

Research on the Assessment of Mathematical Intuitive Imagination Literacy

Abstract: The assessment of high school students' mathematical intuitive imagination literacy can provide an important reference and basis for high school mathematics teachers to develop students' mathematical intuitive imagination literacy. However, there is a lack of research on how to assess the mathematical intuitive imagination literacy of high school students. This paper adopted the theoretical analysis approach, relying on lots of literature from the *China National Knowledge Infrastructure*, to discuss and analyze two aspects of the Curriculum Standards for assessing mathematical intuitive imagination literacy: three academic quality levels and four dimensions. The purpose of this paper is to provide a scientific and reasonable solution for assessing mathematical intuitive imagination literacy. Finally, we proposed the way of assessing mathematical intuitive imagination literacy of high school students, the setting of test questions, the selection of scorers, and the giving of evaluation results. This study can provide a reference for more in-depth research on mathematical intuitive imagination literacy in the future.

Keywords: Mathematical intuition, Mathematical imagination, Intuitive imagination literacy, High school students, Assessment

1. INTRODUCTION

The assessment of high school students' mathematical intuitive imagination literacy is not only the need to understand the characteristics and differences in their levels of mathematical intuitive imagination literacy, but also can provide guidance and reference for instructional design that points to the development of intuitive imagination literacy [1]. Therefore, it is necessary to establish a scientific and reasonable way to assess mathematical intuitive imagination literacy. However, the current problem is that there is little has been done on how to assess mathematical intuitive imaginative literacy and few reasonably feasible assessment strategies for high school teachers and educational researchers. Therefore, this study intends to adopt the theoretical analysis approach to research how to assess the mathematical intuitive imagination literacy of high school students to provide a reference for future research on intuitive imagination literacy.

2. ANALYSIS OF MATHEMATICAL INTUITIVE IMAGINATION LITERACY

The Ministry of Education of the People's Republic of China specifies in the Curriculum Standards for General High School Mathematics (2017 Edition) (hereinafter referred to as the Curriculum Standards) that intuitive imagination is the literacy of using geometric intuition and spatial imagination to perceive the forms and changes of things, and using spatial forms, especially figures, to understand and solve mathematical problems.[2]

The Curriculum Standards also specifically divide intuitive imagination literacy into three

academic quality levels and four dimensions. The details are shown in Table 1.

Table 1: The three levels and four dimensions of intuitive imagination literacy

Level	Dimension	Specific Performance
Level 1	Situation and Questions	Be able to abstract the geometric figures of objects in familiar situations and establish the connection between simple figures and objects; experience the relationship between figures and figures, figures and quantities.
	Knowledge and Skills	Be able to discover mathematical laws in familiar mathematical situations with the help of properties and transformations (translation, symmetry, and rotation) of figures; be able to describe the positional and metric relations of simple figures and their specific properties.
	Thinking and Expression	Be able to understand mathematical problems through graphical intuition; be able to describe and express familiar mathematical problems with figures, inspire ideas for solving these problems, and experience the combination of numbers and shapes.
	Communication and Reflection	Be able to use graphical intuition to communicate in daily life.
Level 2	Situation and Questions	Be able to imagine and construct corresponding geometric figures in a related situation; be able to propose mathematical questions with the help of figures, discover the relationship between figures and figures, figures and quantities, and explore the laws of motion of figures.
	Knowledge and Skills	Be able to master the basic methods of studying the relationship between figures and figures, figures and quantities; be able to explore mathematical laws with the help of properties of figures and solve practical or mathematical problems.
	Thinking and Expression	Be able to propose mathematical problems through intuitive imagination; be able to explore ideas for solving problems with figures; be able to form the idea of combining numbers and shapes and experience the role and significance of geometric intuition.
	Communication and Reflection	Be able to use intuitive imagination to explore mathematical problems during communication.
Level 3	Situation and Questions	Be able to propose mathematical questions with the help of figures and intuitive imagination in an integrated situation.
	Knowledge and Skills	Be able to make integrated use of the relationship between figures and figures, figures and quantities, to understand the connection between the various branches of mathematics; be able to establish the connection between mathematics and other disciplines with the help of intuitive imagination and form intuitive models of the theoretical system.

	Thinking and Expression	Be able to express complex mathematical problems intuitively through imagination, reflect the essence of mathematical problems, and form ideas for solving problems.
	Communication and Reflection	Be able to use intuitive imagination to explore the essence of problems and the connection with mathematics during communication.

2.1 Analysis of Relevant Contents of Level 1

2.1.1 Situation and Questions

Familiar contexts are things and phenomena that are directly related to the content, common in everyday life, mathematics, and other sciences, and within the students' existing knowledge and cognitive level. [3]

Abstracting the geometric figures of objects means discarding the properties of the object other than its geometric features, imagining it as a regular geometric figure, and expressing it graphically.

Establishing the connection between simple figures and objects means matching the shapes of objects with geometric figures, imagining the corresponding geometric figures based on the objects, and associating the geometric figures with the corresponding objects.

Experiencing the relationship between figures and figures, figures and quantities means focusing on the understanding of the essential properties of geometric figures and to describe the structural features of geometric figures that they reflect, [4] in which the structural features of geometric figures include the quantitative and positional relationships of vertices, lines (angles) and faces.

2.1.2 Knowledge and Skills

Familiar mathematical situations are things and phenomena that are directly related to the content, common in the mathematical discipline, within the cognitive structure that students already have, and that are characteristic of the mathematical discipline. [3, 5-6]

Discovering mathematical laws with the help of properties and transformations (translation, symmetry, and rotation) of figures means studying the relationship between the basic elements of a figure, [7] or applying a certain change of position to the given figures (or part of them) according to a determined law, and then to analyze the relationship between the relevant figures in the new figure. Finally, some general commonality in the figure is concluded.

Describing the positional and metric relations of simple figures and their specific properties means using the mathematical language to express the relative position of a figure concerning a reference figure, its distance and length, and the relationship between its elements, for figures with a single constituent element.

2.1.3 Thinking and Expression

Understanding mathematical problems through graphical intuition means clarifying the conditions of mathematical problems with the help of figures, and clarifying the relationships between mathematical concepts and knowledge.

Describing and expressing familiar mathematical problems with figures, and inspiring ideas for solving these problems means transforming textual descriptions into figures for mathematical problems that are described in words and are included in the students' existing cognitive structure, so as to deepen their understanding of these problems and obtain ideas for solving them.

Experiencing the combination of numbers and shapes means experiencing and appreciating the mathematical idea of using the geometric intuition of shapes to clarify certain relationships between numbers through the process of describing and expressing familiar mathematical problems using figures.

2.1.4 Communication and Reflection

Using graphical intuition to communicate in daily life means visually explaining and communicating mathematical concepts, conclusions, applications, and ideas about figures by means of mathematical language in the various activities that students regularly perform. [2]

2.2 Analysis of Relevant Contents of Level 2

2.2.1 Situation and Questions

A related situation is a life situation, mathematical situation, or scientific situation that is based on the familiar situation, associated with the familiar situation, and can be created through the familiar situation. [2-3]

Imagining and constructing corresponding geometric figures means to allow students to fully explore the intrinsic connection between the problem and the figure, relate the problem to a familiar figure according to its intrinsic connection or structural features of the number equation, and construct the figure according to its geometric features. [8-9]

Proposing mathematical questions with the help of figures means generating new questions and expressing them through the exploration of problem situations based on the characteristics and laws of figures. [10]

Discovering the relationship between figures and figures, figures and quantities means finding out the relationship between the division, combination, and position of figures, as well as the

transformation and correspondence between figures and quantities through the observation of figures.

Exploring the laws of motion of figures means observing and analyzing the processes of the manipulation related to figures, inferring and exploring the laws contained in them, and drawing or guessing the general conclusions. [11]

2.2.2 Knowledge and Skills

Mastering the basic methods of studying the relationship between figures and figures, figures and quantities means clarifying some basic methods of studying geometric problems in secondary school, to clarify their meaning, formation process, what problems they can solve, and how to apply them. Also, to be able to apply these methods flexibly and to reach the level of proficiency in solving problems. [11]

Exploring mathematical laws with the help of properties of figures means discovering and applying the mathematical laws contained in a set of figures with a specific relationship through observation, analysis, and reasoning, and then summarizing and inferring general mathematical conclusions. [12]

Solving practical or mathematical problems means analyzing and solving problems whose elements, properties, and relationships are related to mathematics or problems with realistic situations that need to consider practical conditions. [13]

2.2.3 Thinking and Expression

Proposing mathematical problems through intuitive imagination means exploring the problem situations with the help of geometric intuition and spatial imagination to generate and express valuable mathematical problems. [10]

Exploring ideas for solving problems with figures means observing and thinking about the figures in the problem situations, introducing obvious (or hidden) properties from them, and finding ways to solve the problems by making appropriate combinations according to the conclusions to be proved. [14]

Forming the idea of combining numbers and shapes means having a system of ideas and concepts to solve mathematical problems based on the correspondence and mutual transformation between numbers and shapes. [15]

Experiencing the role and significance of geometric intuition means experiencing and appreciating how the use of figures to describe and analyze problems brings about results and impacts on solving mathematical problems. [16]

2.2.4 Communication and Reflection

During communication means in the process of exchanging verbal information between people based on mathematical knowledge.

Using intuitive imagination to explore mathematical problems means exploring and discussing various aspects of mathematical questions such as their conditions, solution methods, and the essence of mathematics through the process of geometric intuition and spatial imagination.

2.3 Analysis of Relevant Contents of Level 3

2.3.1 Situation and Questions

An integrated situation is a synthesis of related situations based on related situations, which contains more basic information and familiar elements of mathematics. [3]

Proposing mathematical questions with the help of figures and intuitive imagination means exploring the problem situations with the help of geometric intuition and spatial imagination to generate valuable mathematical problems and express them. [10]

2.3.2 Knowledge and Skills

Making integrated use of the relationship between figures and figures, figures and quantities means fully and comprehensively analyzing the relationships between graphs in terms of division, combination, and position, as well as the relationships between figures and quantities in terms of transformation and correspondence, to achieve certain purposes.

Understanding the connection between the various branches of mathematics means appreciating the relationship between the contents of different mathematical knowledge after classification, such as study objects and research methods. [17]

Establishing the connection between mathematics and other disciplines with the help of intuitive imagination means forming an understanding of the connections between the knowledge and theoretical systems elaborated in different aspects of each discipline through geometric intuition and spatial imagination.

Forming intuitive models of the theoretical system means developing a physical or abstract object that people use more familiar and observable images from experience to represent highly generalized, ordered multi-level mathematical theoretical knowledge. [18]

2.3.3 Thinking and Expression

Expressing complex mathematical problems intuitively through imagination means transforming the literal and symbolic language of the problem into a graphical language through geometric means for visualization, thus making the complex and abstract problems concise and visual. [19]

Reflecting on the essence of mathematical problems means expressing the process of occurrence and development of the objectively existing concepts and principles behind mathematical problems. [20]

Forming ideas for solving problems means grasping the main points of the problem, fully exploring the implicit conditions, correctly identifying the thinking method for solving the problem, and clarifying the steps. [21]

2.3.4 Communication and Reflection

During communication means in the process of exchanging verbal information between people based on mathematical knowledge.

Using intuitive imagination to explore the essence of problems and the connection with mathematics means enabling students to understand in depth the process of occurrence and development of concepts and principles behind mathematical problems through geometric intuition and spatial imagination, and to build a system of mathematical knowledge and thinking. [20]

3. ANALYSIS OF MATHEMATICAL INTUITIVE IMAGINATION LITERACY ASSESSMENT

According to the above analysis of the relevant content in the curriculum standards, we divided the 12 aspects consisting of the three levels and four dimensions of mathematical intuitive imagination literacy into 21 exam points, and the number of exam points corresponding to each aspect is shown in Table 2.

Table 2: The number of exam points for the assessment

Level Dimension	Situation and Questions	Knowledge and Skills	Thinking and Expression	Communication and Reflection	Total
Level 1	2	2	2	1	7
Level 2	2	2	3	1	8
Level 3	1	2	2	1	6
Total	5	6	7	3	21

3.1 Analysis of Level 1 Assessment

3.1.1 Situation and Questions

From the above analysis, the Level 1, Situation and Questions dimension can be set for 2 exam points: the first is *abstracting the geometric figures of objects in familiar situations and establishing the connection between simple figures and objects*; the second is *experiencing the relationship between figures and figures, figures and quantities*.

For the first exam point, it can be examined in the following way: give students some common things or phenomena in life and study, and ask them to draw the regular geometric figures of them. Afterward, see whether students can draw the corresponding geometric figures and whether these figures are reasonable.

For the second exam point, it can be examined in the following way: give students some geometric figures and ask them to distinguish the mathematical relationships between different figures and between figures and quantities. Afterward, see whether students can tell the relationship between them and see whether what they say is reasonable.

To facilitate the assessment, it can be carried out in the following way: combine the two above-mentioned ways, that is, follow up the question of the first exam point and then add a question asking students to observe the structural features of geometric figures and to identify the quantity and position relations in the corresponding geometric figures.

As a result, for the Level 1, Situation and Questions dimension, there could be set 1 large question including 2 sub-questions, for a total of 20 points, with each sub-question carrying 10 points. For the first sub-question, 5 questions can be set with 2 points each; for the second sub-question, 5 sub-questions can also be set with an average of 2 points each. Eventually, students' current situation on the Level 1, Situation and Questions dimension is judged based on their scores on each question.

3.1.2 Knowledge and Skills

From the above analysis, the Level 1, Knowledge and Skills dimension can be set for 2 exam points: the first is *discovering mathematical laws in familiar mathematical situations with the help of properties and transformations (translation, symmetry, and rotation) of figures*; the second is *describing the positional and metric relations of simple figures and their specific properties*.

For the first exam point, it can be examined in the following way: provide students with some mathematical figures that students have studied or been exposed to, ask them to observe the properties and transformation process of the figures, and indicate the mathematical laws found. Afterward, see whether students can clearly recognize the properties and transformation process of the figures and write the corresponding mathematical laws, and whether they write them reasonably.

For the second exam point, it can be examined in the following way: on the one hand, give students certain three-dimensional figures, ask them to observe from different positions and draw the figures observed in different positions, then see whether students can draw the figures seen in different positions and whether the drawing is reasonable; on the other hand, give students certain plane geometry figures, ask them to analyze according to the known conditions and find out the unknown metric relationship. Afterward, see if the students can solve the answer and if the answer is correct.

To facilitate the assessment, it can be carried out in the following way: using the two methods mentioned above, give the graphs separately and set questions according to the content of the exam point, that is, the first and the second exam point are independent of each other.

As a result, for the Level 1, Knowledge and Skills dimension, there could be set 2 large questions, for a total of 20 points, with each question carrying 10 points. For the first large question, 3 sub-questions can be set; for the second large question, 2 sub-questions can be set, and points are assigned according to the number of questions. Eventually, students' current situation on the Level 1, Knowledge and Skills dimension is judged based on their scores on each question.

3.1.3 Thinking and Expression

From the above analysis, the Level 1, Thinking and Expression dimension can be set for 2 exam points: the first is *understanding mathematical problems through graphical intuition*; the second is *describing and expressing familiar mathematical problems with figures, inspiring ideas for solving these problems, and experiencing the combination of numbers and shapes*.

For the first exam point, it can be examined in the following way: give students some mathematical problems that they rarely or never meet normally, ask them to draw the corresponding geometric figures based on textual descriptions, through visual imagination, and solve mathematical problems with the help of geometric figures. After that, see whether the students can solve the mathematical problems based on geometric figures and whether the answers are correct.

For the second exam point, it can be examined in the following way: give students some common, non-graphical representations of mathematical problems and ask them to describe and express these problems using appropriate graphs. Afterward, see whether they are able to express the given problems graphically and see whether their expressions are reasonable.

To facilitate the assessment, it can be carried out in the following way: using the two methods mentioned above, give the graphs separately and set questions according to the content of the exam point, that is, the first and the second exam point are independent of each other.

As a result, for the Level 1, Thinking and Expression dimension, there could be set 2 large questions including 6 sub-questions, for a total of 20 points, with each large question carrying 10 points. For the first large question, 2 sub-questions can be set; for the second large question, 4 sub-questions can be set, and points are assigned according to the number of specific questions and the difficulty of the questions. Eventually, students' current situation on the Level 1, Thinking and Expression dimension is judged based on their scores on each question.

3.1.4 Communication and Reflection

From the above analysis, the Level 1, Communication and Reflection dimension can be set for one exam point, that is, *using graphical intuition to communicate in daily life*.

For this exam point, it can be examined in the following way: give students a familiar life scene and ask them to observe the shapes in the scene and then describe them in mathematical language. Afterward, see whether the students can describe the corresponding geometric figures and whether the descriptions are reasonable.

As a result, for the Level 1, Communication and Reflection dimension, there could be set 1 large questions including 2 sub-questions, for a total of 20 points, with each sub-question carrying 10 points. For the first large question, 5 sub-questions can be set with 2 points each; for the second large question, 5 sub-questions can be set with an average of 2 points each. Eventually, students' current situation on the Level 1, Communication and Reflection dimension is judged based on their scores on each question.

3.2 Analysis of Level 2 Assessment

3.2.1 Situation and Questions

From the above analysis, the Level 2, Situation and Questions dimension can be set for 2 exam

points: the first is *imagining and constructing corresponding geometric figures in a related situation*; the second is *proposing mathematical questions with the help of figures, discovering the relationship between figures and figures, figures and quantities, and exploring the laws of motion of figures*.

For the first exam point, it can be examined in the following way: provide students with a geometric background and ask them to determine the three views of a three-dimensional geometry based on the problem's intrinsic connections or structural features of the figure, and draw the constructed three-dimensional geometry based on the requirements of the problem. Afterward, see if the students can draw the corresponding geometric figures and if the drawings are reasonable.

For the second exam point, it can be examined in the following way: firstly, ask students to propose mathematical problems of different difficulties according to the geometric figures provided, and see whether they can propose mathematical problems and whether the problems are reasonable; secondly, analyze the figures according to the characteristics of three-dimensional figures and known conditions, and find out the position and quantity relations of the figures contained in them, and see whether students can find out the relations between the figures and whether the findings are reasonable; thirdly, provide students with information about the beginning and end states of the figures, and ask them to describe the transformation process of the figures according to the known conditions, and then see whether they can describe the transformation process and whether the description is reasonable.

To facilitate the assessment, it can be carried out in the following way: combine the two above-mentioned ways partly, that is, follow up the question of the first exam point and then add a question asking students to propose a corresponding mathematical problem based on the constructed graph.

As a result, for the Level 2, Situation and Questions dimension, there could be set 2 large questions including 5 sub-questions, for a total of 20 points, with each large question carrying 10 points. For the first large question, 3 sub-questions can be set; for the second large question, 2 sub-questions can be set, and points are assigned according to the number of specific questions and the difficulty of the questions. Eventually, students' current situation on the Level 2, Situation and Questions dimension is judged based on their scores on each question.

3.2.2 Knowledge and Skills

From the above analysis, the Level 2, Knowledge and Skills dimension can be set for 2 exam points: the first is *exploring mathematical laws with the help of properties of figures*; the second *solving practical or mathematical problems*.

For the first exam point, it can be examined in the following way: provide students with a set of graphs with certain quantitative relationships and ask them to answer questions based on the laws revealed by the graphs, and eventually summarize the general laws of the graphs from the particular to the general. Afterward, see whether the students can summarize the corresponding mathematical laws and whether the summary is reasonable.

For the second exam point, it can be examined in the following way: provide a purely mathematical problem and a practical problem, and ask students to prove or answer them based on the graph and its known conditions. Afterward, see whether they can complete the proof and answer, and whether the proof and answer process is reasonable.

To facilitate the assessment, it can be carried out in the following way: using the two methods mentioned above, give the graphs separately and set questions according to the content of the exam point, that is, the first and the second exam point are independent of each other.

As a result, for the Level 2, Knowledge and Skills dimension, there could be set 2 large questions including 5 sub-questions, for a total of 20 points. For the first large question, 3 sub-questions can be set; for the second large question, 2 sub-questions can be set, and points are assigned according to the number of questions. Eventually, students' current situation on the Level 2, Knowledge and Skills dimension is judged based on their scores on each question.

3.2.3 Thinking and Expression

From the above analysis, the Level 2, Thinking and Expression dimension can be set for 3 exam points: the first is *proposing mathematical problems through intuitive imagination*; the second is *exploring ideas for solving problems with figures*; the third is *forming the idea of combining numbers and shapes and experiencing the role and significance of geometric intuition*.

For the first exam point, it can be examined in the following way: describe some of the geometric features of the figure in words, let students construct the geometric figure through visual imagination and propose mathematical problems on this basis. Afterward, see whether students can propose corresponding mathematical problems and whether the problems are reasonable.

For the second exam point, it can be examined in the following way: ask students to draw the corresponding graphs based on the known conditions of the question, conclude with the graphs, and prove their conclusions or write the process of their answers. After that, see whether they can write the conclusion and its proof or answer process with the help of graphs and whether the writing is reasonable.

For the third exam point, it can be examined in the following way: let students talk about their feelings about the role and meaning of geometric intuition and their understanding of the idea of combining numbers and shapes in the process of solving problems.

To facilitate the assessment, it can be carried out in the following way: use the first method above to set separate questions and combine the latter two methods. That is, the questions of the first exam point are independent, and the questions of the third exam point follow upon the basis of the second exam point, and then add to let students talk about their understanding of geometric intuition and the idea of combining numbers and shapes according to the process of solving the problems.

As a result, for the Level 2, Thinking and Expression dimension, there could be set 2 large questions including 4 sub-questions, for a total of 20 points. For the first large question, 2 sub-questions can be set; for the second large question, 4 sub-questions can be set, and points are assigned according to the number of questions. Eventually, students' current situation on the Level 2, Thinking and Expression dimension is judged based on their scores on each question.

3.2.4 Communication and Reflection

From the above analysis, the Level 2, Communication and Reflection dimension can be set for one exam point, that is, *using intuitive imagination to explore mathematical problems during communication*.

For this exam point, it can be examined in the following way: provide students with a mathematical situation, ask them to observe and imagine the given figure according to the conditions and requirements of the question, and then write their thinking process of solving the problem. Afterward, see whether students can write the corresponding thinking process and whether the process is reasonable.

As a result, for the Level 2, Communication and Reflection dimension, there could be set 1 large questions including 2 sub-questions, for a total of 20 points, with each sub-question carrying 10 points. The points are assigned according to the number of specific questions and the difficulty of the questions. Eventually, students' current situation on the Level 2, Communication and Reflection dimension is judged based on their scores on each question.

3.3 Analysis of Level 3 Assessment

3.3.1 Situation and Questions

From the above analysis, the Level 3, Situation and Questions dimension can be set for one exam point, that is, *proposing mathematical questions with the help of figures and intuitive imagination in an integrated situation.*

For this exam point, it can be examined in the following way: provide students with a real-world situation that contains a lot of mathematical information and geometric elements, ask them to abstract mathematical questions from it, and represent these questions visually with figures. Afterward, see whether students can come up with appropriate mathematical questions and whether these questions are reasonable.

As a result, for the Level 3, Situation and Questions dimension, there could be set 1 large questions including 3 sub-questions, for a total of 20 points, and points are assigned according to the number of specific questions and the difficulty of the questions. Eventually, students' current situation on the Level 3, Situation and Questions dimension is judged based on their scores on each question.

3.3.2 Knowledge and Skills

From the above analysis, the Level 3, Knowledge and Skills dimension can be set for 2 exam points: the first is *making integrated use of the relationship between figures and figures, figures and quantities to understand the connection between the various branches of mathematics*; the second is *establishing the connection between mathematics and other disciplines with the help of intuitive imagination and forming intuitive models of the theoretical system.*

For the first exam point, it can be examined in the following way: provide students with questions that include both graphs and multiple branches of mathematics, and ask them to identify the branches of mathematics involved and the connections between them. Later, see whether the students can identify the corresponding branches of mathematics and their connections, and whether the results is reasonable.

For the second exam point, it can be examined in the following way: give students some questions that combine mathematical graphics with other subjects, ask them to state the relationship between mathematics and other subjects reflected in the questions, and give examples that has the above relationships. Afterward, see whether the students can state the relationship between them and give examples, as well as see whether what they say is reasonable.

To facilitate the assessment, it can be carried out in the following way: using the two methods mentioned above, give the graphs separately and set questions according to the content of the exam point, that is, the first and the second exam point are independent of each other.

As a result, for the Level 3, Knowledge and Skills dimension, there could be set 2 large questions including 7 sub-questions, for a total of 20 points, with each large question carrying 10 points. For the first large question, 5 sub-questions can be set; for the second large question, 2 sub-questions can be set, and points are assigned according to the number of questions. Eventually, students' current situation on the Level 3, Knowledge and Skills dimension is judged based on their scores on each question.

3.3.3 Thinking and Expression

From the above analysis, the Level 3, Thinking and Expression dimension can be set for 2 exam points: the first is *expressing complex mathematical problems intuitively through imagination*; the second is *reflecting the essence of mathematical problems, and forming ideas for solving problems*.

For the first exam point, it can be examined in the following way: provide students with a complex real-world problem situation, ask them to analyze the conditions of the problem, and draw mathematical graphs based on the problem. Afterward, see whether the students can draw the corresponding graphs and whether the drawings are reasonable.

For the second exam point, it can be examined in the following way: ask students to write the essence of the problem based on the problem statement and the graph, and write the steps to solve the problem with the graph. After that, see if students can write the corresponding steps to solve the problem and whether the steps are reasonable.

To facilitate the assessment, it can be carried out in the following way: combine the two above-mentioned ways, that is, follow up the question of the first exam point and then add a question asking students to analyze the problem with the graph, summarize its mathematical essence, and write the steps to solve the problem.

As a result, for the Level 3, Thinking and Expression dimension, there could be set 1 large question including 2 sub-questions, for a total of 20 points. The points are assigned according to the difficulty of the questions. Eventually, students' current situation on the Level 3, Thinking and Expression dimension is judged based on their scores on each question.

3.3.4 Communication and Reflection

From the above analysis, the Level 3, Communication and Reflection dimension can be set for one exam point, that is, *using intuitive imagination to explore the essence of problems and the connection with mathematics.*

For this exam point, it can be examined in the following way: give students a familiar life scene and ask them to observe the shapes in the scene and then describe them in mathematical language. Afterward, see whether the students can describe the corresponding geometric figures and whether the descriptions are reasonable.

To facilitate the assessment, it can be carried out in the following way: based on the questions already completed through intuitive imagination, ask students to write the mathematical essence and the mathematical knowledge used in each of these questions.

As a result, for the Level 3, Communication and Reflection dimension, there could be set 1 large questions including 4 sub-questions, for a total of 20 points, with each sub-question carrying 5 points. Eventually, students' current situation on the Level 3, Communication and Reflection dimension is judged based on their scores on each question.

3.4 Analysis of Overall Assessment

3.4.1 Assessment questions and score settings

According to the above analysis about the three levels and four dimensions of mathematical intuitive imagination literacy, when preparing the test paper, 18 test questions can be set, with 240 points. The questions corresponding to each part of the 12 parts composed of the three levels and four dimensions account for 20 points, and the test time is set at 120 minutes.

3.4.2 Scorers and score calculation

To ensure the reliability of the scores, each question should be evaluated by multiple scorers, and then the average of the scores rated by multiple scorers is taken as the final score of each question. Among them, the scorers should have strong mathematical intuitive imagination and rich pedagogical knowledge. Therefore, it is proposed to invite a master of education majoring in mathematics and a high school math teacher as scorers for this scoring.

Two scorers should rate each question separately. If the score difference is less than or equal to 3, the average of the two scores will be taken as the final score of the question; if the score difference is more than 3, an additional high school mathematics teacher will be invited to score the question,

and the above two scorers will re-score the question, and finally, the average of the three scores will be taken as the final score of the question. After that, the final scores of the 18 questions were added together as the final score of this test paper.

3.4.3 Giving of evaluation results

From the perspective of the three levels and four dimensions of mathematical intuitive imagination literacy as a whole, the total score of this test paper was 240. Students are considered to have a low level of intuitive imagination literacy if their scores are between 144 - 180 (60% - 75% of the total score); students are considered to have a moderate level of intuitive imagination literacy if their scores are between 181 - 216 (76% - 90% of the total score); and students are considered to have a high level of intuitive imagination literacy if their scores are between 217 - 240 (91% - 100% of the total score).

4. CONCLUSION AND SUGGESTION

From the definition of intuitive imagination, it can be seen that the assessment of high school students' intuitive imagination can be based on their achievement of the three levels and four dimensions, and this process can be achieved by analyzing the students' scores on the questions. Thus, the way of assessing the mathematical intuitive imagination of high school students, the setting of test questions, the selection of scorers, and the giving of evaluation results should closely follow the provisions of the Curriculum Standards for the specific performance of the three levels and four dimensions, and make them the basis and core of the assessment. Only in this way can the final evaluation results obtained be more convincing and ensure good reliability and validity.

For high school mathematics teachers, they can refer to the above analysis of each section of the mathematical intuitive imagination literacy assessment, and correspondingly select or adapt practice questions, high-stakes questions, etc. In turn, a reasonable test paper with detailed scoring criteria can be developed. On this basis, the test subjects are asked to complete the test paper. After that, appropriate scorers are selected to mark the test strictly according to our predetermined scoring methods and scoring criteria. Finally, the students are graded according to their final scores.

This study provides a feasible method for evaluating the mathematical intuitive imagination literacy of high school students, and it can provide a reference for more in-depth research on mathematical intuitive imagination literacy in the future. At the same time, the questions given in this assessment are mainly drawing questions, answer questions, and proof questions, and the types of questions are not rich enough. Therefore, it is necessary to add different types of questions in the future to enrich the examination methods and difficulty levels of the questions, in order to find more detailed and comprehensive assessment ideas.

REFERENCES

- 1 . Zheng XJ, Chen QH, Ke YM. Construction of a model for assessing mathematical intuitive imagination literacy of high school students. *Educational Research and Experiment*. 2021;40(03):71-78.
- 2 . Ministry of Education of the People's Republic of China. *General High School Mathematics Curriculum Standards (2017 Edition)*. Beijing: People's Education Press; 2018.
- 3 . Luo R. Research on the creation of question situations based on the core literacy of high school mathematics - through the perspective of familiar situations, related situations and integrated situations. *Research of Mathematic Teaching-Learning*. 2021;40(05):22-26+67.
- 4 . Yang RJ, Zhan JF. Reinforcing the process of abstracting geometric figures from physical objects--and the interface between teaching graphics in primary and middle schools. *Middle School Mathematics Teaching and Learning*. 2014;13(19):1-3.
- 5 . Zhang JX. Research on the creation of contextual problems of high school mathematics in the perspective of core mathematical literacy. Chongqing: Chongqing Normal University; 2020.
- 6 . Zhang Y. The "disproportionality" and "appropriateness" of mathematical situations teaching. *Journal of Teaching and Management*. 2022;39(13):45-47.
- 7 . Chen LH, Cao JF. 2021 Mid-Term Examination "the properties of graphs" topic proposition analysis. *Mathematics Education in China*. 2022;13(Z1):68-78+96.
- 8 . Wu RJ. Talking about several methods of solving problems in junior high school mathematics. *Mathematics, physics, and chemistry study (junior high school version)*. 2014;13(08):53-54.
- 9 . Zhang YY. Constructing geometric figures to solve problems. *Journal of Bijie Teachers College*. 2001;8(02):82-85.
- 10 . Bi JY, Hu YP, Ma YH, Han LN. Research on creating situations and proposing problems in mathematics teaching. *Collection of research results of "Research on the Development of Teachers' Teaching Ability" (Volume 4)*. 2017;4:550-551.
- 11 . Zeng JP. Study the mathematics syllabus and talk about the meaning of "know", "understand", "master" and "apply" of the syllabus knowledge. *Journal of Tongren University*. 2000;2(01):59-60.
- 12 . Wu LK. Guidance on the methods of solving junior high school mathematics regular exploration problems. *Research on the Curriculum, Teaching Materials, and Teaching (Chinese Education Research)*. 2021;20(Z6):45-47.
- 13 . Xing SQ. Mathematical problems and their solution strategies in the perspective of academic

- quality in high school mathematics. *Education Science Forum*. 2022;29(04):43-45.
- 14 . Zhuang WL. Explore problem solving ideas using basic shapes. *High School Mathematics Teaching*. 1999;20(06):33-30.
 - 15 . Chen JB. Use the idea of combining numbers and shapes to solve objective questions in the college entrance examination. *Mathematics Teaching*. 2015;61(10):43-46.
 - 16 . Ministry of Education of the People's Republic of China. *Compulsory education mathematics curriculum standard (2011 Edition)*. Beijing: Beijing Normal University Publishing Group; 2012.
 - 17 . Xie JQ, Yu JG. Let high school students understand the branches of mathematics in chapter introductions. *Primary and Middle School Mathematics (High School Edition)*. 2013;6(05):4-6.
 - 18 . Song L. *Intuitive Models: A Bridge between Arithmetic and Algorithms*. Beijing Education. 2017;24(06):53-54.
 - 19 . Lin, Y. The effective use of geometric intuition in the simplification of complex problems. *Teachers Today*. 2013;2(04):28-30.
 - 20 . Lu Y. Highlight the essence of the problem according to the conditions of association - the analysis and reflection of the analytic geometry questions in the national paper B of the 2021 college entrance examination in mathematics. *High School Mathematics Teaching*. 2021;42(04):19-22
 - 21 . Cai JS. Talking about the formation of problem solving ideas. *Secondary School Mathematics Research*. 2004;3(06):14-15.