

# Magnetic field dependent thermosolutal convection flow between two squeezing plates

## Abstract

The effect of magnetic field dependent thermosolutal convection and MFD viscosity of the fluid dynamic are studied between two squeezing plates. The unsteady constitutive expression of mass conservation modified Navier-Stokes, Maxwell and MFD thermosolutal convection are coupled as a system of ordinary differential equations. The corresponding solution for momentum as well as solution for the magnetic field equations are determined by the PCM method. The validity and accuracy of PCM results is proved by the comparison of the PCM solution with numerical solver package BVP4c. It has been shown the magnetic Reynold number caused to decrease the flow, magnetic field distribution and temperature of the fluid. Also x-component and y- component of magnetic field have opposite behavior with increase in MFD viscosity. Application of the study include automotive magneto-rheological shock absorber, novel aircraft landing gear system, heating up are cooling processes and biological sensor system etc.

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## Nomenclature

$p$	pressure	$C$	dimensional concentration
$x, y, z$	Cartesian coordinate	$\kappa$	thermal conductivity
$U_x$	x-component of velocity( $m. s^{-1}$ )	$\mu$	dynamic viscosity
$U_y$	y-component of velocity( $m. s^{-1}$ )	$\nu$	kinematic viscosity
$U_z$	z-component of velocity( $m. s^{-1}$ )	$\sigma$	electrical conductivity
$t$	time(s)	$\rho$	fluid density
$T_u$	temperature at upper plate( $k$ )	$\alpha$	positive constant

$T_l$	temperature at lower plate( $k$ )	$\eta$	similarity variable
$C_u$	concentration at upper plate	$\theta$	dimensionless variable
$C_l$	concentration at lower plate	$\varphi$	dimensionless variable
$p_r$	prandtl number( $\nu/k$ )	'	derivative w.r.t $\eta$
$N_b$	squeezing parameter	*	dimensionless variable
D(t)	distance between two plates	$u$	fluid condition on upper plate
$c_p$	specific heat of fluid( $J/kgK$ )	$l$	fluid condition on upper plate
$H_m$	Hartmann number	$U$	fluid velocity( $m.s^{-1}$ )
$E_c$	Eckert number	$T$	fluid temperature
$m_\eta$	magnetic field dependent viscosity	$P_m$	pyro magnetic coefficient
$N_{em}$	magnetic Reynold number	$P_s$	salinity magnetic coefficient
$N_C$	strength of magnetic field		

## Introduction

Research based studies show that magnetic fields can reduce the heart attack risk by streamlining the blood flow. Tao and Huang showed that when magnetic field is applied parallel to blood direction, it significantly reduced blood viscosity. Ferro hydrodynamics (FHD) is mechanics of fluid motion under magnetic polarization. Magnetic fluids are involved in heat transfer processes via Ferro fluids. Liquid cooled are example of such processes, in which Ferro fluids carry heat away from coils. [1]. This invention increases the coil's amplifying capacity, which contributes to high fidelity sound generation in loud speakers. Another crucial application of magnetic field is in the field of drug designing where magnetic field is used to carry drug to desired target sites in the human body, a drop of Ferro fluid in human body can be detected by the magnetic field [2]. According to Lenz's law when conducting fluid move in magnetic field then magnetic field changes its motion. In MHD, the motion changes the field and vice versa [3, 4]. Alloys have important applications in heat transfer processes such a steel, brass and bronze are few typical examples. Alloys have applications in various fields including powder technology, aerospace technology and in surgeries. Recently Aamir et al. [5] studied the Theremosolutal convection between coaxial contracting disc and investigated the fluid dynamic aspects of these magnetic

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field dependent discs. They studied their problem in cylindrical coordinate system and analyzed the flow field, temperature variation, skin fraction and Nusselt number.

### Literature Review

The study of squeezing flow has its origin in the 19<sup>th</sup> century and continues to receive considerable attention due to practical application in physical and biophysical area. Stoke problems of a convective flow past a perpendicular infinite plates in a spinning systems in occurrence of inconsistent magnetic field was studied by Mutua et al. [6]. He concluded that velocity field and temperature depend upon all of the parameters. The effect of these parameters changes the rate of heat transfer as well as the skin friction. It was observed that that increasing magnetic parameter (M) and Eckert number (Ec) elevated the free convection heating and as free convection cooling velocity profile. Seth [7] examined heat transfer and MHD flow on a porous flat plate along with mass transfer. Fluid velocity component was observed to be raised as the Hall parameter and value of time was increased. Unsteady flow between two permeable squeezed plates with thermal radiation has been studied by Victor [8]. For analysis he used Galerkin finite element method. He concluded that the fluid velocity and temperature distribution are strongly depended on Radiation parameter and on Prandtl number. In his study he also concluded that Grashof number and Magnetic parameter have a negligible effect on temperature distribution. Inclined magnetic field applied on couple flow inside two parallel porous plates was studied by Simon [9]. He considered the lower plate is absorbent and motionless. He found that fluid velocity decrease with a rise in magnetic field. For the solution of the magnetic squeeze film problem Rashidi et al. [10] used the DTM-Pade combined method to determine convergence, stability and versatility. The important effect of changing the electrical conductivity of a viscous, incompressible and electrically conducting fluid on the free convection flow was observed. Furthermore, it was noted that heat transfer on the isothermal non conducting plate was also influenced by changing the electrical conductivity. Shorma et al. [11] studied natural convection flow and heat generation in electrically conductivity flow. It was reported that suitable magnetic field can change aerodynamic force along with heat transfer rates in convenient manner. This type of MHD studies are linked with fission reactor development where the plasma is restricted by use of strong magnetic field [12]. MHD effects in the so-called blanket was studied by Morley [13]. The blanket is present between the magnetic field coils and plasma. Blanket is used for the avoiding radiation damages. Sohail et al [14] explored the way in

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which flow between two squeezing plates is effected by changing the magnetic field. Recently Aamir et al. [15] studied the thermo-solutal convection between coaxial contracting disc and investigated the fluid dynamic aspects of these magnetic field dependent discs. They studied their problem in cylindrical coordinate system then analyzed the flow field, temperature variation, skin fraction and nusselt number.

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### Problems formulation

Suppose that an incompressible and axisymmetric viscous fluid divided by the distance of  $y = l\sqrt{1 - \alpha t}$  between two parallel squeezing plates, where  $l$  is length equal to the separation of plates at  $t=0$ . For  $\alpha > 0$  both plates are pressed until they reach at  $t = \frac{1}{\alpha}$  both are separated at  $\alpha < 0$ . Lower plate is stationary and upper plate are squeezed. It is considered to be the optimal conductor for both surfaces, according to Hughes. The electrical forces are much smaller than the magnetic forces and consequently are ignored in the present problem. The applied magnetic field produced to induced magnetic field  $(B_x, B_y, B_z)$

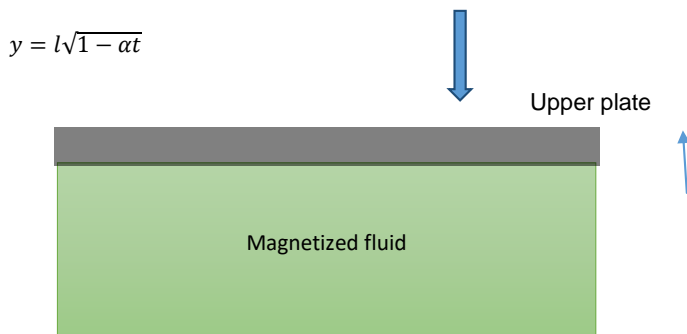
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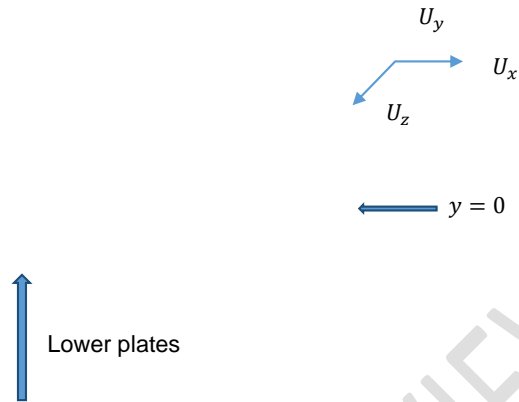
$$H_x = \frac{\alpha x E_0}{(\mu_2(1-\alpha t))} \quad H_y = \frac{-x E_0}{\mu_1(1-\alpha t)^{0.5}} \quad H_z = \frac{x F_0}{\mu_2(1-\alpha t)}$$

$E_0$  and  $F_0$  are dimensionless  $\mu_1$  and  $\mu_2$  are the external and inner media magnetic permeability between the two plates, respectively. Incompressible fluid is presumed to have a variable viscosity given by  $\mu = \mu_0(1 + \partial \cdot B)$  in the absence of the magnetic field  $\mu_0$  is the viscosity of the fluid.  $\partial_x = \partial_y = \partial_z$  isotropic was taken for the variance coefficient of viscosity  $\partial$ . The component of the  $\mu$  are  $\mu_x = \mu_0(1 + \partial B_x)$ ,  $\mu_y = \mu_0(1 + \partial B_y)$  and  $\mu_z = \mu_0(1 + \partial B_z)$  further  $B = \mu_p(M + H)$  where  $M$  is magnetization  $H$  is magnetic field and  $\mu_p$  is magnetic permeability. The effect of shear viscosity dependency is not taken into account since it has a marginal effect on a high field mono dispersive device. The equations governing viscous fluid flow and heat transfer are:

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### Geometry of problem





### 3.2 Basic governing equations

Continuity equation

$$\nabla \cdot U = 0 \quad (1)$$

Maxwell equation

$$\nabla \cdot B = 0, \quad \nabla \times B = 0 \quad (2)$$

Navier stoke equation

$$\rho \left[ \frac{\partial U}{\partial t} + (U \cdot \nabla) U \right] + \nabla p - \mu \nabla^2 U - \frac{1}{\mu_2} (\nabla \times B) \times B = 0 \quad (3)$$

Magnetic field equation

$$\frac{\partial B}{\partial t} - \nabla \times (U \times B) - \frac{1}{\sigma \mu_2} \nabla^2 B = 0 \quad (4)$$

Solute concentration equation is

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$$[\rho_0 C_{s,h} - \mu_0 H \cdot (\frac{\partial M}{\partial T})] \frac{DT}{Dt} + \mu_0 T (\frac{\partial M}{\partial T}) \frac{DH}{Dt} = K_1 \nabla^2 T \quad (5)$$

Solute concentration

$$[\rho_0 C_{s,h} - \mu_0 H \cdot (\frac{\partial M}{\partial C})] \frac{DC}{Dt} + \mu_0 C (\frac{\partial M}{\partial C}) \frac{DH}{Dt} = K'_1 \nabla^2 C \quad (6)$$

U is fluid velocity,  $\sigma$  is electrical conductivity,  $\rho_0$  is reference density,  $\rho$  is the density of fluid, p is pressure of fluid,  $\mu$  is variable dynamic viscosity,  $C_{s,h}$  is specific heat of constant volume, T is temperature, C is solute concentration, M is magnetization,  $K_l$  is thermal conductivity,  $K'_l$  solute conductivity.

Boundary condition are chosen as:

$$\begin{aligned} U_x = 0, U_y = 0, U_z = \frac{\sigma_1 x}{1 - \alpha t} \\ B_x = 0, B_y = 0, B_z = 0, \\ T = T_l, C = C_l \text{ at } Y = 0 \\ U_x = 0, U_y = \frac{-\alpha l}{2\sqrt{1 - \alpha t}}, U_z = \frac{\sigma_u x}{1 - \alpha t}, \\ B_x = 0, B_y = \frac{-xE_0}{\sqrt{1 - \alpha t}}, B_z = \frac{xF_0}{1 - \alpha t}, \\ T = T_u, C = C_u \text{ at } y = l\sqrt{1 - \alpha t}. \end{aligned} \quad (7)$$

where  $U_x, U_y, U_z$  are respectively the x, y and z components of the velocity,  $\alpha, T_l$  and  $C_l$  represent the thermal diffusivity, temperature at lower plate and concentration at lower plate respectively, while temperature and concentration at upper plate are represented by  $T_u, C_u$  respectively,  $c_p$  specific heat at constant pressure. Introducing the following similarity transformations, the partial differential equations (1) to (6) are reduced to a system of ordinary differential equations.

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$$\begin{aligned}
U_x &= \frac{\alpha x}{2(1-at)} m(\eta), & U_y &= \frac{-al}{\sqrt{1-at}} m(\eta) \quad , \\
U_z &= \frac{\sigma_l x}{1-at} n(\eta) \quad , & B_x &= \frac{\alpha x E_0}{2L^2(1-at)} h(\eta) \quad , \\
, B_y &= \frac{-aE_0}{l\sqrt{1-at}} h(\eta) & , B_z &= \frac{2vx F_0}{al(1-at)} k(\eta) \\
\theta &= \frac{T-T_u}{T_l-T_u}, & \varphi &= \frac{c-c_u}{c_l-c_u}, & \eta &= \frac{y}{l\sqrt{1-at}} \quad . \quad (8)
\end{aligned}$$

The partial differential equations (3.1) to (3.6) are transformed in the form of the following ordinary differential equations.

$$m'''' = \frac{N_b}{m_\eta} [2m'' + \eta m''' - 2mm''' + 2N_c^2 N_{em} (2hh' + \eta hh'' + 2h^2 m'' - 2hmh'')] \quad (9)$$

$$n'' = \frac{1}{m_\eta} [N_b(2n + \eta n' + m'n - 2mn') - N_c N_a (h'k - 2hk')] \quad (10)$$

$$h'' = N_{em} (h + \eta h' + hm' - mh') \quad (11)$$

$$k'' = N_{em} (2k + \eta k' - 2mk') + \frac{N_c N_{em}}{N_a} (hn') \quad (12)$$

$$\theta'' = N_b P_r (\eta \theta' - 2m\theta') - P_m \nabla T H_m E_c (2(\eta h\theta' - 2mh\theta') + (h\theta + \eta h'\theta - 2mh'\theta')) \quad (13)$$

$$\varphi'' = N_b P_r (\eta \varphi' - 2m\varphi') - P_c \nabla T H_m E_c (2(\eta h\varphi' - 2mh\varphi') + (h\varphi + \eta h'\varphi - 2mh'\varphi')) \quad (14)$$

Boundary conditions are reduced to

$$\begin{aligned}
 m(0) = 0, m'(0) = 0, n(0) = 1, \theta(0) = 1, \varphi(0) = 1, h(0) = 0, k(0) = 0, \\
 m(1) = 0.5, m'(1) = 0, n(1) = 1, \theta(1) = 0, \varphi(1) = 0, h(1) = 1, k(1) = 1.
 \end{aligned}
 \tag{15}$$

where  $N_b$  is rotational Reynold number,  $N_{em}$  is magnetic Reynold number,  $N_c$  is strength of magnetic field in y direction,  $N_d$  strength of magnetic field in z direction,  $m_\eta$  is magnetic field dependent viscosity parameter.  $P_r, E_c, H_m, P_m, P_s$  are Prandtl number, Eckert number, Hartman number, pyro magnetic coefficient and salinity magnetic coefficient respectively.

After solving these equations by PCM methods and BVP4C methods we get a different table and graph are

$\eta$	Numerical results					PCM results				
	$m(\eta)$	$n(\eta)$	$h(\eta)$	$\theta(\eta)$	$\varphi(\eta)$	$m(\eta)$	$n(\eta)$	$h(\eta)$	$\theta(\eta)$	$\varphi(\eta)$
.1	0.0143	0.9171	0.9000	0.9063	0.9157	0.0141	0.9170	0.900	0.9066	0.9157
.2	0.0525	0.8411	0.1805	0.8121	0.8308	0.0522	0.8411	0.1805	0.8127	0.8308
.3	0.1087	0.7725	0.2722	0.7170	0.7452	0.1083	0.7724	0.2722	0.7179	0.7452
.4	0.1767	0.7112	0.3659	0.6190	0.6568	0.1762	0.7112	0.3659	0.6198	0.6568
.5	0.2508	0.6575	0.4620	0.5234	0.5705	0.2500	0.6575	0.4620	0.5237	0.5705
.6	0.3247	0.6113	0.5613	0.4241	0.4805	0.3240	0.6112	0.5613	0.4249	0.4805
.7	0.3926	.5725	0.6644	0.3227	0.3881	0.3920	0.5724	0.6644	0.3232	0.3881
.8	0.4485	0.5411	0.7716	0.2186	0.2923	0.4480	0.5410	0.7716	0.2180	0.2923
.9	0.4863	0.5170	0.8834	0.1112	0.1920	0.4860	0.5170	0.8834	0.1119	0.1920

TABLE.1:Computation for  $m(\eta), n(\eta), h(\eta), k(\eta), \theta(\eta)$  with  $N_b = 0.1, N_c = 0.2, N_d = 0.5, P_r = 0.2, H_m = 1, P_m = 0.2, P_s = 1, N_{em} = 0.3, E_c = 2, S=1, m_\eta=0.3$  and various values of  $\eta$ .

	Numerical results	PCM results
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$m_\eta$	$m''(0)$	$-n'(0)$	$-\theta'(0)$	$m''(0)$	$-n'(0)$	$-\theta'(0)$
1	3.0232	0.2524	1.1545	3.0172	0.2521	1.1554
2	3.0146	0.1292	1.1545	3.0086	0.1291	1.1554
3	3.0117	0.869	1.1545	3.0057	0.0867	1.1554
4	3.0103	0.0654	1.1545	3.0043	0.0653	1.1554

TABLE.2:Computation for  $m''(\eta)$ ,  $-n'(\eta)$ ,  $-h'(\eta)$ ,  $-k'(\eta)$ ,  $-\theta'(\eta)$  with  $N_b = 0.2, N_c = 0.1, N_d = 1, P_r = 2, H_m = 1, P_m = 1, P_s = 0.5, N_{em} = 0.5, E_c = 2, S=1$  and various of  $m_\eta$

$N_{em}$	Numerical results			PCM results		
	$m''(0)$	$-n'(0)$	$-\theta'(0)$	$m''(0)$	$-n'(0)$	$-\theta'(0)$
.5	3.0117	0.0869	1.1545	3.0057	0.0867	1.1554
1	3.0118	0.0863	1.1334	3.0058	0.0863	1.1342
2	3.0118	0.0853	1.1000	3.0058	0.0853	1.1005
3	3.0118	0.0846	1.0751	3.0058	0.0847	1.0756

TABLE.3:Computation for  $m''(\eta)$ ,  $-n'(\eta)$ ,  $-h'(\eta)$ ,  $-k'(\eta)$ ,  $-\theta'(\eta)$  with  $N_b = 0.2, N_c = 0.1, N_d = 1, P_r = 2, H_m = 1, P_m = 0.2, P_s = 0.5, E_c = 2, m_\eta = 3, S=1$  and various values of  $N_{em}$ .

$E_c$	Numerical results			PCM results		
	$m''(0)$	$-n'(0)$	$-\theta'(0)$	$m''(0)$	$-n'(0)$	$-\theta'(0)$
0.1	3.0127	0.1421	1.0015	3.0068	0.1421	1.0015
0.5	3.0127	0.1421	1.0087	3.0068	0.1421	1.0088
1	3.0127	0.1421	1.0176	3.0068	0.1421	1.0178
1.5	3.0127	0.1421	1.0265	3.0068	0.1421	1.0268

TABLE.4:Computation for  $m''(\eta)$ ,  $-n'(\eta)$ ,  $-h'(\eta)$ ,  $-k'(\eta)$ ,  $-\theta'(\eta)$  with  $N_b = 0.1, N_c = 2, N_d = 1, P_r = 0.2, H_m = 1, P_m = 0.2, P_s = 1, N_{em} = .3, m_\eta = 3, S=1$  and various values of  $E_c$ .

	Numerical results	PCM results
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Pm	$m''(0)$	$-n'(0)$	$-\theta'(0)$	$m''(0)$	$-n'(0)$	$-\theta'(0)$
0.1	3.0188	0.1241	1.0114	3.0148	0.1249	1.0114
0.5	3.0188	0.1241	1.0684	3.0148	0.1249	1.0688
1	3.0188	0.1241	1.1370	3.0148	0.1249	1.1377
1.5	3.0188	0.1241	1.2023	3.0148	0.1249	1.2029

TABLE.5:Shows  $n(\eta)$  and  $k(\eta)$  with various values of  $N_d$  and constant values of  $N_b = 0.1, N_c = 5, N_{em} = 1, P_r = 1, H_m = 10, P_m = 1, P_s = 1, m_\eta = 2, E_c = 0.1$  3.0127 0.1420 -0.8986 -0.9030 1.0355 1.0176

Ps	Numerical results			PCM results		
	$m''(0)$	$-n'(0)$	$-\phi'(0)$	$m''(0)$	$-n'(0)$	$-\phi'(0)$
.1	3.0188	0.1241	1.0116	3.0148	0.1249	1.0114
.5	3.0188	0.1241	1.0672	3.0148	0.1249	1.0688
1	3.0188	0.1241	1.1296	3.0148	0.1249	1.1377
1.5	3.0188	0.1241	1.1854	3.0148	0.1249	1.1998

TABLE.6:Computation for  $m''(\eta), -n'(\eta), -h'(\eta), -k'(\eta), -\theta'(\eta)$  with  $N_b = 0.2, N_c = 0.1, P_r = 2, H_m = 1, E_c = 2, m_\eta=3, P_m=.5, N_{em}=1, S=1$  and various values of  $P_s$ .

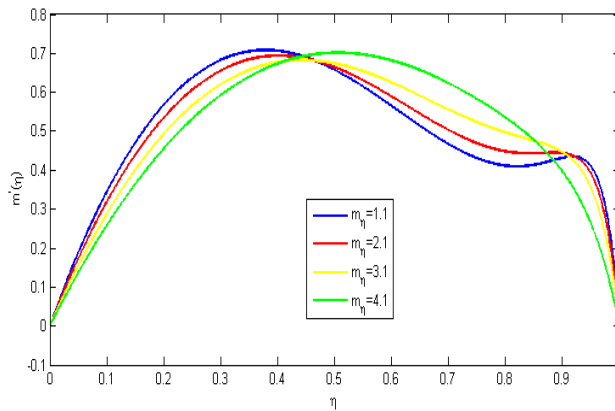


Figure 1a: Shows the effect of  $m'(\eta)$ , for various values of  $m_\eta$  and constant values of  $N_b = 0.2, N_c = 8, N_d = 1, P_r = 2, H_m = 1, P_m = 9, P_s = 1, N_{em} = 13, E_c = 0.3$ .

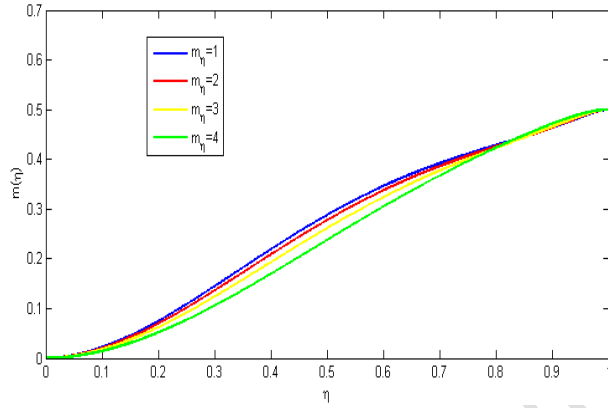


Figure 1b: Shows the effect of  $m(\eta)$ , for various values of  $m_\eta$  and constant values of  $N_b = 0.2, N_c = 8, N_d = 1, P_r = 2, H_m = 1, P_m = 9, P_s = 1, N_{em} = 13, E_c = 0.3$ .

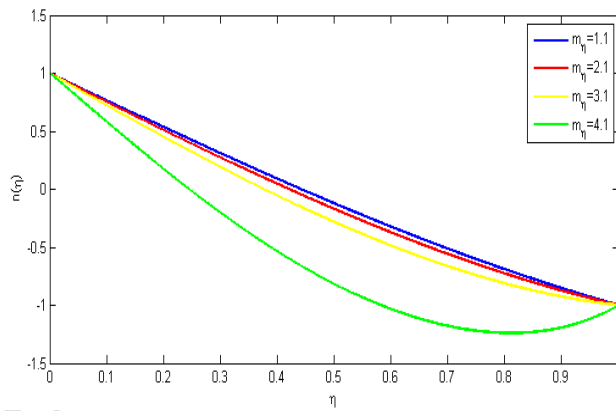


Figure 2a: Shows the  $n(\eta), h(\eta), k(\eta)$  and  $\theta(\eta)$ , various values of  $m_\eta$  and constant values of  $N_b = .01, N_c = 3, N_d = 2, P_r = .2, H_m = 1, P_m = 1, P_s = .5, N_{em} = 0.5, E_c = 0.3$ .

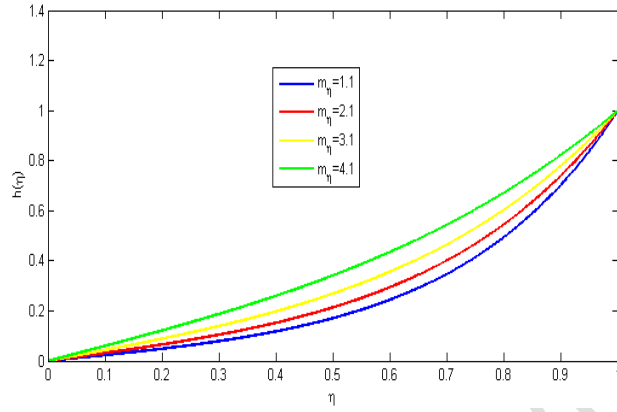


Figure 2b: Shows the  $k(\eta)$  with various values of  $m_\eta$  and constant values of  $N_b = .01, N_c = 3, N_d = 2, P_r = .2, H_m = 1, P_m = 1, P_s = .5, N_{em} = 0.5, E_c = 0.3$ ,

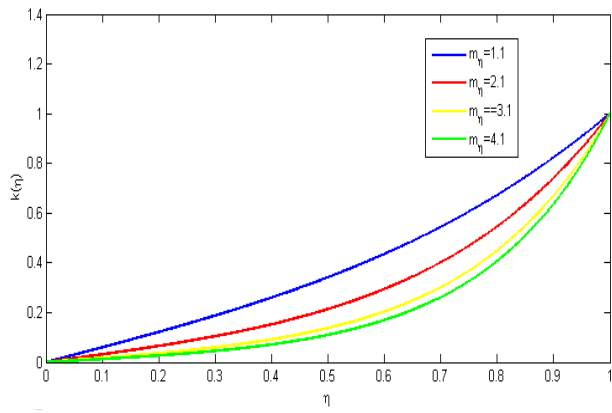


Figure 2c: Shows the  $k(\eta)$  various values of  $m_\eta$  and constant values of  $N_b = .01, N_c = 3, N_d = 2, P_r = .2, H_m = 1, P_m = 1, P_s = .5, N_{em} = 0.5, E_c = 0.3$ .

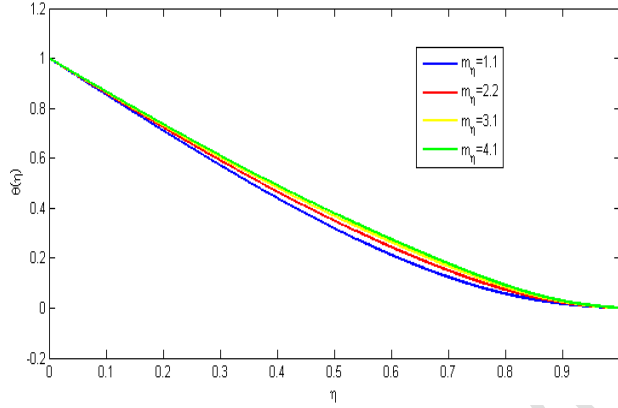


Figure 2d: Shows the  $\theta(\eta)$ , various values of  $m_\eta$  and constant values of  $N_b = .01, N_c = 3, N_d = 2, P_r = .2, H_m = 1, P_m = 1, P_s = .5, N_{em} = 0.5, E_c = 0.3$ .

### Results and discussion

The effect of MFDV  $m_\eta$  on  $m(\eta)$  and  $m'(\eta)$  can be seen in Figure 1. The resistance force of fluid flow increases as the viscosity coefficient rises as a result flow velocity decreases. Figure 1b indicates that as the viscosity of a fluid increases, the fluid flow at the center of the channel decreases that reflect the viscosity function as defined. As the viscosity parameter rises the fluid internal resistance increases as a result the flow decreases. It was also discovered that the fluid flow in the x direction decreases near the lower plate due to inertia and in the central zone it increases and start gradually decrease as it approaches the upper plates. Figure 2(a) shows the largest reduction in the z-component of velocity  $n(\eta)$  near the upper plate for a fixed value of  $m_\eta = 4$ , while for other various values of  $m_\eta$ ,  $h(\eta)$  and  $k(\eta)$  have the opposite behavior.

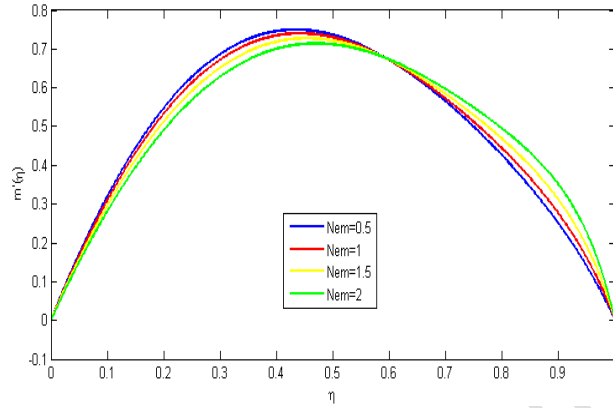


Figure 3a: Shows the effect of  $m'(\eta)$ , at the various values of  $N_{em}$  and constant values of  $N_b = 0.9, N_c = 10, N_d = 0.2, P_r = 0.2, H_m = 1, P_m = 1, P_s = 0.5, m_\eta = 3, E_c = 5$ .

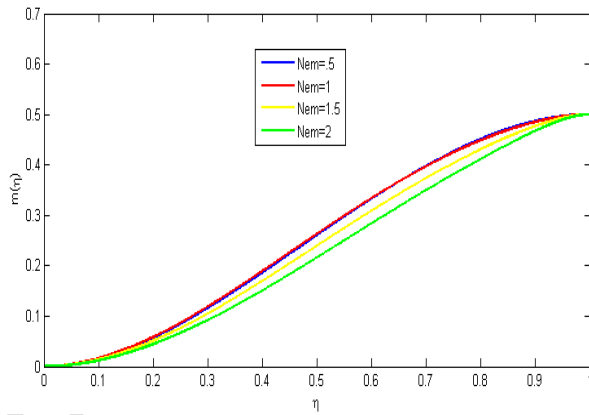


Figure 3 b: Shows the effect of  $m(\eta)$  at the various values of  $N_{em}$  and constant values of  $N_b = 0.9, N_c = 10, N_d = 0.2, P_r = 0.2, H_m = 1, P_m = 1, P_s = 0.5, m_\eta = 3, E_c = 5$ .

Figure 3 shows the effect of magnetic Reynold number on velocity component of  $m(\eta)$  and  $m'(\eta)$ . Figure 3a indicates that as the magnetic Reynold's number increases the velocity component in x-direction decreases in the surrounding of lower plate while it increases near the region of upper plate. This is because the magnetic Reynold number depends upon the squeezing

of upper plate as the upper plate moves away from lower plate the ratio of fluid flux to magnetic diffusivity increases and as a result the velocity of fluid increase from central region to upper plate. Similarly opposite behavior can be seen in Figure 3b where the y-component of velocity increases and its maximum value is achieved at  $N_{em}=2$  for the specific values of other parameters.

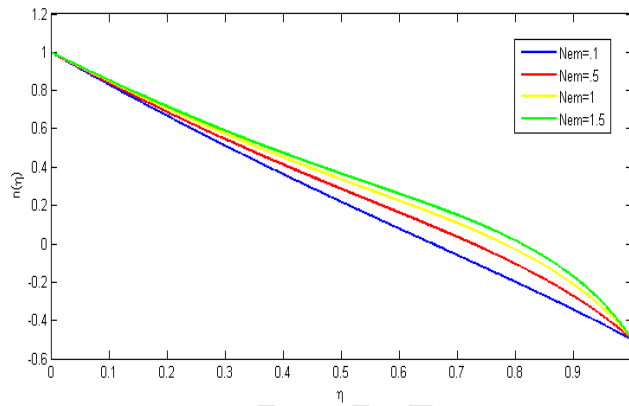


Figure 4(a): shows the effect of  $n(\eta)$ , with various values of  $N_{em}$  and at constant values of  $N_b = 0.9, N_c = 8, N_d = 0.2, P_r = 0.2, H_m = 10, P_m = 1, P_s = .5, m_\eta = 3, E_c = 5$ .

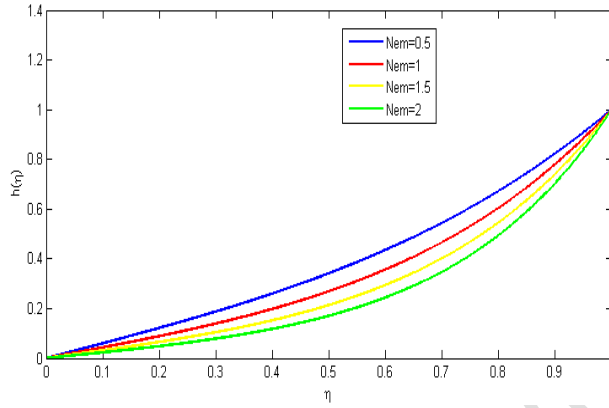


Figure 4b: shows the effect of  $h(\eta)$  with various values Of  $N_{em}$  and at constant values of  $N_b = 0.9, N_c = 8, N_d = 0.2, P_r = 0.2, H_m = 10, P_m = 1, P_s = .5, m_\eta = 3, E_c = 5$ .

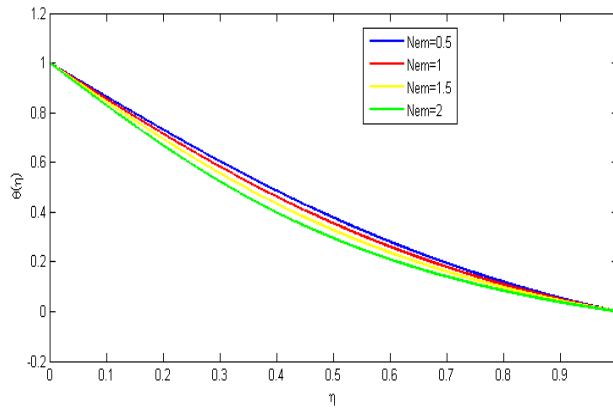


Figure 4c: shows the effect of  $\phi(\eta)$ , with various values Of  $N_{em}$  and at constant values of  $N_b = 0.9, N_c = 8, N_d = 0.2, P_r = 0.2, H_m = 10, P_m = 1, P_s = .5, m_\eta = 3, E_c = 5$ .



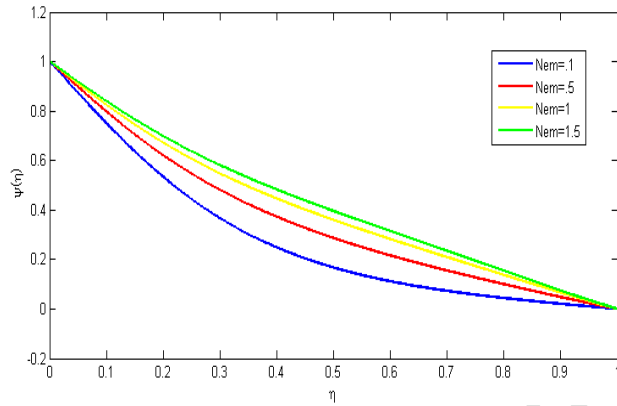


Figure 4d: Shows the effect  $\varphi(\eta)$ , with various values of  $N_{em}$  and at constant values of  $N_b = 0.9, N_c = 8, N_d = 0.2, P_r = 0.2, H_m = 10, P_m = 1, P_s = .5, m_\eta = 3, E_c = 5$ .

Figure 4a shows the effect of magnetic Reynold number on z-component of velocity. From this figure one can observe that on increasing the magnetic Reynold number the z-component of velocity rises. It also observed that on increasing magnetic flux or decreasing diffusivity  $h(\eta), \theta(\eta)$  and  $\varphi(\eta)$  shows same behavior as shown in Figure 4(b-d) and clear increases are observed in  $\theta(\eta)$  and  $\varphi(\eta)$ .

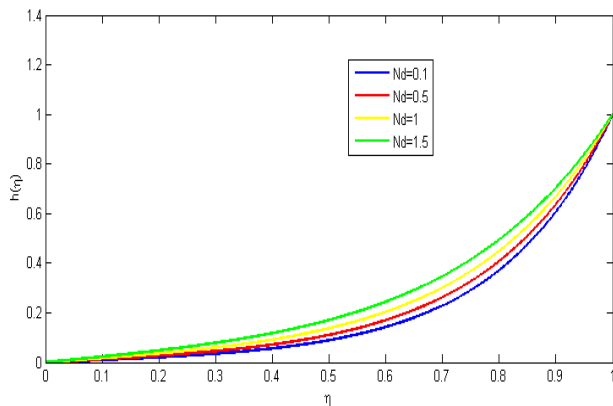


Figure 5a: Shows  $h(\eta)$  and  $k(\eta)$  with various values of  $N_d$  and constant values of  $N_b = 0.1, N_c = 5, N_{em} = 1, P_r = 1, H_m = 10, P_m = 1, P_s = 1, m_\eta = 2, E_c = 0.1$ .

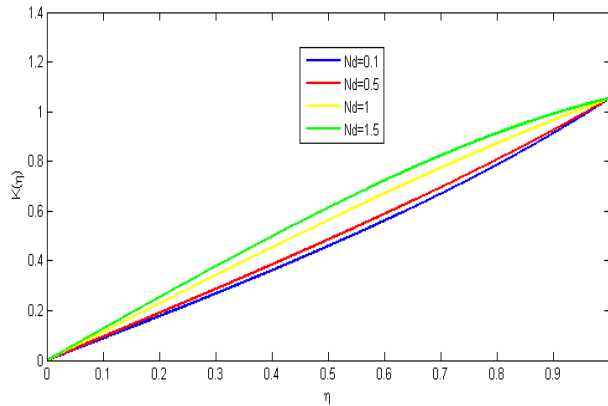


Figure 5b: Shows  $h(\eta)$  and  $k(\eta)$  with various values of  $N_d$  and constant values of  $N_b = 0.1, N_c = 5, N_{em} = 1, P_r = 1, H_m = 10, P_m = 1, P_s = 1, m_\eta = 2, E_c = 0.1$ .

Figure 5 shows the effect of  $N_d$  on magnetic component in y-direction i.e.  $h(\eta)$  and on magnetic component in z-direction i.e.  $k(\eta)$ . As  $N_d$  shows the strength of applied magnetic field along the z axis, so on increasing  $N_d$  the magnetic field in z-direction increases.

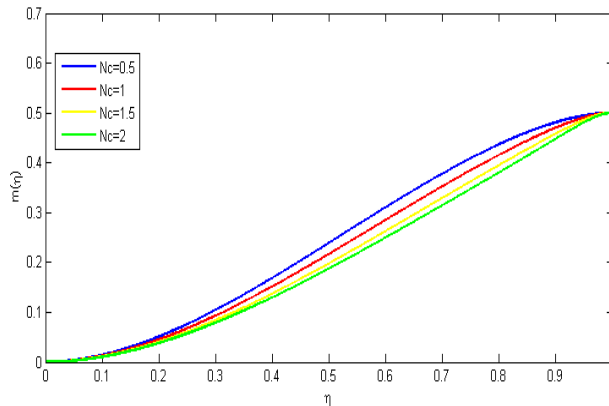


Figure 6a: Shows the graph of  $m'(\eta), m(\eta), n(\eta)$  and  $k(\eta)$  with various values of  $N_c$  and constant values of  $N_b = 0.1, N_d = .5, N_{em} = 3, P_r = .1, H_m = 10, P_m = 1, P_s = 1, m_\eta = .1, E_c = 0.1, N_c = 3$ .

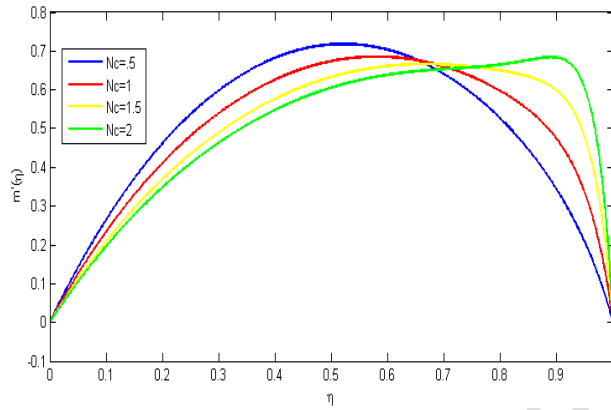


Figure 6b: Shows the graph of  $m'(\eta)$ ,  $m(\eta)$ ,  $n(\eta)$  and  $k(\eta)$  with various values of  $N_c$  and constant values of  $N_b = 0.1, N_d = .5, N_{em} = 3, P_r = .1, H_m = 10, P_m = 1, P_s = 1, m_\eta = .1, E_c = 0.1, N_c = 3$ .

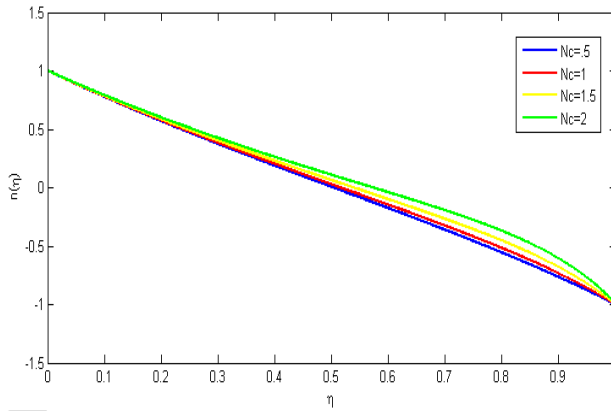


Figure 6c: Shows the graph of  $m'(\eta)$ ,  $m(\eta)$ ,  $n(\eta)$  and  $k(\eta)$  with various values of  $N_c$  and constant values of  $N_b = 0.1, N_d = .5, N_{em} = 3, P_r = .1, H_m = 10, P_m = 1, P_s = 1, m_\eta = .1, E_c = 0.1, N_c = 3$ .

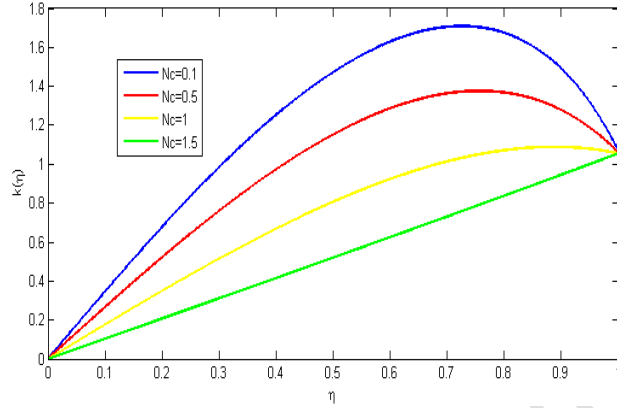


Figure 6d: Shows the graph of  $m'(\eta)$ ,  $m(\eta)$ ,  $n(\eta)$  and  $k(\eta)$  with various values of  $N_c$  and constant values of  $N_b = 0.1, N_d = .5, N_{em} = 3, P_r = .1, H_m = 10, P_m = 1, P_s = 1, m_\eta = .1, E_c = 0.1, N_c = 3$ .

Figure 6 shows the effect of  $N_c$  on velocity component in y-direction i.e.  $m(\eta)$  and on velocity component in x-direction i.e.  $m'(\eta)$ . As  $N_c$  shows the strength of applied magnetic field in y-direction, so on increasing  $N_c$  the velocity component in x-direction i.e.  $m'(\eta)$  decreases in the surrounding of lower plate while it increases near the region of upper plate in figure 6b. Figure 6a indicates that as the velocity component in y direction i.e.  $m(\eta)$  decreases. Figure 6 (c-d) shows that on increasing  $N_c$  the velocity component in z-direction i.e.  $n(\eta)$  near to upper plate are increasing and it decrease the magnetic component in z-direction i.e.  $k(\eta)$ .

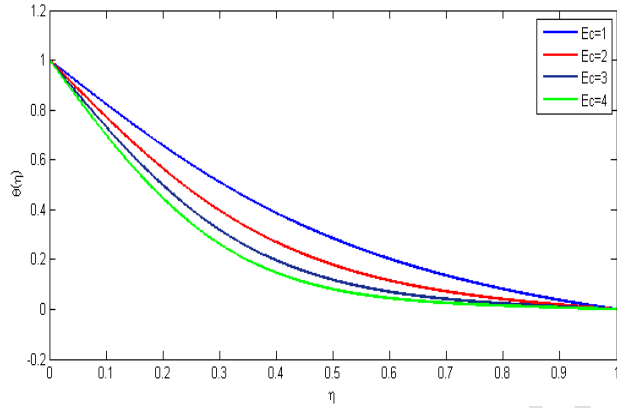


Figure 7a: shows the graph of  $\theta(\eta), \varphi(\eta)$ , with various values of  $E_c$  and constant values of  $N_b = 0.2, N_d = 1, N_{em} = .5, P_r = 10, H_m = 10, P_m = 1, P_s = .5, m_\eta = 2, E_c = 0.1, N_c = 3$ .

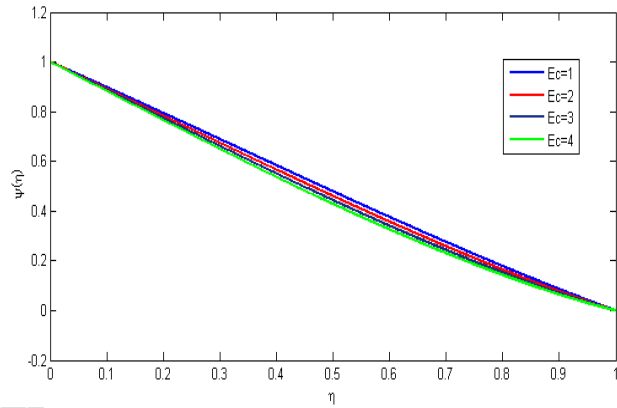


Figure 7b: shows the graph of  $\theta(\eta), \varphi(\eta)$ , with various values of  $E_c$  and constant values of  $N_b = 0.2, N_d = 1, N_{em} = .5, P_r = 10, H_m = 10, P_m = 1, P_s = .5, m_\eta = 2, E_c = 0.1, N_c = 3$ .

The Eckert number  $E_c$  is the ratio of the kinetic energy to the boundary layer enthalpy difference and is used to characterize heat dissipation. Figure 7(a-b) shows the increasing  $E_c$  decreasing both the  $\theta(\eta), \varphi(\eta)$  due to temperature difference.

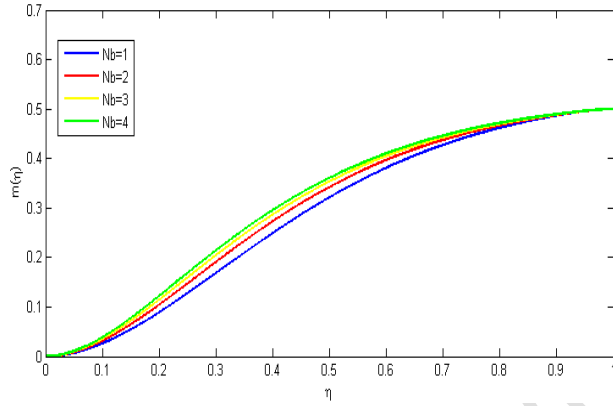


Figure 8a: shows  $m'(\eta)$ ,  $m(\eta)$ ,  $n(\eta)$ ,  $k(\eta)$ , the  $\theta(\eta)$  and  $\phi(\eta)$  with various values of  $N_b$  and constant values of  $N_c = 8$ ,  $N_d = .1$ ,  $N_{em} = 3$ ,  $P_r = 2$ ,  $H_m = 10$ ,  $P_m = 1$ ,  $P_s = .5$ ,  $m_\eta = .1$ ,  $E_c = 0.1$ .

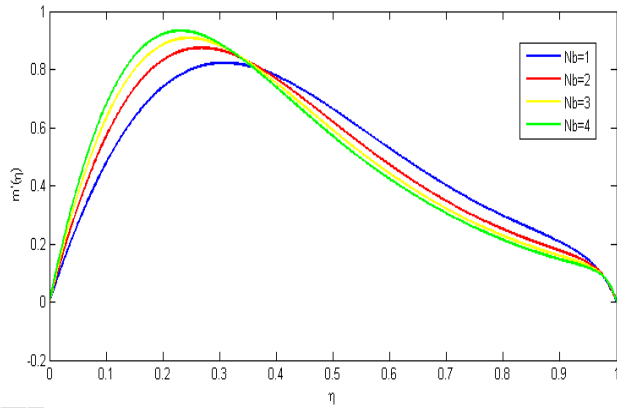


Figure 8b: shows  $m'(\eta)$ ,  $m(\eta)$ ,  $n(\eta)$ ,  $k(\eta)$ , the  $\theta(\eta)$  and  $\phi(\eta)$  with various values of  $N_b$  and constant values of  $N_c = 8$ ,  $N_d = .1$ ,  $N_{em} = 3$ ,  $P_r = 2$ ,  $H_m = 10$ ,  $P_m = 1$ ,  $P_s = .5$ ,  $m_\eta = .1$ ,  $E_c = 0.1$ .

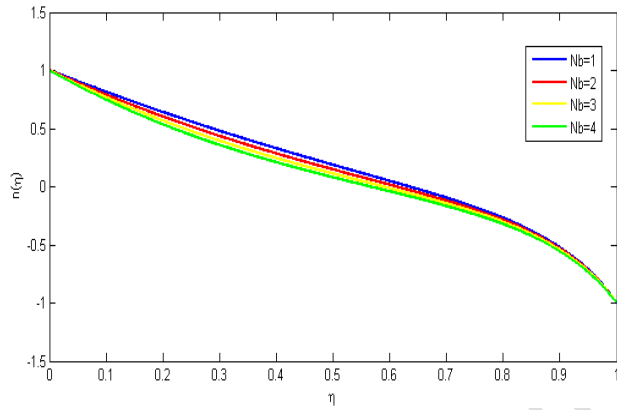


Figure 8c: shows  $m'(\eta)$ ,  $m(\eta)$ ,  $n(\eta)$ ,  $k(\eta)$ , the  $\theta(\eta)$  and  $\phi(\eta)$  with various values of  $N_b$  and constant values of  $N_c = 8$ ,  $N_d = .1$ ,  $N_{em} = 3$ ,  $P_r = 2$ ,  $H_m = 10$ ,  $P_m = 1$ ,  $P_s = .5$ ,  $m_\eta = .1$ ,  $E_c = 0.1$ .

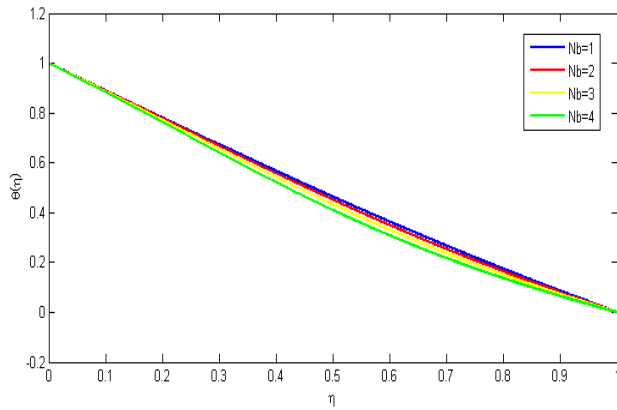


Figure 8d: shows  $m'(\eta)$ ,  $m(\eta)$ ,  $n(\eta)$ ,  $k(\eta)$ , the  $\theta(\eta)$  and  $\phi(\eta)$  with various values of  $N_b$  and constant values of  $N_c = 8$ ,  $N_d = .1$ ,  $N_{em} = 3$ ,  $P_r = 2$ ,  $H_m = 10$ ,  $P_m = 1$ ,  $P_s = .5$ ,  $m_\eta = .1$ ,  $E_c = 0.1$ .

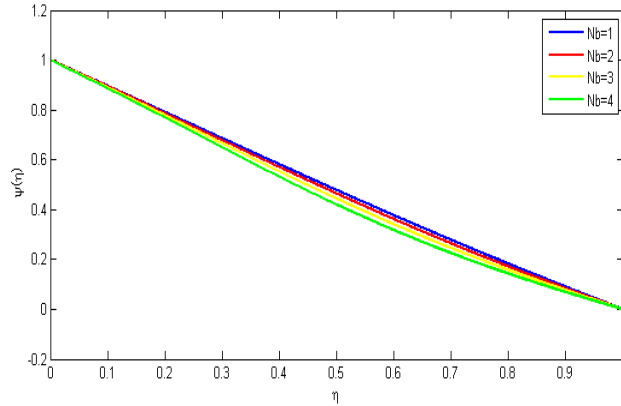


Figure 8e: shows  $m'(\eta)$ ,  $m(\eta)$ ,  $n(\eta)$ ,  $k(\eta)$ , the  $\theta(\eta)$  and  $\phi(\eta)$  with various values of  $N_b$  and constant values of  $N_c = 8$ ,  $N_d = .1$ ,  $N_{em} = 3$ ,  $P_r = 2$ ,  $H_m = 10$ ,  $P_m = 1$ ,  $P_s = .5$ ,  $m_\eta = .1$ ,  $E_c = 0.1$

The squeezing Reynold number  $N_b$  present the ratio between normal velocity of upper body and kinematic viscosity of the fluid. Figure 8 shows the effect of squeezing Reynold number on velocity component of  $m(\eta)$  and  $m'(\eta)$ . Figure 8a indicates that as the squeezing Reynold's number increases the velocity component in x-direction decreases in the surrounding of lower plate while it increases near the region of upper plate. Similarly, opposite behavior can be seen in Figure 3b where the y-component of velocity increases. Figure 8c indicate that the squeezing Reynold number increase the velocity component in z direction decrease. Similarly, same behavior can be seen in Figure 8(d-f) where  $\theta(\eta)$  and  $\varphi(\eta)$  decrease.

Also Table 2 shows increase in viscosity decrease in skin friction  $m''(0)$  and slightly increase heat flux  $-\theta'(0)$ . Similarly, effect of  $N_{em}$  and  $E_c$  is shown in Table 3-4. Table 5-6 shown the effect of  $P_m$  and  $P_s$  on  $m''$ ,  $n'$ ,  $h'$ ,  $\theta'$  and  $\varphi'$ .

### 3.4 Concluding remark

The mathematical formulation for the constitutive expression, Maxwell equations, energy equation and concentration equation for an unsteady of viscous fluid is employed to model the flow between the two squeezing plates in the form of equations (7) - (12), subject to the boundary conditions given in equation (3.13). Analysis of these equations are performed by

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PCM and compared with numerical results obtained from BVP4C Package. The following conclusions are drawn with the help of tabulated and graphical results:

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- Opposite behavior of the velocity along the x-axis and y-axis with increase in MFD viscosity.
- Magnetic field distribution  $h(\eta)$  decrease with increasing Magnetic Reynold number
- Eckert number have same effect on  $\theta(\eta), \psi(\eta)$ .
- Opposite behavior of X component and y component of velocity with increasing Squeezing Reynold number
- Table 2. show that  $m''(0)$  decrease and increase  $-\theta'$  with increasing MFD viscosity

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