Original Research Article

Bidirectional controlled teleportation of two-qubit and three-qubit state via nine-qubit entangled state

Abstract

In this paper, a bidirectional controlled quantum teleportation via a nine-qubit entangled state is given. In this scheme, Alice wants to teleport a two-qubit entangled state to Bob and Bob wants to teleport arbitrary three-qubit state to Alice at the same time. The quantum teleportation is supervised by a controller. Furthermore, in our scheme, the users only need to perform Bell-state and single-qubit measurement and the scheme is efficient and economical.

Keywords: Bidirectional controlled teleportation; nine-qubit entangled state; Bell-base measurement; Single-qubit measurement

1 Introduction

Quantum teleportation (QT) is an important technique to communicate quantum information between users. The first QT scheme was proposed to teleport an unknown quantum state via Einstein-Podolsky-Rosen (EPR) state as quantum channel by Bennett [1]. Then, a lot of QT schemes [2-8] are investigated with differently entangled states as channel. In 2013, Zha et al. [9] proposed a bidirectional quantum teleportation (BQT) scheme to teleport two single-qubit states via a five-qubit entangled cluster state and Li et al. [10] proposed a BQT scheme via a five-qubit composite GHZ-Bell state too. BQT scheme, in which two users can teleport quantum information to each other simultaneously, attracts a large number of scholars to study. Such as, Yan [11] and Sun et al. [12] proposed two BTQ schemes by using six-qubit entangled state respectively. Binayak et al. [13] proposed a BQT scheme to teleport arbitrary two-qubit state via ten-qubit entangled state and Mohammad et al. [14] proposed a BQT schemes to teleport arbitrary number of qubits. On the other hand, Hassanpour and Houshmand [15] proposed another type of BQT scheme to teleport a two-qubit Bell-state via GHZ states. This type of teleportation can transmit more quantum information with less consumption of quantum and classical resource. Later, Yang et al. [16] proposed a scheme for transmitting two three-qubit entangled states and Zhou et al. gave two BQT schemes to teleport two-qubit entangled state [17] or two-qubit entangled state and three-qubit entangled state [18] respectively. Later, Mohammand et al. [19] proposed a BQT scheme to transmit two n-qubit entangled states via (2n+2)-qubit entangled state.

In 2015, Sang [20] investigated an asymmetric bidirectional quantum teleportation via five-qubit entangled cluster state. In this scheme, Alice wants to teleport a two-qubit entangled state to Bob and Bob wants to teleport an arbitrary single-qubit state to Alice. Later, Li et al. [21] proposed an asymmetric bidirectional controlled quantum teleportation (BCQT) scheme via six-qubit entangled state to exchange two-qubit entangled state and single-qubit state. Zhou et al. [22] and Vikram [23] proposed two asymmetric BQT schemes to transmit a three-qubit entangled state and an arbitrary single-qubit state via two sets of GHZ state by performing different operations. Binayak [24] gave a BQT scheme to teleport a three-qubit entangled state and a two-qubit entangled state. In these schemes, they all try to transmit an entangled multi-qubit state and a single-qubit state. Differently, Hong [25] and Zhang et al. [26] investigated two BCQT schemes to teleport an arbitrary two-qubit state and single-qubit state via seven-qubit entangled state and eight-qubit entangled state respectively. Later, Long et al. [27] proposed an BCQT scheme to teleport an arbitrary single-qubit state and three-qubit state via 9-qubit genuine entangled state and Huo et al. [28] proposed a BCQT scheme for teleporting two-qubit state and three-qubit state via eleven-qubit entangled state. Base on those schemes, we try to propose a BCQT scheme for teleporting a three-qubit state and two-qubit Bell state at the same time via a nine-qubit entangled state.

The organization of this paper is outlined as follows. In section 2, we first illustrate a nine-qubit entangled state, which will be utilized as quantum channel in our scheme. Then we introduce a scheme to teleport an arbitrary two-qubit entangled state and three-qubit state. In the end, discussions and conclusions are given briefly in section 3.

2 Bidirectional controlled quantum teleportation of two-qubit and three- qubit state

To teleport two-and three-qubit state, we need to produce a nine-qubit entangled state. The entangled state will be used as quantum channel and shared between agents in advance. This nine-qubit entangled state is given by

$$\begin{aligned}|\phi\rangle = \frac{1}{4}(&|000000000\rangle + |000010010\rangle + |000100101\rangle + |000110111\rangle + |001001001\rangle + |001011011\rangle \\ &+|001101100\rangle + |001111110\rangle + |110000001\rangle + |110010011\rangle + |110100100\rangle + |110110110\rangle \\ &+|111001000\rangle + |111011010\rangle + |111101101\rangle + |111111111\rangle).\end{aligned} \tag{2.1}$$

which can be produced from $|\phi_0\rangle = |000000000\rangle_{123456789}$. All operations to produce the nine-qubit entangled state $|\phi\rangle$ are shown in Figure 1.

Now, let to introduce our quantum teleportation scheme. Suppose there are three agents, Alice, Bob and Charlie in our scheme. Alice wants to teleport an arbitrary two-qubit entangled state $|\psi_1\rangle$ to Bob. At the same time, Bob wants to teleport an arbitrary three-qubit state $|\psi_2\rangle$ to Alice. And Charlie is a supervisor who can terminate the quantum teleportation if she thinks the communication process is not secure. The two quantum states will be communicated between Alice and Bob in our scheme are given by

$$\begin{aligned}|\psi_1\rangle_{A_1A_2} &= a_0|00\rangle + a_1|11\rangle, \\ |\psi_2\rangle_{B_1B_2B_3} &= b_0|000\rangle + b_1|001\rangle + b_2|010\rangle + b_3|011\rangle + b_4|100\rangle + b_5|101\rangle + b_6|110\rangle + b_7|111\rangle,\end{aligned} \tag{2.2}$$

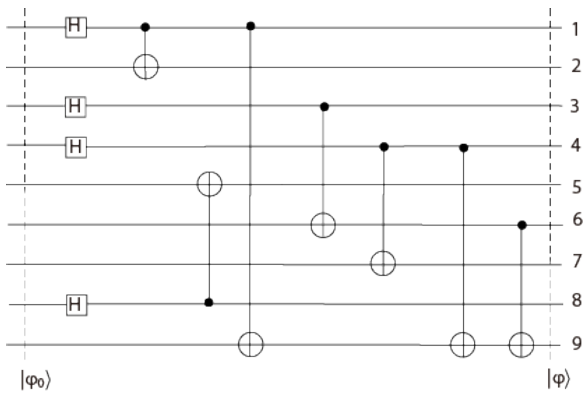

Figure 1: A quantum circuit for producing a nine-qubit entangled state $|\phi\rangle$ from quantum state $|\phi_0\rangle$

where $a_1,\ a_2,\ b_i,\ i=0,1,...,7$ are arbitrary complex coefficients and satisfy the following conditions $|a_0|^2+|a_1|^2=1,\ \sum_{i=0}^{7}|b_i|^2=1.$ To achieve the quantum scheme, Alice should share the nine-qubit entangled state $|\phi\rangle$ as quantum channel with Bob and Charlie in advance. Then the total state of quantum system held by Alice, Bob and Charlie can be described by

$$
\begin{aligned}
&|\Omega\rangle = |\psi_1\rangle_{A_1A_2}\otimes|\phi\rangle_{123456789}\otimes|\psi_2\rangle_{B_1B_2B_3}\\
=&(a_1|00\rangle+b_1|11\rangle)_{A_1A_2}\otimes\frac{1}{4}(|000000000\rangle+|000010010\rangle+|000100101\rangle+|000110111\rangle+|001001001\rangle\\
+&|001011011\rangle+|001101100\rangle+|001111110\rangle+|110000001\rangle+|110010011\rangle+|110100100\rangle+|110110110\rangle\\
+&|111001000\rangle+|111011010\rangle+|111101101\rangle+|111111111\rangle)_{123456789}\otimes(b_0|000\rangle+b_1|001\rangle\\
+&b_2|010\rangle+b_3|011\rangle+b_4|100\rangle+b_5|101\rangle+b_6|110\rangle+b_7|111\rangle)_{B_1B_2B_3}
\end{aligned}
\tag{2.3}
$$

where particles $A_1,\ A_2$, 1, 3, 4 and 5 belong to Alice, particles $B_1,\ B_2\ B_3$, 2, 6, 7 and 8 belong to Bob, and particle 9 belongs to Charlie.

The quantum teleportation scheme consists of the following main steps:

Step 1. Alice performs a CNOT operation on her qubit A_1 and A_2 with qubit A_2 as control qubit and A_1 as target qubit, and then the state $|\psi_1\rangle=(a_0|00\rangle+a_1|11\rangle)_{A_1A_2}$ is changed into $|\psi_1\rangle=(a_0|00\rangle+a_1|01\rangle)_{A_1A_2}=|0\rangle_{A_1}\otimes(a_0|0\rangle+a_1|1\rangle)_{A_2}$. Thereby, we can drop the qubit A_1 because the qubit is not entangled with the qubit A_2 qubit and contains nothing about the quantum information $|\psi_1\rangle$.

Step 2. Alice performs a Bell-state measurement on her particles A_2 and 1, then broadcasts her measurement result via a classical channel. Bob also performs three Bell-state measurements on his own particles $B_16,\ B_27,$ and B_38 and tell the others his measurement result via a classical channel.

To understanding the quantum process easily, we rewrite the Equation (3) the following equation,

$$|\Omega\rangle = \frac{1}{16}[|\Phi^m\rangle_{A_21}|\Phi^i\rangle_{B_38}|\Phi^j\rangle_{B_27}|\Phi^k\rangle_{B_16}(a_0b_0|00000\rangle + (-1)^m a_1b_0|10001\rangle + (-1)^i a_0b_1$$
$$|00010\rangle + (-1)^{i+m}a_1b_1|10011\rangle + (-1)^j a_0b_2|00101\rangle + (-1)^{m+j}a_1b_2|10100\rangle + (-1)^{i+j}a_0b_3$$
$$|00111\rangle + (-1)^{m+i+j}a_1b_3|10110\rangle + (-1)^k a_0b_4|01001\rangle + (-1)^{m+k}a_1b_4|11000\rangle + (-1)^{i+k}$$
$$a_0b_5|01011\rangle + (-1)^{m+i+k}a_1b_5|11010\rangle + (-1)^{j+k}a_0b_6|01100\rangle + (-1)^{m+j+k}a_1b_6|11101\rangle$$
$$+ (-1)^{i+j+k}a_0b_7|01110\rangle + (-1)^{m+i+j+k}a_1b_7|11111\rangle)_{23459}$$
$$+ |\Psi^m\rangle_{A_21}|\Phi^i\rangle_{B_38}|\Phi^j\rangle_{B_27}|\Phi^k\rangle_{B_16}(a_0b_0|10001\rangle + (-1)^m a_1b_0|00000\rangle + (-1)^i a_0b_1|10011\rangle$$
$$+ (-1)^{i+m}a_1b_1|00010\rangle + (-1)^j a_0b_2|10100\rangle + (-1)^{m+j}a_1b_2|00101\rangle + (-1)^{i+j}a_0b_3|10110\rangle$$
$$+ (-1)^{m+i+j}a_1b_3|00111\rangle + (-1)^k a_0b_4|11000\rangle + (-1)^{m+k}a_1b_4|01001\rangle + (-1)^{i+k}a_0b_5$$
$$|11010\rangle + (-1)^{m+i+k}a_1b_5|01011\rangle + (-1)^{j+k}a_0b_6|11101\rangle + (-1)^{m+j+k}a_1b_6|01100\rangle$$
$$+ (-1)^{i+j+k}a_0b_7|111111\rangle + (-1)^{m+i+j+k}a_1b_7|01110\rangle)_{23459}$$
$$+ |\Phi^m\rangle_{A_21}|\Phi^i\rangle_{B_38}|\Phi^j\rangle_{B_27}|\Psi^k\rangle_{B_16}(a_0b_0|001001\rangle + (-1)^m a_1b_0|11000\rangle + (-1)^i a_0b_1|01011\rangle$$
$$+ (-1)^{i+m}a_1b_1|11010\rangle + (-1)^j a_0b_2|01100\rangle + (-1)^{m+j}a_1b_2|111101\rangle + (-1)^{i+j}a_0b_3|01110\rangle$$
$$+ (-1)^{m+i+j}a_1b_3|11111\rangle + (-1)^k a_0b_4|00000\rangle + (-1)^{m+k}a_1b_4|10001\rangle + (-1)^{i+k}a_0b_5$$
$$|00010\rangle + (-1)^{m+i+k}a_1b_5|10011\rangle + (-1)^{j+k}a_0b_6|00101\rangle + (-1)^{m+j+k}a_1b_6|10100\rangle$$
$$+ (-1)^{i+j+k}a_0b_7|00111\rangle + (-1)^{m+i+j+k}a_1b_7|10110\rangle)_{23459}$$
$$+ |\Psi^m\rangle_{A_21}|\Phi^i\rangle_{B_38}|\Phi^j\rangle_{B_27}|\Psi^k\rangle_{B_16}(a_0b_0|11000\rangle + (-1)^m a_1b_0|01001\rangle + (-1)^i a_0b_1|11010\rangle$$
$$+ (-1)^{i+m}a_1b_1|01011\rangle + (-1)^j a_0b_2|11101\rangle + (-1)^{m+j}a_1b_2|011100\rangle + (-1)^{i+j}a_0b_3|11111\rangle$$
$$+ (-1)^{m+i+j}a_1b_3|01110\rangle + (-1)^k a_0b_4|10001\rangle + (-1)^{m+k}a_1b_4|00000\rangle + (-1)^{i+k}a_0b_5$$
$$|10011\rangle + (-1)^{m+i+k}a_1b_5|00010\rangle + (-1)^{j+k}a_0b_6|10100\rangle + (-1)^{m+j+k}a_1b_6|00101\rangle$$
$$+ (-1)^{i+j+k}a_0b_7|10110\rangle + (-1)^{m+i+j+k}a_1b_7|00111\rangle)_{23459}$$
$$+ |\Phi^m\rangle_{A_21}|\Phi^i\rangle_{B_38}|\Psi^j\rangle_{B_27}|\Phi^k\rangle_{B_16}(a_0b_0|00101\rangle + (-1)^m a_1b_0|10100\rangle + (-1)^i a_0b_1|00111\rangle$$
$$+ (-1)^{i+m}a_1b_1|10110\rangle + (-1)^j a_0b_2|00000\rangle + (-1)^{m+j}a_1b_2|10001\rangle + (-1)^{i+j}a_0b_3|00010\rangle$$
$$+ (-1)^{m+i+j}a_1b_3|10011\rangle + (-1)^k a_0b_4|01100\rangle + (-1)^{m+k}a_1b_4|11101\rangle + (-1)^{i+k}a_0b_5$$
$$|01110\rangle + (-1)^{m+i+k}a_1b_5|11111\rangle + (-1)^{j+k}a_0b_6|01001\rangle + (-1)^{m+j+k}a_1b_6|11000\rangle$$
$$+ (-1)^{i+j+k}a_0b_7|01011\rangle + (-1)^{m+i+j+k}a_1b_7|11010\rangle)_{23459}$$
$$+ |\Psi^m\rangle_{A_21}|\Phi^i\rangle_{B_38}|\Psi^j\rangle_{B_27}|\Phi^k\rangle_{B_16}(a_0b_0|10100\rangle + (-1)^m a_1b_0|00101\rangle + (-1)^i a_0b_1|10110\rangle$$
$$+ (-1)^{i+m}a_1b_1|00111\rangle + (-1)^j a_0b_2|10001\rangle + (-1)^{m+j}a_1b_2|00000\rangle + (-1)^{i+j}a_0b_3|10011\rangle$$
$$+ (-1)^{m+i+j}a_1b_3|00010\rangle + (-1)^k a_0b_4|11101\rangle + (-1)^{m+k}a_1b_4|01100\rangle + (-1)^{i+k}a_0b_5$$
$$|11111\rangle + (-1)^{m+i+k}a_1b_5|01110\rangle + (-1)^{j+k}a_0b_6|11000\rangle + (-1)^{m+j+k}a_1b_6|01001\rangle$$
$$+ (-1)^{i+j+k}a_0b_7|11010\rangle + (-1)^{m+i+j+k}a_1b_7|01011\rangle)_{23459}$$
$$+ |\Phi^m\rangle_{A_21}|\Psi^i\rangle_{B_38}|\Phi^j\rangle_{B_27}|\Phi^k\rangle_{B_16}(a_0b_0|00010\rangle + (-1)^m a_1b_0|10011\rangle + (-1)^i a_0b_1|00000\rangle$$
$$+ (-1)^{i+m}a_1b_1|10001\rangle + (-1)^j a_0b_2|00111\rangle + (-1)^{m+j}a_1b_2|10110\rangle + (-1)^{i+j}a_0b_3|00101\rangle$$
$$+ (-1)^{m+i+j}a_1b_3|10100\rangle + (-1)^k a_0b_4|01011\rangle + (-1)^{m+k}a_1b_4|11010\rangle + (-1)^{i+k}a_0b_5$$
$$|01001\rangle + (-1)^{m+i+k}a_1b_5|11000\rangle + (-1)^{j+k}a_0b_6|01110\rangle + (-1)^{m+j+k}a_1b_6|11111\rangle$$
$$+ (-1)^{i+j+k}a_0b_7|01100\rangle + (-1)^{m+i+j+k}a_1b_7|11101\rangle)_{23459}$$
$$+ |\Psi^m\rangle_{A_21}|\Psi^i\rangle_{B_38}|\Phi^j\rangle_{B_27}|\Phi^k\rangle_{B_16}(a_0b_0|10011\rangle + (-1)^m a_1b_0|00010\rangle + (-1)^i a_0b_1|10001\rangle$$
$$+ (-1)^{i+m}a_1b_1|00000\rangle + (-1)^j a_0b_2|10110\rangle + (-1)^{m+j}a_1b_2|00111\rangle + (-1)^{i+j}a_0b_3|10100\rangle$$
$$+ (-1)^{m+i+j}a_1b_3|00101\rangle + (-1)^k a_0b_4|11010\rangle + (-1)^{m+k}a_1b_4|01011\rangle + (-1)^{i+k}a_0b_5$$

$$
\begin{aligned}
&|11000\rangle + (-1)^{m+i+k} a_1 b_5 |01001\rangle + (-1)^{j+k} a_0 b_6 |11111\rangle + (-1)^{m+j+k} a_1 b_6 |01110\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |11101\rangle + (-1)^{m+i+j+k} a_1 b_7 |01100\rangle)_{23459} \\
&+ |\Phi^m\rangle_{A_2 1} |\Phi^i\rangle_{B_3 8} |\Psi^j\rangle_{B_2 7} |\Psi^k\rangle_{B_1 6} (a_0 b_0 |01100\rangle + (-1)^m a_1 b_0 |11101\rangle + (-1)^i a_0 b_1 |01110\rangle \\
&+ (-1)^{i+m} a_1 b_1 |11111\rangle + (-1)^j a_0 b_2 |01001\rangle + (-1)^{m+j} a_1 b_2 |11000\rangle + (-1)^{i+j} a_0 b_3 |01011\rangle \\
&+ (-1)^{m+i+j} a_1 b_3 |11010\rangle + (-1)^k a_0 b_4 |00101\rangle + (-1)^{m+k} a_1 b_4 |10100\rangle + (-1)^{i+k} a_0 b_5 \\
&|00111\rangle + (-1)^{m+i+k} a_1 b_5 |10110\rangle + (-1)^{j+k} a_0 b_6 |00000\rangle + (-1)^{m+j+k} a_1 b_6 |10001\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |00010\rangle + (-1)^{m+i+j+k} a_1 b_7 |10011\rangle)_{23459} \\
&+ |\Psi^m\rangle_{A_2 1} |\Phi^i\rangle_{B_3 8} |\Psi^j\rangle_{B_2 7} |\Psi^k\rangle_{B_1 6} (a_0 b_0 |11101\rangle + (-1)^m a_1 b_0 |01100\rangle + (-1)^i a_0 b_1 |11111\rangle \\
&+ (-1)^{i+m} a_1 b_1 |01110\rangle + (-1)^j a_0 b_2 |11000\rangle + (-1)^{m+j} a_1 b_2 |01001\rangle + (-1)^{i+j} a_0 b_3 |11010\rangle \\
&+ (-1)^{m+i+j} a_1 b_3 |01011\rangle + (-1)^k a_0 b_4 |10100\rangle + (-1)^{m+k} a_1 b_4 |00101\rangle + (-1)^{i+k} a_0 b_5 \\
&|10110\rangle + (-1)^{m+i+k} a_1 b_5 |00111\rangle + (-1)^{j+k} a_0 b_6 |10001\rangle + (-1)^{m+j+k} a_1 b_6 |00000\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |10011\rangle + (-1)^{m+i+j+k} a_1 b_7 |00010\rangle)_{23459} \\
&+ |\Phi^m\rangle_{A_2 1} |\Psi^i\rangle_{B_3 8} |\Phi^j\rangle_{B_2 7} |\Psi^k\rangle_{B_1 6} (a_0 b_0 |01011\rangle + (-1)^m a_1 b_0 |11010\rangle + (-1)^i a_0 b_1 |01001\rangle \\
&+ (-1)^{i+m} a_1 b_1 |11000\rangle + (-1)^j a_0 b_2 |01110\rangle + (-1)^{m+j} a_1 b_2 |11111\rangle + (-1)^{i+j} a_0 b_3 |01100\rangle \\
&+ (-1)^{m+i+j} a_1 b_3 |11101\rangle + (-1)^k a_0 b_4 |00010\rangle + (-1)^{m+k} a_1 b_4 |10011\rangle + (-1)^{i+k} a_0 b_5 \\
&|00000\rangle + (-1)^{m+i+k} a_1 b_5 |10001\rangle + (-1)^{j+k} a_0 b_6 |00111\rangle + (-1)^{m+j+k} a_1 b_6 |10110\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |00101\rangle + (-1)^{m+i+j+k} a_1 b_7 |10100\rangle)_{23459} \\
&+ |\Psi^m\rangle_{A_2 1} |\Psi^i\rangle_{B_3 8} |\Phi^j\rangle_{B_2 7} |\Psi^k\rangle_{B_1 6} (a_0 b_0 |11010\rangle + (-1)^m a_1 b_0 |01011\rangle + (-1)^i a_0 b_1 |11000\rangle \\
&+ (-1)^{i+m} a_1 b_1 |01001\rangle + (-1)^j a_0 b_2 |11111\rangle + (-1)^{m+j} a_1 b_2 |01110\rangle + (-1)^{i+j} a_0 b_3 |11101\rangle \\
&+ (-1)^{m+i+j} a_1 b_3 |01100\rangle + (-1)^k a_0 b_4 |10011\rangle + (-1)^{m+k} a_1 b_4 |00010\rangle + (-1)^{i+k} a_0 b_5 \\
&|10001\rangle + (-1)^{m+i+k} a_1 b_5 |00000\rangle + (-1)^{j+k} a_0 b_6 |10110\rangle + (-1)^{m+j+k} a_1 b_6 |00111\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |10100\rangle + (-1)^{m+i+j+k} a_1 b_7 |00101\rangle)_{23459} \\
&+ |\Phi^m\rangle_{A_2 1} |\Psi^i\rangle_{B_3 8} |\Psi^j\rangle_{B_2 7} |\Phi^k\rangle_{B_1 6} (a_0 b_0 |00111\rangle + (-1)^m a_1 b_0 |10110\rangle + (-1)^i a_0 b_1 |00101\rangle \\
&+ (-1)^{i+m} a_1 b_1 |10100\rangle + (-1)^j a_0 b_2 |00010\rangle + (-1)^{m+j} a_1 b_2 |10011\rangle + (-1)^{i+j} a_0 b_3 |00000\rangle \\
&+ (-1)^{m+i+j} a_1 b_3 |10001\rangle + (-1)^k a_0 b_4 |01110\rangle + (-1)^{m+k} a_1 b_4 |11111\rangle + (-1)^{i+k} a_0 b_5 \\
&|01100\rangle + (-1)^{m+i+k} a_1 b_5 |11101\rangle + (-1)^{j+k} a_0 b_6 |01011\rangle + (-1)^{m+j+k} a_1 b_6 |11010\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |01001\rangle + (-1)^{m+i+j+k} a_1 b_7 |11000\rangle)_{23459} \\
&+ |\Psi^m\rangle_{A_2 1} |\Psi^i\rangle_{B_3 8} |\Psi^j\rangle_{B_2 7} |\Phi^k\rangle_{B_1 6} (a_0 b_0 |10110\rangle + (-1)^m a_1 b_0 |00111\rangle + (-1)^i a_0 b_1 |10100\rangle \\
&+ (-1)^{i+m} a_1 b_1 |00101\rangle + (-1)^j a_0 b_2 |10011\rangle + (-1)^{m+j} a_1 b_2 |00010\rangle + (-1)^{i+j} a_0 b_3 |10001\rangle \\
&+ (-1)^{m+i+j} a_1 b_3 |00000\rangle + (-1)^k a_0 b_4 |11111\rangle + (-1)^{m+k} a_1 b_4 |01110\rangle + (-1)^{i+k} a_0 b_5 \\
&|11101\rangle + (-1)^{m+i+k} a_1 b_5 |01100\rangle + (-1)^{j+i} a_0 b_6 |11010\rangle + (-1)^{m+j+k} a_1 b_6 |01011\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |11000\rangle + (-1)^{m+i+j+k} a_1 b_7 |01001\rangle)_{23459} \\
&+ |\Phi^m\rangle_{A_2 1} |\Psi^i\rangle_{B_3 8} |\Psi^j\rangle_{B_2 7} |\Psi^k\rangle_{B_1 6} (a_0 b_0 |01110\rangle + (-1)^m a_1 b_0 |11111\rangle + (-1)^i a_0 b_1 |01100\rangle \\
&+ (-1)^{i+m} a_1 b_1 |11101\rangle + (-1)^j a_0 b_2 |01011\rangle + (-1)^{m+j} a_1 b_2 |11010\rangle + (-1)^{i+j} a_0 b_3 |01001\rangle \\
&+ (-1)^{m+i+j} a_1 b_3 |11000\rangle + (-1)^k a_0 b_4 |00111\rangle + (-1)^{m+k} a_1 b_4 |10110\rangle + (-1)^{i+k} a_0 b_5 \\
&|00101\rangle + (-1)^{m+i+k} a_1 b_5 |10100\rangle + (-1)^{j+k} a_0 b_6 |00010\rangle + (-1)^{m+j+k} a_1 b_6 |10011\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |00000\rangle + (-1)^{m+i+j+k} a_1 b_7 |10001\rangle)_{23459} \\
&+ |\Psi^m\rangle_{A_2 1} |\Psi^i\rangle_{B_3 8} |\Psi^j\rangle_{B_2 7} |\Psi^k\rangle_{B_1 6} (a_0 b_0 |11111\rangle + (-1)^m a_1 b_0 |01110\rangle + (-1)^i a_0 b_1 |11101\rangle \\
&+ (-1)^{i+m} a_1 b_1 |01100\rangle + (-1)^j a_0 b_2 |11010\rangle + (-1)^{m+j} a_1 b_2 |01011\rangle + (-1)^{i+j} a_0 b_3 |11000\rangle \\
&+ (-1)^{m+i+j} a_1 b_3 |01001\rangle + (-1)^k a_0 b_4 |10110\rangle + (-1)^{m+k} a_1 b_4 |00111\rangle + (-1)^{i+k} a_0 b_5 \\
&|10100\rangle + (-1)^{m+i+k} a_1 b_5 |00101\rangle + (-1)^{j+k} a_0 b_6 |10011\rangle + (-1)^{m+j+k} a_1 b_6 |00010\rangle \\
&+ (-1)^{i+j+k} a_0 b_7 |10001\rangle + (-1)^{m+i+j+k} a_1 b_7 |00000\rangle)_{23459}]
\end{aligned}
\tag{2.4}
$$

where $m,\ i,\ j,\ k = 0,\ 1$, and $|\Phi^0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),\ |\Phi^1\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),\ |\Psi^0\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),\ |\Psi^1\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. The Equation (4) shows that the quantum system will collapse to one of the two hundred and fifty-six possible states with equal probability when Alice and Bob have finished their measurements on their own particles.

For example, we assume that the measurement result of Alice is $|\Psi^1\rangle_{A_21}$ and the outcomes of Bob's measurements are $|\Phi^0\rangle_{B_38},\ |\Phi^1\rangle_{B_27}$ and $|\Phi^0\rangle_{B_16}$. Then, the quantum state held by Alice, Bob and Charlie collapse into the following state

$$\begin{aligned}|\Omega\rangle = (&a_0b_0|10001\rangle - a_1b_0|00000\rangle + a_0b_1|10011\rangle - a_1b_1|00010\rangle - a_0b_2|10100\rangle + a_1b_2\\ &|00101\rangle - a_0b_3|10110\rangle + a_1b_3|00111\rangle + a_0b_4|11000\rangle - a_1b_4|01001\rangle + a_0b_5|11010\rangle\\ &- a_1b_5|01011\rangle - a_0b_6|11101\rangle + a_1b_6|01100\rangle - a_0b_7|11111\rangle + a_1b_7|01110\rangle)_{23459}\end{aligned} \tag{2.5}$$

Step 3. Charlie, as a controller in scheme, could do nothing on her particle 9 to interminate this quantum communicating works if she does not allow Alice and Bob to teleport their quantum information for some secure causes. Otherwise, she has to take a single-qubit measurement on particle 9 under the basis $\{|+\rangle,\ |-\rangle\}$ and broadcasts her measurement result via a classical channel too. Let

$$\begin{aligned}\alpha_0^{\pm} =&(a_0|0\rangle \pm (-1)^m a_1|1\rangle)_2; \qquad \alpha_1^{\pm} = (a_0|1\rangle \pm (-1)^m a_1|0\rangle)_2\\ \beta_0^{\pm} =&(b_0|000\rangle + (-1)^i b_1|001\rangle \pm (-1)^j b_2|010\rangle \pm (-1)^{i+j} b_3|011\rangle \pm (-1)^k\\ &b_4|100\rangle \pm (-1)^{i+k} b_5|101\rangle + (-1)^{j+k} b_6|110\rangle + (-1)^{i+j+k} b_7|111\rangle)_{345}\\ \beta_1^{\pm} =&(b_0|100\rangle + (-1)^i b_1|101\rangle \pm (-1)^j b_2|110\rangle \pm (-1)^{i+j} b_3|111\rangle \pm (-1)^k\\ &b_4|000\rangle \pm (-1)^{i+k} b_5|001\rangle + (-1)^{j+k} b_6|010\rangle + (-1)^{i+j+k} b_7|011\rangle)_{345}\\ \beta_2^{\pm} =&(b_0|010\rangle + (-1)^i b_1|011\rangle \pm (-1)^j b_2|000\rangle \pm (-1)^{i+j} b_3|001\rangle \pm (-1)^k\\ &b_4|110\rangle \pm (-1)^{i+k} b_5|111\rangle + (-1)^{j+k} b_6|100\rangle + (-1)^{i+j+k} b_7|101\rangle)_{345}\\ \beta_3^{\pm} =&(b_0|001\rangle + (-1)^i b_1|000\rangle \pm (-1)^j b_2|011\rangle \pm (-1)^{i+j} b_3|010\rangle \pm (-1)^k\\ &b_4|101\rangle \pm (-1)^{i+k} b_5|100\rangle + (-1)^{j+k} b_6|111\rangle + (-1)^{i+j+k} b_7|110\rangle)_{345}\\ \beta_4^{\pm} =&(b_0|110\rangle + (-1)^i b_1|111\rangle \pm (-1)^j b_2|100\rangle \pm (-1)^{i+j} b_3|101\rangle \pm (-1)^k\\ &b_4|010\rangle \pm (-1)^{i+k} b_5|011\rangle + (-1)^{j+k} b_6|000\rangle + (-1)^{i+j+k} b_7|001\rangle)_{345}\\ \beta_5^{\pm} =&(b_0|101\rangle + (-1)^i b_1|100\rangle \pm (-1)^j b_2|111\rangle \pm (-1)^{i+j} b_3|110\rangle \pm (-1)^k\\ &b_4|001\rangle \pm (-1)^{i+k} b_5|000\rangle + (-1)^{j+k} b_6|011\rangle + (-1)^{i+j+k} b_7|010\rangle)_{345}\\ \beta_6^{\pm} =&(b_0|011\rangle + (-1)^i b_1|010\rangle \pm (-1)^j b_2|001\rangle \pm (-1)^{i+j} b_3|000\rangle \pm (-1)^k\\ &b_4|111\rangle \pm (-1)^{i+k} b_5|110\rangle + (-1)^{j+k} b_6|101\rangle + (-1)^{i+j+k} b_7|100\rangle)_{345}\\ \beta_7^{\pm} =&(b_0|111\rangle + (-1)^i b_1|110\rangle \pm (-1)^j b_2|101\rangle \pm (-1)^{i+j} b_3|100\rangle \pm (-1)^k\\ &b_4|011\rangle \pm (-1)^{i+k} b_5|010\rangle + (-1)^{j+k} b_6|001\rangle + (-1)^{i+j+k} b_7|000\rangle)_{345},\end{aligned} \tag{2.6}$$

the equation (4) can be rewritten into the following simple form

$$
\begin{aligned}
|\Omega\rangle = \frac{1}{16}[&|\Phi^m\rangle_{A_21}|\Phi^i\rangle_{B_38}|\Phi^j\rangle_{B_27}|\Psi^k\rangle_{B_16}(|+\rangle_9\alpha_0^+\otimes\beta_0^+ + |-\rangle_9\alpha_0^-\otimes\beta_0^-)\\
&+|\Psi^m\rangle_{A_21}|\Phi^i\rangle_{B_39}|\Phi^j\rangle_{B_28}|\Phi^k\rangle_{B_17}(|+\rangle_9\alpha_1^+\otimes\beta_0^+ + |-\rangle_9\alpha_1^-\otimes\beta_0^-)\\
&+|\Phi^m\rangle_{A_21}|\Phi^i\rangle_{B_39}|\Phi^j\rangle_{B_28}|\Psi^k\rangle_{B_17}(|+\rangle_9\alpha_0^+\otimes\beta_1^+ - |-\rangle_9\alpha_0^-\otimes\beta_1^-)\\
&+|\Psi^m\rangle_{A_21}|\Phi^i\rangle_{B_39}|\Phi^j\rangle_{B_28}|\Psi^k\rangle_{B_17}(|+\rangle_9\alpha_1^+\otimes\beta_1^+ + |-\rangle_9\alpha_1^-\otimes\beta_1^-)\\
&+|\Phi^m\rangle_{A_21}|\Phi^i\rangle_{B_39}|\Psi^j\rangle_{B_28}|\Phi^k\rangle_{B_17}(|+\rangle_9\alpha_0^+\otimes\beta_2^+ - |-\rangle_9\alpha_0^-\otimes\beta_2^-)\\
&+|\Psi^m\rangle_{A_21}|\Phi^i\rangle_{B_39}|\Psi^j\rangle_{B_28}|\Phi^k\rangle_{B_17}(|+\rangle_9\alpha_1^+\otimes\beta_2^+ + |-\rangle_9\alpha_1^-\otimes\beta_2^-)\\
&+|\Phi^m\rangle_{A_21}|\Psi^i\rangle_{B_39}|\Phi^j\rangle_{B_28}|\Phi^k\rangle_{B_17}(|+\rangle_9\alpha_0^+\otimes\beta_3^+ + |-\rangle_9\alpha_0^-\otimes\beta_3^-)\\
&+|\Psi^m\rangle_{A_21}|\Psi^i\rangle_{B_39}|\Phi^j\rangle_{B_28}|\Phi^k\rangle_{B_17}(|+\rangle_9\alpha_1^+\otimes\beta_3^+ - |-\rangle_9\alpha_1^-\otimes\beta_3^-)\\
&+|\Phi^m\rangle_{A_21}|\Phi^i\rangle_{B_39}|\Psi^j\rangle_{B_28}|\Psi^k\rangle_{B_17}(|+\rangle_9\alpha_0^+\otimes\beta_4^+ + |-\rangle_9\alpha_0^-\otimes\beta_4^-)\\
&+|\Psi^m\rangle_{A_21}|\Phi^i\rangle_{B_39}|\Psi^j\rangle_{B_28}|\Psi^k\rangle_{B_17}(|+\rangle_9\alpha_1^+\otimes\beta_4^+ - |-\rangle_9\alpha_1^-\otimes\beta_4^-)\\
&+|\Phi^m\rangle_{A_21}|\Psi^i\rangle_{B_39}|\Phi^j\rangle_{B_28}|\Psi^k\rangle_{B_17}(|+\rangle_9\alpha_0^+\otimes\beta_5^+ - |-\rangle_9\alpha_0^-\otimes\beta_5^-)\\
&+|\Psi^m\rangle_{A_21}|\Psi^i\rangle_{B_39}|\Phi^j\rangle_{B_28}|\Psi^k\rangle_{B_17}(|+\rangle_9\alpha_1^+\otimes\beta_5^+ + |-\rangle_9\alpha_1^-\otimes\beta_5^-)\\
&+|\Phi^m\rangle_{A_21}|\Psi^i\rangle_{B_39}|\Psi^j\rangle_{B_28}|\Phi^k\rangle_{B_17}(|+\rangle_9\alpha_0^+\otimes\beta_6^+ - |-\rangle_9\alpha_0^-\otimes\beta_6^-)\\
&+|\Psi^m\rangle_{A_21}|\Psi^i\rangle_{B_39}|\Psi^j\rangle_{B_28}|\Phi^k\rangle_{B_17}(|+\rangle_9\alpha_1^+\otimes\beta_6^+ + |-\rangle_9\alpha_1^-\otimes\beta_6^-)\\
&+|\Phi^m\rangle_{A_21}|\Psi^i\rangle_{B_39}|\Psi^j\rangle_{B_28}|\Psi^k\rangle_{B_17}(|+\rangle_9\alpha_0^+\otimes\beta_7^+ + |-\rangle_9\alpha_0^-\otimes\beta_7^-)\\
&+|\Psi^m\rangle_{A_21}|\Psi^i\rangle_{B_39}|\Psi^j\rangle_{B_28}|\Psi^k\rangle_{B_17}(|+\rangle_9\alpha_1^+\otimes\beta_7^+ - |-\rangle_9\alpha_1^-\otimes\beta_7^-)].
\end{aligned}
\tag{2.7}
$$

where $m,\ i,\ j,\ k = 0,\ 1$, and $|\Phi^0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),\ |\Phi^1\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),\ |\Psi^0\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),\ |\Psi^1\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. The equatio[illegible] shows that the system state held by Alice and Bob collapses to a tensor product of α and β when three agents have finished their operations on their own particles.

Step 4. Firstly, Alice and Bob perform some appropriate Pauli operations on their own particles to restruct quantum information. Next, Alice produces a auxiliary qubit ($|0\rangle_{Au}$) and then performs a CNOT operation on her two qubits with qubit 2 as control qubit and auxiliary qubit as targat qubit. Now, Alice and Bob both recover the quantum information from the other one in the end.

Let back to the example above. Suppose Alice and Bob's measurement results are $|\Psi^1\rangle_{A_21},\ |\Phi^0\rangle_{B_38},\ |\Phi^1\rangle_{B_27},\ |\Phi^0\rangle_{B_16}$ and $|+\rangle_9$. Then, the finial state of quantum system held by Alice and Bob is

$$
\begin{aligned}
|\Omega\rangle =&(a_0|1\rangle + a_1|0\rangle)_2 \otimes (b_0|000\rangle + b_1|001\rangle - b_2|010\rangle\\
&- b_3|011\rangle + b_4|100\rangle + b_5|101\rangle - b_6|110\rangle - b_7|111\rangle)_{345}
\end{aligned}
\tag{2.8}
$$

According to the measurement results informed from the other users, Bob performs a unitary operation X_2 and Alice performs a unitary operation $I_4 \otimes Z_5 \otimes I_6$ on their own qubits and Bob restructs the information from Alice. In this time, The state held by Alice is $|\psi\rangle_{2,Au} = a_0|00\rangle + a_1|10\rangle$. When Alice perfroms a CNOT operation on her own two qubits, she also restructs the information form Bob too.

On the other sides, if Charlie does not allow Alice and Bob to communicate their quantum information and then does nothing on her particle 9. But, Alice did not know what was happened in time and finished her operations in step 2. The quantum system state held by them is

$$
\begin{aligned}
|\Omega\rangle = & \frac{1}{\sqrt{2}} a_0 |1\rangle_2 (b_0|0001\rangle + b_1|0011\rangle - b_2|0100\rangle - b_3|0110\rangle + b_4|1000\rangle \\
& + b_5|1010\rangle - b_6|1101\rangle - b_7|1111\rangle)_{3459} \\
& - \frac{1}{\sqrt{2}} a_1 |0\rangle_2 (b_0|0000\rangle + b_1|0010\rangle - b_2|0101\rangle - b_3|0111\rangle + b_4|1001\rangle \\
& + b_5|1011\rangle - b_6|1100\rangle - b_7|1110\rangle)_{3459}
\end{aligned} \tag{2.9}
$$

It is easy to see that the particle 9 is entangled with the other five qubits 2,3,4 and 5. In a word, Alice and Bob could not get anything about the quantum information without Charlie's help.

Obviously, we can always choice appropriate operations to recover quantum information if measurement results are different from the result in the example above.

3 Summary

Tab 1 Comparison of the four schemes

SC	QC	NO	CRC	AQ	TQIT	η
[28]	Eleven-Q	5BM,1SM	11	0	2Q↔ 3Q	5/22
[29]	Five-Q	3BM,1SM,2CN	7	2	2QE↔1Q	3/14
[30]	Nine-Q	2GM,1BM,1SM	9	0	2QE↔3QE	5/18
Our	Nine-Q	4BM,1SM,2CN	9	1	2QE ↔3Q	5/19

In the table, SC denotes the scheme, QC denotes the quantum channel, NO denotes the necessary operations, BM denotes the Bell-state measurement, SM denotes the single-qubit measurement, GM denotes the GHZ-state measurement, CN denotes the CNOT operation, CRC denotes the classical resource consumption, AQ denotes auxiliary qubit, TQIT denotes the type of quantum information transmitted, and QE denotes qubit entangled state, η denotes the intrinsic efficiency.

In this work, we introduce a BCQT scheme by using a nine-qubit entangled state as quantum channel. In this scheme, Alice needs to perform tow CNOT-gate and take a Bell-state measurement, at the same time, Bob needs to take three Bell-state measurements and Charlie takes a single-qubit measurement. The consumption of both quantum and classical resources is 19 bits. We uses the intrinsic efficiency [31] to measure the efficiency of schemes, which is defined as $\eta = \frac{q_s}{q_u+q_t+q_a}$, where q_s is the number of qubits that consist of quantum information to be shared, q_u is the number of the qubit that is used as a channel and q_t is the classical bits transmitted, and q_a is the number of the qubit that is used as auxiliary qubit. The intrinsic efficiency of our scheme reaches $5/19$. Here, we give a comparison of four BCQT schemes in Table 1. The second scheme [29] can teleport two-qubit entangled state and a single-qubit state via five-qubit entangled state as channel, in which users need to perform three Bell state measurements and one single-qubit measurement. The intrinsic efficiency of this scheme is the lowest one. The other three schemes are all invastigated to teleport two-qubit and three-qubit state. The first scheme [28] for transmitting an arbitrary two-qubit state and a three-qubit state, in which they need to perform five Bell-state measurements and one single-qubit measurement, consumes 22 bits and The intrinsic efficiency is 5/22. The third scheme [30] can teleport a two-qubit entangled state and a three-qubit entangled state, they need perform two GHZ state measurements, one Bell state measurement and sing-qubit measurement and consumes 18 bits. The intrinsic efficiency is 5/18 which is the most efficient among four schemes. Our scheme can teleport two-qubit entangled state and arbitrary three-qubit state, in which users need to perform four Bell-state measurements and one single-qubit measurement and consumes 19 bits resource. The intrinsic efficiency is 5/19 which is more efficient than the first and second scheme. However, only Bell state and single-qubit measurements are performed in our scheme, so that our scheme is more simple than the others. In addition, a user in our scheme can transmit an arbitrary three-qubit state, while a user in scheme [30] can only transmit a three-qubit entangled state.

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