Original Research Article

EXPONENTIATED FRECHET DISTRIBUTION WITH APPLICATION IN TEMPARATURE OF ASSAM, INDIA OVERVIEW WITH NEW PROPERTIES AND ESTIMATION

ABSTRACT

In this paper, a new kind of distribution has suggested with the concept of exponentiate. The reliability analysis including survival function, hazard rate function, reverse hazard rate function and mills ratio has been studied here. Its quantile function and order statistics are also included. Parameters of the distribution are estimated by the method of Maximum Likelihood estimation method along with Fisher information matrix and confidence intervals have also been given. The application has been discussed with the 30 years temperature data of Silchar city, Assam, India. The goodness of fit of the proposed distribution has been compared with Frechet distribution and as a result, for all 12 months, the proposed distribution fits better than the Frechet distribution.

Keywords: Type-II Extreme Value distribution; Exponentiated distribution; Reliability analysis; Order statistics; fisher Information matrix; confidence interval; Maximum likelihood Estimation; Application.

Mathematical Subject Classification: Primary: 62E99, 62N99; Secondary: 62P12

1. INTRODUCTION

Probabilistic extreme value theory deals with the stochastic behavior of the maximum and the minimum of random variables. Extreme value distributions are the limiting distributions for the minimum or the maximum of a large collection of random observations from the same arbitrary distribution. Extreme value distributions are usually considered into three families as type-I (Gumbel-type distribution), type-II (Frechet-type distribution) and type-III (Weibull-type distribution). In this paper we will discuss about type-II extreme value distribution i.e. Frechet distribution. Mourice Rene Frechet (1927) introduced Frechet distribution which is defined by its Probability Density Function (PDF) and Cumulative Distribution Function (CDF) as

$$f(y;\alpha,\beta)=\alpha\beta^{\alpha}y^{-(\alpha+1)}\exp\left\{-\left(\frac{\beta}{x}\right)^{\alpha}\right\}\ ;x>0,\alpha,\beta>0, \tag{1.1}$$

$$F(x;\alpha,\beta)=\exp\left\{-\left(\frac{\beta}{x}\right)^{\alpha}\right\}\ ;x>0,\alpha,\beta>0. \tag{1.2}$$

here $\alpha>0$ is a shape parameter $\beta>0$ is a scale parameter.

Ramos P.L. et al (2019) published a full review about Frechet distribution and they have discussed different estimation techniques. Abd-Elfattah and Omima (2009) introduced a generalization technique in Frechet distribution. Krishna et al (2013) discussed Marshall-Olkin Frechet distribution as a new generalization of Frechet distribution.

A random variable X is said to have an exponentiated distribution if its cdf is given by

$$G_{\theta}(X)=[F(x)]^{\theta};\ x,\theta>0, \tag{1.3}$$

and the pdf of X is given by

$$g_{\theta}(x)=\theta[F(x)]^{\theta-1}f(x). \tag{1.4}$$

Gupta et al (1998) first used this technique and introduced exponentiated exponential distribution. Later Gupta and Kundu (2001) studied some properties of this distribution. Hassan et al (2017) discussed exponentiated Lomax distribution and its properties. Nasir et al (2018) obtained the exponentiated Burr XII power series distribution with properties and its applications. Pal et al (2006) studied the exponentiated weibull family as an extension of weibull distribution. Rather and Subramanian (2018) discussed the exponentiated Mukharjee-Islam distribution which shows more flexibility than the classical distribution. The same authors (2020) published an exponentiated Garima distribution and discussed its application with engineering science data.

Many authors have been working on exponentiated Frechet distribution also by using various exponentiated method. Nadarajah and Kotz (2003) introduced an exponentiated Frechet distribution as a generalization of standard Frechet distribution. Mansour et al (2018) discussed the properties and application of Kumaraswami Exponentiated Frechet distribution. Badr M.M. (2019) introduced Beta generalized Exponentiated Frechet distribution and discussed its properties with application. Frechet distribution fits in meteorological data. It is expected and hoped that the Exponentiated Frechet distribution (EFD) will also be a better model.

2. Exponentiated Frechet Distribution (EFD)

On substituting (1.2) and (1.1) in (1.3) and (1.4) we will get the cdf and pdf of Exponentiated Frechet distribution as

$$G(x;\alpha,\beta,\theta)=\left[\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right]^{\theta} \quad x>0;\alpha,\beta,\theta>0 \qquad (2.1)$$

$$g(x;\alpha,\beta,\theta)=\alpha\beta^{\alpha}\theta\, x^{-(\alpha+1)}\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta} \quad x>0;\alpha,\beta,\theta>0 \qquad (2.2)$$

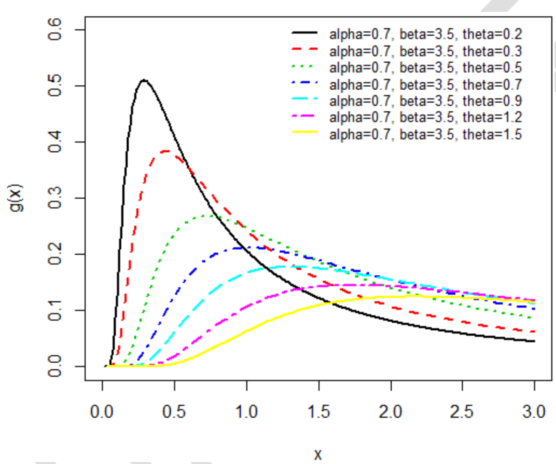
Where α and θ are shape parameter and β is the scale parameter. Further

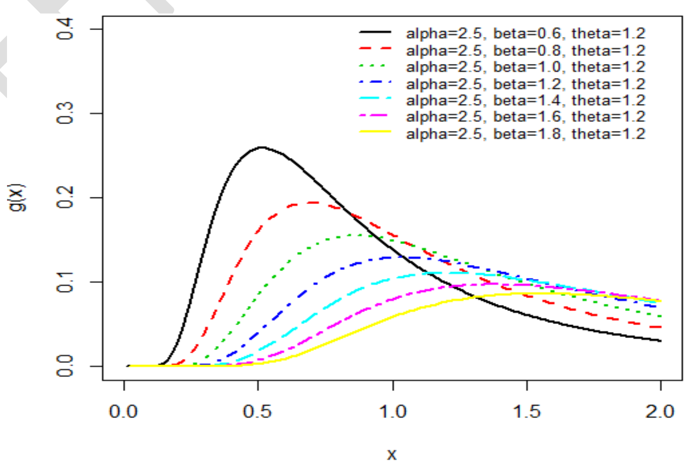
$$\lim_{x\to-\infty} G(x;\alpha,\beta,\theta)=0 \quad \text{and}$$

$$\lim_{x\to\infty} G(x;\alpha,\beta,\theta)=1$$

This shows that EFD is a proper density function.

Graphs of the pdf and the cdf of EFD are shown in fig.1 and fig.2 for varying values of the parameters α, β and θ. The R-software is used for designing the graphs.





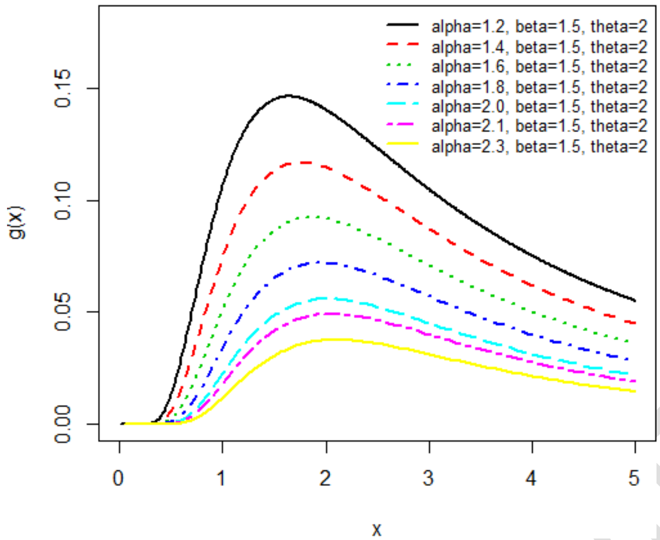
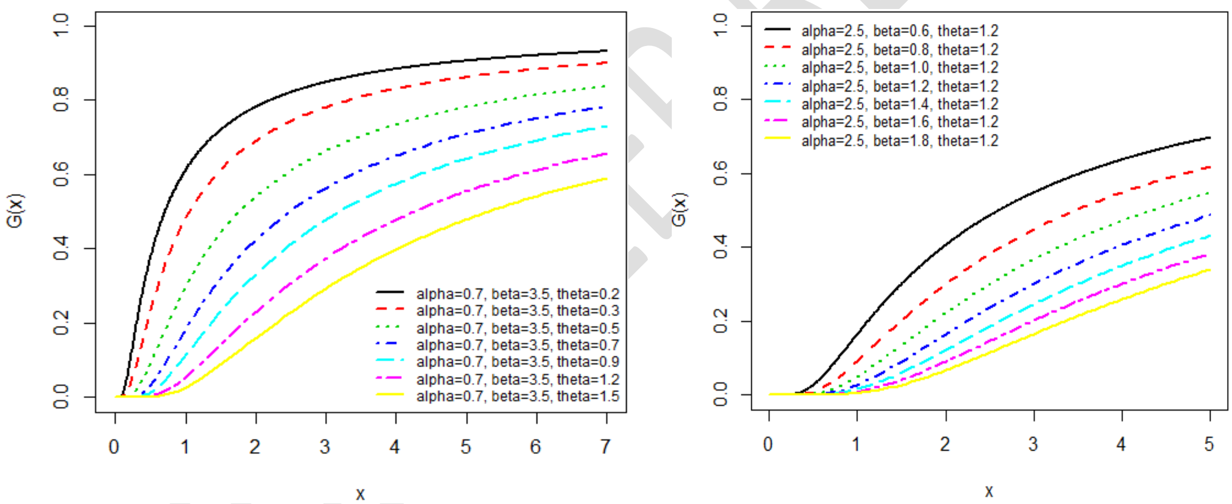


Fig.1: Graphs of the pdf of EFD for varying values of parameters



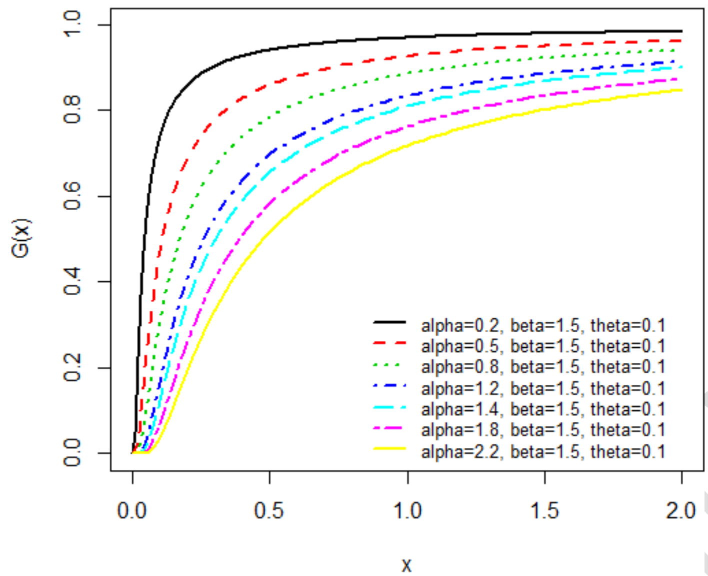


Fig.2: Graphs of the cdf of EFD for varying values of parameters

3. STATISTICAL PROPERTIES

In this section, statistical properties including asymptotic behavior, survival function, hazard function, reverse hazard rate, mills ratio and quintile function of EFD has been studied.

3.1. Asymptotic Behavior

The asymptotic behavior of EFD for $x \to 0$ and $x \to \infty$ are

$$\lim_{x \to 0} g(x;\alpha,\beta,\theta) = \lim_{x \to 0} \left[\alpha\beta^{\alpha}\theta\, x^{-(\alpha+1)} \left\{ \exp\left(-\frac{\beta}{x} \right)^{\alpha} \right\}^{\theta} \right] = 0$$

and $$\lim_{x \to \infty} g(x;\alpha,\beta,\theta) = \lim_{x \to \infty} \left[\alpha\beta^{\alpha}\theta\, x^{-(\alpha+1)} \left\{ \exp\left(-\frac{\beta}{x} \right)^{\alpha} \right\}^{\theta} \right] = 0$$

These results confirm that the proposed distribution has a mode.

3.2. Reliability Properties

The survival function (or the reliability function) is the probability that a subject survives longer than the expected time. The survival function of the EFD is given by

$$S(x) = 1 - G(x) = 1 - \left\{ \exp\left(-\frac{\beta}{x} \right)^{\alpha} \right\}^{\theta}$$

The hazard function (also known as the hazard rate, instantaneous failure rate or force of mortality) is the probability to measure the instant death rate of a subject. Suppose X be a continuous random variable with pdf $g(x)$ and cdf $G(x)$. The hazard rate function of X is defined as

$$h(x)=\lim_{\Delta x\to 0}\frac{P(X<x+\Delta x / X>x)}{\Delta x}=\frac{g(x)}{1-G(x)}$$

The corresponding $h(x)$ of EFD can be obtained as

$$h(x)=\frac{\alpha\beta^{\alpha}\theta\, x^{-(\alpha+1)}\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}}{1-\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}}$$

The reverse hazard rate is the ratio between the probability density function and its distribution function. The reverse hazard function of EFD is given by

$$h_r(x)=\alpha\beta^{\alpha}\theta\, x^{-(\alpha+1)}$$

The mills ratio is the ratio between the survival function and the pdf of a distribution. The mills ratio of EFD is

$$m(x)=\frac{1-\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}}{\alpha\beta^{\alpha}\theta\, x^{-(\alpha+1)}\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}}$$

Graphs of the survival function and the hazard function of EFD are shown in fig.3 and fig.4 for varying values of the parameters α, β and θ.

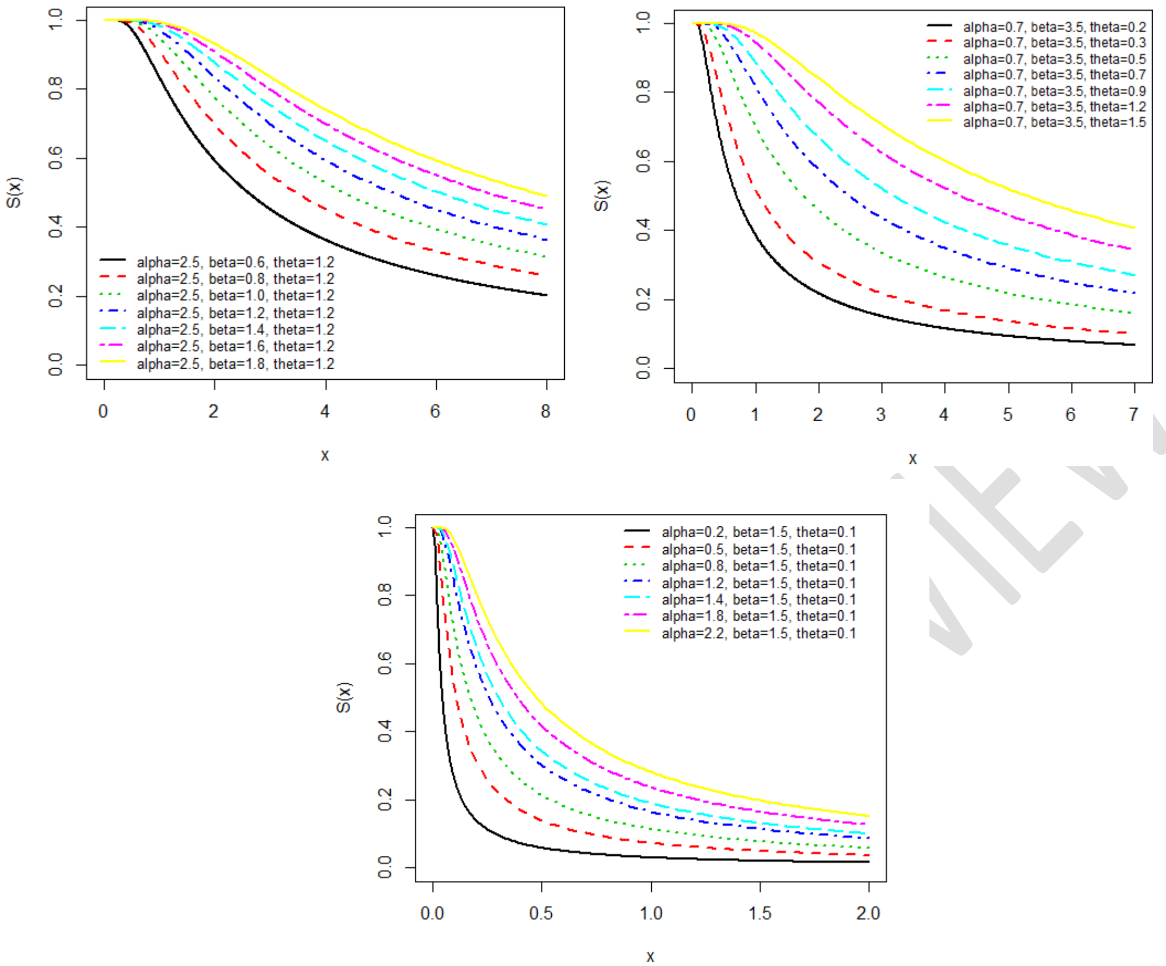
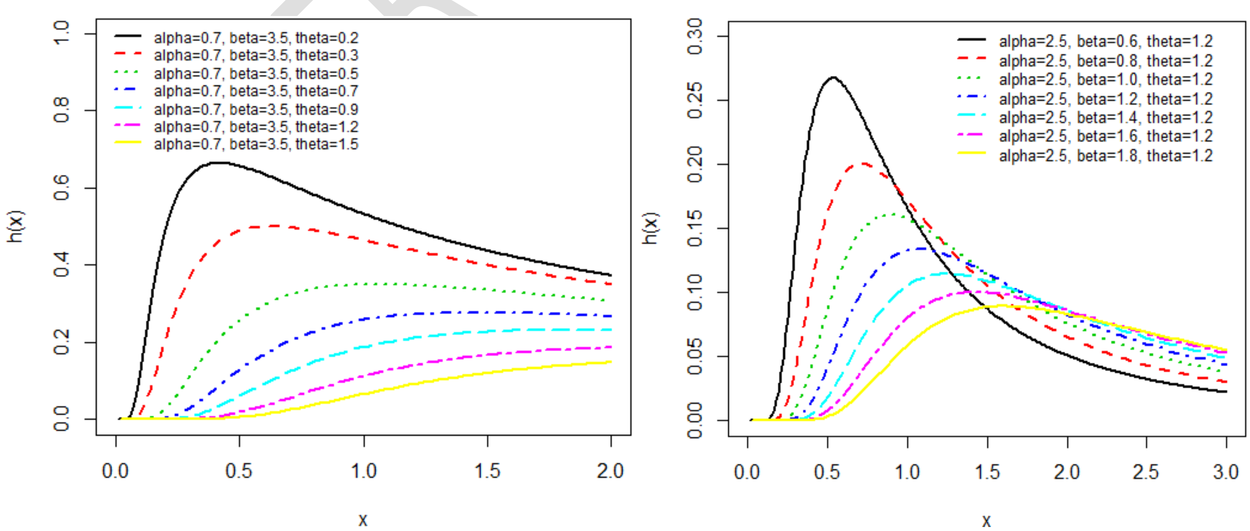


Fig. 3: Graphs of survival function of EFD for varying values of parameters



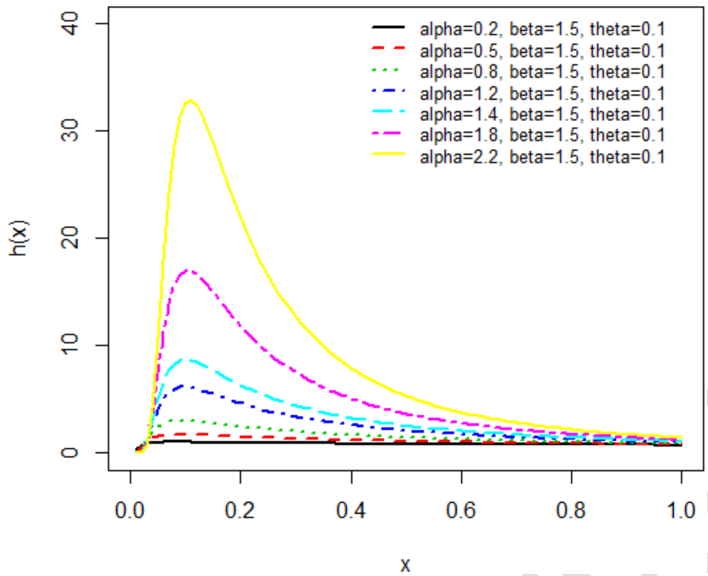


Fig. 4: Graphs of Hazard rate function of EFD for varying values of parameters

3.4. Quantile Function

The quantile function is defined as

$$Q(u) = G^{-1}(u)$$

Therefore, the corresponding quantile function for EFD can be expressed as

$$Q(u) = -\frac{\beta}{(\log u)^{\frac{1}{\alpha\theta}}}$$

Let U has the uniform $U(0,1)$ distribution. Taking $u = 0.5$, the median of EFD can be obtained as

$$Q(0.5) = -\frac{\beta}{(\log 0.5)^{\frac{1}{\alpha\theta}}}$$

Thus, the formula for generating random samples from EFD for simulating random variable X is given by

$$X = Q(u) = -\frac{\beta}{(\log u)^{\frac{1}{\alpha\theta}}}$$

4. Statistical Properties

In this section, we will discuss about the various structural properties of proposed Exponentiated Frechet Distribution (EFD).

4.1. Moments

Using (2.2), the r^{th} moment about origin of EFD is given by

$$E\left(X^r\right)=\int_0^\infty x^r g(x)dx=\int_0^\infty \alpha\beta^\alpha \theta x^{r-(\alpha+1)}\left\{\exp\left(-\left(\frac{\beta}{x}\right)^\alpha\right)\right\}^\theta dx$$

Taking $\left(\frac{\beta}{x}\right)^{\alpha\theta}=t$ and thus $x=\beta t^{-\frac{1}{\alpha\theta}}$ and $dx=\beta\frac{-1}{\alpha\theta}t^{-\frac{1}{\alpha\theta}-1}dt$, we get

$$E\left(X^r\right)=\mu_r{}'=\beta^r\Gamma\left[\frac{1}{\theta}\left(1-\frac{r}{\alpha}\right)\right] \qquad (4.1.1)$$

By putting $r=1,2,3,4$ in (4.1.1), we can get the first four moments about origin as

$$E(X)=\mu_1{}'=\beta\Gamma\left[\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right] \qquad E\left(X^2\right)=\mu_2{}'=\beta^2\Gamma\left[\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right]$$

$$E\left(X^3\right)=\mu_3{}'=\beta^3\Gamma\left[\frac{1}{\theta}\left(1-\frac{3}{\alpha}\right)\right] \qquad E\left(X^4\right)=\mu_4{}'=\beta^4\Gamma\left[\frac{1}{\theta}\left(1-\frac{4}{\alpha}\right)\right]$$

From above we can observed that

$$\mu_2'-\mu_1'=\beta\left[\beta\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\}-\Gamma\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]$$

$$\mu_3'-\mu_2'=\beta^2\left[\beta\Gamma\left\{\frac{1}{\theta}\left(1-\frac{3}{\alpha}\right)\right\}-\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\}\right]$$

As a consequence, we can settled the recurrence relation of raw moments for EFD as

$$\mu_{r+1}'=\beta^r\left[\beta\Gamma\left\{\frac{1}{\theta}\left(1-\frac{r+1}{\alpha}\right)\right\}-\Gamma\left\{\frac{1}{\theta}\left(1-\frac{r}{\alpha}\right)\right\}\right]+\mu_r'$$

The first four central moment of EFD is given by

$$\mu_1=0$$

$$\mu_2=\beta^2\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\}-\Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]$$

$$\mu_3=\beta^3\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{3}{\alpha}\right)\right\}-3\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\}\Gamma\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}+2\,\Gamma^3\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]$$

$$\mu_3 = \beta^4 \left[\begin{array}{l} \Gamma\left\{\frac{1}{\theta}\left(1-\frac{4}{\alpha}\right)\right\} - 4\Gamma\left\{\frac{1}{\theta}\left(1-\frac{3}{\alpha}\right)\right\}\Gamma\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\} + 6\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\}\Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\} \\ -3\Gamma^4\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\} \end{array} \right]$$

The corresponding variance and other related properties are given below

$$\text{Variance } \left(\sigma^2\right) = \beta^2\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\} - \Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]$$

$$\text{Standard deviation } \left(\sigma\right) = \beta\sqrt{\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\} - \Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]}$$

$$\text{Co-efficient of variation } \left(\frac{\sigma}{\mu_1'}\right) = \frac{\sqrt{\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\} - \Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}}}{\Gamma\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}}$$

$$\text{Co-efficient of dispersion } \left(\frac{\sigma^2}{\mu_1'}\right) = \frac{\beta\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\} - \Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]}{\Gamma\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}}$$

$$\text{Skewness } \left(\frac{\mu_3^2}{\mu_2^3}\right) = \frac{\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{3}{\alpha}\right)\right\} - 3\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\}\Gamma\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\} + 2\Gamma^3\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]^2}{\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\} - \Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]^3}$$

Kurtosis

$$\left(\frac{\mu_4}{\mu_2^2}\right) = \frac{\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{4}{\alpha}\right)\right\} - 4\Gamma\left\{\frac{1}{\theta}\left(1-\frac{3}{\alpha}\right)\right\}\Gamma\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\} + 6\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\}\Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]^2 - 3\Gamma^4\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}}{\left[\Gamma\left\{\frac{1}{\theta}\left(1-\frac{2}{\alpha}\right)\right\} - \Gamma^2\left\{\frac{1}{\theta}\left(1-\frac{1}{\alpha}\right)\right\}\right]^2}$$

4.2. Incomplete Moments

Let us suppose that X is a random variable follows EFD, then the rth incomplete moment denoted as $I(x;r)$ can be defined as follows

$$I(x;r)=\int_0^z x^r g(x)dx$$

The rth incomplete moment of EFD is given as

$$I(x;r)=\beta^r\,\gamma\left[\left\{\frac{1}{\theta}\left(1-\frac{r}{\alpha}\right)\right\},\left(\frac{\beta}{x}\right)^{\alpha\theta}\right]$$

4.3. Harmonic Mean

Let us suppose that X is a random variable follows EFD, harmonic mean denoted as $H.M.$ can be defined as follows

$$H.M.=E\left(\frac{1}{X}\right)=\int_0^\infty \frac{1}{x}g(x)dx$$

The Harmonic mean of the proposed model is

$$H.M.=\frac{1}{\beta}\Gamma\left\{\frac{1}{\beta}\left(1+\frac{1}{\alpha}\right)\right\}$$

4.4. Moment Generating Function and related

If X has a EFD, then the moment Generating Function is defined as

$$M_X(t)=E\left(e^{tX}\right)=\int_0^\infty e^{tX}g(x)dx$$

The moment generating function of EFD is given as

$$M_X(t)=\sum_{j=0}^{\infty}\frac{t^j}{j!}\beta^j\Gamma\left\{\frac{1}{\theta}\left(1-\frac{j}{\alpha}\right)\right\}$$

The characteristic function of EFD is given by

$$\Phi_X(t)=M_X(it)=\sum_{j=0}^{\infty}\frac{(it)^j}{j!}\beta^j\Gamma\left\{\frac{1}{\theta}\left(1-\frac{j}{\alpha}\right)\right\}$$

and the Cumulant Generating Function is

$$K_X(t)=\ln\left[\sum_{j=0}^{\infty}\frac{t^j}{j!}\beta^j\Gamma\left\{\frac{1}{\theta}\left(1-\frac{j}{\alpha}\right)\right\}\right]$$

5. Order Statistics of EFD

Let $x_1, x_2,..., x_n$ be the random samples from EFD(α,β,θ). The pdf of i^{th} order statistics is given by

$$g_{i:n}(x)=\frac{n!}{(i-1)!(n-i)!}g_X(x)[G_X(x)]^{i-1}[1-G_X(x)]^{n-i}$$

The pdf of i^{th} order statistics $X_{(i)}$ of EFD is given by

$$g_{i:n}(x)=\frac{n!}{(i-1)!(n-1)!}\alpha\beta^{\alpha}\,\theta\,x^{-(\alpha+1)}\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}\left[\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}\right]^{i-1}\left[1-\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}\right]^{n-i}$$

The pdf of the first order statistic $X_{(1)}$ can be expressed as

$$g_{1:n}(x)=n\alpha\beta^{\alpha}\theta\,x^{-(\alpha+1)}\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}\left[1-\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}\right]^{n-1}$$

The pdf of the highest order statistic $X_{(n)}$ can be expressed as

$$g_{n:n}(x)=n\alpha\beta^{\alpha}\theta\,x^{-(\alpha+1)}\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}\left[\left\{\exp\left(-\frac{\beta}{x}\right)^{\alpha}\right\}^{\theta}\right]^{n-1}$$

6. Maximum Likelihood Estimation and Fisher Information Matrix

Let $x_1, x_2,..., x_n$ be the random samples of size n from a EFD (α,β,θ). The log-likelihood function can be expressed as

$$\log L=\sum_{i=1}^{n}\log g(x_i \setminus \alpha,\beta,\theta)=n(\log\theta+\log\alpha+\alpha\log\beta)-(\alpha+1)\sum_{i=1}^{n}\log x_i-\beta^{\alpha\theta}\sum_{i=1}^{n}x_i^{-\alpha\theta}$$

The maximum likelihood estimate (MLE) $(\hat{\alpha},\hat{\beta},\hat{\theta})$ of (α,β,θ) of EFD are the solutions of the following log- likelihood equations

$$\frac{\partial}{\partial\alpha}\log L=\frac{n}{\alpha}+n\log\beta-\sum_{i=1}^{n}\log x_i-\alpha\beta^{\theta(\alpha-1)}\sum_{i=1}^{n}x_i^{-\alpha\theta}+\theta\beta^{\alpha\theta}\sum_{i=1}^{n}x_i^{-\alpha\theta}\log x_i=0$$

$$\frac{\partial}{\partial\beta}\log L=\frac{n\alpha}{\beta}-\alpha\theta\beta^{\alpha\theta-1}\sum_{i=1}^{n}x_i^{-\alpha\theta}=0$$

$$\frac{\partial}{\partial\theta}\log L=\frac{n}{\theta}-\theta\beta^{\alpha(\theta-1)}\sum_{i=1}^{n}\log x_i^{-\alpha\theta}-\alpha\beta^{\alpha\theta}\sum_{i=1}^{n}x_i^{-\alpha\theta}\log x_i=0$$

These log-likelihood equation can't be solved analytically and required statistical software with iterative numerical techniques. These equations can be solved using R-software.

The 3×3 observed information matrix of EFD can be presented as,

$$\begin{pmatrix}\hat{\alpha}\\ \hat{\beta}\\ \hat{\theta}\end{pmatrix} \sim \left[\begin{pmatrix}\alpha\\ \beta\\ \theta\end{pmatrix}, \begin{pmatrix} \frac{\partial^2 \log L}{\partial \alpha^2} & \frac{\partial^2 \log L}{\partial \alpha \partial \beta} & \frac{\partial^2 \log L}{\partial \alpha \partial \theta} \\ \frac{\partial^2 \log L}{\partial \beta \partial \alpha} & \frac{\partial^2 \log L}{\partial \beta^2} & \frac{\partial^2 \log L}{\partial \beta \partial \theta} \\ \frac{\partial^2 \log L}{\partial \theta \partial \alpha} & \frac{\partial^2 \log L}{\partial \theta \partial \beta} & \frac{\partial^2 \log L}{\partial \theta^2} \end{pmatrix}\right]$$

The inverse of the information matrix results in the well-known variance-covariance matrix. The 3×3 approximate Fisher information matrix corresponding to the above observed information matrix is given by

$$I^{-1} = -E\begin{bmatrix} \frac{\partial^2 \log L}{\partial \alpha^2} & \frac{\partial^2 \log L}{\partial \alpha \partial \beta} & \frac{\partial^2 \log L}{\partial \alpha \partial \theta} \\ \frac{\partial^2 \log L}{\partial \beta \partial \alpha} & \frac{\partial^2 \log L}{\partial \beta^2} & \frac{\partial^2 \log L}{\partial \beta \partial \theta} \\ \frac{\partial^2 \log L}{\partial \theta \partial \alpha} & \frac{\partial^2 \log L}{\partial \theta \partial \beta} & \frac{\partial^2 \log L}{\partial \theta^2} \end{bmatrix}$$

The solution of the Fisher information matrix will yield asymptotic variance and covariance of the ML estimators for $(\hat{\alpha}, \hat{\beta}, \hat{\theta})$. The approximate 100(1-α)% confidence intervals for (α, β, θ) respectively are $\hat{\alpha} \pm Z_{\frac{\alpha}{2}} \frac{\sigma_{\alpha\alpha}}{n}$, $\hat{\beta} \pm Z_{\frac{\alpha}{2}} \frac{\sigma_{\beta\beta}}{n}$ and $\hat{\theta} \pm Z_{\frac{\alpha}{2}} \frac{\sigma_{\theta\theta}}{n}$, where Z_{α} is the upper $100\alpha^{th}$ percentile of the standard normal distribution.

7. Application of EFD

In this study, monthly mean (maximum) temperature series of Silchar city, Assam, India from January 1988-December 2018 (30 years) which is collected from India Meteorological Department, Pune, India has been analyzed. For the application purpose, the datasets from January to December has been considered.

In order to compare the Exponentiated Frechet Distribution(EFD) with Frechet distribution (FD), we consider the criteria like Bayesian information criterion *(BIC)*, Akaike Information Criterion *(AIC)*, Akaike Information Criterion Corrected *(AICC)* and $-2\log L$. The better distribution corresponds to lesser values of AIC, BIC, $AICC$ and $-2\log L$. The formulae for calculating AIC, *BIC* and *AICC* are as follows:

$$AIC = 2K - 2\log L,\ BIC = k\log n - 2\log L,\ AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

Where *k* is the number of parameters, *n* is the sample size and *-2 logL* is the maximized value of log likelihood function. The ML estimates of the parameters of the considered distributions along with values of $-2\log L, AIC, AICC$ and BIC for the datasets are presented in Table 1.

Table 1: ML estimates of the parameters

Month	Distribution	ML estimates			$-2\log L$	AIC	$AICC$	BIC
		α	β	θ				
January	**EFD**	**0.175749 (0.0335)**	**0.100000 (0.0456)**	**53.72741 (9.7130)**	**123.5671**	**129.5671**	**130.4559**	**133.8690**
	FD	**0.998638 (0.1793)**	**25.21178 (4.5312)**	**-**	**262.2624**	**266.2624**	**266.6910**	**269.1304**
February	**FFD**	**0.257402 (0.0499)**	**0.100000 (0.0385)**	**83.44501 (15.323)**	**110.2884**	**116.2884**	**117.1773**	**120.5903**
	FD	**0.998114 (0.1792)**	**27.62806 (4.9668)**	**-**	**268.0018**	**272.0018**	**272.4304**	**274.8698**
March	**EFD**	**0.183198 (0.0352)**	**0.100000 (0.0449)**	**60.61852 (10.953)**	**130.4332**	**136.4332**	**137.3221**	**140.7352**
	FD	**0.998723 (0.1793)**	**30.70221 (5.5178)**	**-**	**274.4669**	**278.4669**	**278.8955**	**281.3349**
April	**EFD**	**0.270026 (0.0533)**	**0.100000 (0.0382)**	**73.17402 (13.341)**	**128.7770**	**134.7770**	**135.6659**	**139.0789**
	FD	**0.999069 (0.1794)**	**31.32754 (5.6292)**	**-**	**275.6763**	**279.6763**	**280.1048**	**282.5442**
May	**EFD**	**0.103025 (0.0191)**	**0.100000 (0.0578)**	**92.12879 (16.643)**	**113.3843**	**119.3843**	**120.2732**	**123.6863**
	FD	**0.999187 (0.1795)**	**31.6614 (5.6889)**	**-**	**276.3187**	**280.3187**	**280.7473**	**283.1867**
June	**EFD**	**0.304659 (0.0611)**	**0.100000 (0.0365)**	**71.56617 (13.068)**	**137.6811**	**143.6811**	**144.5700**	**147.9831**
	FD	**0.999528 (0.1795)**	**32.2824 (5.7995)**	**-**	**277.4787**	**281.4787**	**281.9073**	**284.3467**

July	**EFD**	**0.177185 (0.0338)**	**0.100000 (0.0455)**	**71.96994 (13.018)**	**124.2184**	**130.2184**	**131.1073**	**134.5204**
	FD	**0.999469 (0.1796)**	**32.59439 (5.8557)**	**-**	**278.0826**	**282.0826**	**282.5111**	**284.9505**
August	**EFD**	**0.249177 (0.0494)**	**0.100000 (0.0398)**	**54.90938 (9.9318)**	**146.3962**	**152.3962**	**153.2851**	**156.6981**
	FD	**0.99962 (0.1795)**	**32.8331 (5.8981)**	**-**	**278.5161**	**282.5161**	**282.9447**	**285.3841**
Septembe r	**EFD**	**0.277857 (0.0555)**	**0.100000 (0.0381)**	**62.24918 (11.302)**	**142.3221**	**148.3221**	**149.2110**	**152.6240**
	FD	**0.999573 (0.1796)**	**32.60451 (5.8572)**	**-**	**278.0889**	**282.0889**	**282.5175**	**284.9569**
October	**EFD**	**0.253127 (0.0498)**	**0.100000 (0.0393)**	**65.35098 (11.863)**	**134.1400**	**140.1400**	**141.0289**	**144.4419**
	FD	**0.999728 (0.1796)**	**31.77596 (5.7079)**	**-**	**276.4742**	**280.4742**	**280.9028**	**283.3422**
Novembe r	**EFD**	**0.17098 (0.0326)**	**0.100000 (0.0462)**	**58.3169 (10.531)**	**129.9273**	**135.9273**	**136.8162**	**140.2292**
	FD	**0.999683 (0.1795)**	**29.61339 (5.3195)**	**-**	**272.1095**	**276.1095**	**276.5381**	**278.9775**
Decembe r	**EFD**	**0.184774 (0.0356)**	**0.100000 (0.0449)**	**43.17822 (7.7877)**	**140.5016**	**146.5016**	**147.3905**	**150.8036**
	FD	**0.998876 (0.1794)**	**26.57575 (4.7758)**	**-**	**265.4995**	**269.4995**	**269.9280**	**272.3674**

It is obvious from above table, that EFD provides much better fit than Frechet distribution for data relating to minimum temperature and hence the proposed distribution can be considered an important distribution for modeling mean maximum temperature data.

8. Conclusion

In this paper Exponentiated Frechet distribution (EFD) has been proposed. Its statistical properties including behavior of pdf, cdf and hazard rate function have been discussed. The distribution of the order statistics has been given. The maximum likelihood estimation for estimating parameters of the proposed distribution has been discussed. The applications of the proposed distribution for modeling data relating to real life datasets like temperature have been explained and the goodness of fit of the EFD and Frechet distribution has been presented for ready comparison.

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