

Generalized Estimators for Finite Population Variance Using Measurable and Affordable Auxiliary Character

Abstract

In this paper, a generalized exponential-type estimator for estimating the population variance using measurable and affordable auxiliary character in single phase sampling has been proposed. Some special cases of the proposed generalized estimator with $\alpha = 1$ and $\alpha = 2$ have also been discussed. The expressions for the mean square error and bias of the proposed generalized estimator have been derived. The efficiency of the proposed generalized estimator has been compared theoretically with the existing estimators and the conditions under which the proposed estimators are better than some existing estimators have also been given. Numerical examples with five real data sets to compute MSEs and PREs were conducted to ascertain the efficiency of the proposed estimators over others and the results showed that both of the proposed estimators were more efficient than the estimators considered in literature. Thus, the estimator with $\alpha = 2$ perform better than the estimator with $\alpha = 1$.

Keywords: Variance estimator, Population variance, Auxiliary character, MSE, Efficiency.

1. Introduction

In survey sampling, the use of auxiliary characters is frequently recognized to increase the accuracy or precision of the population characteristics estimation procedures. Many researchers have considered the use of auxiliary characters in different forms to construct efficient estimators for population parameters (See Sisodia and Dwivedi [1], Murthy [2], Yadav and Kadilar [3], Singh and Tailor [4], Bahl and Tuteja [5], Singh et al. [6], Singh et al [7], Kadilar and Cingi [8], Singh and Solanki [9], Sahai and Ray [10], Srivastava and Jhajj [11], Ahmed et al. [12], Audu and Adewara [13], Audu et al. [14], Muili et al. [15], Khoshnevisan et al. [16], Singh and Audu [17], Ahmed et al.[18] and Audu and Singh [19], Das and Tripathi [20], Das and Tripathi [21], Patel and Rina [22], Rajyaguru and Gupta [23], Rajyaguru and Gupta [24], Archana and Rao [25], Singh et al. [26], Audu et al. ([27]-[32]), Singh et al. [33], Muili et al. [34], Ishaq et al. [35], Audu et al. [36], Zakari et al. [69]). The issue of estimating variance of populations' characters is of great interest and as received attentions of researchers due to difficulties in controlling variabilities in application. Researches in the fields like industries, laboratories, agriculture and biological experiments are associated with this obstacle and therefore the objective results appear to be uncontrollable. As a result, researchers are keen in the variability associated with their products. The issue of constructing efficient population variance estimators with higher predictive ability has been discussed by various authors such as Liu [47], Das & Tripathi [42], Grover [44], Isaki [45], Upadhaya & Singh [60], Singh & Singh [55], Ahmed et al. [37], Al-Jararha & Ahmed [40], Ahmed et al. [38], Kadilar & Cingi [46], Bahl & Tuteja [41], Upadhaya et al. [61], Ahmed and Singh [39], Swain [59], Subramani & Kumarapandiyam [58], Shabbir & Gupta [52], Singh & Malik [53], Yadav et al. [65], Yadav & Kadilar [66], Ahmed et al. ([12], [18]), Muili et al. ([48], [62]-[64]). The aforementioned authors had worked on variance estimation under different sampling procedures.

The current study considered variance estimation thereby proposed an improved ratio-product type-exponential estimator.

2. Materials and Methods

Let $\Omega = (\Omega_1, \dots, \Omega_N)$ be a finite population of size N and let (y_i, x_i) be the values of the study variable Y and the auxiliary variable X on i^{th} unit $\Omega_i, i = 1, \dots, N$. Let \bar{Y} and \bar{X} be population means of Y and X respectively (Zakari et al., [68]). We assumed that the population mean \bar{X} and the population variance S_x^2 of X were known. Let $s_y^2 = (n-1)^{-1} \sum_{i=1}^n (y_i - \bar{y})^2$ and $s_x^2 = (n-1)^{-1} \sum_{i=1}^n (x_i - \bar{x})^2$ be the unbiased estimators of $S_y^2 = (N-1)^{-1} \sum_{i=1}^N (y_i - \bar{y})^2$ and $S_x^2 = (N-1)^{-1} \sum_{i=1}^N (x_i - \bar{x})^2$ respectively and the correlation coefficient between Y and X be ρ_{xy} . Also, let $C_y = S_y / \bar{Y}$ and $C_x = S_x / \bar{X}$ be the coefficients of variation of Y and X .

To obtain the bias and MSE of the estimator, the following error terms are defined:

Let $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$ such that the expectations are $E(e_0) = E(e_1) = 0$, $E(e_0^2) = \theta(\beta_{2(y)} - 1) = V_{40}$, $E(e_1^2) = \theta(\beta_{2(x)} - 1) = V_{04}$, $E(e_0 e_1) = \theta(\lambda_{22} - 1) = V_{22}$ and $\theta = \left(\frac{1}{n} - \frac{1}{N}\right)$, $\beta_{2(y)}$ and $\beta_{2(x)}$ are the population coefficient of kurtosis of y and x respectively.

The conventional unbiased estimator of variance is given by

$$S_y^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2 \quad (1)$$

The variance of \hat{S}_y^2 is given by

$$Var(\hat{S}_y^2) \cong S_y^4 V_{40} \quad (2)$$

Variance estimator denoted by \hat{S}_R^2 following Isaki [45] is given as in (3)

$$\hat{S}_R^2 = s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \quad (3)$$

MSE(\hat{S}_R^2) is given as in (4)

$$MSE(\hat{S}_R^2) \cong S_y^4 [V_{40} + V_{04} - 2V_{22}] \quad (4)$$

The usual regression estimator proposed is given as in (5)

$$\hat{S}_{Reg}^2 = s_y^2 + b_{(s_y^2, s_x^2)} (S_x^2 - s_x^2) \quad (5)$$

where $b_{(s_y^2, s_x^2)}$ is the sample regression slope.

The MSE of \hat{S}_{Reg}^2 is given as in (6)

$$MSE(\hat{S}_{Reg}^2) \cong S_y^4 V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2) \quad (6)$$

Where $\rho_{(s_y^2, s_x^2)} = V_{22} / \sqrt{V_{40} V_{04}}$

Singh et al. [57] suggested difference type estimator for finite population variance as in (7)

$$\hat{S}_d^2 = k_1 s_y^2 + k_2 (S_x^2 - s_x^2) \quad (7)$$

$MSE(\hat{S}_d^2)$ at optimum values $k_{1(opt)} = V_{04} / (V_{04} + V_{40} V_{04} - V_{22}^2)$ and

$k_{2(opt)} = S_x^2 V_{22} / (S_y^2 (V_{04} + V_{40} V_{04} - V_{22}^2))$ is given by

$$MSE(\hat{S}_d^2) \cong \frac{S_y^4 V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2)}{1 + V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2)} \quad (8)$$

The ratio type exponential estimator for the population variance defined by (Bahl & Tuteja [41]) is given as:

$$\hat{S}_{BT}^2 = s_y^2 \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right) \quad (9)$$

The MSE of \hat{S}_{BT}^2 , to first order of approximation, is given as (10)

$$MSE(\hat{S}_{BT}^2) \cong S_y^4 [V_{40} + 4^{-1} V_{04} - V_{22}] \quad (10)$$

Shabbir & Gupta [52] defined variance estimator as in (11)

$$\hat{S}_{SG}^2 = [k_3 s_y^2 + k_4 (S_x^2 - s_x^2)] \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right) \quad (11)$$

where MSE of \hat{S}_{SG}^2 , at optimum values of $k_{3(opt)} = \frac{V_{04}}{8} \left(\frac{8 - V_{04}}{V_{04} + V_{40} V_{04} - V_{22}^2}\right)$ and

$k_{4(opt)} = \frac{S_y^2}{8 S_x^2} \left(\frac{-4V_{40} + V_{04} + 8V_{22} - V_{22} V_{04} + 4V_{40} V_{04} - 4V_{22}^2}{(V_{04} + V_{40} V_{04} - V_{22}^2)}\right)$ is given in (12)

$$MSE(\hat{S}_{SG}^2)_{\min} \cong \frac{S_y^4}{64} \left(\frac{-4V_{04}^2 - 16V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2) (V_{04} - 4)}{1 + V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2)}\right) \quad (12)$$

Swain [59] proposed a generalized estimator for population variance is given as in (13)

$$\hat{S}_{SW}^2 = s_y^2 \left[k \left(\frac{S_x^2}{s_x^2}\right)^q + (1-k) \left(\frac{s_x^2}{S_x^2}\right)^h \right]^\delta \quad (13)$$

Where k , q , h and $\delta = (1, -1)$ are real and free parameters to be chosen suitably. The minimum MSE of \hat{S}_{SW}^2 , to first order of approximation, at optimum value $k = (\delta h + (V_{22} / V_{04})) / (\delta(g + h))$, is given by

$$MSE(\hat{S}_{SW}^2)_{\min} \cong S_y^4 V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2) \quad (14)$$

Yadav et al. [65] defined generalized variance estimator as in (15)

$$\hat{S}_{YG}^2 = \left[k_5 s_y^2 + k_6 (S_x^2 - s_x^2) \right] \left\{ \lambda \left(\frac{aS_x^2 + b}{as_x^2 + b} \right) + (1 - \lambda) \exp \left(\frac{a(S_x^2 - s_x^2)}{a(S_x^2 + s_x^2) + 2b} \right) \right\} \quad (15)$$

where

$$k_{5(opt)} = \left(\frac{1 - 8^{-1} g^2 (1 + 3\lambda + 4\lambda^2) V_{04}}{1 - 4^{-1} g^2 \lambda (1 + 3\lambda) V_{04}^2 + V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2)} \right),$$

$$k_{6(opt)} = \frac{S_y^2}{S_x^2} \left(\frac{1}{2} g (1 + \lambda) + k_{5(opt)} \left(\frac{V_{22}}{V_{04}} - g (1 + \lambda) \right) \right)$$

$$MSE_{\min}(\hat{S}_{YG}^2) \cong S_y^4 \left\{ \left(1 - \frac{1}{4} g^2 (1 + \lambda)^2 V_{04} \right) - \frac{1 - 8^{-1} g^2 (1 + 3\lambda + 4\lambda^2) V_{04}}{1 - 4^{-1} g^2 \lambda (1 + 3\lambda) V_{04}^2 + V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2)} \right\} \quad (16)$$

Where $g = aS_x^2 / (aS_x^2 + b)$.

The minimum MSE of \hat{S}_{YG}^2 , at $(\lambda, a, b) = (1, 1, 0)$, is as in (17)

$$MSE(\hat{S}_{YG}^2)_{\min} \cong S_y^4 \left(\frac{S_y^{-4} MSE(\hat{S}_{Reg}^2) (1 - V_{04})}{1 - V_{04} + S_y^{-4} MSE(\hat{S}_{Reg}^2)} \right) \quad (17)$$

Yadav & Kadilar [66] proposed ratio-product-ratio type estimator for \hat{S}_{YK}^2 , is given as in (18)

$$\hat{S}_{YK}^2 = s_y^2 \left\{ \alpha_1 \left[\frac{(1 - \beta_1) s_x^2 + \beta_1 S_x^2}{\beta_1 s_x^2 + (1 - \beta_1) S_x^2} \right] + (1 - \alpha_1) \left[\frac{\beta_1 s_x^2 + (1 - \beta_1) S_x^2}{(1 - \beta_1) s_x^2 + \beta_1 S_x^2} \right] \right\} \quad (18)$$

where α_1 and β_1 are unknown constants.

MSE of \hat{S}_{YK}^2 at $(\alpha_{1(opt)}, \beta_{1(opt)}) = (1/2, 1/2)$ and $(\alpha_1, \beta_1) = ((V_{04} - V_{22}) / 2V_{04}, 0)$, are given in (19) and (20) respectively

$$MSE(\hat{S}_{YK}^2) \cong S_y^4 V_{40}, \quad (19)$$

$$MSE(\hat{S}_{YK}^2)_{\min} \cong S_y^4 V_{40} (1 - \rho_{(s_y^2, s_x^2)}^2). \quad (20)$$

Singh & Malik [53] proposed an improved estimator for \hat{S}_y^2 , is given as in (21)

$$\hat{S}_{SM}^2 = s_y^2 \left[k_7 + k_8 (S_x^2 - s_x^2) \right] \exp \left(\psi \frac{a(S_x^2 - s_x^2)}{a(S_x^2 + s_x^2) + 2b} \right), \quad (21)$$

Where k_7 and k_8 are suitably chosen constants, ψ takes values +1 and -1 for ratio and product type estimators and a, b be the known population parameters of the auxiliary variables.

$$k_{7(opt)} = \frac{1}{4} \left(\frac{-12V_{04}V_{22} + V_{04}^2 + 16V_{22}^2 - 8V_{04} - 2V_{40}^2}{V_{04}^2 - 4V_{04}V_{22} + 8V_{22}^2 - 2V_{40}V_{04} - 2V_{04} - V_{40}^2} \right)$$

and

$$k_{8(opt)} = \frac{-1}{4S_x^2} \left(\frac{-6V_{04}V_{22} + V_{04}^2 + 8V_{22}^2 - 4V_{04} + 8V_{22} - 8V_{04}V_{22} + 4V_{04}V_{40}}{V_{04}^2 - 4V_{04}V_{22} + 8V_{22}^2 - 2V_{40}V_{04} - 2V_{04} - V_{40}^2} \right)$$

is given by

$$MSE(\hat{S}_{SM}^2)_{\min} \cong \frac{S_y^4}{64} \left[\frac{V_{04} \{V_{04}(V_{04} + 8V_{22}) + 16(V_{04} - 4)MSE(\hat{S}_{Reg}^2) + 16V_{22}(V_{22} - V_{04})\}}{-V_{04}(1 + V_{40} + 2V_{22}) + 4V_{22}^2} \right] \quad (22)$$

Yaqub & Shabbir [67] defined a class of variance estimators as in (23)

$$\hat{S}_{YS}^2 = s_y^2 \left[k_9 + k_{10} (S_x^2 - s_x^2) \right] \left(\frac{aS_x^2 + b}{as_x^2 + b} \right) \left\{ \frac{1}{2} \exp\left(\frac{a(S_x^2 - s_x^2)}{a(S_x^2 + s_x^2) + 2b} \right) + \frac{1}{2} \exp\left(\frac{a(s_x^2 - S_x^2)}{a(s_x^2 + S_x^2) + 2b} \right) \right\} \quad (23)$$

Where k_9 and k_{10} are suitably chosen constants and a and b be the known population parameters of the auxiliary variable.

$$k_{9(opt)} = \frac{V_{04}}{2} \left(\frac{1 + 7(1 - V_{04})}{V_{04}^2 + 4V_{04}(1 - V_{04}) + 4V_{40}V_{04} - 4V_{22}^2} \right)$$

and

$$k_{10(opt)} = \frac{S_y^2}{2S_x^2} \left(\frac{V_{22} + 7V_{22}(1 - V_{04}) - 8V_{04}(1 - V_{04}) + 8V_{40}V_{04} - 8V_{22}^2}{V_{04}^2 + 4V_{04}(1 - V_{04}) + 4V_{40}V_{04} - 4V_{22}^2} \right),$$

is given by

$$MSE(\hat{S}_{YS}^2)_{\min} \cong \frac{S_y^4}{16} \left(\frac{64(1 - V_{04})S_y^{-4}MSE(\hat{S}_{Reg}^2) - V_{04}^2}{V_{04} + 4(1 - V_{04}) + 4S_y^{-4}MSE(\hat{S}_{Reg}^2)} \right) \quad (24)$$

Muneer et al. [50] proposed an improved class of estimators for \hat{S}_y^2 , is given as in (25)

$$\hat{S}_M^2 = s_y^2 \left[k_{11} \left(\frac{S_x^2}{s_x^2} \right) + k_{12} \left(\frac{s_x^2}{S_x^2} \right) \right] \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2} \right) \quad (25)$$

Where k_{11} and k_{12} are suitably chosen constants. The minimum MSE of \hat{S}_M^2 to first order of approximation, at optimum values

$$k_{11(opt)} = \frac{1}{8} \left(\frac{16V_{22}^2 + 6V_{22}V_{40} - 24V_{22}V_{04} - 16V_{40}V_{04} - V_{04}^2 - 16V_{22} - 8V_{04}}{16_{22}^2 - 16V_{22}V_{04} - 4V_{40}V_{04} + V_{04}^2 - 4V_{04}} \right)$$

and

$$k_{12(opt)} = \frac{1}{8} \left(\frac{48V_{22}^2 - 16V_{22}V_{40} - 72V_{22}V_{04} + 16V_{40}V_{04} + 21V_{04}^2 + 16V_{22} - 24V_{04}}{16_{22}^2 - 16V_{22}V_{04} - 4V_{40}V_{04} + V_{04}^2 - 4V_{04}} \right)$$

is given by

$$MSE(\hat{S}_M^2)_{\min} \cong \frac{S_y^4}{16} \left(\frac{64V_{22}^2V_{40} - 48V_{22}^2V_{04} - 128V_{22}V_{40}V_{04} + 48V_{22}V_{04}^2 + 64V_{40}V_{04}^2 - 48V_{40}V_{04}^2 - 9V_{22}^3 - 64V_{22}^2 - 48V_{40}V_{04}^2}{16_{22}^2 - 16V_{22}V_{04} - 4V_{40}V_{04} + V_{04}^2 - 4V_{04}} \right) \quad (26)$$

3.0 Proposed Estimator

In this section, a new generalized mixture estimator is suggested as:

$$\hat{S}_{Prop}^2 = \left\{ d_1 s_y^2 \left[\frac{1}{2} \left(\frac{S_x^2}{s_x^2} + \frac{s_x^2}{S_x^2} \right) \right]^\alpha + d_2 (S_x^2 - s_x^2) \right\} \exp \left\{ \frac{S_x^2 - s_x^2}{S_x^2 + s_x^2} \right\} \quad (27)$$

where d_i ($i = 1, 2$) are unknown constants whose values to be determined later, and α is suitably chosen constant.

Expressing the estimator \hat{S}_{RPR}^2 in terms of e_i ($i = 0, 1$) we can write (27) as

$$\hat{S}_{Prop}^2 = \left\{ d_1 S_y^2 (1 + e_0) \frac{1}{2^\alpha} \left[(1 + e_1)^{-1} + 1 + e_1 \right]^\alpha - d_2 S_x^2 e_1 \right\} \exp \left\{ \frac{-S_x^2 e_1}{2S_x^2 + S_x^2 e_1} \right\} \quad (28)$$

Expanding the RHS of (30) to the first order of approximation, multiplying out and neglecting the terms of e 's greater than two, we get

$$\hat{S}_{Prop}^2 - S_y^2 = (d_1 - 1) S_y^2 + d_1 S_y^2 (e_0 - 2^{-1} e_1 - 2^{-1} e_0 e_1 + A e_1^2) - d_2 S_x^2 (e_1 - 2^{-1} e_1^2) \quad (29)$$

Where $A = [2^{-1} \alpha + 3 \times 2^{-3}]$

By taking expectation of (32), the bias of the proposed generalized estimator, up to the first order of approximation, is given by:

$$Bias(\hat{S}_{Prop}^2) = (d_1 - 1) S_y^2 + d_1 S_y^2 \left(A V_{04} - \frac{1}{2} V_{22} \right) + \frac{d_2}{2} S_x^2 V_{04} \quad (30)$$

By squaring equation (32) and keeping terms only up to the first order of approximation, we have:

$$MSE(\hat{S}_{Prop}^2) = \left\{ \begin{aligned} & (d_1 - 1)^2 S_y^4 + d_1^2 S_y^4 \left[V_{40} - 2V_{22} + \left(2A + \frac{1}{4} \right) V_{04} \right] + d_2^2 S_x^4 V_{04} \\ & - d_1 S_y^4 (2A V_{04} - V_{22}) - 2d_1 d_2 S_y^2 S_x^2 (V_{22} - V_{04}) - d_2 S_y^2 S_x^2 V_{04} \end{aligned} \right\} \quad (31)$$

Minimizing Equation (37) with respect to d_1 and d_2 , we get the optimum values of d_1 and d_2 i.e.

$$d_{1(opt)} = \frac{1 + \left(A - \frac{1}{2} \right) V_{04}}{1 + \left(2A - \frac{3}{4} \right) V_{04} + \left(V_{40} - \frac{V_{22}^2}{V_{04}} \right)} \quad \text{and} \quad d_{2(opt)} = \frac{S_y^2}{S_x^2} \left\{ \frac{1}{2} - d_{1(opt)} \left(1 - \frac{V_{22}}{V_{04}} \right) \right\}$$

using the results of d_1 and d_2 in (37), the minimum MSE of \hat{S}_{Prop}^2 is obtained as:

$$MSE(\hat{S}_{Prop}^2)_{\min} = S_y^4 \left\{ \left(1 - \frac{V_{04}}{4} \right) - \frac{\left[1 + \left(A - \frac{1}{2} \right) V_{04} \right]^2}{\left[1 + \left(2A - \frac{3}{4} \right) V_{04} + \left(V_{40} - \frac{V_{22}^2}{V_{04}} \right) \right]} \right\} \quad (32)$$

Special Cases:

For $\alpha = 1$ this proposed estimator becomes

$$\hat{S}_{Prop(1)}^2 = \left\{ d_1 s_y^2 \left[\frac{1}{2} \left(\frac{S_x^2}{s_x^2} + \frac{s_x^2}{S_x^2} \right) \right] + d_2 (S_x^2 - s_x^2) \right\} \exp \left\{ \frac{S_x^2 - s_x^2}{S_x^2 + s_x^2} \right\} \quad (33)$$

The optimum values of d_1 and d_2 , for this estimator, are given by:

$$d_{1(opt)} = \frac{1 + \frac{3}{8}V_{04}}{1 + V_{04} + \left(V_{40} - \frac{V_{22}^2}{V_{04}}\right)} \text{ and } d_{2(opt)} = \frac{S_y^2}{S_x^2} \left\{ \frac{1}{2} - d_{1(opt)} \left(1 - \frac{V_{22}}{V_{04}}\right) \right\}$$

The minimum mean squared error, up to the first order of approximation, is given as in (34)

$$MSE(\hat{S}_{Prop(1)}^2)_{\min} = S_y^4 \left\{ \left(1 - \frac{V_{04}}{4}\right) - \frac{\left[1 + \frac{3}{8}V_{04}\right]^2}{\left[1 + V_{04} + \left(V_{40} - \frac{V_{22}^2}{V_{04}}\right)\right]} \right\} \quad (34)$$

When $\alpha = 2$, the proposed estimator becomes

$$\hat{S}_{Prop(2)}^2 = \left\{ d_1 s_y^2 \left[\frac{1}{2} \left(\frac{S_x^2}{s_x^2} + \frac{s_x^2}{S_x^2} \right) \right]^2 + d_2 (S_x^2 - s_x^2) \right\} \exp \left\{ \frac{S_x^2 - s_x^2}{S_x^2 + s_x^2} \right\} \quad (35)$$

The optimum values of d_1 and d_2 , for this estimator, are given by:

$$d_{1(opt)} = \frac{1 + \frac{7}{8}V_{04}}{1 + 2V_{04} + \left(V_{40} - \frac{V_{22}^2}{V_{04}}\right)} \text{ and } d_{2(opt)} = \frac{S_y^2}{S_x^2} \left\{ \frac{1}{2} - d_{1(opt)} \left(1 - \frac{V_{22}}{V_{04}}\right) \right\}$$

The minimum mean squared error, up to the first order of approximation, is given as in (36)

$$MSE(\hat{S}_{Prop(2)}^2)_{\min} = S_y^4 \left\{ \left(1 - \frac{V_{04}}{4}\right) - \frac{\left[1 + \frac{7}{8}V_{04}\right]^2}{\left[1 + 2V_{04} + \left(V_{40} - \frac{V_{22}^2}{V_{04}}\right)\right]} \right\} \quad (36)$$

3.1 Theoretical Efficiency comparisons

In this section, theoretical efficiency conditions of proposed estimator over some existing estimators were established

(i) $Var(\hat{S}_y^2) - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1 - V_{40} - 4^{-1}V_{04}) - C^2}{B} \right) < 0 \quad (37)$$

Where $B = 1 + \left(2A - \frac{3}{4}\right)V_{04} + \left(V_{40} - \frac{V_{22}^2}{V_{04}}\right)$ and $C = 1 + \left(A - \frac{1}{2}\right)V_{04}$

(ii) $MSE(\hat{S}_R^2) - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1 - V_{40} - 1.25V_{04} + 2V_{22}) - C^2}{B} \right) < 0 \quad (38)$$

(iii) $[MSE(\hat{S}_{Reg}^2), MSE(\hat{S}_{SW}^2)_{\min} \text{ or } MSE(\hat{S}_{YK}^2)_{\min}] - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1-4^{-1}V_{04} - V_{40}(1-\rho_{(s_y^2, s_x^2)}^2)) - C^2}{B} \right) < 0 \quad (39)$$

(iv) $MSE(\hat{S}_d^2) - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1-4^{-1}V_{04}) - C^2}{B} - \frac{V_{40}(1-\rho_{(s_y^2, s_x^2)}^2)}{S_y^{-4}[1+V_{40}(1-\rho_{(s_y^2, s_x^2)}^2)]} \right) < 0 \quad (40)$$

(v) $MSE(\hat{S}_{BT}^2) - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1-V_{40} - 2^{-1}V_{04} + V_{22}) - C^2}{B} \right) < 0 \quad (41)$$

(vi) $MSE(\hat{S}_{SG}^2)_{\min} - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1-4^{-1}V_{04}) - C^2}{B} - \frac{4V_{04}^2 + 16V_{40}(1-\rho_{(s_y^2, s_x^2)}^2)(V_{04} - 4)}{(V_{40}(1-\rho_{(s_y^2, s_x^2)}^2) - 1)} \right) < 0 \quad (42)$$

(vii) $MSE(\hat{S}_{YG}^2)_{\min} - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1-4^{-1}V_{04}) - C^2}{B} - \frac{S_y^{-4}MSE(\hat{S}_{Reg}^2)(1-V_{04})}{1-V_{04} + S_y^{-4}MSE(\hat{S}_{Reg}^2)} \right) < 0 \quad (43)$$

(viii) $MSE(\hat{S}_{SM}^2)_{\min} - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1-4^{-1}V_{04}) - C^2}{B} - \frac{D}{64E} \right) < 0 \quad (44)$$

Where $D = V_{04}(V_{04}(V_{04} + 8V_{22}) + 16(V_{04} - 4)MSE(\hat{S}_{Reg}^2) + 16V_{22}(V_{22} - V_{04}))$ and

$$E = -V_{04}(1 + V_{40} + 2V_{22}) + 4V_{22}^2$$

(ix) $MSE(\hat{S}_{YS}^2)_{\min} - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1-4^{-1}V_{04}) - C^2}{B} - \frac{64(1-V_{04})S_y^{-4}MSE(\hat{S}_{Reg}^2) - V_{04}^2}{16(V_{04} + 4(1-V_{04})) + 4S_y^{-4}MSE(\hat{S}_{Reg}^2)} \right) < 0 \quad (45)$$

(x) $MSE(\hat{S}_M^2)_{\min} - MSE(\hat{S}_{Prop}^2)_{\min} > 0$, if

$$\left(\frac{B(1-4^{-1}V_{04}) - C^2}{B} - \frac{F}{16G} \right) < 0 \quad (46)$$

Where

$$F = 64V_{22}^2V_{40} - 48V_{22}^2V_{04} - 128V_{22}V_{40}V_{04} + 48V_{22}V_{04}^2 + 64V_{40}V_{04}^2 - 48V_{40}V_{04}^2 - 9V_{22}^3 - 64V_{22}^2 - 48V_{40}V_{04}^2$$

$$\text{and } G = 16V_{22}^2 - 16V_{22}V_{04} - 4V_{40}V_{04} + V_{04}^2 - 4V_{04}$$

4.0 Results and Discussion

To assess the predictive ability of the proposed estimator over other considered estimators, the data sets below were used.

Table 1. Parameters of the Data Sets

Parameters	Data 1 Gujarati [43]	Data 2 Mukherjee et al. [49]	Data 3 Singh and Mangat [54]	Data 4 http://www.osservatorionazionaleirifiuti.it (2004)	Data 5 Murthy [51]
N	23	64	39	103	80
n	5	19	14	40	20
\bar{Y}	39.699	5.549	26.8433	62.6212	51.8264
\bar{X}	1035.065	141.500	106.2	556.5541	11.2646
S_y^2	54.360	2.277	38.9900	8345.7177	336.9757
S_x^2	381735.000	5772.670	124.1286	372300.473	70.6634
$\rho_{(s_y^2, s_x^2)}$	0.8277	0.2971	0.8899	0.6570	0.7941
$\beta_{2(y)}$	2.030	2.773	2.4032	37.1279	2.2667
$\beta_{2(x)}$	2.696	2.341	2.9930	17.8738	2.8664
λ_{22}	2.094	1.458	2.4882	17.2220	2.2209
θ	0.1565	0.0370	0.0458	0.0153	0.0375
V_{40}	0.1612	0.0656	0.0643	0.5525	0.0475
V_{04}	0.2655	0.0496	0.0913	0.2581	0.0699
V_{22}	0.1713	0.0169	0.0682	0.2481	0.0457

The following expression is used for the computations of percentage relative efficiency (*PREs*) of the proposed and considered estimators:

$$PRE = \frac{var(\hat{S}_y^2)}{MSE(i) \text{ or } MSE_{min}(i)} * 100, \quad i = \hat{S}_y^2, \hat{S}_R^2, \hat{S}_{Reg}^2, \hat{S}_d^2, \hat{S}_{BT}^2 \dots \hat{S}_{Prop}^2$$

Table 2. MSE and PRE values of different estimators with respect to \hat{S}_y^2

Estimators	Data 1		Data 2		Data 3		Data 4		Data 5	
	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
\hat{S}_y^2	476.347	100.00	1.763	100.00	97.751	100.00	38517005.7	100.00	5393.752	100.00
\hat{S}_R^2	248.516	191.676	2.186	80.664	29.188	334.895	21940066.5	175.556	2952.369	182.693
\hat{S}_{Reg}^2	149.753	318.089	1.608	109.676	20.303	481.453	21913070.0	175.772	2000.996	269.554
\hat{S}_d^2	142.529	334.209	1.517	116.237	20.036	487.883	16668844.7	231.072	1966.346	274.304
\hat{S}_{BT}^2	166.293	286.451	1.641	107.438	28.771	339.763	25736046.3	149.662	2188.728	246.433
\hat{S}_{SG}^2	131.232	362.981	1.561	112.962	19.329	505.722	17930714.1	214.811	1934.228	278.858
\hat{S}_{SW}^2	149.753	318.089	1.608	109.676	20.303	481.453	21914066.5	175.556	2000.996	269.554
\hat{S}_{YG}^2	140.087	340.036	1.513	116.580	20.009	488.528	15388327.7	250.301	1963.791	274.661
\hat{S}_{YK}^2	149.753	318.089	1.608	109.676	20.303	481.453	21914066.5	175.556	2000.996	269.554
\hat{S}_{SM}^2	112.932	421.800	1.497	117.768	18.612	525.198	24877706.5	154.823	1865.461	289.137
\hat{S}_{YS}^2	125.346	380.026	1.494	118.092	19.316	506.053	14438388.6	266.768	1919.254	281.034
\hat{S}_M^2	51.405	926.643	1.412	124.947	14.456	671.993	13829282.0	278.518	1631.372	330.627
$\hat{S}_{Prop(1)}^2$	44.406	1072.71	0.272	648.162	13.479	725.210	11861495.0	324.723	1608.521	335.324
$\hat{S}_{Prop(2)}^2$	33.285	1431.12	0.250	705.200	3.1788	3075.09	7970518.00	483.243	1091.940	493.960

Note: The minimum *MSE* value of \hat{S}_{YG}^2 is given in Table 3 for $(\lambda, a, b) = (1, 1, 0)$.

Table 3 shows the results of mean square error (*MSE*) values and *PREs* for the proposed and some existing estimators with respect to sample variance. The efficiency comparison of the proposed estimator has been made with the other estimators considered in the study and the results revealed that the performance of the proposed estimators $\hat{S}_{Prop(1)}^2$ and $\hat{S}_{Prop(2)}^2$ is better than all other considered estimators.

5. Conclusion

In this study, a new generalized exponential-type estimator for estimating the finite population variance \hat{S}_y^2 using the auxiliary information in simple random sampling was suggested. Expressions for bias and *MSE* of the estimators were derived up to first degree order approximation. The theoretical efficiency conditions under which the proposed class of estimators are better than the existing estimators were presented. In Table 2, the *MSEs* and *PRE* of the classes of the proposed estimator \hat{S}_{Prop}^2 were compared with other considered estimators numerically. Evidence from the study revealed that the proposed classes of estimators have smaller *MSEs* and higher *PREs* among others which implies that they are more efficient than the existing estimators. Therefore, the new proposed variance estimator is recommended for use in real life applications associated with variabilities.

References

- [1] B. S. V. Sisodia, V.K. Dwivedi. A modified ratio estimator using coefficient of variation of auxiliary variable. *Jour. Ind. Soc. Agr. Statistics*, 33, 2, 13-18, 1981a.
- [2] M. N. Murthy. Product method of estimation. *Sankhya*, 26: 294-307, 1964.
- [3] S. K. Yadav, C. Kadilar. Improved exponential type ratio estimator of population Variance. *Revista Colombiana de Estadística*, 36(1), 145–152, 2013.
- [4] H. P. Singh, R. Tailor. Estimation of finite population mean using known correlation coefficient between auxiliary characters. *Statistica*, 65: 407-418, 2005.
- [5] S. Bahl, P. K. Tuteja. Ratio and product type exponential estimator. *Journal of information and optimization science*, 12(1), 159-163, 1991. Doi; 1080/02522667.1991.10699058
- [6] R. Singh, P. Chauhan, N. Sawan, F. Smarandache. Ratio estimators in simple random sampling using information on auxiliary attribute. *Pak. J. Stat. Oper. Res.* 4(1), 47-53, 2008.
- [7] H. P. Singh, R. Tailor, M. S. Kakran. An improved estimation of population mean using power transformation. *Journal of the Indian Society of Agricultural Statistics*, 58(2):223–230, 2004.
- [8] C. Kadilar, H. Cingi. Ratio estimators for population variance in simple and stratified sampling. *Applied Mathematics and Computation*, 173:1047–1059, 2006.
- [9] H. P. Singh, R. S. Solanki. An efficient class of estimators for the population mean using auxiliary information in systematic sampling. *Journal of Statistical Theory and Practice*, 6(2), 274-285, 2012.
- [10] A. Sahai, S. K. Ray. And efficient estimator using auxiliary information. *Metrika*, 27(4), 271-275, 1980.
- [11] S. K. Srivastava, H.A. Jhaji. A class of estimators of the population mean in survey sampling using auxiliary information. *Biometrika*, 68(1):341-343, 1981.
- [12] A. Ahmed, A. A. Adewara, R. V. K. Singh. Class of ratio estimators with known functions of auxiliary variable for estimating finite population variance. *Asian Journal of Mathematics and Computer Research*, 12(1), 63-70, 2016.
- [13] A. Audu, A. A, Adewara. Modified factor-type estimators under two-phase sampling. *Punjab Journal of Mathematics*, 49(2), 59-73, 2017.

- [14] A. Audu, R. Singh, S. Khare, N. S. Dauran. Almost unbiased estimators for population mean in the presence of non-response and measurement error. *Journal of Statistics & Management Systems*; 24(3), 573-589, 2020. DOI : 10.1080/09720510.2020.1759209.
- [15] J. O. Muili, E. N. Agwamba, Y. A. Erinola, M. A. Yunusa, A. Audu, M. A, Hamzat. Modified ratio-cum-product estimators of finite population variance. *International Journal of Advances in Engineering and Management*, 2(4):309-319, 2020. DOI: 10.35629/5252-0204309319.
- [16] M. Khoshnevisan, R. Singh, P. Chauhan, N. Sawan, F. Smarandache. A general family of estimators for estimating population means using known value of some population parameter(s). *Far East Journal of Theoretical Statistics*, 22(2), 181-191, 2007.
- [17] R. V. K. Singh, A. Audu. Efficiency of ratio estimators in stratified random sampling using information on auxiliary attribute. *International Journal of Engineering Science and Innovative Technology*, 2(1), 166-172, 2013.
- [18] A. Ahmed, R. V. K. Singh, A. A. Adewara. Ratio and product type exponential estimators of population variance under transformed sample information of study and supplementary variables. *Asian Journal of Mathematics and Computer Research*, 11(3), 175-183, 2016.
- [19] A. Audu, R. V. K. Singh. Exponential-type regression compromised imputation class of estimators. *Journal of Statistics and Management Systems*, 1-15, 2020. DOI: 10.1080/09720510.2020.1814501
- [20] A. K. Das, T. P. Tripathi. A class of estimators for co-efficient of variation using knowledge on co-efficient of variation of an auxiliary character. In annual conference of Ind. Soc. Agricultural Statistics. Held at New Delhi, India; 1981.
- [21] A. K. Das, T. P. Tripathi. Use of auxiliary information in estimating the coefficient of variation. *Alig. J. of Statist.*, 12, 51-58, 1992.
- [22] P. A. Patel, S. Rina. A Monte Carlo comparison of some suggested estimators of coefficient of variation in finite population. *Journal of Statistics Science*, 1(2), 137-147, 2009.
- [23] A. Rajyaguru, P. Gupta. On the estimation of the coefficient of variation from finite population-I, *Model Assisted Statistics and Application*, 36(2), 145-156, 2002.
- [24] A. Rajyaguru, P. Gupta. On the estimation of the coefficient of variation from finite population –II, *Model Assisted Statistics and Application*, 1(1), 57-66, 2006.
- [25] V. Archana, A. Rao. Same improved estimators of co-efficient of variation from bivariate normal distribution. a monte carlo comparison. *Pakistan Journal of Statistics and Operation Research*, 10(1), 2014.

- [26] Singh R, Mishra M, Singh BP, Singh P, Adichwal NK. Improved estimators for population coefficient of variation using auxiliary variables. *Journal of Statistics & Management Systems*. 21(7), 1335-1335, 2018.
- [27] A. Audu, R. Singh, S. Khare. Developing calibration estimators for population mean using robust measures of dispersion under stratified random. *STATISTICS IN TRANSITION new series*, 22 (2), pp. 125–142, 2021. DOI: 10.21307/stattrans-2021-019
- [28] A. Audu, O. O. Ishaq, A. Abubakar, K. A. Akintola, U. Isah, A. Rashida, and S. Muhammad. Regression-type Imputation Class of Estimators using Auxiliary Attribute. *Asian Research Journal of Mathematics*, 17(5): 1-13, 2021. DOI: 10.9734/ARJOM/2021/v17i530296
- [29] A. Audu, O. O. Ishaq, U. Isah, S. Muhammed, K. A. Akintola, A. Rashida, A. Abubakar On the Class of Exponential-Type Imputation Estimators of Population Mean with Known Population Mean of Auxiliary Variable. *NIPES Journal of Science and Technology Research* 2(4), pp.1 – 11, 2020. <https://doi.org/10.37933/nipes/2.4.2020.1>
- [30] A. Audu, O. O. Ishaq, J. O. Muili, A. Abubakar, A. Rashida, K. A. Akintola, U. Isah. Modified Estimators of Population Mean Using Robust Multiple Regression Methods. *NIPES Journal of Science and Technology Research* 2(4), 12 – 20, 2020. <https://doi.org/10.37933/nipes/2.4.2020.2>
- [31] A. Audu, O. O. Ishaq, J. O. Muili, Y. Zakari, A. M. Ndatsu, S. Muhammed. On the Efficiency of Imputation Estimators using Auxiliary Attribute. *Continental J. Applied Sciences*, 15 (1), 1-13, 2020. DOI: 10.5281/zenodo.3721046
- [32] A. Audu, O. O. Ishaq, Y. Zakari, D. D. Wisdom, J. Muili and A. M. Ndatsu. Regression-cum-exponential ratio imputation class of estimators of population mean in the presence of non-response. *Science Forum Journal of Pure and Applied Science*, 20, 58-63, 2020. DOI: <http://dx.doi.org/10.5455/sf.71109>
- [33] R. Singh, P. Mishra, A. Audu and S. Khare. Exponential Type Estimator for Estimating Finite Population Mean. *Int. J. Comp. Theo. Stat.*, 7(1), 37-41, 2020.
- [34] J. O. Muili, E. N. Agwamba, Y. A. Erinola, M. A. Yunusa, A. Audu and M. A. Hamzat. A Family of Ratio-Type Estimators of Population Mean using Two Auxiliary Variables. *Asian Journal of Research in Computer Science*, 2020. DOI: 10.9734/AJRCOS/2020/v6i130152
- [35] O. O. Ishaq, A. Audu, A. Ibrahim, H. S. Abdulkadir, K. Tukur. On the linear combination of sample variance, ratio, and product estimators of finite population variance under two-stage sampling. *Science Forum Journal of Pure and Applied Science*, 2020, 20, 307-315. DOI: <http://dx.doi.org/10.5455/sf.90563>

- [36] A. Audu, M. A. Yunusa, O. O. Ishaq, M. K. Lawal, A. Rashida, A. H. Muhammad, A. B. Bello, M. U. Hairullahi and J. O. Muili. Difference-Cum-Ratio Estimators for Estimating Finite Population Coefficient of Variation in Simple Random Sampling. *Asian Journal of Probability and Statistics*, 2021, 13(3), 13-29. DOI: 10.9734/AJPAS/2021/v13i330308
- [37] Ahmed, M. S., Raman, M. S. & Hossain, M. I. Some competitive estimators of finite population variance using multivariate auxiliary information. *Information Management Science*, 2000, 11(1), 49–54.
- [38] Ahmed, M. S., Walid, A. D. & Ahmed, A. O. H. Some estimators for finite population variance under two-phase sampling. *Stat Transit*, 2003, 6(1): 143–150.
- [39] Ahmed, A. and Singh, R. V. K. Improved exponential type estimators for estimating population variance in survey sampling. *International Journal of Advance Research, IJOAR.*, 2015, 33 (1), 1-16.
- [40] Al-Jararha, J. & Ahmed, M. S. The class of chain estimators for a finite population variance using double sampling. *Information Management Science*, 2002, 13(2): 13–18.
- [41] Bahl, S. & Tuteja, R. K. Ratio and product type exponential estimators. *Information Optimization Science*, 1991, 12(1): 159–164.
- [42] Das, A. K. & Tripathi, T. P. Use of auxiliary information in estimating the finite population variance. *Sankhya C.*, 1978, 40(2): 139–148.
- [43] Gujarati, D. M. (2004). *Basic econometrics*. New York: The McGraw-Hill Companies.
- [44] Grover, L. K. A correction note on improvement in variance estimation using auxiliary information. *Communication in Statistics-Theory and Methods*, 2010, 43:101–105.
- [45] Isaki, C. T. Variance estimation using auxiliary information. *Journal of American Statistics Association*, 1983, 78:117–123.
- [46] Kadilar, C. & Cingi, H. Ratio estimators for the population variance in simple and stratified random sampling. *Applied Mathematics and Computation*, 2006, 173(2): 1047–1059.
- [47] Liu, T. P. A general unbiased estimator for the variance of a finite population. *Sankhya C.*, 1974, 36(1): 23–32.
- [48] Muili, J. O., Zakari, Y. & Audu, A. Modified Class of Estimator of Finite Population Variance. *FUDMA Journal of Sciences*, 2019, 3(4): 67-78.

- [49] Mukherjee, C., White, H. & Whyte, M. (1998). *Econometrics and data analysis for developing countries*. Routledge; London.
- [50] Muneer, S., Khalil, A., Shabbir, J. & Narjis, G. (2018). A new improved ratio-product type exponential estimator of finite population variance using auxiliary information. *Journal of Statistical Computation and Simulation*. <https://doi.org/10.1080/00949655.1504947>
- [51] Murthy, M. N. (1967). *Sampling theory and methods*. Calcutta: *Statistical Public Society*.
- [52] Shabbir, J. & Gupta, S. On improvement in variance estimation using auxiliary information. *Communication in Statistical-Theory and Methods*, 2007, 36(1), 2177–2185.
- [53] Singh, R. & Malik, S. Improved estimation of population variance using information on auxiliary attribute in simple random sampling. *Applied Mathematics and Computation*, 2014, 235, 43–49.
- [54] Singh, R. & Mangat, N. S. (1996). *Elements of survey sampling*. Dordrecht, The Netherlands Kluwer Academic.
- [55] Singh, H. P. & Singh, R. Improved ratio type estimator for variance using auxiliary information. *Journal of Indian Society of Agricultural Statistics*, 2011, 54(3), 276–287.
- [56] Singh, H. P. & Solanki, R. S. Improved estimation of finite population variance using auxiliary information. *Communication in Statistics-Theory and Methods*, 2013, 42, 2718–2730.
- [57] Singh, H. P., Upadhyaya, L. N. & Namjoshi, U. D. Estimation of finite population variance. *Curr Science*, 1988, 57, 1331–1334.
- [58] Sumramani, J. & Kumarapandiyam, G. Generalized modified ratio type estimator for estimation of population variance. *Sri-Lankan Journal of Applied Statistics*, 2015, 16(1): 69–90.
- [59] Swain, A. K. P. C. Generalized estimator of finite population variance. *Journal of Statistical Theory and Application*, 2015, 14(1): 45–51.
- [60] Upadhyaya, L. N. & Singh, H. P. An estimator of population variance that utilizes the kurtosis of an auxiliary variable in sample surveys. *Vikram Mathematics Journal*. 199, 19, 14–17.
- [61] Upadhyaya, L. N., Singh, H. P. & Singh, S. A class of estimators for estimating the variance of the ratio estimator. *Journal of Japan Statistical Society*, 2004, 34(1), 47–63.

- [62] Muili, J. O., Audu, A., Singh, R. V. K. and Yunusa, I. A. Improved Estimators of Finite Population Variance using Unknown Weight of Auxiliary Variable. *Annals. Computer Science Series*, 2019, 17(1), 148-153
- [63] Muili, J. O., Audu, A., Singh, R. V. K. Modified Estimator of Finite Population Variance in Simple Random Sampling. *Journal of Science and Technology Research*, 2019, 1(1), 81-90.
- [64] Muili, J. O., Audu, A., Singh, R. V. K. Improved Variance Estimators using Linear Combination of Tri-Mean and Quartile Average. *Annals. Computer Science Series*, 2019, 17(1), 142-147
- [65] Yadav, S. K., Kadilar, C. & Shabbir, J. Improved family of estimators of population variance in simple random sampling. *Journal of Statistical Theory Practice*, 2015, 9(2): 219–226.
- [66] Yadav, S. K. & Kadilar, C. A two-parameters variance estimator using auxiliary information. *Applied Mathematics and Computation*, 2014, 226: 117–122.
- [67] Yaqub, M. & Shabbir, J. (2016). An improved class of estimators for finite population variance. *Hacetatepe Journal of Mathematical Statistics*, 2016, 45(5): 1641–1660.
- [68] Zakari, Y., Muhammad, I. & Sani, N. M. Alternative ratio-product type estimator in simple random sampling. *Communication in Physical Sciences*, 2020, 5(4), 418-426.
- [69] Zakari, Y., Muili, J. O., Tela, M. N., Danchadi, N. S. & Audu, A. Use of unknown weight to enhance ratio-type estimator in simple random sampling. *Lapai Journal of Applied and Natural Sciences*, 2020, 5(1), 74-81.