are concerned with blood groupings and transfusion respectively. Section 4 deals with Boolean algebra and its extension to logical subtraction and division. Section 5 discusses the blood groups and their Boolean representations. The sixth section introduces a function for blood cross-matching and creates a blood compatibility equation required for reaching the right conclusion when carrying out blood cross-matching during blood transfusion. The seventh and last section furnishes instances to show the significance of the blood compatibility equation.

2 Blood Groupings

2.1 Antigens and Antibodies

The main components of blood are:

1. red blood cells (erythrocytes), which carry oxygen around the body
2. white blood cells (leukocytes),, which play a crucial role in the immune system
3. platelets (thrombocytes), which enable blood clotting
4. plasma, which is a yellowish liquid that contains nutrients and salts

It is known in medicine that reactions between the red cells and plasma are related to the presence of antigens on the cells and antibodies in the plasma. **Antigens** are molecules which can be either proteins or sugars. The types and features of antigens can vary between individuals, due to small genetic differences. On the other hand, **antibodies** are protective proteins produced by the immune system in response to the presence of a foreign substance, called an antigen. Antibodies recognize and latch onto antigens in order to remove them from the body [4], [8], [9].

2.2 Blood Groups

Blood group or **blood type** refers to the classification of blood based on inherited differences (polymorphisms) in antigens on the surfaces of the red blood cells. Scientists use two major types of antigens to classify the major blood groups:

1. **ABO** antigens
2. **Rh** antigens

2.3 ABO Blood Grouping

Blood clumping or agglutination occurs when the antigens on the red cells are bound by the antibodies in the plasma. We call the antigens A and B, and depending upon which antigen the red cells express, blood either belongs to blood group **A** or blood group **B**. The blood group depends on which antigens are on the surface of the red blood cells. A third blood group contains red cells that react as if they lack the properties of **A** and **B**, and this group is called **O** after the German word *Ohne*, which means *without*. The fourth blood group, **AB**, contains red cells expressed as both A and B antigens.

Thus, there are 4 main blood groups defined by the **ABO** system:

1. blood group **A** has A antigens on the red blood cells with anti-B antibodies in the plasma
2. blood group **B** has B antigens with anti-A antibodies in the plasma
3. blood group **O** has no antigens, but both anti-A and anti-B antibodies in the plasma
4. blood group **AB** has both A and B antigens, but no antibodies

2.4 Rhesus Factor System

Rh blood group system is a system for classifying blood groups according to the presence or absence of the **Rh** antigen, often called the **Rh factor** on the cell membranes of the red blood cells. The designation **Rh** is derived from the use of the blood of *rhesus* monkeys in the basic test for determining the presence of the **Rh** antigen in human blood. If **Rh** is present in the blood, the blood group or type is **Rh positive** denoted $Rh+$. If it is absent, the blood group is **Rh negative** denoted $Rh-$.

2.5 Unified Blood Typing

It has been noted that the first system of blood grouping is the **ABO** system and has the four main types: Type **O**, Type **A**, Type **B**, Type **AB**. The second system which was discussed in a previous section is the Rhesus system and is classified as Rhesus positive, +, and Rhesus negative, . The two systems combine to define the following eight different blood types:

1. **A** Rh positive (**A**+)
2. **A** Rh negative (**A**−)
3. **B** Rh positive (**B**+)
4. **B** Rh negative (**B**−)
5. **O** Rh positive (**O**+)
6. **O** Rh negative (**O**−)
7. **AB** Rh positive (**AB**+)
8. **AB** Rh negative (**AB**−).

3 Blood Transfusion

3.1 ABO Blood Group Transfusion

The **ABO** blood group is the most important of all the blood group systems. Receiving blood from the wrong **ABO** group can be life-threatening. If **ABO** incompatible red cells are transfused, red cell haemolysis can occur. For example if group **A** red cells are infused into a recipient who is group **O**, the recipient's anti-A antibodies bind to the transfused cells. An **ABO** incompatible transfusion reaction may result in overwhelming haemostatic and complement activation, resulting in shock, renal failure and death. **ABO** typing of donor and recipient blood is done to prevent transfusion of incompatible RBCs. As a rule, blood for transfusion should be of the same **ABO** type as that of the recipient [11].

3.2 Rh Blood Groups Transfusion

The **Rh** antigen poses a danger for the Rh-negative person, who lacks the antigen, if Rh-positive blood is given in transfusion. Adverse effects may not occur the first time Rh-incompatible blood is given, but the immune system responds to the foreign Rh antigen by producing anti-Rh antibodies. If Rh-positive blood is given again after the antibodies form, they will attack the foreign red blood cells, causing them to clump together, or agglutinate. The resulting hemolysis, or destruction of the red blood cells, causes serious illness and sometimes death.

3.3 Unified Blood Type Transfusion

The table below demonstrates blood group compatibility. A person with **AB+** blood type can receive blood from all the major blood types. At the other extreme, people with **O−** blood types can only receive blood from donors with the same blood type.

Blood Group	Gives to these groups	Receives from these groups
O−	All	**O−**
O+	**AB+**, **A+**, **B+**, **O+**	**O−**, **O+**
A−	**AB−**, **AB+**, **A−**, **A+**	**O−**, **A−**
A+	**AB+**, **A+**	**O−**, **O+**, **A−**, **A+**
B−	**AB−**, **AB+**, **B−**, **B+**	**O−**, **B−**
B+	**AB+**, **B+**	**O−**, **O+**, **B−**, **B+**
AB−	**AB−**, **AB+**	**O−**, **A−**, **B−**, **AB−**
AB+	**AB+** only	All

Table 1: Unified Blood Types

4 Extended Boolean Algebra

The present day Boolean algebra is apparently very simple but it deprives us of reaching more important results.This section comprises very little of what is commonly understood by Boolean algebra. The principal object has been to prepare the way for the other parts of the works by stating the grounds of the algebraic notation of Boolean quantities and rendering it familiar by a few examples.

4.1 Boolean Algebra

The year 1854 is of singular importance in the history of science, owing to the emergence of Boolean algebra through the publication of Boole's book entitled *Laws of thought* [12]. This algebra, which is worthy of a crown, had adorned the publications of the erudite *Mathematics Association of America* for some years now [22], [23]. The book is a landmark and remains a valuable classic that is still a mine of information.

First, let us get the drift of this subject of Boolean algebra. In *Laws of Thought* we read Boole [12]:

> Let us conceive, then, of an Algebra in which the symbols x, y, z, etc. admit indifferently of the values 0 and 1, and of these values alone. The laws, the axioms, and the processes, of such an Algebra will be identical in their whole extent with the laws, the axioms, and the processes of an Algebra of Logic. Difference of interpretation will alone divide them. Upon this principle the method of the following work is established.

Boolean algebra is the science in which the letters, as logical symbols, are lawfully treated as quantitative symbols, susceptible only of the values 0 and 1. It arose naturally out of the attempt to apply symbolic methods to logic [24], [25], [26], [27]. Like many other branches of mathematics, it had its origin in the solution of logical problems. It consists of the rules for manipulating the subsets of any universal set. The respective interpretations of the symbols 0 and 1 in the system of Logic are empty set and universal set respectively. To understand the principles in which inquiries of this sort are conducted, it is necessary to become familiar with the plan of notation.

4.2 Boolean Quantity

A **Boolean quantity** is anything capable of existing in only two states. Rhesus factor on the red blood cell, therefore, is a Boolean quantity since we can speak of its presence and absence. It is the same with a proposition which is either true or false and other things of this nature.

4.3 Boolean Values

Boolean values are values used to represent Boolean quantities. They may be categorized into two classes, namely

1. Real Boolean Values
2. Unreal Boolean Values.

The real Boolean values are the values of the two possible states of any Boolean quantity. They are the starting values and expected results of every Boolean operations. They admit of interpretations in Logic and are 0 (zero) and 1 (unity).

Unreal Boolean values are values which are not of the two possible states of a Boolean quantity. “They are of a nature altogether foreign to the province of general reasoning ” (Boole, The Laws of Thought). They are -1 and $\frac{1}{0}$.

4.4 Boolean Constants and Variables

A **Boolean constant** is a fixed Boolean value. Each of the two symbols 0 and 1, also used in ordinary algebra [6], [18], [19], [20], [21], is a Boolean constant.

A **Boolean variable** is a symbol, usually a letter, which represents a quantity that can have any one of the set of Boolean values. The letters representing Boolean quantities are of two kinds, namely

1. known Boolean variables
2. unknown Boolean variables.

Known Boolean variables are those which may be assumed to be of any value whatever. In Boolean algebra they are denoted by the first letters of the English alphabet as A, B, C etc. or as A_1, A_2, A_3 etc. read A one, A two, A three, etc. *Unknown Boolean variables* are those whose value can be determined only by actually performing the operations involved in the solution of the problem. They are generally denoted by the last letters of the alphabet as X, Y, Z etc. or as X_1, X_2, X_3 etc. read X one, X two, X three, etc.

4.5 Boolean Symbol of Relation

The Boolean symbol of relation is the equality sign = which shows that the Boolean quantity before it is equal in value to the Boolean quantity after it.Thus $A = B$ denotes that the Boolean quantity represented by A is equal in value to the Boolean quantity represented by B.

4.6 Complement of Boolean Values

The **complement** of a quantity is the inverse of the quantity. It is indicated by a bar over the quantity (overbar). The complement of 0, written as $\overline{0}$, is 1,i.e $\overline{0} = 1$. Similarly, the complement of 1 is 0, i.e $\overline{1} = 0$.

Let A represents a Boolean value. Its complement, written as $\overline{A}$, represents the inverse of any Boolean value assigned to A. Thus, if $A = 0$, then $\overline{A} = 1$ and if $A = 1$, then $\overline{A} = 0$.

4.7 Boolean Expression and Function

4.7.1 Boolean Expression

A **Boolean expression** is any Boolean value written in algebraic symbols. It is formed by combining Boolean variables and logical connectives or signs.Thus,

$$A\overline{B} + B$$

and

$$\frac{A}{A + \overline{B}}$$

are Boolean algebraic expressions.

The Boolean expressions in the n variables $A, B, C, \ldots$ are defined recursively as follows: $0, 1, A, B, C, \cdots$ are Boolean expressions; if E_1 and E_2 are Boolean expressions, then $E_1 + E_2$, E_1E_2, $E_1 - E_2$, and E_1/E_2 are Boolean expressions.

4.7.2 Boolean Function

A **Boolean function** is a function which assumes Boolean values when its input variables assume Boolean values. It is a relationship in which the Boolean values of a single dependent Boolean variable are determined by the Boolean values of one or more independent Boolean variables. Any Boolean algebraic expression involving a Boolean input variable A is termed a *function of* A, and may be represented under the abbreviated general form $f(A)$. Any Boolean expression involving two Boolean input variables, A and B, is similarly termed a *function of A and B*, and may be represented under the general form $f(A, B)$, and so on for any other case. In the equality

$$Y = A + B,$$

the variable Y is the function of the variables A and B and can also be written as

$$Y = f(A, B)$$

A Boolean function is formed with variables, operators $(+, -,$ etc), parenthesis and equality sign. Its value can be either 0 or 1.

A Boolean function can be expressed algebraically from a given membership table by forming a minterm for each combination of the variables that produces a 1 in the function and then logically adding all those terms.

4.8 Laws of Boolean Algebra

Boolean laws are rules by which logical operations can be expressed symbolically in equation form and be manipulated mathematically. They include

1. Boolean axioms
2. Boolean theorems

4.8.1 Axioms of Boolean Algebra

The **Boolean axioms** are logical assumptions which are taken to be true without proof. They are self-evident logical statements and are also called *Boolean postulates*. They are classified into four viz.

1. logical sum axioms
2. logical difference axioms
3. logical product axioms
4. logical quotient axioms.

The *logical sum axioms* are assumptions about sums of Boolean values. They are as follows.

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

The *logical difference axioms* are assumptions about differences of Boolean values. They are as follows.

$$0 - 0 = 0$$
$$0 - 1 = -1$$
$$1 - 0 = 1$$
$$1 - 1 = \{0, 1\}$$

The *logical product axioms* are assumptions about products of Boolean values. They are as follows.

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1.$$

The *logical quotient axioms* are assumptions about quotients of Boolean values. They are as follows.

$$\frac{0}{0} = \{0, 1\}$$
$$\frac{0}{1} = 0$$
$$\frac{1}{0} = \text{infinity}$$
$$\frac{1}{1} = 1$$

It turns out that Boolean operations do indeed satisfy many of the formal laws of operations satisfied by ordinary operations. Once one has proved the laws habits acquired in ordinary algebra can usually be transferred to Boolean algebra.

Boolean subtraction and division, though at first sight it may seem artificial is important and fundamental. The reader will have to take this in trust at first because a certain amount of groundwork must be done before really interesting applications are possible. After a time, however, it will become obvious that Boolean subtraction and division is the natural process for tackling important problems in Boolean algebra. This view is not contradicted by the fact that some of these problems were done originally without Boolean subtraction and division. Problems solved for the first time in mathematics are usually done by a rather difficult method, the solution then being simplified by a later work. The Greeks and Romans (nil) with difficulty add and multiply numbers without having Arabic numerals, but this is no argument against using Arabic numerals today.

4.8.2 Boolean Theorems of Logical Addition and Multiplication

Annulment Law: A Boolean variable added to 1 equals 1 or multiplied by 0 equals 0.

T1: $1 + A = 1$

T2: $A \cdot 0 = 0$

Identity Law: A Boolean variable added to 0 or multiplied by 1 always equals that variable.

T3: $0 + A = A$

T4: $A \cdot 1 = A$

Idempotent Law: A Boolean variable added to or multiplied by itself is equal to the variable.

T5: $A + A = A$

T6: $A \cdot A = A$

Double Negation Law: A double complement of a variable is always equal to the variable.

T7: $\overline{\overline{A}} = A$

Complement Law: A Boolean variable added to its complement equals 1 or multiplied by its complement equals 0.

T8: $\overline{A} + A = 1$

T9: $A \cdot \overline{A} = 0$

4.8.3 Boolean Theorems of Logical Subtraction and Division

Difference Complement Theorem The complement of difference of two variables is equal to the quotient of their complements.

T10: $\overline{A - A} = \dfrac{\overline{A}}{\overline{A}}.$

Quotient Complement Theorem The complement of quotient of two variables is equal to the difference of their complements.

T11: $\overline{\left(\dfrac{A}{A}\right)} = \overline{A} - \overline{A}.$

Difference of Like Terms Theorem The difference of two like minterms, say A each, is equal to $\{0, A\}$.

T12: $A - A = \{0, A\}.$

4.8.4 Laws of Logical Signs

The laws of logical signs are the same in Boolean Algebra as in Ordinary Algebra:

$$
\begin{aligned}
+ \times + &= + \\
+ \times - &= - \\
- \times - &= + \\
- \times + &= -
\end{aligned}
$$

and

$$
\begin{aligned}
+ \div + &= + \\
+ \div - &= - \\
- \div - &= + \\
- \div + &= -
\end{aligned}.
$$

Hence, like signs produce plus and unlike signs minus.

4.9 Boolean Algebraic Equations

The most useful part of Boolean algebra is that which relates to the solution of problems. This is performed by means of Boolean equations. In many problems it is desirable to solve for some of the variables as functions of the remaining variables.

A **Boolean algebraic equation** is an algebraic expression stating the equality between two Boolean expressions. It consists of two Boolean algebraic expressions connected by the sign of equality. An example is the Boolean equation

$$XA = \overline{B}$$

which states that if the Boolean expression A is logically subtracted from the unknown X, the difference will be equal to the Boolean expression $\overline{B}$. The part that precedes the sign of equality is called the *first member* or *left-hand side* of the equation; the part that follows the sign of equality is called the *second member* or *right-hand side* of tho equation.Thus in the above instance, the first member is $X - A$ and the second member is $\overline{B}$.

4.9.1 Solving Boolean Equations

Solving a Boolean equation is the process of finding the Boolean expression(s) of the unknown which will satisfy the equation. Thus to solve a Boolean equation is to find the Boolean expression(s) of the unknown or to find Boolean expression(s) which being substituted for the unknown will render the two members equivalent. Any non-fractional expression with only positive terms of the unknown by which the equation is satisfied is/are called the *real Boolean root(s)* or *real Boolean solution(s)* of the equation.

4.9.2 Solution Set of Boolean Equations

The **solution set** of a Boolean equation is the set of all possible Boolean expressions that cause the equation to be a true statement or satisfied. If we replace X by AB in the equation $X + A\overline{B} = A$ and simplifies each member, we have

$$
\begin{aligned}
AB + A\overline{B} &= A \\
A(B + \overline{B}) &= A \\
A(1) &= A \\
A &= A
\end{aligned}
$$

which is a true equation. Then AB is a Boolean root of the equation. Again, if we replace X by the simple Boolean expression A, we have

$$A + A\overline{B} = A$$

which simplifies as follows:

$$\begin{aligned} A(1 + \overline{B}) &= A \\ A(1) &= A \\ A &= A \end{aligned}$$

which is a true equation. Then A is another Boolean root of the above equation. Here we bring down the curtain on the subject of Boolean equation. For more information on this subject, the reader is referred to [21].

5 Blood Types and their Boolean Representations

5.1 Blood Types as Sets of Antigens

The universal set **U** of antigens contains the two antigens A and B. We wish to determine all the possible subsets of the universal set of antigens, namely $\mathbf{U} = \{\text{antigen}\, A, \text{antigen}\, B\}$. We know that we can list the set itself, $\{\text{antigen}\, A, \text{antigen}\, B\}$ and the empty set $\{\}$, what about the others? If we are to proceed in a manner that has some order, then we should by taking the elements one at a time. Note that taking zero elements at a time gives us the empty set. Since the set $\{\text{antigen}\, A, \text{antigen}\, B\}$ contains two elements, we would have the following:

Zero at a time	One at a time	Two at a time
$\{\}$	$\{\text{antigen}\, A\}, \{\text{antigen}\, B\}$	$\{\text{antigen}\, A, \text{antigen}\, B\}$

Table 2: Subsets of the Universal Set

These are the four subsets for the given universal set; they correspond to the four blood types, **O**, **A**, **B** and **AB**: $\mathbf{O} = \{\}$, $\mathbf{A} = \{\text{antigen}\, A\}$, $\mathbf{B} = \{\text{antigen}\, B\}$, $\mathbf{AB} = \{\text{antigen}\, A, \text{antigen}\, B\}$.

5.2 Operations on Antigenic Sets

5.2.1 Union of Antigenic Sets

The union of sets **M** and **N** is the set of elements that are elements of either set **M** or set **N**, or both. When we list the elements in the union of two sets, we list all of the elements in set **M** and all of the elements in set **N**, but if an element is in both sets, we list it only once. The notation for **M** union **N** is $\mathbf{M} \cup \mathbf{N}$. With regard to the blood sets of antigens, the union of the sets **A** and **B** is the set of antigens that are elements of at least one of the two sets. Thus, $\mathbf{A} \cap \mathbf{B} = \mathbf{U}$.

5.2.2 Intersection of Antigenic Sets

In set theory, if we have two sets **M** and **N**, then the intersection of **M** and **N** is a set of elements that are members of both **M** and **N**. The notation for **M** intersection **N** is $\mathbf{M} \cap \mathbf{M}$. This resulting set is composed of elements that are common to both **M** and **N**. Examining sets **A** and **B** of antigens we see that there are no antigens common to both. Therefore, the intersection of these two sets is the empty set. Hence, $\mathbf{A} \cap \mathbf{B} = \{\}$ which is blood type **O** and, therefore, $\mathbf{A} \cap \mathbf{B} = \mathbf{O}$. Two sets whose intersection is the empty set is said to be disjoint? Disjoint sets have no elements in common.

5.2.3 Complement of Antigenic Sets

The complement of the set **A** is the set of antigens in the universal set, **U**, that are not in the set **A**. The notation for the complement of **A** is $\overline{\mathbf{A}}$. The set $\overline{\mathbf{A}}$, the complement of **A**, is the set of antigens that are in **U**, but not in **A**. The antigen in $\overline{\mathbf{A}}$ is therefore antigen B. Hence, we have $\overline{\mathbf{A}} = \{\text{antigen}\, B\}$. But the set $\{\text{antigen}\, B\}$ is the same as set **B**. Thus, we have $\mathbf{B} = \overline{\mathbf{A}}$. Similarly, the complement of **B** is the set $\overline{\mathbf{B}} = \{\text{antigen}\, A\}$, which is exactly the same as set **A**. Hence, $\overline{\mathbf{B}} = \mathbf{A}$.

The complement of set **AB** is the set of antigens that are in **U**, but not in **AB**. There is no element in $\overline{\mathbf{AB}}$ and consequently, $\overline{\mathbf{AB}} = \mathbf{O}$. The absence of both antigens is the presence of no antigen.

The set **O** is the empty set and therefore contains no elements, and its complement is the set of antigens in **U** that are not in **O**. In this case, the complement of **O** is all the elements or antigens in **U** which is **AB**. Therefore, $\overline{\mathbf{O}} = \{\text{antigen}\, A, \text{antigen}\, B\}$.

5.3 Boolean Assignment

Ordinary usage assigns a meaning to the symbols $+, -, \times, /$, when these are associated with numbers but in Boolean algebra, we are free to give them any meaning that we please. In Boole's original algebra, the symbols $+, -, \times$ and $/$ are used to express various Boolean processes or operations performed in Boolean variables. The sign $+$ means a union of sets, the sign $\times$ means an intersection of sets, the sign $-$ means the inverse operation to union of sets, and the sign $/$ means the inverse operation to intersection of sets.

Type **A** blood is assigned the Boolean variable A and type **B** blood the Boolean variable B. The negation variable $\overline{A}$ means we have antigen that is complementary to A. Similarly, the negation variable $\overline{B}$ means we have antigen that is complementary to B.

Since type **AB** blood is a union of both types or sets of antigens, A and B, we assigned it the Boolean sum $A + B$. We have shown elsewhere that $B = \overline{A}$. Employing this gives us $A + B = A + \overline{A} = 1$, which is the universal set. It follows from $B = \overline{A}$ that $A = \overline{B}$ so that we can also have $A + B = \overline{B} + B = 1$. Thus, we shall assign the Boolean value 1 to blood type **AB**.

There are no antigens in the red cells of those in blood group **O** and so we assign the Boolean value 0 to blood type **O**.

Table shows the blood types and their boolean representations.

Blood Group	Boolean Representation
O	0
A	A or $\overline{B}$
B	B or $\overline{A}$
AB	$A + B$ or 1

Table 3: Subsets of the Universal Set

Below is a list of the most important of the logical (interpretable) identities and results needed in the rest of the works. Practice with examples will serve to make these identities familiar and this is the best way of learning them.

1. Logical addition of blood types:

   (a) $A + 0 = A$

   (b) $A + 1 = 1$

   (c) $A + A = A$

   (d) $A + B = 1$

These results are demonstrated in the Truth Table 4

A	B	$A+0$	$A+1$	$A+A$	$A+B$
0	1	0	1	0	1
1	0	1	1	1	1

Table 4: Logical addition of blood types

2. Logical multiplication of Blood types:
   (a) $A \cdot 0 = 0$
   (b) $A \cdot 1 = A$
   (c) $AA = A$
   (d) $AB = 0$

These results are shown in the Truth Table 5

A	B	$A \cdot 0$	$A \cdot 1$	AA	AB
0	1	0	0	0	0
1	0	0	1	1	0

Table 5: Logical multiplication of Blood types

3. Logical subtraction of blood types:
   (a) $A - 0 = A$
   (b) $1 - A = \{1, \overline{A}\}$
   (c) $A - A = \{0, A\}$

These results are shown in the Truth Table 6

A	$A-0$	$1-A$	$A-A$
0	0	1	0
1	1	$\{0,1\}$	$\{0,1\}$

Table 6: Logical subtraction of blood types

4. Logical division of blood types
   (a) $\frac{0}{A} = \{0, A\}$
   (b) $\frac{A}{1} = A$
   (c) $\frac{A}{A} = \{1, A\}$.

These results are demonsrated in the Truth Table 7

A	$0/A$	$A/1$	A/A
0	$\{0,1\}$	0	$\{0,1\}$
1	0	1	1

Table 7: Logical division of blood types

5.4 Important Theorems

Theorems not only help to solve mathematical problems easily but their proofs also help to develop a deeper understanding of the underlying concepts.

Theorem 5.1. $\overline{A+B} = 0$.

Proof. We apply De Morgan's theorem:

$$\overline{A+B} = \overline{A}\,\overline{B} = \overline{A}A = 0$$

□

Theorem 5.2. $\overline{AB} = A + B$.

Proof. Apply De Morgan's second theorem: $\overline{AB} = \overline{A} + \overline{B} = B + A = A + B$. □

To understand the rest of this work one must be willing to alter one's ideas about Boolean subtraction and division. Hitherto, for most readers, these are impossible, but now they will be used in our investigation. The extension of Boolean algebra to them is desirable because they help to overcome difficulties. The reader will never forget how much he will gain by their use.

6 Blood Cross-matching Function

Cross-matching involves mixing of donor RBCs with the recipient serum to detect fatal reactions. The *blood cross-matching function* is defined as

$$T(D,R) = D + R,$$

where D is the Boolean variable associated with the donor's blood group or type and R is the Boolean variable associated with the recipient's blood group or type.

6.1 Blood Compatibility Principle

The **principle of compatibility of blood group** asserts that the recipient's blood is compatible with the donor's if the recipient's blood type remains the same after the blood transfusion. It follows from this that to ensure blood compatibility, the cross-matching function equals R. That is, for blood compatibility,

$$T(D,R) = R$$

or

$$D + R = R.$$

Figure 1 depicts the flow chart of the methodology of the cross-matching of the blood types of the donor and recipient.

The fact that $D + R = R$ for compatibility leads to some interesting genetic results.

The Boolean expression AB should not be confounded with the blood type **AB**.

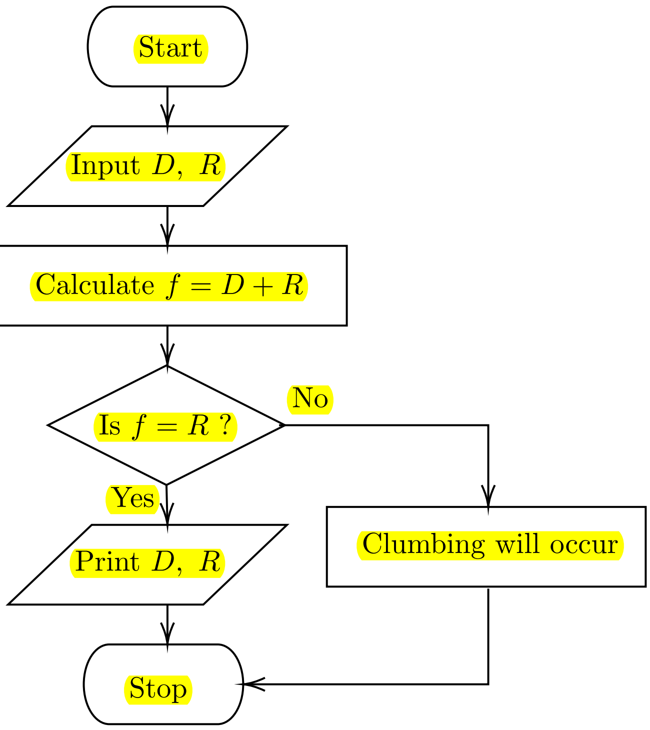

Figure 1: Flow Chart of the Methodology of Blood-Cross Matching

Theorem 6.1. *Every recipient is its own donor:* $D = R$.

Proof. We begin with the blood compatibility equation $D+R = R$. Let $D = R$. we have $R+R = R$ which holds true. Hence every recipient is its own donor. □

Theorem 6.2. *The blood type* **AB** *is a universal recipient.*

Proof. Let us begin with the blood compatibility equation $D + R = R$. If we set $R = 1$, we get $D + 1 = 1$ which is always valid no matter what is assigned to D. Thus, blood group **A** is the universal recipient. □

Theorem 6.3. *The blood type* **O** *is the universal donor.*

Proof. Let us start with the equation $D + R = R$. If we set $D = 0$, we get $0 + R = R$ which is always valid no matter what is assigned to R. Thus, blood group **O** is the universal donor. □

Theorem 6.4. $R - R = \{0, A, B, 1\}R$.

Proof. Let E be any Boolean variable associated with blood antigens. Then $E = \{0, A, B, A + B\}$. Whatever the Boolean variable assigned to E the equation $R + ER = R$ holds good. It follows from this that $R - R = \{0, A, B, A + B\}R$ which, understanding that $A + B = 1$, becomes $R - R = \{0, A, B, 1\}R$.. □

Now the reader is in his element. I believe he is on the edge of his seat.

6.2 Unified Blood Group in Ordered Pair

A unified blood group or type can be represented by an ordered pair (a, b), where a represents the Boolean variable associated with **ABO** type and b represents the Boolean variable associated with **Rh** type. For instance, $(A, 0)$ is the Boolean ordered pair representing the blood type **A**− and $(B, 1)$ is the Boolean ordered pair representing the blood type **B**+; the second members 0 and 1 are the Boolean representations of negative rhesus factor and positive rhesus factor respectively.

6.3 Common Donor for Two or More Recipients

The common donor to two or more recipients $R_1, R_2, R_3 \ldots R_n$ is defined by the function

$$D + R_1R_2R_3 \ldots R_n = R_1R_2R_3 \ldots R_n \tag{6.1}$$

Where $R_1R_2R_3 \ldots R_n$, a logical product, is the blood type common to the n recipients.

7 Illustrative Examples

The very limited extent of this work would admit of noting more than a few instances of what we have been discussing.

Example 7.1. *Determine the blood groups of individuals to whom individuals with blood group* **AB** *can donate blood.*

The donor is type **AB**, written in Boolean Algebra as $A+B$. Let R be the Boolean representation of the blood group of the recipient to whom the donor can donate blood. For compatibility, $D + R = R$. Hence,

$$A + B + R = R$$

which becomes

$$A + B = R - R.$$

But

$$R - R = \{0, A, B, A + B\}R.$$

Thus, we have

$$A + B = \{0, A, B, A + B\}R.$$

The Boolean variable

$$\begin{aligned} R &= \frac{A + B}{\{0, A, B, A + B\}} \\ &= \frac{1}{\{0, A, B, 1\}} \\ &= \{1/0, 1/A, 1/B, 1/1\} \\ &= \{1/0, 1/A, 1/B, 1\}. \end{aligned}$$

The Boolean expressions $1/0$, $1/A$, and $1/B$ are illogical and hence ruled out of court. The only logical expression is 1 which is $A + B$ which in turn represents the blood group **AB**. Thus, we have only one type of donor for **AB** type recipient, namely, the donor whose blood is of type **AB**.

Example 7.2. *Two unconscious patients X and Y whose blood group genotypes are* **AO** *and* **AB** *respectively were transfused with blood from the same donor. Patient X immediately showed signs of difficulty in breathing while patient Y showed no negative reaction. What is the genotype of the blood the patients X and Y were transfused with?.*

For X the Boolean representation is A and for Y the Boolean representation is $A+B$. Applying the compatibility equation for Y: $D+R=R$, we have

$$D+A+B=A+B$$

which becomes

$$D=(A+B)-(A+B)$$

which in turn becomes

$$D=A-A+B-B=\{0,A\}+\{0,B\}.$$

This simplifies to

$$D=\{0,A,B,A+B\}.$$

Let us combine each of D with A to see which will add up to a different blood group from A. We have $0+A=A$, $A+A=A$, $B+A=1$, $A+B+A=1$. Hence, the Boolean summations that give a blood group different from A are $A+B$ and $A+B+A$. Thus, the blood group donor is either **B** or **AB**. The genotype is either **BO**, **BB** or **AB**.

Example 7.3. *Determine the blood groups of the possible donors that can give blood safely to a recipient with blood group* **B**.

Let $R=A$. By Theorem 6.4 the blood groups of the donors of blood type **B** can be found:

$$\begin{aligned}D &= R-R\\ &= \{0,A,B,1\}R\\ &= \{0,A,B,1\}B\\ &= \{0\cdot B, A\cdot B, B\cdot B, 1\cdot B\}\\ &= \{0, A\cdot\overline{A}, B, B\}\\ &= \{0,0,B,B\}\\ &= \{0,B\}\end{aligned}$$

.

From this result we conclude that the possible donors that can give blood safely to a recipient with blood group **B** are those belonging to groups **O** and **B**.

Example 7.4. *Determine the common donor's blood type to the recipients with blood types* **A** *and* **AB**.

Let $R_1=A$ and $R_2=A+B=1$. Applying $D+R_1R_2=R_1R_2$, we get

$$D+A\cdot 1=A\cdot 1$$

which becomes

$$D=A-A=\{0,A\}.$$

The donor common to both type **A** and type **AB** are type **O** and type **A**.

Example 7.5. *Two individuals of blood groups* **A** *and* **X** *where* **X** *is unknown are to be given a blood transfusion. If the donor's blood group compatible with both recipients is* **A**, *determines* **X**.

We start with the blood compatibility equation $D+R_1R_2=R_1R_2$. Setting $D=A, R_1=A$ and $R_2=X$ we obtain $A+AX=AX$ which becomes $A=AX-AX$. This equation becomes $A=(A-A)X$. Making X the subject of this equation furnishes

$$X=\frac{A}{A-A}=\frac{A}{\{0,A\}}=\{A/0,A/A\}.$$

Now the fraction $A/0$ is illogical. Therefore, we have $X=A/A$ which becomes $X=\{1,A\}$. Thus, we conclude that X is either type **AB** or type **A**.

Example 7.6. *Let us suppose a nurse caring for two patents who need a blood transfusion. The patients have been tested and found to have blood types* **A**+ *and* **AB**−. *What do they have in common?*

To solve this problem, we first express the blood type in Boolean language, viz **A**+ represents $(A, 1)$ and **AB** represents $(A + B, 0)$. The product of $(A, 1)$ and $(A + B, 0)$ gives

$$\begin{aligned}(A, 1)(A + B, 0) &= (A(A + B), 1 \times 0)\\ &= (A + AB, 0)\\ &= (A, 0).\end{aligned}$$

The result $(A, 0)$ is interpreted as antigen A is on the red blood cells of both blood types. The Rhesus factor common to both of them is that of negative.

Example 7.7. *A nurse is concerned about the type of blood that a patient is to receive. Which blood type may a patient with blood type* **O**+ *safely receive?*

Let us start with with the blood compatibility equation $D+R = R$. The Boolean representation of **O**+ is $(0, 1)$. Thus we have $D + (0, 1) = (0, 1)$ which becomes

$$D = (0, 1) - (0, 1)$$

which in turn becomes

$$D = (0 - 0, 1 - 1) = (0, \{0, 1\}).$$

Thus D is either $(0, 0)$ and $(0, 1)$.

Now, $(0, 0)$ is the Boolean representation of blood type **O**− and $(0, 1)$ of blood type **O**+. Thus, the patient with blood type **O**+ can receive blood safety from donors with blood types **O**− and **O**+.

Example 7.8. *A patient is brought to the emergency department following a motor vehicle accident and has lost a large volume of blood. The patient blood type is* **AB**−. *Which blood type may this patient safely receive in transfusion.*

We start with the compatibility equation $D+R = R$. Setting $R = (A+B, 0$ gives $D+(A+B, 0) = (A + B, O)$ which becomes

$$\begin{aligned}D &= (A + B, 0) - (A + B, 0)\\ &= (A + B - A - B, 0 - 0)\\ &= (A - A + B - B, 0)\\ &= (\{0, A\} + \{0, B\}, 0)\\ &= (\{0 + 0, 0 + B, A + 0, A + B\}, 0)\\ &= (\{0, B, A, A + B\}, 0).\end{aligned}$$

Hence, we have D as $(0, 0), (B, 0), (A, 0)$ and $(A + B, 0)$. Thus, the blood types the patient with **AB**− can receive safely are **O**−, **B**−, **A**− and **AB**−.

Example 7.9. *A donor of blood group* **Y** *gives blood safely to two recipients of blood groups* **AB**− *and* **X**. *If* **B**− *is common to the recipients' blood groups, find* **X** *and* **Y**.

This problem is a sitting duck. The antigenic ordered pairs representing the blood groups **AB**− and **B**− are respectively $(A+B, 0)$ and $(B, 0)$. The antigenic equation for the common blood group to the two recipients is $X(A + B, 0) = (B, 0)$ which is the same as $X(1, 0) = (B, 0)$. Thus

$$X = \frac{(B, 0)}{(1, 0)} = \left(\frac{B}{1}, \frac{0}{0}\right) = (B, \{0, 1\}).$$

It follows that X equals $(B, 0)$ and $(B, 1)$ which represent respectively blood groups $\mathbf{B}-$ and $\mathbf{B}+$.

Now we have to find the blood group represented by **Y**. To achieve this we apply the blood compatibility equation $D + R = R$. Setting $D = Y$ and $R = X(1, 0) = (B, 0)$, we get

$$Y + (B, 0) = (B, 0)$$

which becomes

$$Y = (B, 0) - (B, 0) = (B - B, 0 - 0) = (\{0, B\}, 0).$$

Therefore, Y is $(0, 0)$ and $(B, 0)$ which represent respectively the blood groups $\mathbf{O}-$ and $\mathbf{B}-$.

8 Conclusion

In this work, we discussed a methodology for cross-matching the blood types of a recipient and a donor. The method is based on the extended Boolean algebra which includes the logical operations of subtraction and division.

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