# Holographic Dark Energy and Dark Matter Interaction in Anisotropic Bianchi Type-V Universe

D. Gemici-Deveci[a], E. Aydiner[b]

[a]*Department of Opticianry, Vocational School of Health Services, Altınbaş University, Tr-34144, İstanbul, Turkey*
[b]*Department of Physics, Faculty of Science, Istanbul University, Fatih Tr-34134, İstanbul, Turkey*

**Abstract**

In this study, we consider an holographic dark energy and dark matter interacting model in the Bianchi Type-V universe with a stretched exponential scale factor. We obtain the Hubble, shear, deceleration, and equation of state parameters based on the presented model and give the numerical solutions. We show that the anisotropy in the early universe plays an important role in the time evolution of the universe. Furthermore, we show that an interacting anisotropic model with stretched exponential scale factors can explain all epochs of the universe.

*Keywords:* Time evolution of the universe, dark energy, dark matter, holographic dark energy.

## 1. Introduction

Recently, the dark energy (DE) and also holographic dark energy (HDE) and dark matter (DM) interacting models getting more important topics in cosmology. Indeed, it is shown in the literature that the interacting models have potential to solve many problems of the cosmology such as singularity, cosmic coincidence and cosmological constant problems [1]-[20]. Additionally, in a more recent study, Aydiner *et al* also show that the late time transition can be explained based on DE and DM interactions [2]. Although no one knows that the physical origin of the dark sector however it plays more important role on the cosmic evolution of the universe.

On the other hand, except the theoretical studies, in the some observational results of Planck collaboration indicates that the dark sector makes contribution to significant part of the total energy of the universe [21]. In addition according to analyzing of the observational data, most cosmological models implicitly assume that the dark sector of the universe only interact gravitationally, but there are also some studies which assume non-gravitationally interaction [6, 22]. The mentioned couplings and the cosmological parameter constraints in some cosmological models are researched [23, 24, 25]. The energy decaying between dark sector has also been showed by different observational data [26, 27, 28]. This observational findings indicate that the physical interactions in dark sector can help us to understanding of the universe dynamics.

We know that the recent observational studies like Cosmic Background Explorers (COBE) [29], Wilkinson Microwave Anisotropy Probe (WMAP) [30, 31] and Planck collaboration [21] reveal that the universe spatially flat, homogeneous and isotropic at the late times as predicted by Friedmann–Lemaître–Robertson–Walker (FLRW) model. However, WMAP data shows that the early universe may not be exactly uniform [32, 33]. In this sense, the anisotropy and interaction of dark sector in the weak anisotropic universe can play important role on the evolution and formation of the universe. More recently, the HDE and DE interacting models including anisotropy to examine the accelerated expansion of late time of the universe are also studied in Refs. [9, 34, 35, 36, 37, 38, 39, 40]. The HDE is a special case of the DE model, which based on the use of various horizons as the radius of the universe depending on the holographic principle tied up with the thermodynamics of black holes and string theory.

Based on these motivation, in this study, we consider HDE and DM interactions in a anisotropic universe model with a special scale factor which has stretched exponential form. Before the presented our model we discuss two points. Our HDE and DM interacting model based on Bianchi Type-V model. All Bianchi models have considered various anisotropic universe metrics. All of them imply that the universe is anisotropic. For example, Bianchi Type-I cosmological model considering a mixture of a perfect fluid and reduces it to flat FLRW soon after inflation. Bianchi Type-II, IV, V, VI0, VII0, VIII, and IX models describe anisotropic space-time and isotropy at late stages of the universe even for ordinary matter. Type-III is missing because it turns out to be a special case of Type-VI. Types-I, VII0, V, VIII and IX have received particular attention as they contain the isotropic FLRW flat, flat, open, open and closed universes, respectively. Other types can be arbitrarily close to, but not exactly, isotropic. These models can be used to analyze aspects of the physical universe which pertain or which may be affected by anisotropy in the rate of expansion, for example, the cosmic microwave background radiation, nucleosynthesis in the early universe and the question of isotropization of the universe itself. Most cosmological models assume that the mater in the universe can be described by dust (a pressureless distribution) or at the early stages of universe viscous effects do play a role. However, bulk viscous models have prime roles in getting inflationary phases of the universe. Unlike other types, Bianchi Type-V universe model not to be in thermal equilibrium in their early stages. Type-V spatially homogenous with perfect fluid cosmological model which contains both viscosity and heat flow [41, 42, 43, 44]. Therefore, in this study, we consider a spatially homogenous anisotropy Bianchi Type-V space-time in which the source of matter distribution is viscous fluid with a cosmological constant.

Additionally, unlike previous HDE and DM interaction model, we consider a stretched exponential scale factor which is given by [45, 46];

$$a = \exp\left[\frac{H_0 T_0}{\beta}\left(\frac{t}{T_0}\right)^{\beta} - 1\right] \tag{1}$$

where $H_0$ is the current value of Hubble parameter, $T_0$ the current age of the universe and $\beta$ is a constant. Clearly, this equation has a stretched exponential form for $\beta < 1$ values and for $\beta = 1$ reduces to exponential form. Actually, one can see that the scale factor in Eq. (1) for all $\beta$ values describe early time and late time exponential expansion periods. However, the time evolution of a stretched form is very different from exponential type. A stretched exponential function for short time period decays faster than normal exponential function, however it slowly down more than exponential one for large time. This interesting behavior can be used to explain the difference of the early and late time inflationary dynamics of the universe. In general, it is assumed that the universe, after 'big-bang', suddenly expands in a very short time, which faster than the late time exponential expansion. Therefore, to explain the early time and late time exponential expanding difference and the early time expanding problems such as flatness and horizon can be solved by using stretched scale factor. Furthermore, the parameter indicates not only spatial disordered but also represents entropic restriction in the considering system, which is consistent not thermal behaviour of the Bianchi V. For this reason, we prefer to study the form of scale factor given in Eq. (1) in the present work. We should note here that the $\beta$ exponent characterizes interactions or disorders between components in an inhomogeneous system. We can say that the irregularity or interactions in the system increase for $\beta < 1$ values, while $\beta = 1$, the system is homogeneous and anisotropic.

In summary, in this study, considering HDE and DM interacting in Bianchi Type-V universe with a scale function in Eq. (1), we discuss the time evaluation of the cosmological parameters depend on $\beta$ parameter and we compare our results with the results of the other theoretical studies in the literature.

The outline of this study is the following: In Section 2, we have consider HDE and DM interaction model in the anisotropic Bianchi Type-V universe with a stretched exponential form of the scale factor. In this section, we obtain some of the cosmological parameters. In Section 3, we give the numerical results and compare with the theoretical and recent observational results. Finally, we present our conclusion in Section 4.

## 2. Holographic Dark Energy Dark Matter Interaction

The holographic principle was firstly suggested by Hooft [47] and Susskind [48] for the black hole physics that contains the effective local quantum field theories with degrees of freedom. In this principle, the entropy scales extensively for an effective quantum field theory in a box of size $L$ with UV cut-off represented by $\lambda$ and the principle assumes that the entropy of a system scales with the surface area not volume, not with its volume. When applying the principle to the DE problem a new DE model is showed up and it is called the HDE model. According to this HDE model, the volume space can seem as 2D line encoded on the lower boundary to the region, so IR cut-off related to DE can be said the size of the corresponding horizon. The energy density of the HDE model is defined by

$$\rho_{DE} = 3c^2 M_p^2 H^2 \tag{2}$$

where $M_p$ is the Planck mass, $c$ is the speed velocity and $H$ is the Hubble parameter with function of time and then depends on $L$ with $H = L^{-1}$ and can be find $H = \dot{a}/a$. In this study, we have chosen the scale of $M_p c = 1$ and also we have chosen an apparent horizon with the radius of $L$.

$$L = ar(t) \tag{3}$$

where $a$ is the average scale factor that indicates the relative size of the universe with the function of time. In this study, we consider spatially homogeneous and anisotropic Bianchi Type-V universe that can be described by the line element

$$ds^2 = -dt^2 + R_1^2 dx^2 + e^{2\alpha x}(R_2^2 dy^2 + R_3^2 dz^2) \tag{4}$$

$R_1$, $R_2$, and $R_3$ are the corresponding metric functions of cosmic time $t$ where $\alpha$ is a constant. The average scale factor of the space-time can be defined as $a{=}(R_1 R_2 R_3)^{1/3}$ in case of the corresponding metric functions, therefore the average Hubble's parameter can be interpreted as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_2}{R_2}\right) \tag{5}$$

over dot indicates the derivative with respect to the cosmic time $t$ and $H_i = \dot{R}_i/R_i$. The Einstein's field equations in natural units ($8\pi G = 1$ and $c = 1$);

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} = R = -T_{ij} \tag{6}$$

where $g_{ij}u^i u^j = 1$, $u^i = (1, 0, 0, 0)$ is the four velocity vector and $R_j$ is the Ricci tensor, $R$ is the Ricci scalar, $T_{ij}$ is the energy-momentum tensor. From the energy conservation

$$T_i^{(DM)j} - T_i^{(DE)j} = 0\ . \tag{7}$$

where $T_{ij}^{(DE)}$ and $T_{ij}^{(DM)}$ are the energy-momentum tensor of DE and DM. These are given by

$$T_i^{(DM)j} = diag[-\rho_{DM}, P^{DM}, P^{DM}, P^{DM}] \tag{8a}$$

$$T_i^{(DE)j} = diag[-\rho_{DE}, P_x^{DE}, P_y^{DE}, P_z^{DE}] \tag{8b}$$

with the relation of $P_{DE} = w_{DE}\rho_{DE}$ the energy momentum tensor of DE is given by

$$T_i^{(DE)j} = diag[-1, w_{DE_x}, w_{DE_y}, w_{DE_z}]\rho_{DE} \tag{9}$$

where $w$ is the EoS parameter of DE with the components of $w_{DE_x}, w_{DE_y}$ and $w_{DE_z}$ are function of the time and they are the directional of $w$ along $x, y$ and $z$ coordinate axes, respectively. By the modifying EoS parameter we assume that the skewness parameters of $\gamma_x$, $\gamma_y$ and $\gamma_z$ and retyping (9)

$$T_i^{(DE)j} = diag[-1, (w_{DE} + \gamma_x), (w_{DE} + \gamma_y), (w_{DE} + \gamma_z)] \tag{10}$$

$\gamma$ denotes the deviation from $w_{DE}$ on $x$, $y$ and $z$ axes. Using of Eqs. (4), (6), (7) and (10) the corresponding field equations are derived as

$$\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \frac{\dot{R}_3\dot{R}_1}{R_3R_1} - \frac{3\alpha^2}{R_12} = (\rho_{DE} + \rho_{DM}) \tag{11a}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} - \frac{\alpha^2}{R_1^2} = -(w_{DE} + \gamma_z)\rho_{DE} \tag{11b}$$

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} - \frac{\alpha^2}{R_1^2} = -(w_{DE} + \gamma_x)\rho_{DE} \tag{11c}$$

$$\frac{\ddot{R}_3}{R_3} + \frac{\ddot{R}_1}{R_1} + \frac{\dot{R}_3\dot{R}_1}{R_3R_1} - \frac{\alpha^2}{R_1^2} = -(w_{DE} + \gamma_y)\rho_{DE} \tag{11d}$$

$$\frac{2\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = 0 \ . \tag{11e}$$

By using Eq. (11e) we can get;

$$R_1^2 = R_2R_3 \ . \tag{12}$$

Assuming $R_2 = R_3^{\varsigma}$ and the relation between the scale factor, and $\varsigma$ is a positive constant and preserves the anisotropic character of the space-time. The metric functions are assumed a power-law relation with cosmic time in an attempt to find exact solutions of the set of field equations. The metric functions are calculated by using Eq. (12)

$$R_1 = a, \quad R_2 = a^{\frac{2\varsigma}{\varsigma+1}}, \quad R_3 = a^{\frac{2}{\varsigma+1}} \tag{13}$$

where the scale factor $a$ is given in Eq. (1). The directional Hubble parameters can be calculated by $H_i = \dot{R}_i/R_i$ by using Eq. (13);

$$H_1 = H_0t^{-1+\beta}T_0^{1-\beta} \tag{14a}$$

$$H_2 = \frac{H_0 \varsigma t^{-1+\beta} {T_0}^{1-\beta}}{1+\varsigma} \tag{14b}$$

$$H_3 = \frac{H_0 t^{-1+\beta} {T_0}^{1-\beta}}{1+\varsigma} \tag{14c}$$

the time dependent Hubble parameter is calculated by putting (14a)-(14c) to (5)

$$H = H_0 t^{-1+\beta} T_0^{1-\beta} \tag{15}$$

the time dependent deceleration parameter $q = -a.\ddot{a}/\dot{a}$ can be found by using (1)

$$q = -\left( \frac{\beta - 1}{H_0 T_0} \left( \frac{t}{T_0} \right)^{-\beta} + 1 \right). \tag{16}$$

The time dependent skewness parameters can be calculated by putting (13) to (11b), (11c) and (11d)

$$\begin{aligned} \gamma_x = \exp\left[ \frac{-H_0(-1+t^\beta)}{{T_0}^{1-\beta}} \right] \left[ \frac{\alpha^2 {T_0}^2}{3{H_0}^2} \right] \\ -2\frac{\exp\left[ (\frac{H_0(1-t^2)}{2T_0})(1+\frac{2\varsigma}{1+\varsigma}) \right] \varsigma t^{-2+2\beta} {T_0}^{4-2\beta}}{9(1+k)} \\ -2\frac{\exp\left[ (\frac{H_0(-1+t^2)}{2T_0})(\frac{4\varsigma}{1+\varsigma}) \right] \varsigma t^{-2+\beta}(-1+\beta) {T_0}^{3-\beta}}{9H_0(1+\varsigma)} \\ -2\frac{\exp\left[ (\frac{H_0(-1+t^\beta){T_0}^{1-\beta}}{2T_0})(\frac{-4}{1+\varsigma}) \right] t^{-2+\beta}(-1+\beta) {T_0}^{3-\beta}}{9H_0(1+\varsigma)} - w_{DE} \end{aligned} \tag{17a}$$

$$\begin{aligned} \gamma_y = \exp\left[ \frac{-H_0(-1+t^\beta)}{{T_0}^{1-\beta}} \right] \left[ \frac{\alpha^2 {T_0}^2}{3{H_0}^2} - \frac{(-1+\beta)}{H_0 {T_0}^{-3+\beta}} \right] \\ -2\frac{\exp\left[ (\frac{H_0(1-t^2)}{2T_0})(1+\frac{2\varsigma}{1+\varsigma}) \right] t^{-2+2\beta} {T_0}^{4-2\beta}}{9(1+\varsigma)} \\ -2\frac{\exp\left[ (\frac{H_0(-1+t^2)}{2T_0})(\frac{4}{1+\varsigma}) \right] t^{-2+\beta}(-1+\beta) {T_0}^{3-\beta}}{9H_0(1+\varsigma)} - w_{DE} \end{aligned} \tag{17b}$$

$$\begin{aligned} \gamma_z = \exp\left[ \frac{-H_0(-1+t^\beta)}{{T_0}^{1-\beta}} \right] \left[ \frac{\alpha^2 {T_0}^2}{3{H_0}^2} - \frac{(-1+\beta)}{H_0 {T_0}^{-3+\beta}} \right] \\ -2\frac{\exp\left[ (\frac{H_0(1-t^2)}{2T_0})(1+\frac{2\varsigma}{1+\varsigma}) \right] \varsigma t^{-2+2\beta}}{9(1+\varsigma){T_0}^{-4+2\beta}} \\ -2\frac{\exp\left[ (\frac{H_0(-1+t^2)}{2T_0})(\frac{4\varsigma}{1+\varsigma}) \right] \varsigma t^{-2+\beta}(-1+\beta)}{9H_0(1+\varsigma){T_0}^{-3+\beta}} - w_{DE}\ . \end{aligned} \tag{17c}$$

The physical quantities named as cosmological parameters that are important in cosmology. Here $\theta$ is the expansion scalar and it formulates as;

$$\theta = 3H = 3H_0 t^{-1+\beta-1} T_0^{1-\beta} \ . \tag{18}$$

The scalar expansion $\theta > 0$ indicates that the model yields expanding in nature. The shear scalar which is a measure of how the expansion rate differs in the different directions is represented as $\sigma^2$

$$\sigma^2 = \frac{1}{2}(\sum_{i=1}^{n} H_i^2 - \frac{1}{3}\theta^2) \tag{19}$$

can be calculated by using (15) and (18)

$$\sigma^2 = \frac{H_0^2(-1+k)^2 t^{-2+2\beta} T_0^{2-2\beta}}{(1+k)^2} \ . \tag{20}$$

As can be seen the rate of $\sigma^2/\theta^2$ does not depending on the cosmic time that it implies that the model does not pass isotropy. The corresponding average anisotropy parameter is the measure of the magnitude of anisotropy in the expansion rate is given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}(\frac{H_i - H}{H})^2 \tag{21}$$

using Eqs. (14a)-(14c),(15) and (21)

$$\Delta = \frac{2(-1+\varsigma)^2}{3(1+\varsigma)^2} \ . \tag{22}$$

In order to obtain the coupling interaction equation we follow theoretical procedure. Based on the FLRW metric the interaction between DE and DM is represented by the continuity equation which is the energy balance equations of DE and DM [1, 16, 50, 49]. We assume that the universe in which the DE and DM are interacting with each other and the total energy density satisfies the continuity equation, therefore we can rewrite Eq. (7) by using Eq. (11a)-Eq. (11e) and Eq. (12) to find the continuity equation as

$$\dot{\rho}_{DE} + \dot{\rho}_{DM} + \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right)(\rho_{DE}(1+w_{DE}) + \rho_{DM}) + \left(\gamma_x\frac{\dot{R}_1}{R_1} + \gamma_y\frac{\dot{R}_2}{R_2} + \gamma_z\frac{\dot{R}_3}{R_3}\right)\rho_{DE} = 0 \tag{23}$$

we can rewrite by separating the equation of (23) for the non-interaction case

$$\dot{\rho}_{DE} + 3H(1+w_{DE})\rho_{DE} + (\gamma_x H_x + \gamma_y H_y + \gamma_z H_z)\rho_{DE} = 0 \tag{24a}$$

$$\dot{\rho}_{DM} + 3H\rho_{DM} = 0 \tag{24b}$$

for the interaction case

$$\dot{\rho}_{DE} + 3H(1 + w_{DE})\rho_{DE} + (\gamma_x H_x + \gamma_y H_y + \gamma_z H_z)\rho_{DE} = -Q \tag{25a}$$

$$\dot{\rho}_{DM} + 3H\rho_{DM} = Q \tag{25b}$$

where $Q$ represents the overall conservation of the energy-momentum tensor which indicates a transfer from DE component to DM component and vice versa. There are different $Q$ definition in the literature [16, 17, 20, 51]. If we choose to take the coupling term depending on the density of DE;

$$Q = 3b^2 H\rho_{DE} \tag{26}$$

where $b$ is the coupling constant. Then we have used Eqs. (2), (25a) and (26) to get the corresponding EoS parameter of DE;

$$w_{DE} = -1 - b^2 - \frac{2\dot{H}}{3H^2} - \frac{(\gamma_x H_x + \gamma_y H_y + \gamma_z H_z)}{3H} \tag{27}$$

and by using Eqs. (14b), (14c), (17a), (17b) and (17c) the directional EoS Parameters can be obtained as

$$\begin{aligned} w_{DE} = {} & -1 - b^2 + \frac{-2t^\beta(1+\beta){T_0}^{-1+\beta}}{3H_0} - \frac{1}{3t^2}e^{-4\eta}{T_0}^{-2\beta} - 3e^{-4\eta}t^2 w{T_0}^{2\beta} \\ & + e^{2\eta}{T_0}^\beta(-2H_0 t^\beta(-1+\beta)T_0 + 9t^2\alpha^2{T_0}^\beta) - \frac{4e^{3\eta(3-\frac{2\varsigma}{1+\varsigma})}\varsigma t^{2\beta}}{(1+\varsigma)^2{H_0}^{-2}{T_0}^{-2}} - \\ & \frac{2e^{3\eta(\frac{1+3\varsigma}{1+\varsigma})}(1+3\varsigma)t^{2\beta}}{(1+\varsigma)^2{H_0}^{-2}{T_0}^{-2}} - \frac{2e^{\frac{4\eta\varsigma}{1+\varsigma}}(-1+\beta)t^\beta}{(1+\varsigma)^2{H_0}^{-1}{T_0}^{-\beta-1}}((1+3\varsigma)^2(1+\varsigma^2)) \end{aligned} \tag{28}$$

where $\eta = H_0(-1+t^\beta){T_0}^{-1+\beta}\beta^{-1}$.

It can be seen from Eq. (28) that the directional EoS parameter depends on the cosmological time and the coupling constant.

## 3. Numerical Results and Discussion

In the previous section, we consider a HDE and DM interacting model in Bianchi Type-V universe with a stretched exponential scale function in Eq. (1). We obtain some cosmological parameters depend on $\beta$ such as Hubble, deceleration, skewness, shear and EoS parameters. Here, we give the obtained numerical solutions and compare our results with the observational and theoretical results of the isotropic universe models.

Firstly, we give the time evolution of the scale factor $a$ and Hubble parameter $H$ for various $\beta$ values in Fig. 1. We set $H_0 = 0.07$ Gyr$^{-1}$ and $T_0 = 13.77$ Gyr in both figures. We mentioned in introduction that the scale factor $a$ in

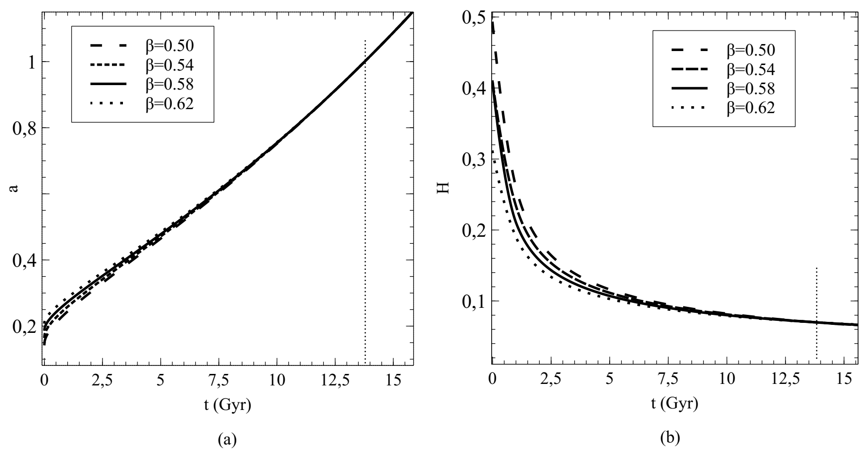


Figure 1: Time evolution of the scale factor in (a) and Hubble parameter in (b) for various $\beta$ values. We set $H_0 = 0.07$ Gyr$^{-1}$ and $T_0 = 13.77$ Gyr in both figures. The vertical dot line indicates the present time.

Eq. (1) for all $\beta < 1$ values describe early time and late time exponential expansion periods. Indeed, as seen from Fig. 1(a) that in early stage i.e. shortly after 'big-bang', the scale factor behaves faster than exponential which indicates the universe suddenly expands in a very short time. This inflationary dynamics rapid expansion cause to flat universe metric. This initial phase can called early time inflationary expansion. If we assume that $\beta$ represents the Bianchi anisotropy, one can see anisotropy effects plays an dominant role at the initial phase up to intermediate time in the cosmic evolution of the universe. Interestingly, in the intermediate time evolution is relatively power law expansion as seen from Fig. 1(a) and the $\beta$ effects on the dynamics disappears around almost $t > 7.5$ Gyr up to almost 10 Gyr. However, for $> 10$ Gyr scale factor goes to a new regime which faster than subsequent previous regime but it slow than early initial times expansion dynamics. This last period may represents the late time acceleration phase of the universe. This also indicates that the stretched exponential scale factor may represent the anisotropic Bianchi Type V model which contain initial thermal non-equilibrium and eat low at the early time of the universe. Obtained results are also good agreement for late time transition with the theoretical results in Ref. [2] in which they show that DM and DE interaction can explain the catastrophic transition from power law expanding to the inflationary expansion at the late time. Similarly, for the same parameter we plot Hubble parameter $H$ versus cosmic time in Fig. 1(b) for various $\beta$ values. It can be seen Hubble parameter is getting the same value but not the constant value at the present time. If we remember the Hubble parameter is depended the scale factor by $\dot{a}/a$, the expansion of the universe causes increasing behavior of the scale factor, therefore it causes $\dot{a} > 0$. Because of the accelerated expansion of the Universe the value of $\dot{a}$ increases. It means that the Hubble parameter is decreasing with time. Therefore, it can be said that in an anisotropic universe model where the interactions of DE and DM are taken into consideration, the Universe will continue to expand in the late times. Overlapping of these curves is occuring approximately at the 6.650 Gyr. It can be said that accelerated expansion is independent from the Hubble parameter and the present age of the Universe. As seen from this figure that the anisotropy effects play important role at the early times, however, the anisotropy effects nearly disappear after for $> 10$ Gyr and Hubble parameter converge to $H_0 = 0.07$ Gyr$^{-1}$ as expected. This is also good agreement with theoretical calculations from observational data [52, 53]. The dynamics of the average anisotropy parameter is calculated in Eq. (22). It depends on the value of $\varsigma$ which is a positive constant and preserves the anisotropic character of the space-time. Therefore the anisotropy does not decay at the late times of the Universe, so it is possible to say that this model is arbitrarily close to, but not exactly, isotropic.

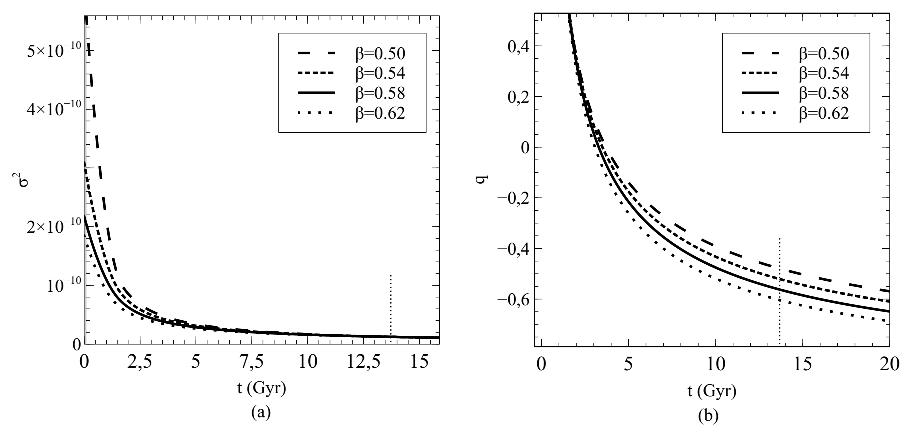


Figure 2: Time evolution of the shear scalar in (a) and deceleration parameter in (b) for various $\beta$ values. We set $H_0 = 0.07$ Gyr$^{-1}$ and $T_0 = 13.77$ Gyr in both figures. The vertical dot line indicates the present time.

In Fig. 2, we present the time evolution of the shear scale factor $\sigma$ and deceleration parameter $q$ for various $\beta$ values. We set $H_0 = 0.07$ Gyr$^{-1}$ and $T_0 = 13.77$ Gyr in both figures. The shear scalar $\sigma$ indicates the deviation from the average expansion rate of the universe [54]. In our model, as seen from Fig. 2(a), the anisotropy and thermal flow in the early times play an important role on the expansion dynamics as well the Hubble parameter. However, when anisotropy effects nearly disappear and the universe up to thermal equilibrium, shear parameter also reach to a saturated values like the Hubble constant. Generally, it is assumed that this parameter approaches to a constant non-zero value in the $\lim_{t\to\infty} \sigma^2/\theta = 0^+$. This refers to an isotropic and flat universe. The obtained result is consistent with the other theoretical studies [55, 56]. Interestingly, the results in Fig. 2(b) the early time anisotropy effects determine the future time evolution of the universe. The dimensionless deceleration parameter measures velocity rate of the cosmic expansion. As seen form Fig. 2(b), values of the deceleration parameter $q$ decreases and reach to negative value with time as expected.

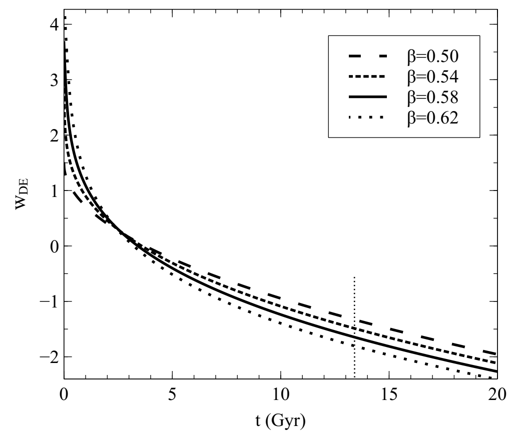


Figure 3: Time evolution of the EoS parameter of DE for various $\beta$ values. We set $w = -1$, $H_0 = 0.07$ Gyr$^{-1}$ and $T_0 = 13.77$ Gyr. The vertical dot line indicates the present time.

Finally, we plot the time evolution of the EoS parameter of DE $w_{DE}$ for various $\beta$ values in Fig. 3. Here, we set $w = -1$, $H_0 = 0.07$ Gyr$^{-1}$ and $T_0 = 13.77$ Gyr. One can see more interesting physical behavior in Fig. 3. The time crossover around $t_c = 3.21$ Gyr time can be seen more apparently. Indeed, this point can be regarded as phase transition point in time from early time inflationary expansion to the intermediate regime of the cosmic time evolution of the universe. We can state that the anisotropy effects dominates the time evolution below $t_c$. However, these effects weaken and nearly disappears above $t_c$. The time evolution of the EoS parameter of DE $w_{DE}$, in general, obey to expected behavior in FLRW cosmology. It decrease slowly and reach up the present value i.e -1. However, we see that the initial anisotropy effects determine the EoS value. Figure says that EoS parameter can converge to -1 value only if the universe has strong anisotropy effects. In fact, as seen from figure $w_{DE}$ can take -1 value for $\beta = 0.5$ values. This clearly indicates the anisotropy affects and non-thermal dynamics should be strong in the beginning of the universe. On the other hand, it is very difficult to predict the late time transition evidences form this figure. To obtain more detail and precise results we need more detail studies on the model and its results.

## 4. Conclusion

In this study, we consider a HDE and DM interacting model in Bianchi Type V universe with a stretched exponential scale function in Eq. (1). We obtain some cosmological parameters depend on $\beta$ such as Hubble, shear, deceleration and EoS parameters. We present the obtained numerical solutions and compare our results with the theoretical results of the isotropic universe models to discuss or explain, particularly, early and late time expansion of the universe.

We suggest and conclude that the stretched exponential scale factor can represent the different epochs of the universe time evolution. Indeed, we see that this type of the scale function is more suitable than a single type scale factor which has exponential or trigonometric form. On the other hand, we study HDE and DM interacting model in isotropic Bianchi Type V model considering the stretched exponential form of the scale factor. We also suggest that the exponent $\beta$ denotes anisotropy effects in the early time universe. Hence, combining this scale factor and Bianchi Type V model, we obtain the time evolution behavior of the Hubble, shear, deceleration and EoS parameters in the presence of the HDE and DM interactions.

We show that the anisotropy can play an important role on the time evolution of the universe. Our numerical results are strongly provide this simple idea. We also reveal that the presence of the anisotropy determine the future of the universe. Particularly, we find that anisotropy effects leads to a rapid expansion in the early universe after 'big-bang', which solve the flatness and horizon problems. This epoch continues to approximately 3.21 Gyr in the cosmic time under strong anisotropy effects. Interestingly, our results also emphasize that our universe goes into a different accelerating phase around $> 10$ Gry in the cosmic time as can be seen both figures. This form of the scale factor in the model could describe all of the cosmic history including inflationary stage as well as the epoch of the late time cosmic expansion.

In summarize, the results of the presented model, surprisingly, proposes some solutions to important problems of cosmology. Therefore, we, finally, conclude has potential which can lead to new discussions and studies in this area.


[1] E. Aydiner, Chaotic universe model, Scientific Reports, (1) (2018) 721.

[2] E. Aydiner, I. Basaran-Oz, T. Dereli and M. Sarisaman, Late time transition in the evolution of the Universe, (2021) arXiv:Researchsquare.

[3] P. Brax and J. Martin, The supergravity quintessence model coupled to the minimal supersymmetric standard model, Journal of Cosmology and Astroparticle Physics 11 (2006).

[4] Y. L. Bolotin, A. Kostenko, O. Lemets and D. Yerokhin, Cosmological evolution with interaction between dark energy and dark matter, International Journal of Modern Physics D, 24 (03) (2015).

[5] L. Amendola, G. C. Campos and R. Rosenfeld, Consequences of dark matter-dark energy interaction on cosmological parameters derived from type Ia supernova data, Physical Review D, 75 (2007).

[6] B. Wang, E. Abdalla and F. Atrio-Barandela, Dark matter and dark energy interactions: theoretical challenges, cosmological implications and observational signatures, Reports on Progress in Physics, 79 (9) (2016).

[7] J.H. He and B. Wang, Effects of the interaction between dark energy and dark matter on cosmological parameters, Journal of Cosmology and Astroparticle Physics, 06 (2008) 010.

[8] R.-G.Cai, N. Tamanini and T. Yang, Reconstructing the dark sector interaction with LISA, Journal of Cosmology and Astroparticle Physics, 05 (2017) 031.

[9] W. Yang, S. Pan, Supriya and A. Paliathanasis, Cosmological constraints on an exponential interaction in the dark sector, Monthly Notices of the Royal Astronomical Society, 482 (1) (2019).

[10] D. Tocchini-Valentini and L. Amendola, Stationary dark energy with a baryon-dominated era: Solving the coincidence problem with a linear coupling, Physical Review D, 65 (2002).

[11] L. Amendola and C. Quercellini, Tracking and coupled dark energy as seen by the Wilkinson Microwave Anisotropy Probe, Physical Review D, 68 (2003) .

[12] S. del Campo, R. Herrera and D. Pavon, Interacting models may be key to solve the cosmic coincidence problem, Journal of Cosmology and Astroparticle Physics, (01) (2009) 020.

[13] S. del Campo, R. Herrera and D. and Pavón, Interaction in the dark sector, Physical Review D, 91 (2015) 123539.

[14] L. P. Chimento, Linear and nonlinear interactions in the dark sector, Physical Review D, 81 (2010) 043525.

[15] C. Miguel and L. Samuel, Holographic approach for dark energy–dark matter interaction in curved FLRW spacetime, Classical and Quantum Gravity, 35 (35) (2018).

[16] D. Gemici-Deveci and E. Aydiner, Quadratic interaction effect on the dark energy density in the universe, Chinese Physics B, 26 (10) (2017).

[17] M. Setare, Interacting holographic dark energy model in non-flat universe, Physics Letters B, 642 (1-2) (2006) .

[18] C.-Y. Sun and R.-H.Yue, New interaction between dark energy and dark matter changes sign during cosmological evolution, Physical Review D, 85 (4) (2012).

[19] J. Binder and G. Kremer, Model for a Universe described by a non-minimally coupled scalar field and interacting dark matter, General Relativity and Gravitation, 38 (5) (2006)

[20] D. Gemici-Deveci and E. Aydiner, Influence of Linear Plus Quadratic Interactions Between Dark Components of the Universe on Thermodynamics, Gravitation and Cosmology, 25 (1) (2019).

[21] P. A. Ade, N. Aghanim, M. Arnaud, M. Ashdown, J. Aumont, C. Baccigalupi, A. Banday, R. Barreiro, J. Bartlett, N. Bartolo, et al., Planck 2015 results-xiii. cosmological parameters, Astronomy & Astrophysics, 594 (2016) A13.

[22] A. A. Costa, X.-D.Andre Xu, B. Wang, E. G. Ferreira and E. Abdalla, Testing the interaction between dark energy and dark matter with Planck data, Physical Review D, 89 (10) (2010).

[23] C. Van De Bruck and J. Mifsud, Searching for dark matter-dark energy interactions: going beyond the conformal case, Physical Review D, 97 (2) (2018)

[24] Z.-K. Guo, N. Ohta and S. Tsujikawa, Realizing scale-invariant density perturbations in low-energy effective string theory, Physical Review D, 75 (2) (2007)

[25] T. S. Koivisto and N. J. Nunes, Coupled three-form dark energy, Physical Review D, 88 (1) (2013).

[26] S. Micheletti, E. Abdalla and B. Wang, Field theory model for dark matter and dark energy in interaction, Physical Review D, 79 (12) (2009).

[27] E. Abdalla, L. R. Abramo and L. Sodré Jr an B. Wang, Signature of the interaction between dark energy and dark matter in galaxy clusters, Physics Letters B, 673 (2) (2009).

[28] S. Kumar and R. C. Nunes, Observational constraints on dark matter–dark energy scattering cross section, European Physical Journal C, 77 (11) (2017).

[29] G. F. Smoot, C. L. Bennett, A. Kogut, E. Wright, J. Aymon, N. Boggess, E. Cheng, G. De Amici, S. Gulkis, M. Hauser, et al., Structure in the COBE differential microwave radiometer first-year maps, The Astrophysical Journal, 306 (1992) L1-L5.

[30] D. N. Spergel, L. Verde, H. V. Peiris, E. Komatsu, M. Nolta, C. L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut and others, First-year Wilkinson Microwave Anisotropy Probe (WMAP)* observations: determination of cosmological parameters, The Astrophysical Journal Supplement Series, 148 (1) (2003).

[31] C. Bennett, R. Hill, G. Hinshaw, M. Nolta, N. Odegard, L. Page, D. Spergel, J. Weiland, E. Wright, M. Halpern, et al., First-Year Wilkinson Microwave Anisotropy Probe (WMAP)* Observations: Foreground Emission, The Astrophysical Journal Supplement Series, 148 (1) (2003).

[32] G. Hinshaw, D. Spergel, L. Verde, R Hill, S. Meyer, C. Barnes, C. Bennett, M. Halpern, N. Jarosik, A. Kogut and others, First-Year Wilkinson Microwave Anisotropy Probe (WMAP)* Observations: The Angular Power Spectrum, The Astrophysical Journal Supplement Series, 148 (1) (2003) .

[33] G. Hinshaw, J. Weiland, R Hill, N. Odegard, D. Larson, C. Bennett, J. Dunkley, B. Gold, M. Greason, N. Jarosik, et al. A. Kogut and others, Five-year wilkinson microwave anisotropy probe* observations: data processing, sky maps, and basic results,The Astrophysical Journal Supplement Series, 180 (2) (2009).

[34] S. Pan, W. Yang, C. Singha and E. N. Saridakis, Observational constraints on sign-changeable interaction models and alleviation of the H0 tension, Physical Review D, 100 (8) (2019).

[35] X. Xu, Y.-Z. Ma and A. Weltman, Constraining the interaction between dark sectors with future HI intensity mapping observations, Physical Review D, 97 (2018) 083504.

[36] R. von Marttens, L. Casarini, D. Mota and W. Zimdahl, Cosmological constraints on parametrized interacting dark energy, Physics of the Dark Universe, Physical Review D, 97 (2018) 100248.

[37] R. G. Kim and C. H. Ri, Chang, Observational constraints on cosmic parameters of holographic dark energy model with varying interaction, Astrophysics and Space Science, 365 (7) (2020).

[38] M. Rao, M. V. Santhi, T. Vinutha, et. al., Exact Bianchi type II, VIII and IX string cosmological models in Saez-Ballester theory of gravitation, Astrophysics and Space Science, 314 (1-3) (2008).

[39] S. Sarkar, Holographic dark energy with linearly varying deceleration parameter and escaping big rip singularity of the Bianchi type-V universe, Astrophysics and Space Science, 352 (2) (2014).

[40] M. Santhi, V. Rao and Y. Aditya, Anisotropic magnetized holographic Ricci dark energy cosmological models, Canadian Journal of Physics, 95 (4) (2017).

[41] C. Collins, Tilting at cosmological singularities, Communications in Mathematical Physics, 39 (2) (1974).

[42] D. L. Farnsworth, Some new general relativistic dust metrics possessing isometries, Journal of Mathematical Physics, 8 (12) (1967).

[43] R. Bali nad B. Meena, Conformally flat tilted Bianchi Type-V cosmological models in general relativity, Pramana, 62 (5) (2004).

[44] A. Pradhan and A. Rai, Tilted bianchi type v bulk viscous cosmological models in general relativity, Astrophysics and Space Science, 291 (2) (2004).

[45] N. P. Silva, Model for the Expansion of the Universe, Progress in Physics, 10 (2) (2014).

[46] N. P. Silva, The Universe's Expansion Factor and the Hubble's Law, Progress in Physics, 13 (1) (2016).

[47] G. 'T. Hooft, Dimensional reduction in quantum gravity, arXiv gr-qc/9310026.

[48] L. Susskind, The world as a hologram, Journal of Mathematical Physics, 36 (11) (1995)

[49] J. Valiviita, M. Elisabetta and R. Maartens, Large-scale instability in interacting dark energy and dark matter fluids, Journal of Cosmology and Astroparticle Physics 2008.07 (2008): 020.

[50] T. Clemson, et al., Interacting dark energy: Constraints and degeneracies, Physical Review D 85.4 (2012): 043007.

[51] L. P. Chimento, Linear and nonlinear interactions in the dark sector, Physical Review D 81.4 (2010) 043525.

[52] Y. Chen, K. Suresh and R. Bharat, Determining the Hubble constant from Hubble parameter measurements, The Astrophysical Journal 835.1 (2017) 86.

[53] S. Cao, R. Joseph and R. Bharat, Using Pantheon and DES supernova, baryon acoustic oscillation, and Hubble parameter data to constrain the Hubble constant, dark energy dynamics, and spatial curvature, Monthly Notices of the Royal Astronomical Society 504.1 (2021) 300-310.

[54] C. Collins and J. Wainwright, Role of shear in general-relativistic cosmological and stellar models, Physical Review D, 27 (1983) 1209-1218.

[55] S. Kumar and C. Singh, Bianchi type-V perfect fluid space-time models in general relativity, Astrophysics and Space Science, 312 (1) (2007).

[56] C. Singh, S. Ram and M. Zeyauddin, Bianchi type-V perfect fluid space-time models in general relativity, Astrophysics and Space Science, 315 (1) (2008).