Original Research Article

Geospatial Interpolation of Gold Anomalies: Kriging Verse Inverse Distance Weight

Abstracts

Accurate estimation of the mining process is vital for the optimal allocation of mineral resources. The development of any country is precisely connected with the management of mineral resources. Therefore, the forecasting of mineral resources contributed much to management, planning, and a maximum allocation of mineral resources. However, it is challenging because of its multiscale variability, nonlinearity, non -stationarity, and high irregularity. In this paper, the kriging and Inverse Distance Weights (IDW) were proposed as a spatial interpolation method to predict and identify Au anomalies at Wassa-Amenfi. The prediction accuracy of the spatial interpolation models was improved by reducing the complexity of the production of mineral resource time-series data by using the Box-Cox data transformation method. The performance of these methods is evaluated using root mean square error (RME) as evaluation measures. It is concluded that Kriging method outperformed better than IDW. The region with the highest gold concentration, according to the IDW model results, is located in the center, whereas the kriging model shows a random distribution of extreme gold concentration across the research area.

Keywords: Inverse Distance Weight, Kriging, gold, Interpolation

1.0 Background

Mining projects relating to precious metals such as gold (Au) deposits, require accurate information on tonnage and grade to ensure credible resource estimates and good project feasibility [1,2,3]. This requirement has become very crucial in recent years, due to the negative impacts of increasing depletion of high profit deposits, unstable world market prices, cost of production and weak legislation, among others factors [4]. Notably, many companies failed to live up to expectation due to poor grade and tonnage estimations, as well as ineffective geological controls [5,6, 7,8]. Accurate mineral resource and mineral reserve evaluations, therefore, form the basis on which economic decisions are made on mining project estimation have been mainly by polygonal method and a factor applied for error correction [9].

Though historically, these methods might have worked satisfactorily, there are also many drawbacks due to inherent assumptions that leads to overestimation [10].

Kriging and Inverse Distance Weight (IDW) models are a set of widely used statistical tools for spatial prediction and interpolation. These tools are the fundamental components of a class of spatial statistical methods known as geostatistics. Geostatistics has its origins in the mining industry and early methodological developments were motivated by applications such as estimating the amount of metal in an orebody and how to selectively mine the orebody based on a set of observations at known locations. Given the nature of spatially dependent processes, geostatistics based predictions draw more heavily on nearby observations than on observations farther away. In addition, the uncertainty measure associated with the prediction of the process at an unobserved location depends on the proximity of observed values of the process. The inverse-distance weighting (IDW) method, a deterministic spatial interpolation model, is one of the more popular methods adopted by geoscientists and geographers partly because it has been implemented in many GIS packages. The general premise of this method is that the attribute values of any given pair of points are related to each other, but their similarity is inversely related to the distance between the two locations. However, many studies, especially in the spatial interaction literature, have revealed that the decline in spatial relationship between any two locations is not simply proportional to distance [11]. In addition, IDW cannot estimate the variances of predicted values unsampled locations as compared to what geostatistical methods such as kriging can provide [12]. The primary goal of a geostatistical analysis is often prediction of the spatial process at locations where the process is not observed. Spatial prediction can be performed on a fine grid covering the spatial region of interest and, therefore, can be used to produce an exhaustive map of predicted values. To achieve these estimations, predictions and interpolations, both methods of Kriging and Inverse Distance Weighting (IDW) were employed after data transformation using the Box-Cox method.

2.0 METHODS

2.1 Sample data and organization

2750 soil samples were collected on a grid basis from the research region using the soil auger drilling method, with an average depth of 3 m in Ghana's Wassa -Amenfi, for the study. The gold concentration in parts per billion in these samples was determined by the Ghana Geological Survey Department using graphite furnace-atomic absorption spectrometry (GF-AAS) [14,15].

2.2 Study area

Wassa -Amenfi West District is located in Ghana's Western region. The district is located in the region's center. It is located between latitudes 5^0, 30^1 N and 6^0, 15^1N, and longitudes 10^0, 45^1W and 2^0, 11^1W. Wassa Amenfi East District is to the west, Mpohor Wassa East District is to the east, Prestea Huni Valley District is to the south, and Upper Denkyira West and East District is to the north. The district covers 1,558 square kilometers, or around 7.5 percent of the Western region's total land area. Upper Birimian, Lower Birimian, and Granites are the three major geological soil formations found in the district. The district's granite deposits make most of it rich in minerals like gold. (District analytical report by Ghana Statistical Service). The geological settings of the research region are depicted in Figure 1.

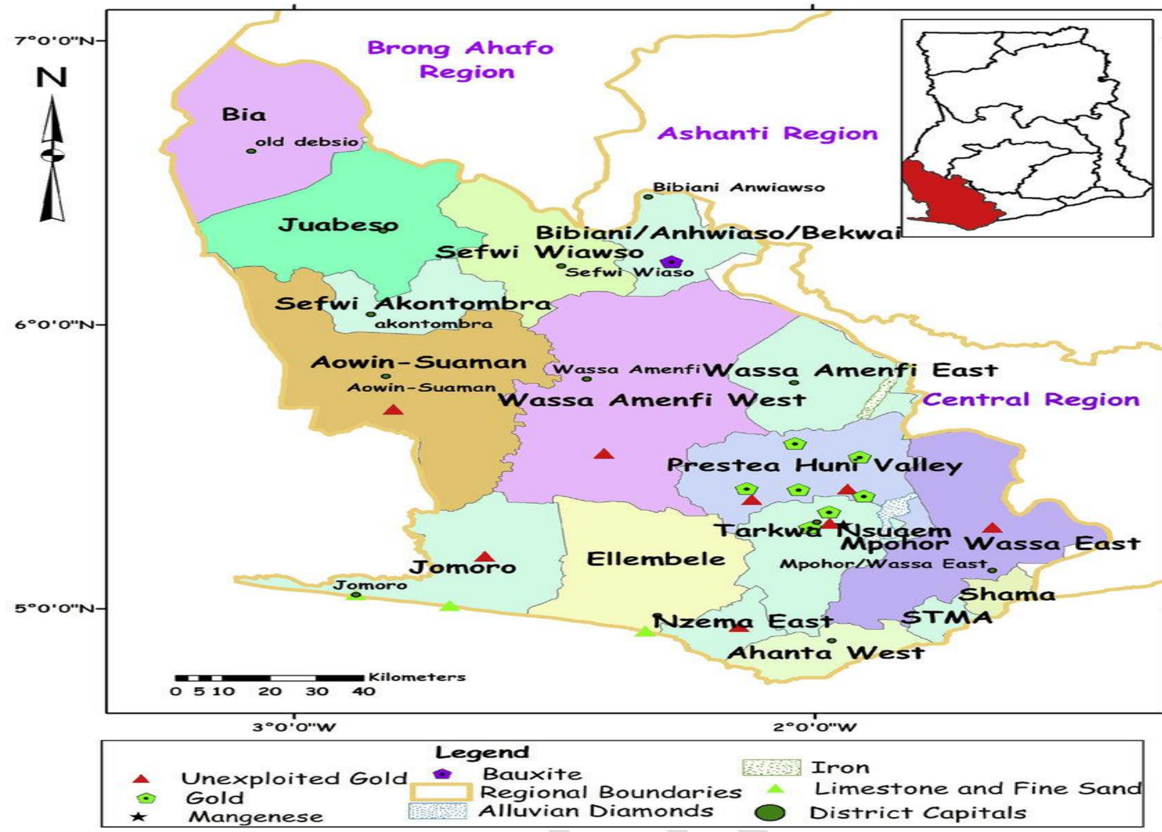

Figure 1*:* Geological map of Ghana showing the Wassa -Amenfi

2.3 Model Formulation

2.3.1 Semi variogram

The plotting of the semi variogram is the necessary step before Kriging [16], [17]. Semi variogram is a graphical device used to model the gold spatial continuity for the data set. The aim of kriging however is to estimate the value of the gold as a variable, at one or more unsampled points or over larger blocks, from more or less sparse sample data on a given support because the data may be distributed in one, two or three dimensions. Calculations in several directions, such as, along strike, across strike and down dip directions give an insight on the structural and geometric controls on the orebody [18]. The variogram represents the variance between sample pairs as a function of distance (lag) between the samples in a particular direction [19]. An experimental variogram, γ (h) * can be defined as [20]:

$\gamma^*(h) = \frac{1}{2n}\sum_{i=1}^{n} \{Z(x_i) - Z(x_i + h\}^2$ (1)

Where $Z(x_i)$ is value of Au concentration at x_i , $Z(x_i + h)$ is grade of sample at distant h from point x_i and n is the number of sample pairs.

2.3.2 Kriging Methods

Consider a collection of random variables $\{Z(s): s \in \mathcal{D} \subset \mathbb{R}^d\}$, which we refer to as a random spatial process. The mean function of the process Z(.) is defined as be $E[Z(s)] \equiv \mu_Z(s)\ \ s \in \mathcal{D}$, and the covariance function is defined as

$$C_Z(s_i, s_j) \equiv \mathrm{cov}\big(Z(s_i), Z(s_j)\big) \ (2)$$

for any $s_i \in \mathcal{D}$ and $s_j \in \mathcal{D}$. Both functions exist if we suppose that var (Z(s)) < ∞, exists for every $s \in \mathcal{D}$.

Spatial processes of the type described above serve as the foundation for statistical studies of geostatistical data. The kriging method is given by:

$Z_{new} = w_1 Z_1 + w_2 Z_2 + \ldots + w_i Z_j$ + $\mathcal{E}_{new}$, where w's are weight that applied to the Au concentration. Estimation of w's:

$$w= \begin{pmatrix} cov(z_1, z_1) & cov(z_1, z_2) \ldots. & cov(z_1, z_i) \\ cov(z_2, z_1) & cov(z_2, z_2) \ldots. & cov(z_2, z_i) \\ cov(z_3, z_1) & cov(z_3, z_2) \ldots.. & cov(z_3, z_i) \end{pmatrix}^{-1} \begin{pmatrix} cov(z_{new}, z_1) \\ cov(z_{new}, z_2) \\ cov(z_{new}, z_3) \end{pmatrix} (3)$$

2.3.3 Inverse Distance Weight

The method can be used to locate mineralization [21]. IDW concept was derived from Shepard's method of spatial interpolation [22], the weight λ assigned to each of the known data points around the unknown area, is determined from the equation.

$Z_{new} = \frac{\lambda_1 Z_1 + \lambda_2 Z_2 + \ldots \lambda_i Z_j}{\lambda_1 + \lambda_2 + \cdots \lambda_i}$, (4) where I, j=1,2… and $\lambda_i = \frac{1}{d_i{}^p}$ (5)

Where d is the distance between the unknown point and the closest data points, as is obvious from the equation, the value of λ is inversely proportional to the distance from the unknown data point. The exponent, p, is assigned to increase the weight of the closest points and decrease the influence of the farthest points, as a result, the higher the value of p, the greater is the difference between the farthest and closest points. As p approaches 0, the weights get more similar.

2.4 Model Validation

To test how adequate the various variogram models were, a process of cross validation was carried out by point kriging, as described by [23]. The procedure involved removing each sample value in the data set and re-estimating the sample value by point kriging from the remaining data using the test model.

3.0 RESULTS AND DISCUSSIONS

3.1 Sample data distribution analysis

This section's descriptive analysis highlights the most essential aspects of the data collected. The table 1 below provides a short overview of all the factors under investigation.

Table 1: Summary Statistics for raw Au.

Mean	Maximum	Minimum	Std.	Skewness	Kurtosis
5.24	990.0	1.0	25.11	27.12	46.45

In general, the index's maximum and lowest Au contents are considerably distinct. The standard deviation is also high, indicating that Au content varies greatly. Positive skewness is also apparent, showing that the right tail is especially severe, indicating non-symmetric yields for the Au concentration.

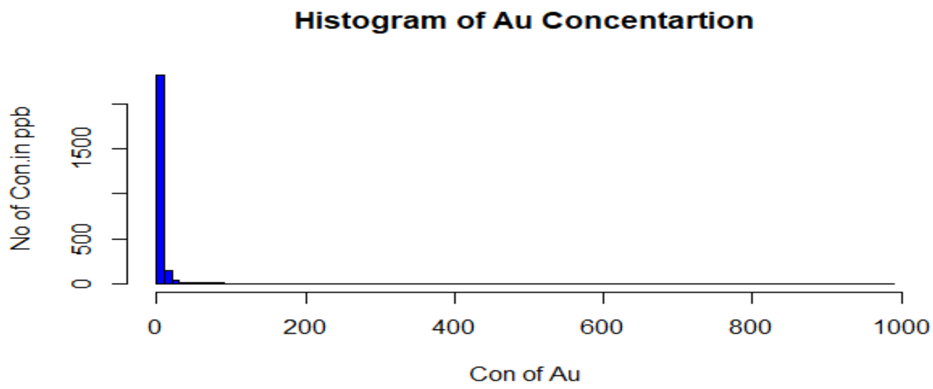

Figure 1: Histogram of Au concentration

The histograms presented for these datasets (Fig.2) also showed that the data was not regularly distributed. The histogram has a high kurtosis and is extremely positively skewed (Fig.2); the high kurtosis indicates that Au concentrations were localized to a specific segment of the research area. The dataset also showed positive skewness, which implies that the concentrations of the elements in the study region are never uniform; the bulk of the samples will have averagely equal values, but a few samples would have extremely high concentrations. Those samples might therefore indicate point anomalies for the respective elements, whereas the background concentrations could reflect background values for the respective element within the research region [24]. According to [25], the very nature of geochemical data makes them rather spatially dependent and as such inherently non-normal.

3.2 Box-Cox transformation

Thus, Box-Cox transformation was applied to the dataset to obtain optimal parameter that will aid in data transformation. (Figure 3).

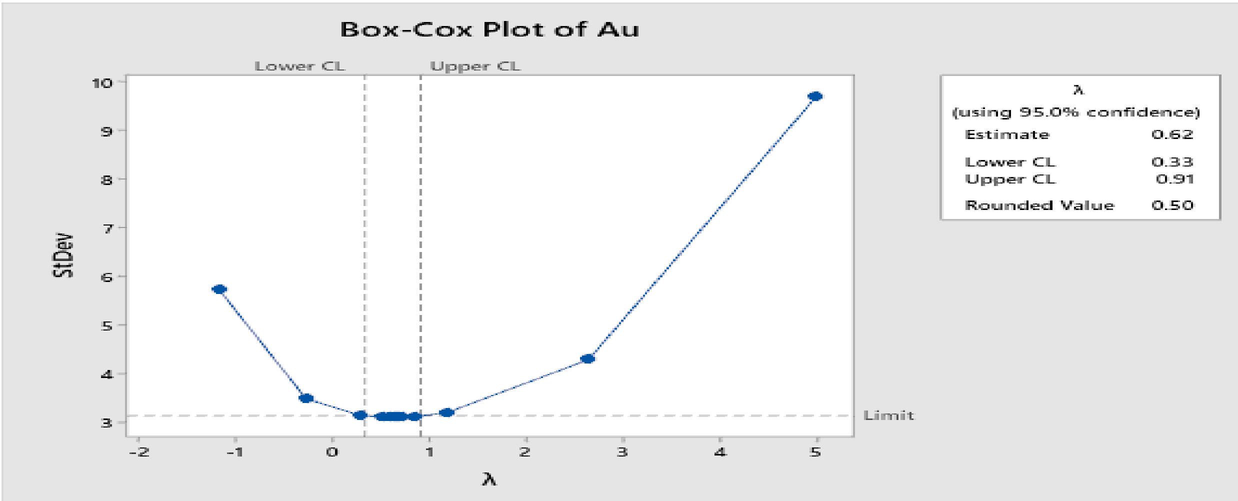

Figure 3: Box – Cox Plot of Au

In figure 3, the estimate value for the optimal (λ) was 0.62. The obtained confidence intervals do not include 0 indicating that logarithm transformation would not be appropriate for the datasets. Furthermore, the rounded value of 0.5 falls within the confidence interval, implying that the best normality transformation would be achieved by square rooting the Au datasets. Thus, square rooting transformation was applied to the same dataset and the summary of the descriptive statistics is shown in table 2**.**

Table 2: Summary Statistics for transformed Au

Mean	Maximum	Minimum	Std.	Skewness	Kurtosis
1.72	31.46	1.00	1.52	1.42	2.45

The difference between table 1 and 2 is questionable, looking at the gold raw data, the range here is 30.46 since the minimum value is 1.00 with the maximum value of 31.46. After the BC transformation, the mean is now 1.72 instead of 5.24 before transformation. The standard deviation is now 1.516 instead of 25.11 prior to transformation. The skewness and the kurtosis are now 1.416 and 2.45 respectively instead of 27.12 and 46.45 for both, before transformation. Indeed, the data has been transformed to near normal since the skewness and kurtosis of a normal distribution is 0 and 3 respectively. In fact, no dataset is exactly normally distributed, instead, it is only necessary for the data to be near normal [26].

3.3 Gold Spatial Distribution

The methods of Kriging and Inverse Distance Weighting (IDW) were employed in analyzing the spatial distribution of the gold in the study area and to delineate the gold anomalous zones within**.** The method of Kriging is dependent on the type of theoretical model that best fit the semi variogram, Fig. 4 is the semi variogram fitted with gaussian curve. The fitted semi-variogram model had a nugget effect of 0.1 and a nugget-to-sill variance of 0.52 with a range of influence of 530m. The banded nature of the mineralization, which is primarily due to the presence of impurities, specifically SiO_2 and Al_2O_2, caused the moderately high nugget-to-sill variance.[27]

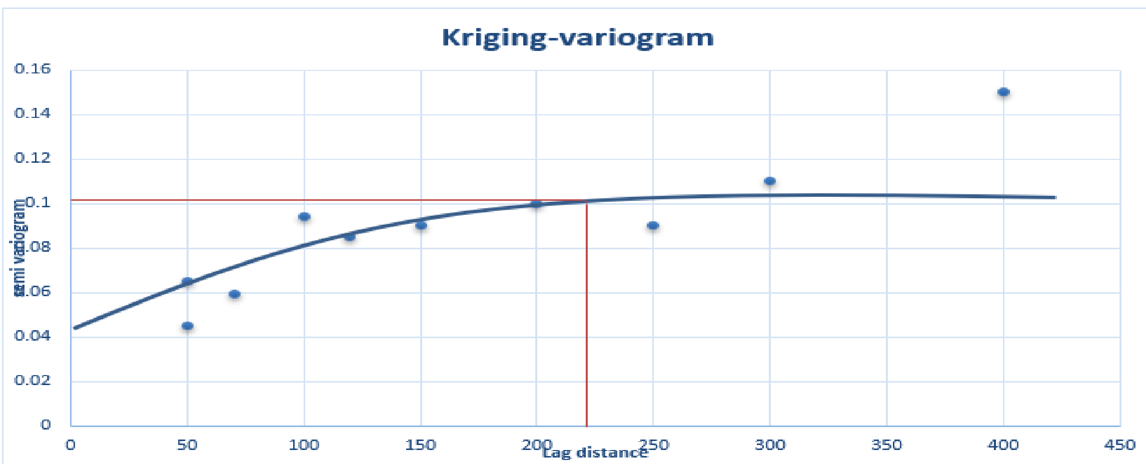

Figure 4: semi-variogram with fitted gaussian model

This was then used to generate the contour map in Fig. 5 and the 3d surface map in Fig. 6. However, Fig. 7 and 8 were generated using the method of IDW. This is important since it allows for comparison between the two results produced by each of the two distinct methods of geospatial modelling. It gives more confidence to the result

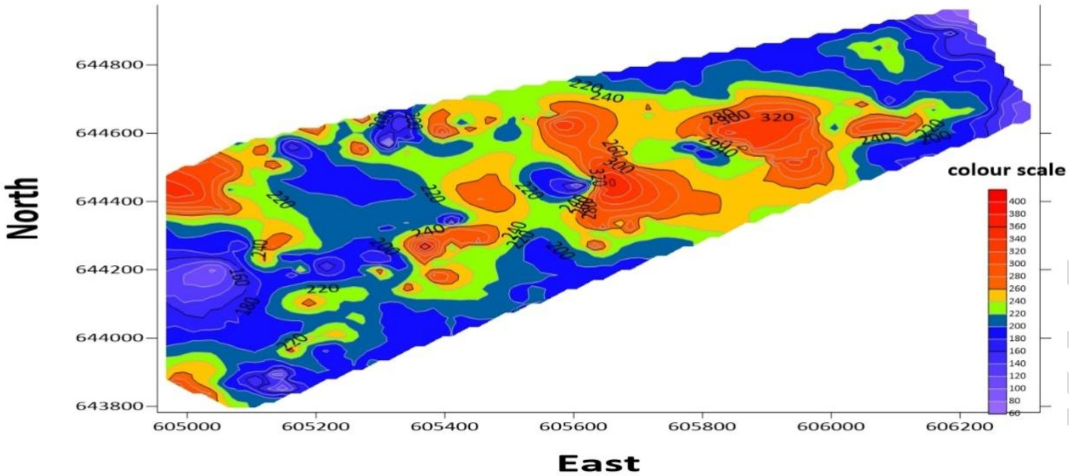

Figure 5: Contour map for Au using Kriging

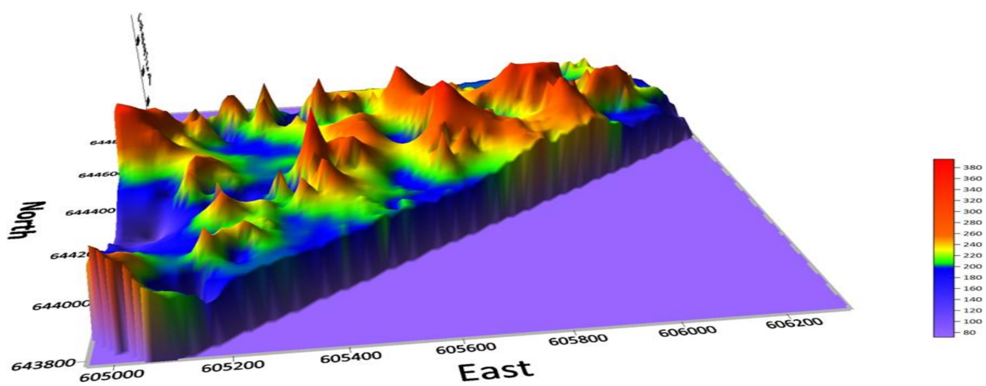

Figure 6: Surface map for Au using Kriging.

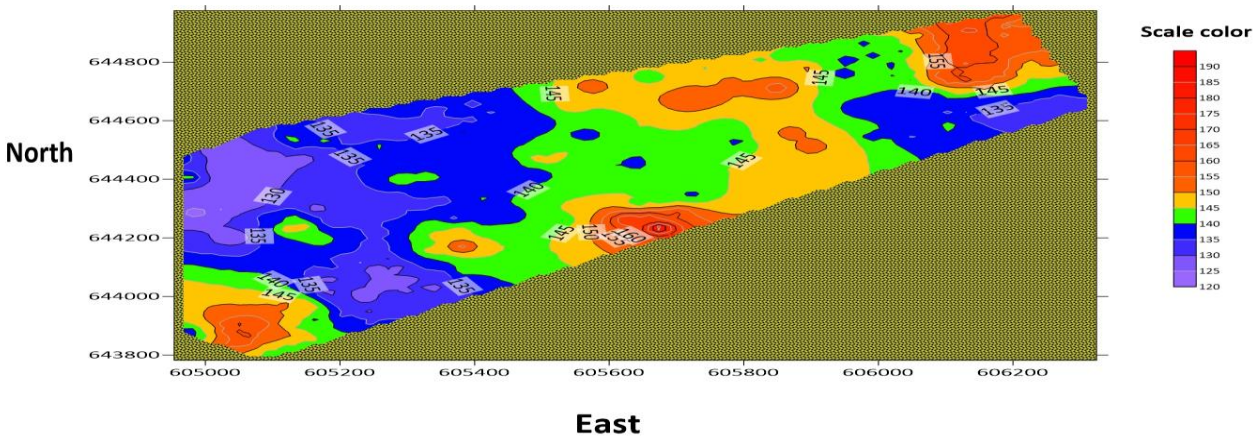

Figure 7: Contour map for Au using IDW.

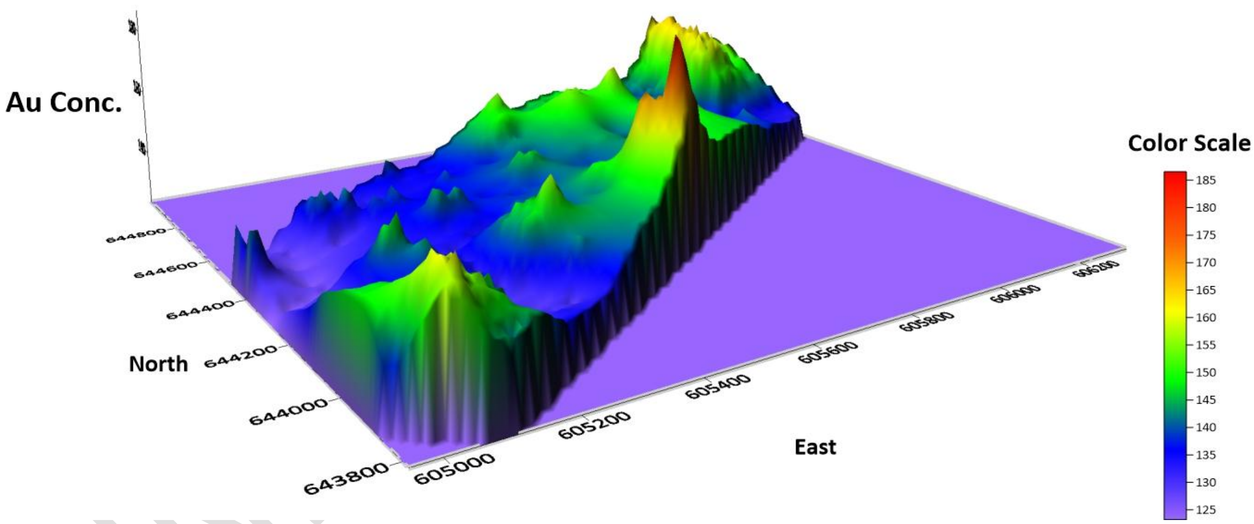

Figure 8:3D Surface map for Au using Kriging.

3.4 Model Performance Test

However, because the Kriging and IDW methods use different approaches to map and interpolate Au concentration, they were compared to determine the best predictive model. The result from table 3 indicates that the Kriging method outperformed the IDW method in terms of predictive power due to the dataset's strong spatial correlation structure.

Table 3: Model Comparison

	Kriging Model	**IDW Model**
RME	5.895	7.50
R^2	0.82	0.74

4.0 CONCLUSION

The results of both methods in analyzing the gold spatial distribution showed some discrepancy, indicating different anomalous zones. Both methods of Kriging and Inverse Distance Weighting (IDW) have therefore proven effective in modeling the gold spatial distribution within the Wassa-Amenfi study area. On the color scale, red indicates higher gold concentrations, while light blue indicates lower gold concentrations. The region with the highest gold concentration, according to the IDW model (Figure 5 and 6) results, is located in the center, whereas the kriging model (Figure 7 and 8) shows a random distribution of extreme gold concentration across the research area. The random identification of Au on the study might due to the occurrence of secondary dispersion mechanism which might have consistently alter the primary constituent.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

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