Review Article

Making Mathematics More Friendly; A Thoughtful Review of Problem-Solving Concept as a Tool

Abstract

Successful learning of mathematical concepts and skills is a function of the approaches and strategies that teachers use in their teaching. How mathematics is taught, to a large extent, is influenced by the perceptions that teachers have about the subject and of what they believe good teaching to be. Teachers have to give up the notion that the topics at hand cannot be learned by the students unless the teacher covers them and adopts a proper teaching method that encourages active learning and better understanding. Adopting problem-solving as an instructional technique and creating a healthy learning atmosphere creates an active learning environment that makes learning more enjoyable for both students and teachers, as well as promoting critical thinking among students.

Keywords: Problem-solving, mathematics learning, constructivist learning, teaching and learning

Introduction

Fundamentally, mathematics is a human activity that cuts across everything all over the world. This human activity is demonstrated right from childhood. Students possess a natural curiosity and interest in mathematics and come to school with an understanding of mathematical concepts and problem-solving strategies that they have discovered through exploration of the world around them. Identifying these fundamental activities, mathematics educators are to provide an experience that will continue to foster students' understanding and appreciation of mathematics to improve on their performance. This can be done by providing mathematics activities in which students are encouraged to explore and make sense of mathematical patterns and relationships that will help them develop mathematical knowledge to solve problems and explore new ideas in the classroom and the technological world.

The need to apply mathematics to solve everyday life problems calls for the introduction of problem-solving as a component of the mathematics curriculum. Problem-solving mathematics curriculum encourages the use of mathematics in a novel or complex situations (Atteh *et al.*, 2014). It also emphasizes the teaching of mathematics content through processes that students encounter in real-world situations. A problem-solving mathematics curriculum includes using and applying mathematics in practical tasks, in real-life problems, and within mathematics itself. It covers a wide range of situations from routine mathematical problems to problems in unfamiliar contexts and open-ended investigations that make use of the relevant mathematics and thinking processes (Singapore, Ministry of Education, 2000).

In Ghana, the Education Reform Review Committee (MOE, 2007) recommended a problem-solving curriculum for pre-university education and this curriculum took effect in 2007. This current pre-university problem-solving mathematics syllabus requires the use of mathematics in solving everyday problems. Specifically, it recommended the application of appropriate mathematical problem-solving strategies in the teaching and learning of mathematics (MOE, 2007). Helping students to acquire the techniques of problem-solving, therefore, becomes one of the main objectives of teaching mathematics in pre-university institutions in Ghana.

This makes problem-solving a primary goal of every mathematics instruction and an integral part of all mathematical activity. A Problem-solving curriculum is not peculiar to mathematics education in Ghana alone. In America, the National Council of Teachers of Mathematics (1989; 2000) had earlier endorsed the inclusion of problem-solving in school mathematics. The NCTM (2000) specifically stressed that all "instructional programs should enable all students to build new mathematical knowledge through problem-solving; solve problems that arise in mathematics and other contexts; apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem solving"(p. 52). Problem-solving is very important in the study of mathematics. It offers opportunities for students to engage in meaningful mathematical discourse, including analyzing various representations of and justifications for their solutions.

However, teaching and learning mathematics have attracted considerable attention among scholars and educators due to the potential effect on students' academic performance. The recommended instructional processes in the mathematics classroom today includes problem-solving which allows learners to participate in practical mathematical debate, such as evaluating and justifying different representations of their solutions and solution processes.

The paper sought to explore scholarly articles and other research studies under designed themes of the researcher in teaching and learning through problem-solving in mathematic classroom.

Techniques for Encouraging students interest

The teaching and learning of mathematics have become a major concern in almost all countries around the world and the assessments of Ghanaian students have shown students poor academic achievements in mathematics (Anamuah-Mensah & Mereku, 2005). In the view of Cai and Moyer (2008), some of the mathematics classrooms used by the teachers often do not prepare students for algebraic thinking and conceptual understanding. Having a conceptual understanding of mathematics enables students to meaningfully operate upon rules and procedures, and provides a strong basis for effective problem-solving. The National Research Council (1989) proposed that effective teachers are those who can stimulate students to learn mathematics. National Research Council (1989) further concluded that a greater proportion of students find mathematics difficult and they are unable to apply what they have learned to their real-life situations because of the way the subject is taught. Nabie (2001) pointed out that the traditional method of "define and explain" technique in teaching mathematical concepts does not enable learners to apply what they learn to new situations or solve mathematical problems. He then encouraged teachers to adopt pragmatic teaching methods to facilitate mathematical understanding and to enable learners to overcome learning difficulties. To overcome various challenges and difficulties of learners, a variety of teaching and learning strategies has been introduced for use in mathematics classrooms moving away from the teacher-centered approach to a more pupils-centered approach (Fujii, 2008).

Effective classroom management techniques are the techniques we should plan to use every day with every mathematics lesson, they should be our priorities as teachers because we need to spend most of our time putting first things first to develop classroom management that works. As mathematics teachers, there are good habits we need to start to develop, they are the way we conduct business with our students, helping them to learn in a very specific context of the classroom.

One of the most important dimensions of effective classroom management is the classroom climate we create for, and with, our students. Classroom climate shows the emotional or psychological climate that exists in the classroom, and as teachers, we play a major role in

establishing a climate that promotes learning, or, if we are not careful, a climate that goes against learning. We should value core principles such as respect, honesty, integrity, and certainly, if we fail to abide by them, we have no hope of achieving a classroom climate that will consistently promote effective learning. And classroom environment that supports thinking must be one in which students feel safe enough to share their formative thoughts (Mckeown and Beck, 1999).

A good classroom climate should show:

- ✓ the conducive atmosphere and positive relationship, where both teachers and students, agree to help and support each other in the classroom,
- ✓ clear and explicit routines and procedures for students understanding and not just what they are doing, but to know how and why those routines and procedures are being followed,
- ✓ teachers encouraging students to grow as learners and as human beings, and all students feel safe to share their concerns.

Another technique that mathematics teachers need to master for effective classroom management is our activities. Our basic priority as educators should be creating activities that give students something to do that will result in learning (Vergara, Fernández & Lorenzo, 2019). Everything that happens in the classroom should be driven by learning, and learning cannot happen unless students actively engage with it. According to Ball (2003), children acquire knowledge about things consciously or unconsciously by exploring their environment through play and interacting with materials in their miniature world. He further stated that learning by exploring the environment enables children to learn better even as building their mathematical meanings and an inquiry community. This suggests that we need to choose carefully the activities we offer students. There should be a direct link between the quality of learning experience we provide for our students and the level of engagement on the part of students.

Healthy classroom activities should;

- ✓ have a clear and explicit structure and purpose,
- ✓ have clear and specific learning outcomes,
- ✓ be suitable to the age of learners and their learning development stage as well,
- ✓ be appropriate for use in the classroom, and safe for both students and teachers,
- ✓ be engaging, enjoyable, interesting, and appeal to different learning styles.

Teaching should be geared towards the premise of creating a classroom environment where students feel safe yet challenged, relaxed yet alert, are motivated to take risks without fear of

ridicule, productive and successful. This will transform the classroom into a problem-solving environment in which developing an approach to thinking about mathematical issues would be valued more highly than memorizing algorithms and using them to get their answers right.

Teachers Preparation before Lesson

To be able to teach all students according to today's expectations, teachers need to understand subject matter deeply and flexibly so that they can help students construct effective cognitive maps, connect one concept to another and resolve misunderstandings (Shulman, 1987). This suggests that teachers need to see how things relate to daily life and different fields (Atteh and Andam, 2019). This kind of understanding provides the basis for Pedagogical Content Knowledge which helps teachers to make ideas available to students. It is believed that when students are participating in an interactive learning atmosphere that the teacher is aware of their thoughts, concepts, and experiences to help correct them (Atteh and Andam, 2019).

This means that teachers need to have a detailed understanding of the mathematical content at hand, the pedagogical concepts of problem-solving and instructional materials that guide the nature and direction of problem-solving, to teach mathematics through problem-solving. A study by Shulman (2000) posited that teaching is much more than simply communicating concepts and ideas to learners, but it includes taking out the accumulated ideas and experiences that students come to the classroom and working on these ideas and experiences together with students in a way that refines, reorganizes, co-constructs and restores these ideas and experiences in a meaningful and understandable way. He further reiterated that teaching involves making an individual’s internal and external capabilities, which can only be done if teachers engage students in classroom discourse.

Research has proposed three content knowledge domains for teaching which comprises subject matter content knowledge (SMCK), pedagogical content knowledge (PCK), and curricular knowledge (CK) (Shulman, 1986; Atteh & Andam, 2019). Subject Matter Content Knowledge is the sum and structure of information which is naturally in the teacher's mind (Shulman, 1986). Shulman claims that the knowledge of the subject matter of the teachers' curriculum should not be limited to knowledge of facts and procedures; but also an understanding of the subject matter's conceptual and syntactic structures. Grasping the SMCK is how the discipline's basic concepts and principles are organized for incorporating its facts. Therefore, teachers will only be able to use suitable resources to teach mathematics

well when they understand the network of fundamental concepts and principles of problem-solving in a holistic way (Atteh and Andam, 2019).

Evidence of pedagogical content knowledge, according to Shulman (1986), is the knowledge of a teacher on how to successfully integrate pedagogy with content in the classroom. It involves knowing what approaches match the content, knowing how to organize content elements for better teaching. Additionally, PCK involves knowledge of teaching methods that integrate effective conceptual representations to tackle learner challenges and assumptions and facilitate clear understanding; and awareness of what the students bring to the learning situation; information that could be either facilitative or dysfunctional for the specific learning task at hand (Atteh and Andam, 2019). Pedagogical content knowledge, according to Harris, Mishra, and Koehler (2009), requires basic information about how students learn, teaching strategies, evaluation techniques, and awareness of various learning theories.

The mathematics curriculum is represented by a wide range of programs at a specified grade level designed to teach mathematics topics (Shulman, 1986). This includes a wide range of available educational resources concerning the subject matter to be treated and the collection of features that govern the use of particular learning materials in different circumstances. The curriculum is regarded as a collective whole containing learners, students, teaching and learning methodologies, expected and unexpected interactions, results, and potential outcomes within the learning institution (Atteh and Andam, 2019). This confirms that curriculum is the organized and directed learning activities and desired results, designed under the auspices of the school through the systematic reconstruction of knowledge and experience, for the continuous and wilful growth of personal-social competence of the learner (Mereku and Agbemaka, 2009). This suggests that Curricular Knowledge is needed for teachers to answer the question of where? when? and how? to determine a topic to be delivered to students at all levels.

Actions encouraging problem-solving

It is believed that different teaching approaches in classrooms influence the outcomes for students in different ways (Granström, 2006). Most math teachers are concerned about enhancing the teaching–learning process, and as a result, they seek out strategies and implement creative methodologies that encourage students to actively participate in class and reinforce previously learned material in order to obtain better academic results (Verschaffel, Depaepe &de Corte, 2015). In classroom settings where students are allowed and encouraged

to cooperate with classmates, teachers give the students more opportunities to understand and succeed. Good teaching involves communication and building relationships with students (Oppendekker& Van Damme, 2006).

Boaler (2002) in his study reported that practices such as working through textbook exercises or discussing and using mathematical ideas were important processes for the development of flexible mathematical knowledge. For him, students who had worked in textbooks performed well in similar textbook situations but these students found it difficult to use mathematics in open, applied, or discussion-based situations. However, the students who had learned mathematics through group-based projects were more able to apply their knowledge in a range of situations. Reynolds and Muijs (1999) are of the view that effective teaching is signified by a high number of opportunities to learn— where the opportunity to learn consists of factors such as length of school day and year, and the number of hours for mathematics classes. It also includes the quality of classroom management, especially time-on-task. The teacher's achievement is improved when the teacher creates classrooms that include:

- ✓ substantial emphasis on academic instruction and students' engagement in academic tasks (Cooney, 1994),
- ✓ whole-class instruction (Reynolds and Muijs, 1999),
- ✓ effective question-answer and individual practices (Borich, 1996),
- ✓ minimal disruptive behaviour (Bunch-Crump & Lo, 2017).
- ✓ high teacher expectations (Clarke, 1997),
- ✓ substantial feedback to students (Borich, 1996).

In successful teaching, teachers are actively asking a lot of questions, and students are involved in a class discussion. And besides active discussion, students are kept involved in the lesson and the teacher has a chance to continually monitor students' understanding of the concept being taught.

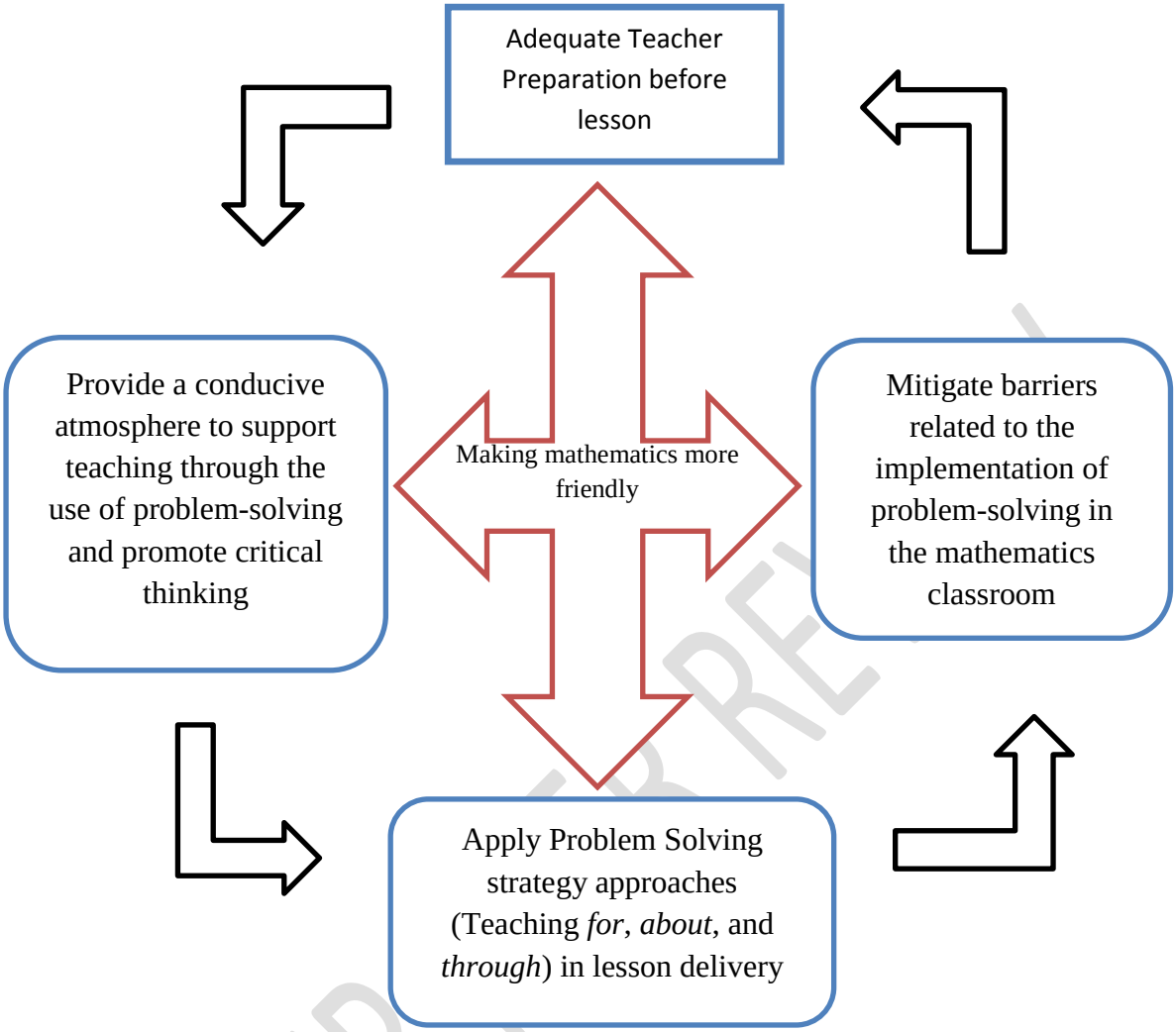

Figure 1. A conceptual framework for making math friendly through problem-solving

In the view of Foong (2002), three problem-solving approaches to teaching mathematics have been identified in problem-solving literature. These are: teaching *for* problem-solving, teaching *about* problem-solving, and teaching *through* problem-solving.

Teaching for problem-solving

Foong (2002) describes teaching *for* problem-solving as teaching that involves students learning mathematical content so that they can apply it to solve problems related to that content area. Here, teachers provide students with the necessary skills and knowledge that are needed to solve mathematical problems. The pivot to mathematical teaching and learning is linking mathematical concepts to real-life experiences which is an integral part of the problem-solving process (Kilpatrick, Swafford, & Findell, 2001). Additionally, Foong (2002)

further stated that in teaching for problem-solving, the emphasis is placed on learning mathematics for the main purpose of applying it to solve problems in a wide range of situations after learning a particular topic. That is, to solve problems students must make meaningful connections between what they know and the new problem (Hiebert and Carpenter, 1992). In the view of Anderson (2000), problems are usually related to the mathematical content just studied and students are provided with a variety of applications in which that mathematics may be used.

Teaching about problem-solving

According to Polya (1985) teaching *about* problem-solving includes guidance about the problem-solving process and instruction about a variety of problem-solving strategies and it often includes the recommendations of Polya‘s problem-solving strategies. Anderson (2000) stresses that in teaching about problem-solving students learn to use a variety of problem-solving strategies or heuristics, such as make a list, draw a diagram, act it out, and solve a similar problem and guess and check. The study of Yelon (1996) outlines ten powerful instructional principles that teachers can adopt in their classroom instructions when teaching about problem-solving. These are:

1. Meaningfulness: motivate students by helping them to connect the topic to be learned to their past, present, and future.
2. Prerequisites: assess students’ level of knowledge and skill and adjust instruction carefully so students are ready to learn the material at the level.
3. Open communication: Be sure students find out what they need to know so they can focus on what to learn.
4. Organize Essential Ideas: Help students focus on and structure the most important ideas, to be able to learn and recall those ideas.
5. Learning Aids: Help students use devices to learn quickly and easily.
6. Novelty: Vary the instructional stimuli to keep students’ attention.
7. Modeling: Show students how to recall, think, act and solve a problem so that they are ready to practice.
8. Active Practice: Provide practice in recalling, thinking, performing, and solving a problem so that students apply and perfect their learning.

9. Pleasant conditions and Consequences: Make learning pleasant, so that students associate comfort with what is learned; and make learning satisfying, so that students keep learning and using what is learned.
10. Consistency: make objectives, tests, practice, content, and explanation consistent, so that students will learn what they need and will use what they have learned outside the instructional settings.

In Foong, (2002) teaching about problem-solving involves using non-routine problems to teach thinking skills and problem-solving heuristics. And the emphasis is on using heuristic strategies to approach and solve unfamiliar problems that are usually not domain-specific to any topics in the syllabus.

Teaching through problem-solving

According to Anderson (2000) teaching through problem-solving starts with a problem, teachers pose problems to challenge students' knowledge thus providing a need for the students to organize their understanding to resolve the problem. And this approach makes problem-solving a means rather than an end. Teaching through problem-solving involves creating an environment where students can discuss their views on a problem and explain their methods of inquiry and generalizations to their classmates (Chapman, 2008). According to Van de Walle (2004), teaching through problem-solving requires students to read a problem carefully, analyze it for whatever information it has, and then examine their mathematical knowledge to see if they can come up with a strategy that will help them find a solution. Norton, McRobbie, & Cooper (2002) also see the teaching of mathematics through problem-solving as an approach in which teachers see themselves as guides, listeners, and observers rather than authorities and dispensers of knowledge and information.

The National Council of Teachers of Mathematics (NCTM, 1989) sees teaching *through* problem-solving as a vehicle for students to construct, evaluate and refine their theories about mathematics and the theories of others. Also, in the view of Resnick (1987) teaching through a problem-solving approach contributes to the practical use of mathematics by helping people to develop the facility to be adaptable when, for instance, technology breaks down. This author further expresses the belief that 'school should focus its efforts on preparing people to be good adaptive learners' so that they can perform effectively when situations are unpredictable and task demands change (p.18).' Thus it can also help people to transfer into new work environments at this time when most are likely to be faced with several career

changes during a working lifetime (NCTM, 1989). Teaching through problem-solving is a means of developing mathematical thinking as a tool for daily living, and that problem-solving ability lies 'at the heart of mathematics' (Cockcroft, 1982). Also because it is how mathematics can be applied to a variety of unfamiliar situations.

Resnick (1987) describes the discrepancies which exist between the algorithmic approaches taught in schools and the 'invented' (problem-solving) strategies which most people use in the workforce to solve practical problems which do not always fit neatly into a taught algorithm. And she says, most people have developed 'rules of thumb' for calculating, for example, quantities, discounts, or the amount of change they should give, and these rarely involve standard algorithms. Teaching or training in problem-solving techniques equips people more readily with the ability to adapt to such situations.

There have been many studies on problem-solving techniques proposed over the years (Verschaffel *et al.*, 1999; Cai, 2003; Atteh *et al.*, 2017). Verschaffel *et al.*, (1999) planned, introduced, and assessed a learning environment that emphasized the acquisition by students of an overall metacognitive strategy to solve mathematical problems through five-stage. These include building a mental representation of the problem, decide how to solve the problem, execute the necessary calculations, interpret the outcome and formulate an answer and evaluate the solution. In a similar study by Atteh *et al.*, (2017), they suggested a problem-solving strategy to solve mathematical problems through four stages which include; holistic understanding of the problem (What are the known and the unknown? and What are the conditions?), identify method(s) for the solution (Make sketches/diagram when needed and develop an appropriate method(s) for solution), apply the method(s) for the solution (Step by step applications of the method(s) developed), and check the accuracy of the solution (Check the validity of the method(s) used, and introduce new method(s) if the method(s) applied is found invalid). Teachers should provide students with more opportunities to participate in problem-solving where critical thinking happens to provide the students with the greatest benefit.

Barriers for Practicing Problem Solving

Barriers related to the school curriculum

According to Zanzali‘s (2003) study, to document constraints that teachers face in implementing the aspirations of the curriculum identifies the influence of examination on

what and how mathematics is taught to students as a problem teachers encounter towards the use of the problem-solving method in teaching. Its implementation in the classroom has become an issue of concern to teachers since the major aim of every good teacher is to help students pass their examination and the trend of examination parallels teaching through problem-solving. Landsman and Gorski, (2007) suggest that the current educational trend standardizes curricula and focuses on test scores undermines instructors' ability to address critical thinking in the classroom. The emphasis on "teaching to the test" distracts the learning process from student-centered instruction and emphasizes the content. If the focus is on learning, students should be given the freedom to explore content, analyze resources, and apply information.

However, Anderson, Sullivan, and White (2004) identified textbooks and assessment regimes used in the school and the schedule for mathematics lessons as hindrances to the teaching of mathematics through problem-solving in their study of the influence of perceived constraints on teachers' problem-solving beliefs and practices. Hence, conservative teaching methods by other teachers in the school as well as parents' demands for preparation of their wards for competitive examinations were other factors identified as impediments to the implementation of problem-solving instructions in the mathematics classroom. Saleh (2009) proposes through a study that limited time for mathematics lessons and problem-solving methods not needed in answering examination questions were some of the reasons hindering the teaching of mathematics through problem-solving. The teachers in this study hold the view that teaching mathematics through problem-solving was time-consuming. In some situations, teachers expressed a lack of resource materials available for teaching problem-solving in their mathematics classrooms (Foong, Yap, and Koay, 1996).

In school, so many things do compete for time. Among these are the mandatory school curricula activity, the external assessment procedures, and the workload in the school curriculum. In the view of Traiton and Midgett (2001), pressure on teachers to increase examination scores of their students makes them stick to textbooks routine of teaching mathematics instead of using problem-solving approaches.

Barriers related to teachers

According to Ellison (2009) teachers shun teaching problem-solving because they are uncomfortable with their problem-solving skills. Taplin (1998) also states that even though problem-solving is emphasized in the mathematics curriculum all over the world, teachers

still do not know how best to teach problem-solving skills. Foong *et al.,* (1996) indicated that several teachers have expressed concerns about their ability to think of the right question at the right time to engage students in discussions when using a problem-solving approach to teach the prescribed content of the mathematics curriculum. Hence, they observe that a large number of teachers expressed their concern that they lacked skills to teach mathematical problem-solving.

Foong *et al.,* (1996) were also concerned with teachers' ability to communicate new concepts to their students to understand using the varied methods suggested in the curricular reforms. The teachers agreed in this study that the problem-solving approach places greater demands on the teacher and noted that the teacher needs a thorough understanding of the subject matter to be treated. In the view of Saleh (2009) Mathematics teachers seem to be more confident in teaching methods that they had experienced in their school life and through that, teachers who do not experience problem-solving methods in their professional training tend not to emphasize problem-solving approaches in teaching. Xenofontos (2007) indicated that elementary school mathematics teachers are trained as generalists and in most cases do not have the strong mathematics background required to teach using problem-solving strategies in teaching. These teachers may not possess enough knowledge to anticipate anything other than limited curricular objectives or teaching styles and hence may be handicapped in realizing a problem-solving orientation.

Likewise, McIntosh, Jarrett, and Peixotto (2000) state that the lack of mathematical expertise to understand different approaches that students might use to solve a problem and to identify promising problem-solving approaches. Hence, they explain that teaching mathematics through problem-solving is difficult among teachers because they have inadequate subject matter knowledge, pedagogical content knowledge, and personal problems. The teachers often provide a strong rationale for not including problem-solving activities in their mathematics instructions to include:

- ✓ it takes too much time to teach,
- ✓ it is too demanding,
- ✓ not measured and tested in public examinations

Parents' expectations in terms of examination results put pressure on teachers to teach for examination instead of adopting problem-solving strategies to teach for conceptual understanding (Anderson 2000). It is common to see teachers teaching according to the

examination syllabus to the detriment of teaching for conceptual understanding which is the benchmark of problem-solving.

Barriers related to students

Research by Fletcher and Santoli (2003) suggested that the vocabulary of mathematics is not usually taught in schools and if students are not reading good textbooks, then they have no place to understand mathematics terms. It is therefore important to emphasize vocabulary instruction as part of mathematics programs if students have to learn mathematics through problem-solving entirely. Students' inability to read and comprehend poses yet another problem to teachers when teaching mathematics using problem-solving strategies. In Anderson (2000) he stated that the diversity in classrooms, students' comprehension of language, and their attitudes and beliefs towards mathematics are potential factors militating against the implementation of the problem-solving mathematics curriculum. Students are sometimes so tuned to some laid down procedures in solving mathematics problems. These students resist teachers' initiatives or plan to adopt problem-solving approaches to teaching mathematics. They prefer to be told mathematics rather than to be guided by the teacher to explore and construct their understanding.

Saleh (2009) confirms that students' knowledge base is a determinant in teaching mathematics through problem-solving and concluded that problem-solving is not good for students with low ability. However, students with high ability levels understand problem-solving better. It is, therefore, relatively easy to teach such students mathematics using problem-solving. On the other hand, students with low ability could also perfect their problem-solving skills if they are exposed to a problem-solving instructional strategy more frequently (Adesoji, 2008).

Summary and Conclusion

Teachers need opportunities to examine mathematical ideas and make connections through peer evaluation and team teaching in instructional situations. But requiring teachers to take more courses will not enhance their knowledge of mathematics in schools and improve their teaching. It is believed that providing a socially constructivist learning atmosphere and instruction content will help wipe out negative behaviors, and students' assumptions about problem-solving (Murat and Memnum, 2008). Therefore, such beliefs that a mathematical problem has only one way of solution, one correct answer, and ordinary students can never

solve an unusual problem correctly should be erased from the minds of students. In furtherance, Ali *et al.*, (2010) suggested the transformation of textbooks of mathematics into problem-based learning form. Considering the fact that many teachers rely on textbooks as the main source of teaching resources, and the availability of problem-solving questions in textbooks on mathematics would enable them to teach mathematics using problem-solving approaches.

When teachers think about what is supposed to happen in a classroom, it's important to remember what kind of learning will promote problem-solving. Implementing problem-solving involves a clear dedication to student-centered learning that both teachers and learners may find challenging and unfamiliar. Therefore it is obvious that creativity and proper planning can overcome other barriers in the application of problem-solving. Besides, problem-solving in any mathematics classroom can be achieved by modifying the teaching and integrating constructive learning strategies. This demonstrates that while there is no guarantee that students will learn, if they are inspired and the information is conveyed in a well-structured manner using engaging and appropriate teaching and learning techniques and resources, students will perform better.

DISCLAIMER

The products used for this research are commonly and predominantly use products in our area of research and country. There is no conflict of interest between the author and producers of the products because we do not intend to use these products as an avenue for any litigation but the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by the personal effort of the author.

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