Additive Seasonality in Time Series using Row and Overall Sample Variances of the Buys-Ballot Table

Abstract: This study considers test for seasonality in the additive model using the Buys-Ballot table and the nature of trending curves (linear, quadratic and exponential). The test is applied to the periodic and overall sample variances of the Buys-Ballot table to detect the presence and absence of seasonal indices in time series.
Keywords: Additive Seasonality, Periodic Variance, Overall Variance, Buys-Ballot table, Trending Curves, Seasonal Indices

1 INTRODUCTION

It is important to note in time series that, the presence of a seasonal component in a time series is true and detection of seasonal periods are quiet easy. Good examples are; for monthly time series data, one time period is one twelfth of a year, it displays specific pattern that repeats every 12 observations, so **s** = 12. Similarly, for quarterly series, it is expected that between observations, four time point apart will yield s = 4. Therefore, s = 1 is for one year time period, s = 2 for observations recorded half yearly, s = 3 for time series data recorded three times in a year, s = 6 for observations recorded six time in a year are other common values s. Seasonality in time series data can be observed as a pattern that repeats every k elements. Some graphical methods are used to detect the presence of seasonal effect in time series are: (1) a run sequence plot (Chambers et al [1]). (2) a seasonal sub-series plot [2]; (3) multiple box plots [1]; (4) the autocorrelation plot [3]. Davey and Flores [4] added statistical tests for seasonality. Kendall and Ord [5] studied test of seasonality in time series analysis. Chatfield [6] presented the use of Buys-Ballot table for detecting the presence of trend and seasonal component in time series.

A seasonal time series with *m* rows and *s* columns, *m* represents the period while *s* represents the seasons. This two-dimensional arrangement of a series is called the Buys-Ballot table (see Table 1). For details of the procedure see Iwueze and Nwogu [7], Nwogu et al. [8], Dozie et al. [9], Dozie and Ihekuna [10], Dozie and Nwanya [11], Dozie [12], Dozie and Uwaezuoke [13] and Dozie and Ibebuogu [14]

Iwueze and Nwogu [7] indicated that, for the trending curves (linear, quadratic and exponential), the seasonal variances depends only on the trend parameters for the additive model. Also, their study show that, if the seasonal variances are functions of trend parameters only, then the suitable model structure is additive. It is the series with seasonal effect in the seasonal variances of the Buys-Ballot table that make the proper model structure to be multiplicative.

For additive model and selected trending curves (linear, quadratic and exponential) studied in this paper, the periodic and overall variances contain both the trending series of the original time series and seasonal indices. Hence, the test for seasonality using the periodic and overall variances for selected trending curves for detection of the presence and absence of seasonal indices has been developed and the model structure is additive.

2 Methodology

The summary of the Buys-Ballot estimates of the periodic and overall variances with error variances with linear trend component obtained by Dozie and Uwaezuoke [13] given in equations (4) and (5) while that of quadratic and exponential trend components by Iwueze and Nwogu [7] listed in equations (6), (7), (8) and (9) for additive model.

Table 1: Buys-Ballot Table

Rows (i)	Columns j								
	1	2	$\cdots$	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	$\cdots$	X_j	$\cdots$	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	$\cdots$	X_{s+j}	$\cdots$	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	$\cdots$	X_{2s+j}	$\cdots$	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	$\cdots$	$X_{(i-1)s+j}$	$\cdots$	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	$\cdots$	$X_{(m-1)s+j}$	$\cdots$	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	$\cdots$	$T_{.j}$	$\cdots$	$T_{.s}$	$T_{..}$		

$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\cdots$	$\bar{X}_{.j}$	$\cdots$	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	$\cdots$	$\hat{\sigma}_{.j}$	$\cdots$	$\hat{\sigma}_{.s}$			$\hat{\sigma}_{x}$

Where, n = ms = length of the series, m is period and s is seasons

2.1 Periodic and Overall Variances

The Buys-Ballot estimates for periodic variances are listed in equations (4), (6) and (8) and that of overall variances are given in (5), (7) and (9) for selected trending curves for the purposes of detection of presence seasonal effects

2.2 Linear Trending Curve $\left(a+bt\right)$

$$\sigma_{i.}^{2}=b^{2}s\left(\frac{s+1}{2}\right)+\frac{2b}{s-1}\sum_{j=1}^{s}jS_{j}+\frac{1}{s-1}\sum_{j=1}^{s}S_{j}^{2}+\sigma_{1}^{2} \tag{4}$$

$$\sigma_{x}^{2}=\frac{b^{2}n\left(n+1\right)}{12}+\frac{m}{n-1}\sum_{j=1}^{s}S_{j}^{2}+\frac{2bm}{n-1}\sum_{j=1}^{s}jS_{j}+\sigma_{1}^{2} \tag{5}$$

2.3 Quadratic Trending Curve $\left(a+bt+ct^{2}\right)$

$$\sigma_{i.}^{2}=\left\{\begin{array}{l}\frac{s\left(s+1\right)}{180}\left\{\left(2s-1\right)\left(8s-11\right)c^{2}-30\left(s-1\right)bc+15b^{2}\right\}+\\ \frac{1}{s-1}\left\{\sum_{j=1}^{s}S_{j}^{2}+2\left[b-2cs\right]C_{1}+2cC_{2}\right\}\\ +\left\{\frac{s^{2}\left(s+1\right)}{3}\left[bc-c^{2}\left(s-1\right)+\frac{4csC_{1}}{s-1}\right]\right\}i+\left[\frac{s^{2}\left(s+1\right)c^{2}}{3}\right]i^{2}\end{array}\right\} \tag{6}$$

$$\sigma_{..}^{2}=\frac{nc^{2}}{n-1}\left\{\begin{array}{l}\frac{\left(n^{2}-s^{2}\right)\left(2n-s\right)\left(8n-11s\right)}{180}+\frac{\left(s^{2}-1\right)\left(2s+1\right)\left(8s-1\right)}{180}\\ +\frac{\left(n-s\right)\left(s+1\right)\left(6n^{2}+7ns-n+s^{2}+5s+6\right)}{36}\end{array}\right\} \tag{7}$$

$$+\frac{bcn\left(n+1\right)^{2}}{6}+\frac{b^{2}n\left(n+1\right)}{12}+\frac{n}{s\left(n-1\right)}\left\{\sum_{j=1}^{s}S_{j}^{2}+2\left[b+c\left(n-s\right)\right]C_{1}+2cC_{2}\right\}$$

Where $C_{1}=\sum_{j=1}^{s}jS_{j},\ C_{2}=\sum_{j=1}^{s}jS_{j}$

2.4 Exponential Trending Curve (be^{ct})

$$\sigma_{i.}^2 = b^2 e^{2c[(i-1)s+1]}\left[\left(\frac{1-e^{2cs}}{1-e^{2c}}\right) - \frac{1}{s}\left(\frac{1-e^{cs}}{1-e^{c}}\right)\right] + \sum_{j=1}^{s} S_j^2 + 2be^{c(i-1)s}\sum_{j=1}^{s} e^{cj} S_j \qquad (8)$$

$$\sigma_{..}^2 = \frac{b^2 e^{2c}}{n-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2c}}\right) - \frac{1}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)^2\right] + \frac{m}{m-1}\sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1}\sum_{j=1}^{s} e^{cj} S_j \qquad (9)$$

3 Test for Seasonality in Time Series Analysis

3.1 Properties of Additive Model

(1) $\bar{X}_{.j} - \bar{X}_{..}$ (10)

(2) $\sum_{j=1}^{s}\left(\bar{X}_{.j} - \bar{X}_{..}\right) = 0$ (11)

(3) $\sigma^2\left(\bar{X}_{.j} - \bar{X}_{..}\right)$ (12)

For properties of some selected trending curve, we observe the following before test of seasonality in the additive model (i) equation (10) contains trending parameters and seasonal effect at season j. (ii) for equation (11) the difference between the column mean and overall mean is equal to zero (iii) equation (12) depends on the trending curves of the original series (iv) for equations (10) and (11), a time series data should be de-trended before test of seasonality.

3.2 Application of Test for seasonality in the Buys-Ballot table

Matched pairs of data are applied to the periodic and overall variances of the Buys-Ballot For the matched pairs of data, $(X_i, Y_i), \quad i = 1, 2, \ldots n$, define $d_i = X_i - Y_i$. For identification of the presence and absence of seasonal indices in time series data. Let X_i represents periodic and overall variances in the presence of seasonal indices and denote Y_i represents periodic and overall variances in the absence of seasonal indices (Nwogu *et al.* [15])

3.3 For linear trend, in the presence of seasonal effect, the periodic variance is obtained as

$$X_i(L) = \sigma_{i.}^2(L) = b^2 s\left(\frac{s+1}{2}\right) + \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 + \sigma_1^2 \qquad (13)$$

When there is no seasonal indices, $S_j = 0 \forall j = 1,2,...,s,$ therefore,

$$Y_i(L) = b^2 s\left(\frac{s+1}{2}\right) + \sigma_1^2 \qquad (14)$$

and

$$d_i = X_i(L) - Y(L)_i = \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 + \qquad (15)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

3.4 The overall variance is obtained as

$$X_i(L) = \sigma_x^2(L) = \frac{b^2 n(n+1)}{12} + \frac{m}{n-1}\sum_{j=1}^{s} S_j^2 + \frac{2bm}{n-1}\sum_{j=1}^{s} jS_j + \sigma_1^2 \qquad (16)$$

When there is no seasonal indices, $S_j = 0 \forall j = 1,2,...,s,$ therefore

$$Y_i(L) = \frac{b^2 n(n+1)}{12} + \sigma_1^2 \qquad (17)$$

and

$$d_i(L) = X_i(L) - Y_i(L) = +\frac{m}{n-1}\sum_{j=1}^{s} S_j^2 + \frac{2bm}{n-1}\sum_{j=1}^{s} jS_j \qquad (18)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

3.5 For Quadratic trend, in the presence of seasonal effect, the periodic variance is obtained as

$$X_i(Q)=\sigma_{i.}^2(Q)=\left\{\begin{array}{l}\frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2-30(s-1)bc+15b^2\right\}+\\ \frac{1}{s-1}\left\{\sum_{j=1}^{s}S_j^2+2\left[b-2cs\right]C_1+2cC_2\right\}\\ +\left\{\frac{s^2(s+1)}{3}\left[bc-c^2(s-1)+\frac{4csC_1}{s-1}\right]\right\}i+\left[\frac{s^2(s+1)c^2}{3}\right]i^2\end{array}\right\} \qquad (19)$$

When there is no seasonal indices, $S_j=0\forall j=1,2,...,s,\quad C_1=C_2=\sum_{j=1}^{s}S_j^2=0.$ thus

$$Y_i(Q)=\left\{\begin{array}{l}\frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2-30(s-1)bc+15b^2\right\}+\\ \\ +\left\{\frac{s^2(s+1)}{3}\left[bc-c^2(s-1)\right]i\right\}+\left[\frac{s^2(s+1)c^2}{3}\right]i^2\end{array}\right\} \qquad (20)$$

$$d_i(Q)=X_i(Q)-Y_i(Q)\left\{\begin{array}{l}\frac{1}{s-1}\left\{\sum_{j=1}^{s}S_j^2+2\left[b-2cs\right]C_1+2cC_2\right\}\\ +\left\{\left[+\frac{4csC_1}{s-1}\right]\right\}i\end{array}\right\} \qquad (21)$$

Which is zero under null hypothesis ($H_o:S_j=0$)

3.6 The overall variance is obtained as

$$X_i(Q)=\sigma_{i.}^2(Q)=\frac{nc^2}{n-1}\left\{\begin{array}{l}\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}\\+\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\end{array}\right\} \quad \textbf{(22)}$$

$$+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s}S_j^2+2\left[b+c(n-s)\right]C_1+2cC_2\right\}$$

When there is no seasonal indices, $S_j=0\forall j=1,2,...,s,\quad C_1=C_2=\sum_{j=1}^{s}S_j^2=0.$ thus

$$Y_i(Q)=\frac{nc^2}{n-1}\left\{\begin{array}{l}\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}\\+\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\end{array}\right\} \quad (23)$$

$$+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+$$

$$d_i(Q)=X_i(Q)-Y_i(Q)=\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s}S_j^2+2\left[b+c(n-s)\right]C_1+2cC_2\right\} \quad (24)$$

Which is zero under null hypothesis ($H_o:S_j=0$)

3.7 For exponential trend, in the presence of seasonal effect, the periodic variance is obtained as

$$X_i(E)=\sigma_{i.}^2(E)=b^2e^{2c[(i-1)s+1]}\left[\left(\frac{1-e^{2cs}}{1-e^{2c}}\right)-\frac{1}{s}\left(\frac{1-e^{cs}}{1-e^{c}}\right)\right]+\sum_{j=1}^{s}S_j^2+2be^{c(i-1)s}\sum_{j=1}^{s}e^{cj}S_j \quad (25)$$

When there is no seasonal indices, $S_j=0\forall j=1,2,...,s,$ therefore

$$Y_i(E)=b^2e^{2c[(i-1)s+1]}\left[\left(\frac{1-e^{2cs}}{1-e^{2c}}\right)-\frac{1}{s}\left(\frac{1-e^{cs}}{1-e^{c}}\right)\right] \quad (26)$$

$$d_i(E)=X_i(E)-Y_i(E)=\sum_{j=1}^{s}S_j^2+2be^{c(i-1)s}\sum_{j=1}^{s}e^{cj}S_j \quad (27)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

3.8 The overall variance is obtained as

$$X_i(E) = \sigma_{..}^2(E) = \frac{b^2 e^{2c}}{n-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2c}}\right) - \frac{1}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)^2\right] + \frac{m}{m-1}\sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1}\sum_{j=1}^{s} e^{cj} S_j \qquad (28)$$

When there is no seasonal indices, $S_j = 0 \forall j = 1, 2, ..., s,$ therefore

$$Y_i(E) = \frac{b^2 e^{2c}}{n-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2c}}\right) - \frac{1}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)^2\right] \qquad (29)$$

$$d_i = X_i(E) - Y_i(E) = \frac{m}{m-1}\sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1}\sum_{j=1}^{s} e^{cj} S_j \qquad (30)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

Table 2: Estimates in the presence of seasonal indices for row variances

Linear Trending Curve $(a+bt)$	$\frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 +$
Quadratic Trending Curve $(a+bt+ct^2)$	$\left\{\begin{array}{l}\frac{1}{s-1}\left\{\sum_{j=1}^{s} S_j^2 + 2[b-2cs]C_1 + 2cC_2\right\} \\ +\left\{\left[+\frac{4csC_1}{s-1}\right]\right\} i\end{array}\right\}$
Exponential Trending Curve (be^{ct})	$\sum_{j=1}^{s} S_j^2 + 2be^{c(i-1)s}\sum_{j=1}^{s} e^{cj} S_j$

Where $C_1 = \sum_{j=1}^{s} jS_j$, $C_2 = \sum_{j=1}^{s} jS_j$

Table 3: Estimates in the absence of seasonal indices for row variance

Linear Trending Curve $(a+bt)$	$b^2 s\left(\frac{s+1}{2}\right) + \sigma_1^2$

Quadratic Trending Curve $\left(a+bt+ct^2\right)$	$\left\{\begin{array}{l}\frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2-30(s-1)bc+15b^2\right\}+\\+\left\{\frac{s^2(s+1)}{3}\left[bc-c^2(s-1)\right]i\right\}+\left[\frac{s^2(s+1)c^2}{3}\right]i^2\end{array}\right\}$
Exponential Trending Curve (be^{ct})	$b^2e^{2c[(i-1)s+1]}\left[\left(\frac{1-e^{2cs}}{1-e^{2c}}\right)-\frac{1}{s}\left(\frac{1-e^{cs}}{1-e^{c}}\right)\right]$

Table 4: Estimates in the presence of seasonal indices for overall variance

Linear Trending Curve $(a+bt)$	$\frac{m}{n-1}\sum_{j=1}^{s}S_j^2+\frac{2bm}{n-1}\sum_{j=1}^{s}jS_j$
Quadratic Trending Curve $\left(a+bt+ct^2\right)$	$\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s}S_j^2+2\left[b+c(n-s)\right]C_1+2cC_2\right\}$
Exponential Trending Curve (be^{ct})	$\frac{m}{m-1}\sum_{j=1}^{s}S_j^2+\frac{2b}{n-1}\sum_{j=1}^{s}e^{cj}S_j$

Where $C_1=\sum_{j=1}^{s}jS_j$, $C_2=\sum_{j=1}^{s}jS_j$

Table 5: Estimates in absence of seasonal indices for overall variance

Linear Trending Curve $(a+bt)$	$\frac{b^2n(n+1)}{12}+\sigma_1^2$
Quadratic Trending Curve $\left(a+bt+ct^2\right)$	$\frac{nc^2}{n-1}\left\{\begin{array}{l}\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}\\+\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\end{array}\right\}$ $+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+$
Exponential Trending Curve (be^{ct})	$\frac{b^2e^{2c}}{n-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2c}}\right)-\frac{1}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)^2\right]$

Additive seasonality in time series is applied using matched pairs of data in the Buys-Ballot table for selected trending curves shown in equations (4), (5), (6), (7), (8) and (9). The test has been developed using the periodic and overall variances of the Buys-Ballot table. The estimates for the data in the presence of seasonal indices for periodic and overall variances are listed in Tables 2 and 4. While the estimates for series without seasonal indices for periodic and overall variances are given in Tables 3 and 5. The Buys-Ballot estimates obtained and listed in equations (15), (18), (21), (24), (27) and (30) indicate that, all the trending curves are functions of the seasonal indices only when the trend parameters are removed, while that of equations (14), (17), (20), (23), (26) and (29) are functions of trend parameters.

4 Concluding Remarks

This study has examined the test for seasonality in additive model using the Buys-Ballot table for selected trending curves. The emphasis is to construct test for the identification of series with and without seasonal effect in time series data. The test has been developed using the periodic and overall variances of Buys-Ballot table for selected trending curves. The model structure is additive. The study indicates that, when the trend dominates the time series, the presence of the seasonal indices will be difficult to detect. Hence, it important to de-trend time series data before constructing test for seasonality in additive model as indicated by Tables 2 and 4 for selected trending curves while that of Tables 3 and 4 are functions of trending parameters of the original time series when seasonal indices are removed.

References

[1] Chambers, J. M., Cleveland, W. S., Kleiner, B., & Tukey, P. A. (1983). Graphical methods for data analysis. Bellmont, CA: Wadsworth.

[2] Cleveland, W. S. (1993). Visualizing data. Murray Hill, NJ: AT & T Bell Laboratories.

[3] Box, G. E. P., Jenkins, G. M., & Reinsel, G. C. (1994). Time series analysis: Forecasting
and control, Englewood Cliffs, NJ: Prentice Hall

[4] Davey, A. M., & Flores, B. E. (1993). Identification of seasonality in time series: A note,
Mathematical and Computing Modelling, 18(6), 73-81.

[5] Kendall, M. & Ord, J. K. (1990). Time series, 3rd ed., Griffin, London

[6] Chatfield, C. (2004). *The analysis of time Series: An introduction*. Chapman and Hall,/CRC Press, Boca Raton.

[7] Iwueze, I. S. & Nwogu, E.C. (2014).*Framework for choice of models and detection of seasonal effect in time series.*Far East Journal of Theoretical Statistics 48(1), 45– 66

[8] Nwogu, E.C, Iwueze, I.S. Dozie, K.C.N. & Mbachu, H.I (2019). Choice between mixed and multiplicative models in time series decomposition. International Journal of Statistics and Applications 9(5), 153-159

[9] Dozie, K.C.N, Nwogu, E.C & Nwanya, J.C (2020). Buys-Ballot technique for analysis of time Series model. International Journal of Scientific Research and Innovative Technology 7(1), 63-78

[10] Dozie, K.C.N, Ihekuna S.O (2020). Buys-Ballot estimates of quadratic trend component and seasonal indices and effect of incomplete data in time series. International Journal of Science and Healthcare Research 5(2), 341-348

[11] Dozie, K.C.N, Nwanya J.C (2020). Comparison of mixed and multiplicative models when trend-cycle component is liner. Asian Journal of Advanced Research and Report 12(4), 32-42

[12] Dozie, K. C. N (2020). Estimation of seasonal variances in descriptive time series analysis. Asian Journal of Advanced Research and Reports 10(3), 37 – 47

[13] Dozie, K.C.N & Uwaezuoke M.U (2021). Procedure for estimation of additive time series model. International Journal of Research and Scientific Innovation 8(2), 251-256

[14] Dozie, K.C.N & Ibebuogu C.C (2021). Road traffic offences in Nigeria: an empirical analysis using buys-ballot approach. Asian Journal of Probability and Statistics 12(1), 68-78

[15] Nwogu, E.C, Iwueze, I.S & Nlebedim, V.C (2016). Some tests for seasonality in time series data. Journal of Modern Applied Statistical Methods 15(2), 382-399