

The Application of Homotopy Perturbation Method: An Overview

ABSTRACT

As an effective method for solving linear and nonlinear equations, the homotopy perturbation method is usually applied to solving relevant problems. We analyze 74 studies on the application of the homotopy perturbation method and present a comprehensive review of them with the conclusion obtained: (1) Homotopy perturbation method is generally applied to solving the problems of ordinary differential equations; (2) Homotopy perturbation method is usually combined with the technology of transform when it is used to solve more complicated equations; (3) By comparing homotopy perturbation method with other similar methods, many researchers sought that homotopy perturbation method is rapidly convergent, highly accurate, computational simple; (4) Some studies point out that when homotopy perturbation method is applied, some parameters including the number of terms, time span, time step must be prescribed carefully. Finally, two suggestions on the further study of the application of the HPM are provided.

Keywords: Homotopy perturbation method, Models, Application mode, Application performance, Shortcoming

1. INTRODUCTION

The homotopy perturbation method (HPM), proposed by Chinese mathematician He in 1998, is an analytical technique for differential equations at first [1]. The basic principle of HMP is combining the traditional perturbation method and the homotopy technique to transform a difficult solving problem into a simple one. Therefore, in recent years, this method has been applied extensively to various models in the field of physics and engineering including not only the differential equations [5,6,24,25,35,75].

Although there are numerous publications on the application of the HPM, there is not a comprehensive review of the studies on them yet. To this end, this research is to collect the studies on the application of the HPM and analyze them to make the situation of the previous studies clear and give some suggestions to further study.

In order to elaborate current situation of the application of the HPM to the largest degree, we shall consider some significant problems of the application of the method: What models did HPM apply to? Which models were solved most usually by the HPM method? How was the

HPM method used for solving problems? What was the effect of the HPM method in the application? And is there any weakness of the HPM in the case when it was utilized?

In this paper, we try to answer these questions based on the references, so a detailed introduction of the application of the HPM is presented, which offers some suggestions to further investigate its application.

2. BASIC IDEA OF THE HPM

To illustrate the basic ideas of the HPM method, we consider the following nonlinear differential equation which can be integral or differential equations [2]

$$L(u) = 0. \quad (1)$$

Where L is any integral or differential operator. We construct a convex homotopy functional $H(u, p)$ with an embedding parameter (homotopy parameter) $p \in [0, 1]$ as follows

$$H(u, p) = (1 - p)F(u) + pL(u).$$

(2)

Where $F(u)$ is a functional operator with a solution v_0 , which can be obtained easily.

It is clear that for $H(u, p) = 0$, we have two boundary conditions

$$H(u, 0) = F(u), H(u, 1) = L(u).$$

This shows that $H(u, p)$ traces an implicitly defined curve continuously from a starting function $H(v_0, 0)$ to the final function $H(f, 1)$. The embedding parameter monotonically increases from zero to unity as the trivial problem $F(u) = 0$ continuously deforms the original problem $L(u) = 0$. Expanding u in Taylor series with respect to p , one has

$$u = \sum_{i=0}^{\infty} p^i u_i = u_0 + pu_1 + p^2u_2 + \dots$$

(3)

If $p \rightarrow 1$, then (3) corresponds to (2) and becomes the approximate solution of the form

$$f = \lim_{p \rightarrow 1} u = u_0 + u_1 + u_2 + \dots$$

(4)

In general, series (4) is convergent for most of the cases, and also the rate of convergence is dependent on the operator L . For more details on the convergence of the HPM, we refer to [3,4].

3. THE APPLICATION OF THE HPM

We take “homotopy perturbation method”, “homotopy perturbation technique” and “homotopy perturbation approach” as keywords to search for literature on the websites of China National Knowledge Infrastructure, Baidu Academic Search, Google Academic Search and the

websites of web of SCI. Then 107 papers were found. And after excluding the literature irrelevant to the application of the HPM, there were 74 papers finally. We analyzed these researches on the solved models, patterns, performance and the shortcomings of the application of the HPM.

3.1 The models solved by the HPM

Once the HPM was generated, it was applied to solve a Lighthill equation and a Duffing equation, which belong to nonlinear ordinary differential equations [1]. Then, HPM was also applied to solve various types of models governed by equations.

For example, Ji-Huan He et al. applied the HPM to the generalized N/MEMS oscillators, Duffing oscillator, Fangzhu oscillator, nonlinear oscillators with coordinate-dependent mass, which were all elucidated by ordinary differential equations(ODEs) [5-8]. And there was much research on its application to partial differential equations(PDEs) [2,9-15]. Gurmeet Kaur et al. applied the HPM to the fragmentation as well as aggregation population balance equation[12]. Sumit Gupta et al. applied it for solving convection-diffusion equations [13]. Hassan Kamil Jassim used it to obtain the solution of the Newell-Whitehead Segel equation [18]. Additionally to the models governed by ODEs and PDEs, the fractional equations can be also solved by the HPM [21,34-49]. Yasir KHAN et al. extended the application of the HPM to obtain the analytical solutions to Klein-Gordon fractional partial differential equation [2]. Chun-Fu WEI applied it to solve the non-linear and singular fractional Lane-Emden type equations [20]. Asma Ali Elbeleze et al. tried to obtain the solution of the fractional Black-Scholes equation [36]. Other models such as integral equations and delay differential equations were considered to be solved by the HPM in recent years[54-68]. Samad Noeiaghdam et al. utilized it to study the second kind of linear Volterra integral equations with discontinuous kernel [51]. The solution of a delay differential equation was presented using the HPM and some numerical illustrations were given by Fatemeh Shakeri, Mehdi Dehghan [54].

Based on the analysis above, we conclude that the models of application of the HPM comprise ODEs, PDEs, fractional equations, integral equations, and other models. Thus we divided the 74 pieces of research into five types according to the models they solve. The numbers of the studies with different kinds of models solved by the HPM are shown in Table 1.

Table 1. The numbers of the researches with different models

Model type	ODEs	PDEs	Fractional equations	Integral equations	Other models
Number of the researches	27	26	18	4	5

**In some studies, HPM is applied to solve multiple model problems in one paper.*

As can be seen from Table 1, the HPM method is applied to solving ODEs the most times, followed by PDEs and fractional equations, and it is rarely applied to solving integral equations or other models.

3.2 The way the HPM method is applied

Purely HPM was applied to various problems without any change or modification in the early time. We had found that 39 pieces of research among the selected literature applied HPM only [2,6-8,59-69].

However, there is a prevailing trend to the improvement of the current HPM in many ways. The HPM coupled with the variational iteration method by Yong-Ju Yang and Shun-Qin Wang was utilized to study local fractional nonlinear oscillators [37]. In the research of Ali Demir and his team, the homotopy was constructed based on the decomposition of a source function [11]. And a new method called multistage HPM which treated the HPM as an algorithm in a sequence of intervals for finding accurate approximate solutions was proposed and applied to some problems [56,70-72].

The hybrid methods combining the HPM with other approaches have been attractive for solving more complicated problems, for example, the combination of it with the technology of certain transform. Since the Laplace transform enables differential equations to be an algebraic equation that is easy to solve, it has been incorporated into HPM in numerous applications, such as to find the solutions of the time-fractional wave equations, the convection-diffusion equations, the Klein-Gordon equations, the Newell-Whitehead-Segel equations and so on [5,16,18,25,27,34,38,50,73]. Additionally, there were other transform patterns combined with HPM. For instance, a method derived by combining Elzaki transform and HPM was presented and applied to the system of nonlinear PDEs [19,49,33]. The Sumudu transform was also considered to be an additional technique to HPM for some nonlinear fractional equations [40,41,47].

3.3 The performance of the application of the HPM

In order to investigate the performance of HPM in solving problems, we analyzed the comments on its application in the found research. Yasir KHAN et al. mentioned that the method was capable of reducing the volume of the computational work as compared to the classical methods [34]. Erdem Cuce and Pinar Mert Cuce's results have revealed that the HPM was a very practical and reliable approach that presents an accurate approximate solution [59]. M. Fathizadeh et al. concluded that it gave series solutions that converged very rapidly in finding the analytical solutions for boundary value problems [10]. The research of Yong-Ju Yang and Shun-Qin Wang showed that it worked well on the local fractional differential equations, which avoided cumbersome computational works [37]. Moreover, HPM was also demonstrated to be simple and accurate extremely for the exact numerical solution in a wide range of values of oscillation amplitude [26]. When applied to Volterra's integro-differential equation, HPM did not require small parameters in the equations, which may eliminate the limitations of the standard perturbation methods [60]. It was also found that HPM was applied in a direct way without using linearization, transformation, discretization or restrictive assumptions [2,10,13,14,59,70,74].

At the same time, some researchers made comparisons of the HPM with other methods to obtain the particular properties of the application of HPM to better use it in certain cases. A detailed comparative study between perturbation method and HPM showed that the errors of HPM increased with a lower rate and it became more accurate as the rate of nonlinearity was

higher [17]. Compared with Adomian decomposition method, HPM did not use Adomian polynomials to find the analytical solutions for integral equations and needed less computation [22,52,66]. And few approaches for a generalized oscillatory system for N/MEMS comprising energy balance method and spreading residual harmonic balance method are considered for comparison with HPM by Naveed Anjum and Ji-Huan He, the results revealed that HPM gave better accuracy than other methods [5].

According to the analysis above, we can conclude some common **characteristics** of the application of HPM listed here: HPM (1) has a rapid speed to calculate; (2) has less computational volume compared to other methods; (3) has similar high accuracy with other methods or even better; (4) can solve the equations without additional assumptions.

3.4 The shortcomings of the application of the HPM.

Although HPM has many advantages in application, the shortcomings can be inevitable, which are supposed to be learned so that one may avoid these problems or further improve the algorithm when applying HPM.

Throughout the 74 **pieces of research** selected, only a small number of them have pointed out the drawbacks of the HPM. It found that more components of **an** expression obtained by the HPM must be taken into account to achieve more accurate solutions, which may make the solving process complicated [14,61]. M.S.H. Chowdhury and I. Hashimstill noted that care had to be taken on the choice of time span, time step, and the number of terms used when HPM was applied to highly chaotic systems such as the Chen system [15]. When the HPM was employed to a sixth-order boundary value problem involving a parameter c , the numerical results showed that the solutions obtained were in poor agreement with the exact solutions for large values of c [52]. L. Cveticanin found that the theoretical reasons why the HPM was successfully used for solving pure nonlinear differential equations were hard to explain [26].

4. CONCLUSION

By an analysis as shown in the previous section, some conclusions can be summarized about the application of HPM as follows:

- (1) The models of application of HPM include ODEs, PDEs, fractional equations, integral equations and other models, in which the number of ODEs is the largest followed closely by that of PDEs.
- (2) Half of the **studies** on the application of the HPM offer an improvement of the method. And the hybrid methods combining HPM with the technology of transform have become increasingly popular as they can intensify its capability of solving complicated equations.
- (3) **Regarding** the performance of the application of HPM, many **researchers** described it as a rapidly convergent, highly accurate, computational simple method when compared to other similar methods.
- (4) The disadvantage of the application of HPM is that the number of terms of the expansion of intermediate process, time span, time step should be chosen carefully, otherwise HPM is not feasible.

Therefore, we put forward two suggestions based on the conclusions above: First, since there is little research on using the HPM to solve integral equations and fractional equations, we suggest researchers use it to solve these two kinds of equations more; Second, it is a promising subject for the researchers to further study how to select several important parameters, such as the number of terms of the expansion of intermediate process, time span, and time step, to make the convergence range of the HPM larger.

In summary, in this study, we have selected and analyzed 74 studies in the four essential aspects of the HPM involving the models it tried to solve, its application mode, its application performance and its shortcomings, and four conclusions are obtained. Furthermore, we provide two suggestions for the application of the HPM.

REFERENCES

- [1] He JH. Homotopy perturbation technique. *Computer methods in applied mechanics and engineering*. 1999; 178(3-4): 257-262.
- [2] Mohyud-din ST, Noor MA. Homotopy perturbation method for solving partial differential equations. *Zeitschrift für Naturforschung A*. 2009; 64(3-4): 157-170.
- [3] Biazar J, Ghazvini H. Convergence of the homotopy perturbation method for partial differential equations. *Nonlinear Analysis: Real World Applications*. 2009; 10(5): 2633-2640.
- [4] Turkyilmazoglu M. Convergence of the homotopy perturbation method. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2011; 12(1-8): 9-14.
- [5] Anjum N, He JH. Homotopy perturbation method for N/MEMS oscillators. *Mathematical Methods in the Applied Sciences*. 2020.
- [6] He JH. Homotopy perturbation method for bifurcation of nonlinear problems. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2005; 6(2): 207-208.
- [7] He JH, El-dib YO. Homotopy perturbation method for Fangzhu oscillator. *Journal of Mathematical Chemistry*. 2020; 58(10): 2245-2253.
- [8] Wu Y, He JH. Homotopy perturbation method for nonlinear oscillators with coordinate dependent mass. *Results Phys*. 2018; 10: 270-271.
- [9] Odibat ZM. A new modification of the homotopy perturbation method for linear and nonlinear operators. *Applied Mathematics and Computation*. 2007; 189(1): 746-753.
- [10] Fathizadeh M, Madani M, Khan Y, Faraz N, Yıldırım A, Tutkun S. An effective modification of the homotopy perturbation method for MHD viscous flow over a stretching sheet. *Journal of King Saud University-Science*. 2013; 25(2): 107-113.
- [11] Demir A, Erman S, Özgür B, Korkmaz E. Analysis of the new homotopy perturbation method for linear and nonlinear problems. *Boundary Value Problems*. 2013; 2013(1): 1-11.
- [12] Kaur G, Singh R, Singh M, Kumar J, Matsoukas T. Analytical approach for solving

- population balances: a homotopy perturbation method. *Journal of Physics A: Mathematical and Theoretical*. 2019; 52(38): 385201.
- [13] Gupta S, Kumar D, Singh J. Analytical solutions of convection-diffusion problems by combining Laplace transform method and homotopy perturbation method. *Alexandria Engineering Journal*. 2015; 54(3): 645-651.
- [14] Ganji DD, Sadighi A. Application of He's homotopy-perturbation method to nonlinear coupled systems of reaction-diffusion equations. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2006; 7(4): 411-418.
- [15] He, JH. Application of homotopy perturbation method to nonlinear wave equations. *Chaos, Solitons & Fractals*. 2005; 26(3): 695-700.
- [16] Singh J, Kumar D, Rathore S. Application of homotopy perturbation transform method for solving linear and nonlinear Klein-Gordon equations. *Journal of Information and Computing Science*. 2012; 7(2): 131-139.
- [17] Ganji DD, Rajabi A. Assessment of homotopy-perturbation and perturbation methods in heat radiation equations. *International Communications in Heat and Mass Transfer*. 2006; 33(3): 391-400.
- [18] Jassim HK. Homotopy perturbation algorithm using Laplace transform for Newell-Whitehead-Segel equation. *International Journal of Advances in Applied Mathematics and Mechanics*. 2015; 2(4): 8-12.
- [19] Elzaki TM, Biazar J. Homotopy perturbation method and Elzaki transform for solving system of nonlinear partial differential equations. *World Applied Sciences Journal*. 2013; 24(7): 944-948.
- [20] Altaie SA, Jameel AF, Saaban A. Homotopy perturbation method approximate analytical solution of fuzzy partial differential equation. *IAENG International Journal of Applied Mathematics*. 2019; 49(1): 22-28.
- [21] Momani S, Odibat Z. Homotopy perturbation method for nonlinear partial differential equations of fractional order. *Physics Letters A*. 2007; 365(5-6): 345-350.
- [22] He JH. Homotopy perturbation method for solving boundary value problems. *Physics letters A*. 2006; 350(1-2): 87-88.
- [23] Biazar J, Eslami M, Ghazvini H. Homotopy perturbation method for systems of partial differential equations. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2007; 8(3): 413-418.
- [24] Bera PK, Sil T. Homotopy perturbation method in quantum mechanical problems. *Applied Mathematics and Computation*. 2012; 219(6): 3272-3278.
- [25] Gómez-Aguilar JF, Yépez-Martínez H, Torres-Jiménez J, Córdova-Fraga T, Escobar-Jiménez RF, Olivares-Peregrino VH. Homotopy perturbation transform method for nonlinear differential equations involving to fractional operator with exponential kernel. *Advances in Difference Equations*. 2017; 2017(1): 1-18.

- [26] Cveticanin L. Homotopy-perturbation method for pure nonlinear differential equation. *Chaos, Solitons & Fractals*. 2006; 30(5): 1221-1230.
- [27] Liu ZJ, Adamu MY, Suleiman E, He JH. Hybridization of homotopy perturbation method and Laplace transformation for the partial differential equations. *Thermal Science*. 2017; 21(4): 1843-1846.
- [28] Gorji M, Ganji DD, Soleimani S. New application of He's homotopy perturbation method. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2007; 8(3): 319-328.
- [29] Mirzazadeh M, Ayati Z. New homotopy perturbation method for system of Burgers equations. *Alexandria Engineering Journal*. 2016; 55(2): 1619-1624.
- [30] Edalatpanah SA, Rashidi MM. On the application of homotopy perturbation method for solving systems of linear equations. *International Scholarly Research Notices*. 2014.
- [31] Nourazar SS, Soori M, Nazari-golshan A. On the exact solution of Newell-Whitehead-Segel equation using the homotopy perturbation method. *Australian Journal of Basic and Applied Sciences*. 2011; 5(8): 1400-1411.
- [32] Shqair M. Solution of different geometries reflected reactors neutron diffusion equation using the homotopy perturbation method. *Results in Physics*. 2019; 12: 61-66.
- [33] Elzaki TM, Kim H. The solution of Burger's equation by Elzaki homotopy perturbation method. *Applied Mathematical Sciences*. 2014; 8(59): 2931-2940.
- [34] Khan Y. A coupling method of homotopy perturbation and Laplace transformation for fractional models. *University "Politehnica" of Bucharest Scientific Bulletin, Series A: Applied Mathematics and Physics*. 2012; 74(1): 57-68.
- [35] Ateş I, Zegeling PA. A homotopy perturbation method for fractional-order advection-diffusion-reaction boundary-value problems. *Applied Mathematical Modelling*. 2017; 47: 425-441.
- [36] El-Sayed AM, Elsaid A, El-Kalla IL, Hammad D. A homotopy perturbation technique for solving partial differential equations of fractional order in finite domains. *Applied Mathematics and Computation*. 2012; 218(17): 8329-8340.
- [37] Yang YJ, Wang SQ. An improved homotopy perturbation method for solving local fractional nonlinear oscillators. *Journal of Low Frequency Noise, Vibration and Active Control*. 2019; 38(3-4): 918-927.
- [38] Jleli M, Kumar S, Kumar R, Same B. Analytical approach for time fractional wave equations in the sense of Yang-Abdel-Aty-Cattani via the homotopy perturbation transform method. *Alexandria Engineering Journal*. 2020; 59(5): 2859-2863.
- [39] Wei CF. Application of the homotopy perturbation method for solving fractional Lane-Emden type equation. *Thermal Science*. 2019; 23(4): 2237-2244.
- [40] Elbeleze AA, Kiliçman A, Taib BM. Homotopy perturbation method for fractional Black-

- Scholes European option pricing equations using Sumudu transform. *Mathematical problems in engineering*. 2013.
- [41] Singh J, Kumar D, Kiliçman A. Homotopy perturbation method for fractional gas dynamics equation using Sumudu transform. In: *Abstract and Applied Analysis*. 2013.
- [42] Wang Q. Homotopy perturbation method for fractional KdV-Burgers equation. *Chaos, Solitons & Fractals*. 2008; 35(5): 843-850.
- [43] Zhang X, Zhao J, Liu J, Tang B. Homotopy perturbation method for two dimensional time-fractional wave equation. *Applied Mathematical Modelling*. 2014; 38(23): 5545-5552.
- [44] Yang XJ, Srivastava HM, Cattani C. Local fractional homotopy perturbation method for solving fractal partial differential equations arising in mathematical physics. *Romanian Reports in Physics*. 2015; 67(3): 752-761.
- [45] Zhang Y, Cattani C, Yang XJ. Local fractional homotopy perturbation method for solving non-homogeneous heat conduction equations in fractal domains. *Entropy*. 2015, 17(10): 6753-6764.
- [46] Odibat Z, Momani S. Modified homotopy perturbation method: application to quadratic Riccati differential equation of fractional order. *Chaos, Solitons & Fractals*. 2008; 36(1): 167-174.
- [47] Kumar D, Singh J, Kumar S. Numerical computation of fractional multi-dimensional diffusion equations by using a modified homotopy perturbation method. *Journal of the Association of Arab Universities for Basic and Applied Sciences*. 2015; 17: 20-26.
- [48] Sakar MG, Uludag F, Erdogan F. Numerical solution of time-fractional nonlinear PDEs with proportional delays by homotopy perturbation method. *Applied Mathematical Modelling*. 2016; 40(13-14): 6639-6649.
- [49] Jena RM, Chakraverty S. Solving time-fractional Navier–Stokes equations using homotopy perturbation Elzaki transform. *SN Applied Sciences*. 2019; 1(1): 1-13.
- [50] Gupta S, Kumar D, Singh J. Analytical solutions of convection–diffusion problems by combining Laplace transform method and homotopy perturbation method. *Alexandria Engineering Journal*. 2015; 54(3): 645-651.
- [51] Noeiaghdam S, Dreglea A, He J, Avazzadeh Z, Suleman M, Fariborzi Araghi M A, et al. Error estimation of the homotopy perturbation method to solve second kind Volterra integral equations with piecewise smooth kernels: Application of the CADNA library. *Symmetry*. 2020; 12(10): 1730.
- [52] Noor MA, Mohyud-din ST. Homotopy perturbation method for solving sixth-order boundary value problems. *Computers & Mathematics with Applications*. 2008; 55(12): 2953-2972.
- [53] Yildirim A. Solution of BVPs for fourth-order integro-differential equations by using homotopy perturbation method. *Computers & Mathematics with Applications*. 2008; 56(12): 3175-3180.

- [54] Shakeri F, Dehghan M. Solution of delay differential equations via a homotopy perturbation method. *Mathematical and computer Modelling*. 2008; 48(3-4): 486-498.
- [55] Biazar J, Ghanbari B. The homotopy perturbation method for solving neutral functional–differential equations with proportional delays. *Journal of King Saud University-Science*. 2012; 24(1): 33-37.
- [56] Compean FI, Olvera D, Campa FJ, De Lacalle LL, Elias-Zuniga A, Rodriguez C A. Characterization and stability analysis of a multivariable milling tool by the enhanced multistage homotopy perturbation method. *International Journal of Machine Tools and Manufacture*. 2012; 57: 27-33.
- [57] Ghorbani A, Saberi-nadjafi J. He's homotopy perturbation method for calculating adomian polynomials. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2007; 8(2): 229-232.
- [58] Noor MA, Noor KI, Khan S, Waseem M. Modified homotopy perturbation method for solving system of linear equations. *Journal of the Association of Arab Universities for Basic and Applied Sciences*. 2013; 13(1): 35-37.
- [59] Cuce E, Cuce PM. A successful application of homotopy perturbation method for efficiency and effectiveness assessment of longitudinal porous fins. *Energy Conversion and Management*. 2015; 93: 92-99.
- [60] El-shahed M. Application of He's homotopy perturbation method to Volterra's integro-differential equation. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2005; 6(2): 163-168.
- [61] Dehghan M, Rahmani Y, Ganji DD, Saedodin S, Valipour MS, Rashidi S. Convection–radiation heat transfer in solar heat exchangers filled with a porous medium: homotopy perturbation method versus numerical analysis. *Renewable Energy*. 2015; 74: 448-455.
- [62] He JH, El-dib YO, Mady AA. Homotopy perturbation method for the fractal toda oscillator. *Fractal and Fractional*. 2021; 5(3): 93.
- [63] Sheikholeslami M, Ashorynejad HR, Ganji DD, Yıldırım A. Homotopy perturbation method for three-dimensional problem of condensation film on inclined rotating disk. *Scientia Iranica*. 2012; 19(3): 437-442.
- [64] Chun C, Sakthivel R. Homotopy perturbation technique for solving two-point boundary value problems—comparison with other methods. *Computer Physics Communications*. 2010; 181(6): 1021-1024.
- [65] Nourazar SS, Soori M, Nazari-golshan A. On the exact solution of Burgers-Huxley equation using the homotopy perturbation method. *Journal of Applied Mathematics and Physics*. 2015; 3: 285-294.
- [66] Rafei M, Ganji DD, Daniali H. Solution of the epidemic model by homotopy perturbation method. *Applied mathematics and computation*. 2007; 187(2): 1056-1062.
- [67] Ganji DD. The application of He's homotopy perturbation method to nonlinear equations

- arising in heat transfer. *Physics letters A*. 2006; 355(4-5): 337-341.
- [68] Yao S, Cheng Z. The homotopy perturbation method for a nonlinear oscillator with a damping. *Journal of Low Frequency Noise, Vibration and Active Control*. 2019; 38(3-4): 1110-1112.
- [69] Saedodin S, Shahbabaie M. Thermal analysis of natural convection in porous fins with homotopy perturbation method (HPM). *Arabian Journal for Science and Engineering*. 2013; 38(8): 2227-2231.
- [70] Chowdhur MSH, Hashim I. Application of multistage homotopy-perturbation method for the solutions of the Chen system. *Nonlinear Analysis: Real World Applications*. 2009; 10(1): 381-391.
- [71] Hashim I, Chowdhury MSH. Adaptation of homotopy-perturbation method for numeric-analytic solution of system of ODEs. *Physics Letters A*. 2008; 372(4): 470-481.
- [72] Hashim I, Chowdhury MSH, Mawa S. On multistage homotopy-perturbation method applied to nonlinear biochemical reaction model. *Chaos, Solitons & Fractals*. 2008; 36(4): 823-827.
- [73] Filobello-Nino U, Vazquez-Leal H, Sarmiento-Reyes A, Cervantes-Perez J, Perez-Sesma A, Jimenez-Fernandez VM et al. Laplace transform-homotopy perturbation method with arbitrary initial approximation and residual error cancelation. *Applied Mathematical Modelling*. 2017; 41: 180-194.
- [74] Mahmoodabadi MJ. Epidemic model analyzed via particle swarm optimization based homotopy perturbation method. *Informatics in Medicine Unlocked*. 2020; 18: 100293.
- [75] Abu-Saleem M, Al-omeri W. Topological folding on the chaotic projective spaces and their fundamental group. *Missouri Journal of Mathematical Sciences*. 2019; 31: 130-135.