***Original Research Article***

# Research on the Cognition of Pre-Service High School Mathematics Teachers for Implementing Mathematical Abstract Literacy


**ABSTRACT**

Currently, mathematical abstract literacy has attracted extensive attention from all walks of life. Many relevant problems about it have been studied except the cognition of implementing mathematical abstract literacy of pre-service high school mathematics teachers. To address this gap, this study investigates 51 pre-service high school mathematics teachers through open-ended interviews in order to learn about the cognitive situation of pre-service high school mathematics teachers for implementing mathematical abstract literacy. After analyzing, it can be found that about implementing mathematical abstract literacy: 1. Pre-service high school mathematics teachers do not have a comprehensive cognition; their cognition, both overall and in different dimensions, is not half of the number of recommendations for implementation made by previous authors, and the subjects they realize for implementation are limited to both teachers and students. 2. Pre-service high school mathematics teachers do not have a very reasonable cognition; there is a certain one-sidedness, and some of the ideas lack feasibility or effectiveness, which need to be further explored and considered. Therefore, it is suggested that: 1. Experts and teachers who are responsible for educating teachers should increase emphasis on mathematical abstraction literacy and provide pre-service teachers with more opportunities to practice teaching. 2. Pre-service teachers should take the initiative to study and research, focus on combining theory and practice, and continuously reflect, adjust and improve.



**Keywords: Teachers, Students, Pre-service teachers, Mathematical abstract**


## 1. INTRODUCTION

Mathematical abstraction refers to the literacy of mathematical research objects obtained through the abstraction of quantity relations and spatial forms. As the first of the six core literacies, mathematical abstract literacy is both the basic idea of mathematics and the support of people's mathematical thinking ability, reflecting the essential characteristics of mathematics and always running through the whole process of mathematics generation, development, and application [1], which is of great significance for cultivating high school students' innovative spirit and practical ability. Therefore, it is necessary to develop students' awareness of mathematical abstraction and mathematical abstract ability in high school mathematics education, and teachers

should pay attention to the implementation of mathematical abstract literacy. However, many scholars and teachers have extensively discussed and studied the present situation of high school students' cognition of mathematical abstract literacy, then they found that the current level of mathematical abstract literacy among high school students is not high [2], which showed that mathematical abstract literacy has not been well implemented in actual teaching. What are the reasons for this? How to develop students' mathematical abstract literacy? This is a question worthy of our study.

## 2. LITERATURE REVIEW

Currently, there have been a number of studies on this issue of implementing mathematical abstract literacy in high school mathematics teaching.

### 2.1 The Situation of High School Students' Mathematical Abstract Literacy Level

After analyzing the responses to a high school examination question, Chen found that high school students have shortcomings in the abstraction of the definition of new concepts and the establishment of the model of the equivariant series, which is due to the lack of students' mathematical core literacy, especially the lack of mathematical abstract literacy [3]. Zhang found through interviews with teachers that students have differences in their understanding of concepts or rules, lack of awareness of summarizing and generalizing mathematical ideas and methods, and lack of learning awareness [4]. A study conducted by Dang on the level of mathematical abstract literacy of high school students found that in terms of the level dimension, a higher majority of senior science students could reach Level 1 and Level 2, and very few students could reach Level 3; in terms of the cognitive dimension, students scored higher under the situation and question dimension, but lower under the knowledge and skills and the communication and reflection dimensions [5]. Xie used various methods such as a questionnaire survey to investigate the current situation of mathematical abstract literacy of senior students and found that students currently do not pay much attention to mathematical abstract literacy, and most of them think they cannot apply their mathematical knowledge [6]. Yang combined the assessment system of mathematical abstract literacy of high school students with students' performance also got similar conclusions, and she also found that high school students' mathematical abstract literacy was slightly lacking in the following aspects: first, the abstraction of mathematical concepts and rules; second, the application of mathematical propositions and models; third, the summary of mathematical methods and ideas; and fourth, the cognition of mathematical structures and systems [7].

### 2.2 The Factors Affecting Students' Mathematical Abstract Literacy

Qin and Yan found a strong correlation between students' mathematics achievement and mathematical abstract literacy, and students with good academic grades performed better overall in mathematical abstract literacy level; meanwhile, there was no

correlation between the level of mathematical abstract literacy and ethnicity or gender [8]. However, by testing the mathematical abstract literacy of high school students, Zheng et al. found that there was a significant difference in the level of mathematical abstract literacy between boys and girls, as well as an uneven level of mathematical abstract literacy among students from different geographical areas in urban areas, counties, and towns [9]. Ge conducted a study from the perspective of metacognition and found that there is a significant positive relationship between mathematical metacognition and mathematical abstract ability, and among the three main factors of mathematical metacognition, mathematical metacognitive monitoring has the greatest influence on mathematical abstract ability [10]. In addition, Zhang concluded that, in addition to students' own factors, the school's attention to teachers' professional development, the school learning environment, teachers' attention to the dynamics of mathematics curriculum reform, teachers' awareness of mathematical abstraction literacy, and the difficulty of developing mathematical abstract literacy are all important influencing factors for implementing mathematical abstract literacy [4].

### 2.3 Strategies for Cultivating High School Students' Mathematical Abstract Literacy

Most scholars have focused their attention on this aspect of strategies for developing mathematical abstract literacy of high school students, and their research can be broadly divided into two dimensions: "general teaching" and "classroom teaching".

In the dimension of "general teaching", Fang believes that in order to master mathematical abstraction and improve the level of concept teaching, the most practical thing is to focus on the formation of mathematical core concepts, take mathematical thinking method education as the core, select rich typical examples, and let students experience the complete process of mathematical abstraction [11]. Yang suggested that students' abstract ability can be developed in three areas: the teaching of mathematical concepts, the teaching of formulas and theorems, and the teaching of problem-solving [12]. Dong et al. emphasized that cultivating students' mathematical abstract literacy should be based firstly on deep learning to perceive mathematical abstract ideas, secondly on constructing mathematical models with the help of disciplinary integration, in addition to applying constructive theory to internalize mathematical structures [13]. Deng pointed out that it is necessary to grasp the essence of mathematical content and set up mathematical inquiry activities of textbooks to develop students' mathematical abstract literacy on this basis [14].

In the dimension of "classroom teaching", Liu suggested creating the basic conditions necessary for the formation of mathematical abstract ability, guiding students to develop methods of mathematical abstraction, abstracting the connections between knowledge from problem situations, and constructing knowledge models and systems, as well as strengthening students' practical and perceptual abilities of mathematical abstraction [15]. An proposed that teachers should pay more attention to cultivating

students' awareness of using mathematical knowledge to solve practical problems, reserve effective "blank space" in classroom teaching, and guide students in training and summarizing [16]. Based on the current information technology background, Shang suggested that information technology can be used to develop students' mathematical abstract ability, to show abstract mathematical concepts to students in a concrete way, so that students can intuitively feel the concrete form of mathematical problems in the process of active operation, and then stimulate students' initiative of independent learning [17]. Kang and Liu made corresponding suggestions for the teaching mode, and they pointed out that students' mathematical abstract literacy could be cultivated by adopting a flipped classroom teaching model, adopting a project-based learning model, and conducting mathematical experiments [18]. Wang et al. argued that in teaching, teachers should not only guide students through the process of observation, analysis, and abstraction but also conduct effective variation training so that students can better grasp the essence and laws of the problem. Also in the teaching process, attention should be paid to teaching students according to their abilities and giving them opportunities to develop at different levels [19]. We compiled the cultivation strategies proposed by 28 academic journals and 22 dissertations, and finally summarized 73 points of content. The details are shown in Table 1.

**Table 1. Coding of cultivation strategies proposed by previous authors**

| Primary Indicators | Secondary Indicators | Code | Content |
|---|---|---|---|
| General Teaching | E Teachers | E1 | Evaluate students in a comprehensive and integrated manner |
| | | E2 | Emphasis on developing students' awareness of using mathematical knowledge to solve actual problems |
| | | E3 | Strengthen students' mathematical reading comprehension skills |
| | | E4 | Use perceptual materials and visual materials |
| | | E5 | Make reasonable use of past knowledge and experience |
| | | E6 | Use the correct mathematical language to express |
| | | E7 | Promote students' mathematical abstract literacy through concept teaching |
| | | E8 | Promote students' mathematical abstract through theorem (law) teaching |
| | | E9 | Promote students' mathematical abstract literacy through exercises teaching |
| | | E10 | Focus on the process of teaching mathematics |
| | | E11 | Focus on the connection of mathematical knowledge |
| | | E12 | Follow the principle of gradual progress, starting from low, from easy to difficult |
| | | E13 | Guide students to experience the whole process of |

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| | | | mathematical abstraction |
| | | **E14** | Set up mathematical inquiry activities |
| | | **E15** | Build a chain of mathematical structure problems |
| | | **E16** | Select inquiry questions to trigger deep thinking |
| | | **E17** | Inspire students to abstract through appropriate means |
| | | **E18** | Teach on the basis of teacher-student interaction |
| | | **E19** | Use positive and negative examples and make appropriate comparisons |
| | | **E20** | Give students the opportunity to experience mathematical abstraction in person |
| | | **F1** | Possess some reading comprehension skills |
| | | **F2** | Develop the habit of abstract generalization |
| | | **F3** | Acquire the necessary methods of mathematical abstraction |
| | | **F4** | Deepen symbolic language understanding |
| | | **F5** | Perceive mathematical ideas |
| | | **F6** | Improve problem solving skills |
| | | **F7** | Build mathematical models |
| | | **F8** | Express relevant mathematical problems proficiently |
| | | **F9** | Learn to abstract the commonality of mathematical problems |
| | **F Students** | **F10** | Develop an awareness of mathematical abstract literacy |
| | | **F11** | Be able to abstract relevant mathematical knowledge to form a knowledge system |
| | | **F12** | Replace complex expressions with simple symbols |
| | | **F13** | Focus on the practical application of mathematical knowledge |
| | | **F14** | Have a proper understanding and appreciation of mathematical abstract literacy |
| | | **F15** | Realize the structure and system of mathematical knowledge |
| | | **F16** | Perceive the generality and general method of mathematics |
| | | **F17** | Form abstract thinking in depth from analogy |
| | | **G1** | Focus on effective “blank space” |
| | | **G2** | Make correct and adequate use of variation |
| **Classroom Teaching** | | **G3** | Create diverse contexts to provide students with a rich abstract background |
| | **G Teachers** | **G4** | Respect for individual student knowledge and teach according to the student's needs |
| | | **G5** | Focus on the role of assimilation in the meaning construction of mathematical abstraction |
| | | **G6** | Fully integrate information technology into teaching |

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| | **G7** | Use project-based learning to implement the overall design of the unit |
| | **G8** | Be expert in using different types of teaching language |
| | **G9** | Use teaching tools at the right time |
| | **G10** | Focus on different types of questions |
| | **G11** | Cite appropriate examples |
| | **G12** | Make the most of graphics |
| | **G13** | Actively initiate and maintain class discussions |
| | **G14** | Emphasis on step-by-step demonstrations |
| | **G15** | Give practice instructions at the right time |
| | **G16** | Guide students in multiple ways |
| | **G17** | Adopt diverse classroom teaching methods |
| | **G18** | Focus classroom instruction on four areas that reflect mathematical abstract and literacy |
| | **G19** | Focus on guiding students to grasp the connotation of mathematical concepts in concept teaching |
| | **G20** | Focus on guiding high school students to build a mathematical knowledge framework in review sessions |
| | **G21** | Emphasis on training students to summarize topic types |
| | **G22** | Capture the essence of mathematical content in the textbook |
| **H Students** | **H1** | Experience the whole process of abstraction from concrete examples reasonably |
| | **H2** | Be good at abstracting and summarizing the ideas and methods |
| | **H3** | Summarize by yourself |
| | **H4** | Leverage mathematical experiences to facilitate deep learning |
| | **H5** | Use the combination of numbers and shapes to turn abstraction into intuition |
| | **H6** | Understand the basic process of mathematical abstraction |
| | **H7** | Be able to abstract problem patterns |
| | **H8** | Think positively |
| | **H9** | Realize hands-on manipulation of real-world problems |
| | **H10** | Use flipped classrooms for independent inquiry learning |
| | **H11** | Focus on math learning exchange |
| | **H12** | Read the textbook vividly and deeply, and avoid discussing the topic on the subject |
| | **H13** | Leverage graphics and multimedia fully |

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| **H14** | Transform the learning methods of mathematical knowledge |

### 2.4 The assessment methods of mathematical abstract literacy of high school students

In order to study the current situation of mathematical abstract literacy of high school students, Zheng et al. constructed an evaluation framework of mathematical abstract literacy based on the division of three levels of mathematical abstract literacy in the Curriculum Standards and the mathematical ability cluster of PISA, which involved six indicators, including three content dimensions of context, cognition, and expression and three-level dimensions of reproduction, connection, and creation [9]. Yu proposed the theoretical concept of classifying mathematical literacy assessment into three forms of knowledge understanding, knowledge transfer, and knowledge innovation concerning Bloom's learning assessment model, PISA learning assessment model, and SOLO learning assessment model [20]. According to the Curriculum Standards and Yu's conception, at the practical level, Jiang simplified the assessment of mathematical abstract literacy into a three-stage progressive form in the shape of a pyramid, of which the bottom is the understanding of basic knowledge through mathematical abstraction, the middle is the formation of knowledge transfer through mathematical abstraction, and the top is designed to achieve knowledge innovation through mathematical abstraction [21].

From the above studies, we can see that many scholars have studied mathematical abstract literacy in terms of its situation, influencing factors, and other aspects, and have especially made a lot of suggestions on the cultivation strategies of mathematical abstract literacy. However, it can be also seen that few people have conducted research on teachers' mathematical abstract literacy, and the research on teachers' cognition of implementing mathematical abstract literacy is in a gap. Through previous studies, it can be found that teachers' cognition of implementing mathematical abstract literacy will affect students' mathematics learning and development to a large extent, so studying this issue is of great practical importance to the implementation of mathematical abstract literacy. Therefore, the purpose of this study is to find out the current pre-service high school mathematics teachers' cognition of implementing mathematical abstract literacy through investigation.

Thus, the main issues of this study are:

1. What areas do the current pre-service high school mathematics teachers focus on regarding the implementation of mathematical abstract literacy?
2. Is the cognition of implementing mathematical abstract literacy comprehensive among current pre-service high school mathematics teachers?
3. Is the cognition of implementing mathematical abstract literacy reasonable among current pre-service high school mathematics teachers?

## 3. RESEARCH METHODS

### 3.1 Participants

In order to faithfully reflect the pre-service high school mathematics teachers' cognition for implementing mathematical abstract literacy, this study used whole group sampling method and selected 51 masters students of education majoring in mathematics of grade 2021 from the School of Mathematics and Statistics of Shandong Normal University as the survey sample that includes 23 male students and 28 female students, all of whom hold high school mathematics teacher qualification certificates and have the intention to go to high school for employment in the future. Their average age of them is 23.3 years old.

### 3.2 Instrument

In this study, we conducted open-ended interviews with an interview outline containing a total of 10 questions, of which the two most prominent questions were, "How do you think mathematical abstract literacy can be implemented in high school?" and "How do you think mathematical abstract literacy can be implemented in mathematics classrooms?" The open-ended interview method is adopted because it is fast, convenient, flexible, not restricted by written language, and facilitates in-depth investigations to obtain the most direct information. The reason for choosing the above questions is to find out the pre-service high school mathematics teachers' real cognitive situation for implementing mathematical abstract literacy.

### 3.3 Data Collection

In order to ensure the reliability of the research, the open-ended interview method was used to interview 51 masters of education one by one individually, and the interview content was recorded during the whole process after the consent of the other party was sought.

### 3.4 Data Processing

Firstly, we converted the interview recording content into text form, removed um, ah, and other discourse markers, strictly followed the original words of the interview to sort out. The core ideas they expressed were further extracted and categorized into two primary indicators: "general teaching" and "classroom teaching", and according to the subject of implementation, each primary indicator was divided into two secondary indicators: "teachers" and "students", which were represented by A, B, C, and D respectively. Then we processed relevant data with the software of SPSS.

## 4. RESULTS

### 4.1 Cognitive Focus

The core ideas expressed by pre-service high school mathematics teachers were extracted and summarized into a total of 48 points.

For the dimension of “general teaching”, there are 23 items, 10 of which are implemented by teachers and 13 by students. In the aspect of teachers, pre-service teachers' cognition mainly focused on "have the awareness of developing students’ mathematical abstraction", accounting for 19.61% of the total number of them, and followed by this are "develop students to use mathematical symbols for expression and communication", "play a leading role", and "teach in integration with related knowledge", accounting for 11.76% separately. In the aspect of students, pre-service teachers' cognition mainly focused on "learn to express in mathematical language", accounting for 37.25% of the total number, and followed by this is "learn to abstract practical problems into mathematical problems", accounting for 17.65%. Thus, for the “general teaching” dimension, pre-service teachers were able to recognize the importance of students and teachers in implementing mathematical abstract literacy. They realized that in order to implement mathematical abstract literacy, students need to have a solid knowledge of mathematics, learn to abstract practical problems into mathematical problems, and be able to express them in mathematical language; at the same time, teachers need to have the awareness of developing students' mathematical abstraction and play a leading role.

For the dimension of “classroom teaching”, there are 25 items, 12 of which are implemented by teachers and 13 by students. In the aspect of teachers, pre-service teachers' cognition mainly focused on "combine with examples and relate to the real-life", accounting for 52.94% of the total number of them, and followed by this is "permeate core literacy in the whole process of teaching", accounting for 17.65% of the total number. In the aspect of students, pre-service teachers' cognition mainly focused on "participating in the whole process from concrete to abstract", accounting for 35.29% of the total number, and followed by this is "think independently", accounting for 13.73%. Thus, for the “classroom teaching” dimension, pre-service teachers were able to realize the importance of students and teachers in implementing abstract mathematical literacy. They were aware that teachers need to link core literacy to real-life situations in classroom teaching and learning, and they also emphasized students' participation in the whole process of abstraction. The details are shown in Table 2.

**Table 2. Statistics of cognitive focus**

| Primary Indicators | Secondary Indicators | Code | Content | Number | Percentage |
|---|---|---|---|---|---|
| **General** | **A** | **A1** | Have the awareness of developing | 10 | 19.61 |

| | | | | | |
|---|---|---|---|---|---|
| **Teaching** | **Teachers** | | students' mathematical abstraction | | |
| | | **A2** | Conduct demonstration | 2 | 3.92 |
| | | **A3** | Develop students to use mathematical symbols for expression and communication | 6 | 11.76 |
| | | **A4** | Play a leading role | 6 | 11.76 |
| | | **A5** | Teach in integration with related knowledge | 6 | 11.76 |
| | | **A6** | Develop students' initiative in mathematical abstraction | 4 | 7.84 |
| | | **A7** | Teach students according to their level of thinking development | 4 | 7.84 |
| | | **A8** | Construct the corresponding index system for evaluation | 1 | 1.96 |
| | | **A9** | Deal with the relationship between the four skills and core literacy | 1 | 1.96 |
| | | **A10** | Give students the freedom to work. | 1 | 1.96 |
| | | **B1** | Abstract with the help of physical models | 5 | 9.80 |
| | | **B2** | Use mathematical models to solve problems | 7 | 13.73 |
| | | **B3** | Have a solid knowledge base in mathematics | 8 | 15.69 |
| | | **B4** | Learn to express in mathematical language | 19 | 37.25 |
| | | **B5** | Look at the problems from the perspective of mathematics | 2 | 3.92 |
| | **B Students** | **B6** | Learn to abstract practical problems into mathematical problems | 9 | 17.65 |
| | | **B7** | Start mathematical abstraction from imitation | 1 | 1.96 |
| | | **B8** | Learn to abstract different categories of content | 8 | 15.69 |
| | | **B9** | Convert complex problems into simple ones | 2 | 3.92 |
| | | **B10** | Have a holistic perception of things | 3 | 5.88 |
| | | **B11** | Develop good thinking habits | 2 | 3.92 |
| | | **B12** | Acquire the correct method of mathematical abstraction | 2 | 3.92 |
| | | **B13** | Understand the connection between knowledge and reality | 2 | 3.92 |
| **Classroom Teaching** | **C Teachers** | **C1** | Provide students with problem situations | 8 | 15.69 |
| | | **C2** | Combine with examples and relate to | 27 | 52.94 |

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| | | the real life | | |
| | **C3** | Teaching according to students' level | 1 | 1.96 |
| | **C4** | Stimulate students' interest in learning | 3 | 5.88 |
| | **C5** | Cite reasonable examples | 6 | 11.76 |
| | **C6** | Permeate core literacy in the whole process of teaching | 9 | 17.65 |
| | **C7** | Assign homework with the reality of life | 4 | 7.84 |
| | **C8** | Choose the appropriate teaching method according to the teaching content | 7 | 13.73 |
| | **C9** | Improve classroom teaching efficiency and put students at the center | 1 | 1.96 |
| | **C10** | Teach with the help of modern multimedia technology | 2 | 3.92 |
| | **C11** | Use teaching tools appropriately | 2 | 3.92 |
| | **C12** | Grasp the textbook accurately | 3 | 5.88 |
| | **D1** | Communicate more in class | 3 | 5.88 |
| | **D2** | Participate in the whole process from concrete to abstract | 18 | 35.29 |
| | **D3** | Think independently | 7 | 13.73 |
| | **D4** | Leverage visual forms and intuitive thinking | 4 | 7.84 |
| | **D5** | Clarify the meaning and significance of mathematical abstraction | 4 | 7.84 |
| **D** | **D6** | Guess boldly | 1 | 1.96 |
| **Students** | **D7** | Ask more questions | 1 | 1.96 |
| | **D8** | Reinforce through exercises | 5 | 9.80 |
| | **D9** | Find and grasp the pattern of things | 4 | 7.84 |
| | **D10** | Summarize independently | 4 | 7.84 |
| | **D11** | Practice in person | 5 | 9.80 |
| | **D12** | Abstract the mathematical ideas involved in things | 2 | 3.92 |
| | **D13** | Be able to make abstract concepts and theorems concrete | 1 | 1.96 |

### 4.2 Cognitive Comprehensiveness

Matching preservice teachers' cognition of implementing mathematical abstract literacy with the content of previous studies reveals that current pre-service teachers realize 31 points of the content of previous studies, accounting for 42.47% of the total.

For the dimension of "general teaching", 37 points were proposed, of which 14 points were recognized by the pre-service teachers, accounting for 37.84%. Among them, pre-service teachers recognized 6 points on the teacher dimension, accounting for 30.00% of the total number of points in this section, and 8 points on the student dimension, accounting for 47.06% of the total number of points in this section.

For the dimension of "classroom teaching", 36 points were proposed, of which 17 points were recognized by the pre-service teachers, accounting for 47.22%. Among them, pre-service teachers recognized 8 points on the teacher dimension, accounting for 36.36% of the total number of points in this section; and 9 points on the student dimension, accounting for 64.29% of the total number of points in this section, and pre-service teachers only recognized more than half of the points in this section, which is relatively comprehensive.

It can be seen that the current pre-service teachers' cognition of implementing mathematical abstract literacy is not comprehensive, whether from the overall perspective or two different dimensions of "general teaching" and "classroom teaching", and it is not half of the number of implementation suggestions put forward by previous authors. In addition, the subjects they recognize for implementation are limited to teachers and students. The details are shown in Table 3.

**Table 3. Statistics of cognitive comprehensiveness**

| Primary Indicators | Secondary Indicators | Recognizing Points | Total Points | Percentage | Recognizing Points | Total Points | Percentage |
|---|---|---|---|---|---|---|---|
| **General Teaching** | **Teachers** | 6 | 20 | 30.00 | 14 | 37 | 37.84 |
| | **Students** | 8 | 17 | 47.06 | | | |
| **Classroom Teaching** | **Teachers** | 8 | 22 | 36.36 | 17 | 36 | 47.22 |
| | **Students** | 9 | 14 | 64.29 | | | |
| **Total Points** | | | | | 31 | 73 | 42.47 |

### 4.3 Cognitive Reasonableness

The pre-service teachers' cognition of implementing mathematical abstract literacy was summarized as 48 points, of which 31 points were similar to the suggestions made by the previous authors, accounting for 64.58% of the total. For the dimension of "general teaching", the pre-service teachers' statements were summarized as 23 points, of which 14 points were similar to the previous suggestions, accounting for 60.87%; for the dimension of "classroom teaching", the pre-service teachers' statements were summarized as 25 points, of which 17 points were similar to the previous suggestions, accounting for 68.00%.

In terms of specific contents, the contents of "participate in the whole process from concrete to abstract" and "learn to express in mathematical language", which were

recognized by the pre-service teachers, were more consistent with the implementation suggestions made by the previous authors; while for the contents of "combine with examples and relate to the real-life" and "permeate core literacy in the whole process of teaching" are not reflected in the implementation suggestions of the previous authors, although the number of teachers who recognize them is relatively large.

It can be seen that the pre-service high school mathematics teachers' cognition of implementing mathematical abstract literacy was not very reasonable, although most of the contents they proposed could match the implementation suggestions made by the previous authors, the number of people who recognized the corresponding contents was generally small; at the same time, some of the implementation measures that pre-service teachers could concentrate on were unreasonable compared with the previous recommendations and still existed in a certain one-sided way. The details are shown in Table 4 and Table 5.

**Table 4. Statistics of cognitive reasonableness**

| Primary Indicators | Secondary Indicators | Recognizing Points | Total Points | Percentage | Recognizing Points | Total Points | Percentage |
|---|---|---|---|---|---|---|---|
| **General Teaching** | **Teachers** | 6 | 10 | 60.00 | 14 | 23 | 60.87 |
| | **Students** | 8 | 13 | 61.54 | | | |
| **Classroom Teaching** | **Teachers** | 8 | 12 | 66.67 | 17 | 25 | 68.00 |
| | **Students** | 9 | 13 | 69.23 | | | |
| **Total Points** | | | | | 31 | 48 | 64.58 |

**Table 5. Statistics of matching situation**

| Primary Indicators | Secondary Indicators | Code | Content | Number | Percentage |
|---|---|---|---|---|---|
| **General Teaching** | **E Teachers** | **E1** | Evaluate students in a comprehensive and integrated manner | 1 | 1.96 |
| | | **E2** | Emphasis on developing students' awareness of using mathematical knowledge to solve actual problems | 10 | 19.61 |
| | | **E3** | Strengthen students' mathematical reading comprehension skills | 0 | 0.00 |
| | | **E4** | Use perceptual materials and visual materials | 0 | 0.00 |
| | | **E5** | Make reasonable use of past knowledge and experience | 0 | 0.00 |
| | | **E6** | Use the correct mathematical language to express | 6 | 11.76 |
| | | **E7** | Promote students' mathematical abstract literacy through concept teaching | 0 | 0.00 |

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| | **E8** | Promote students' mathematical abstract through theorem (law) teaching | 0 | 0.00 |
| | **E9** | Promote students' mathematical abstract literacy through exercises teaching | 0 | 0.00 |
| | **E10** | Focus on the process of teaching mathematics | 0 | 0.00 |
| | **E11** | Focus on the connection of mathematical knowledge | 0 | 0.00 |
| | **E12** | Follow the principle of gradual progress, starting from low, from easy to difficult | 4 | 7.84 |
| | **E13** | Guide students to experience the whole process of mathematical abstraction | 6 | 11.76 |
| | **E14** | Set up mathematical inquiry activities | 0 | 0.00 |
| | **E15** | Build a chain of mathematical structure problems | 0 | 0.00 |
| | **E16** | Select inquiry questions to trigger deep thinking | 0 | 0.00 |
| | **E17** | Inspire students to abstract through appropriate means | 0 | 0.00 |
| | **E18** | Teach on the basis of teacher-student interaction | 0 | 0.00 |
| | **E19** | Use positive and negative examples and make appropriate comparisons | 0 | 0.00 |
| | **E20** | Give students the opportunity to experience mathematical abstraction in person | 1 | 1.96 |
| | **F1** | Possess some reading comprehension skills | 0 | 0.00 |
| | **F2** | Develop the habit of abstract generalization | 2 | 3.92 |
| | **F3** | Acquire the necessary methods of mathematical abstraction | 2 | 3.92 |
| | **F4** | Deepen symbolic language understanding | 0 | 0.00 |
| **F** | **F5** | Perceive mathematical ideas | 0 | 0.00 |
| **Students** | **F6** | Improve problem solving skills | 0 | 0.00 |
| | **F7** | Build mathematical models | 7 | 13.73 |
| | **F8** | Express relevant mathematical problems proficiently | 19 | 37.25 |
| | **F9** | Learn to abstract the commonality of mathematical problems | 9 | 17.65 |
| | **F10** | Develop an awareness of mathematical abstract literacy | 0 | 0.00 |
| | **F11** | Be able to abstract relevant | 8 | 15.69 |

| | | | | | |
|---|---|---|---|---|---|
| | | | mathematical knowledge to form a knowledge system | | |
| | | **F12** | Replace complex expressions with simple symbols | 2 | 3.92 |
| | | **F13** | Focus on the practical application of mathematical knowledge | 2 | 3.92 |
| | | **F14** | Have a proper understanding and appreciation of mathematical abstract literacy | 0 | 0.00 |
| | | **F15** | Realize the structure and system of mathematical knowledge | 0 | 0.00 |
| | | **F16** | Perceive the generality and general method of mathematics | 0 | 0.00 |
| | | **F17** | Form abstract thinking in depth from analogy | 0 | 0.00 |
| | | **G1** | Focus on effective “blank space” | 0 | 0.00 |
| | | **G2** | Make correct and adequate use of variation | 0 | 0.00 |
| | | **G3** | Create diverse contexts to provide students with a rich abstract background | 8 | 15.69 |
| | | **G4** | Respect for individual student knowledge and teach according to the students’ needs | 2 | 3.92 |
| | | **G5** | Focus on the role of assimilation in the meaning construction of mathematical abstraction | 0 | 0.00 |
| | | **G6** | Fully integrate information technology into teaching | 2 | 3.92 |
| **Classroom Teaching** | **G Teachers** | **G7** | Use project-based learning to implement the overall design of the unit | 0 | 0.00 |
| | | **G8** | Be expert in using different types of teaching language | 0 | 0.00 |
| | | **G9** | Use teaching tools at the right time | 2 | 3.92 |
| | | **G10** | Focus on different types of questions | 0 | 0.00 |
| | | **G11** | Cite appropriate examples | 6 | 11.76 |
| | | **G12** | Make the most of graphics | 0 | 0.00 |
| | | **G13** | Actively initiate and maintain class discussions | 0 | 0.00 |
| | | **G14** | Emphasis on step-by-step demonstrations | 0 | 0.00 |
| | | **G15** | Give practice instructions at the right time | 0 | 0.00 |
| | | **G16** | Guide students in multiple ways | 0 | 0.00 |
| | | **G17** | Adopt diverse classroom teaching | 7 | 13.73 |

| | | | | |
|---|---|---|---|---|
| | | methods | | |
| | **G18** | Focus classroom instruction on four areas that reflect mathematical abstract and literacy | 0 | 0.00 |
| | **G19** | Focus on guiding students to grasp the connotation of mathematical concepts in concept teaching | 0 | 0.00 |
| | **G20** | Focus on guiding high school students to build a mathematical knowledge framework in review sessions | 0 | 0.00 |
| | **G21** | Emphasis on training students to summarize topic types | 0 | 0.00 |
| | **G22** | Capture the essence of mathematical content in the textbook | 3 | 5.88 |
| **H Students** | **H1** | Experience the whole process of abstraction from concrete examples reasonably | 18 | 35.29 |
| | **H2** | Be good at abstracting and summarizing the ideas and methods | 2 | 3.92 |
| | **H3** | Summarize by yourself | 4 | 7.84 |
| | **H4** | Leverage mathematical experiences to facilitate deep learning | 0 | 0.00 |
| | **H5** | Use the combination of numbers and shapes to turn abstraction into intuition | 1 | 1.96 |
| | **H6** | Understand the basic process of mathematical abstraction | 4 | 7.84 |
| | **H7** | Be able to abstract problem patterns | 4 | 7.84 |
| | **H8** | Think positively | 7 | 13.73 |
| | **H9** | Realize hands-on manipulation of real-world problems | 5 | 9.80 |
| | **H10** | Use flipped classrooms for independent inquiry learning | 0 | 0.00 |
| | **H11** | Focus on math learning exchange | 3 | 5.88 |
| | **H12** | Read the textbook vividly and deeply, and avoid discussing the topic on the subject | 0 | 0.00 |
| | **H13** | Leverage graphics and multimedia fully | 0 | 0.00 |
| | **H14** | Transform the learning methods of mathematical knowledge | 0 | 0.00 |



## 5. DISCUSSION

### 5.1 Cognitive Focus

From the above data analysis, it can be seen that for implementing mathematical abstract literacy, current pre-service high school mathematics teachers have recognized two implementation subjects, teachers and students, from different dimensions. For the "general teaching" dimension, pre-service teachers realized that in order to implement mathematical abstract literacy, students need to have a solid knowledge of mathematics, learn to abstract practical problems into mathematical problems, and be able to express them in mathematical language; at the same time, teachers need to have the awareness of developing students' mathematical abstraction and play a leading role. For the "classroom teaching" dimension, they were aware that teachers need to link the core literacy to real-life situations in classroom teaching and learning, and they also emphasized students' participation in the whole process of abstraction. From this we can see that current pre-service high school mathematics teachers can realize the importance of teachers and students for implementing mathematical abstraction literacy; students should experience the whole process of mathematical abstraction and master the methods of mathematical abstraction; teachers should play a leading role in linking the teaching process with real-life.

### 5.2 Cognitive Comprehensiveness

According to the above data analysis, it can be seen that the current pre-service teachers' cognition of implementing mathematical abstract literacy is not comprehensive, whether from the overall perspective or from two different dimensions of "general teaching" and "classroom teaching", and it is not half of the number of implementation suggestions put forward by previous authors. In addition, the subjects they recognize for implementation are limited to teachers and students. From this, we can see that current pre-service high school mathematics teachers do not have a comprehensive cognition of implementing mathematical abstract literacy. This conclusion coincides with the findings of previous studies. Teng has conducted a study on this issue and found that teachers are not sufficiently aware of abstract thinking development methods, and most of them knew about disciplinary literacy but did not really understand its concept [22].

### 5.3 Cognitive Reasonableness

According to the above data analysis, it can be seen that although most of the contents they proposed could match the implementation suggestions made by the previous authors, the number of people who recognized the corresponding contents was generally small; at the same time, some of the implementation measures that pre-service teachers could concentrate on were unreasonable compared with the previous recommendations and still existed in a certain one-sided way. From this, we can see that pre-service high school mathematics teachers do not have a very reasonable cognition for implementing mathematical abstraction literacy. Similar findings were found in previous studies. In the study, Zhang found that high school mathematics teachers were less concerned about the dynamics of the new mathematics curriculum

reform, resulting in a lack of in-depth understanding of mathematical abstract literacy and thus no feasible and effective strategies for developing students' mathematical abstract literacy.

## 6. CONCLUSION AND RECOMMENDATIONS

It has been shown that teachers' cognition for implementing mathematical abstract literacy directly affects the implementation effect of mathematical abstract literacy. The current implementation effect of mathematical abstract literacy in the high school mathematics classroom is not good, so is it that teachers' cognition for implementing mathematical abstract literacy is not comprehensive? Is it not reasonable? In this study, we investigated pre-service high school mathematics teachers' cognitive situation of implementing mathematical abstract literacy. Through investigation and analysis, it can be seen that pre-service high school mathematics teachers do not have a comprehensive cognition for implementing mathematical abstract literacy; their cognition, both overall and in different dimensions, is not half of the number of recommendations for implementation made by previous authors, and the subjects they realize for implementation are limited to both teachers and students. In addition, pre-service high school mathematics teachers do not have a very reasonable cognition; there is a certain one-sidedness, and some of the ideas lack feasibility or effectiveness, which need to be further explored and considered.

According to the above conclusions, it is recommended: 1. The teachers and experts who educate pre-service high school mathematics teachers should pay more attention to mathematical abstract literacy, strengthen pre-service teachers' training in this area, and provide them with more opportunities to practice teaching; 2. Pre-service high school mathematics teachers themselves should take the initiative to study and research, seize the opportunities of teaching exercises, focus on combining theory and practice, and constantly reflect, adjust and improve.

The subjects of this study were 51 master students of education students in the same grade at the same institution, so the limitation of this research is that the sample size was small and did not involve other types of pre-service high school mathematics teacher groups. Therefore, in order to find more detailed and comprehensive results, it is necessary to expand the scope of the research sample and adopt a variety of research methods to conduct a more in-depth study.

## FOUNDING

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