The Simulation of Linear Block Algorithm for Modeling Third Order Highly Stiff Problem

Abstract

*In this research, (*remove ⟶ *we and replace it with effort have made to) have proposed the new block method for direct simulation of third order initial value problems without reduction to a system of first order ordinary differential equation, to address the weaknesses in reduction method. The method is derived using the linear block method through interpolation and collocation. The basic properties of the block method were recovered and was found to be consistent, convergent and zero-stability. The new block method is been applied to model third order initial value problems of ordinary differential equations without reducing the equations to their equivalent systems of first order ordinary differential equations. The result obtained on the process on some sampled modeled third order linear problems give better approximation is converges faster to the approximate solutions than the (existing methods which existing method ?, please specify).*

Keywords**:** block method, direct simulation, linear block algorithm, model, reduction.

1 Introduction

Numerical analysis is the area of mathematics which provides convenient methods for modeling mathematical problems and to find out useful material from available solutions which are not expressed in tractable forms. Such problems begin for the most part, from real world applications of algebra, geometry, calculus and they include variables which change consistently.

Conventionally, numerical solutions to third order ordinary differential equations of the form

$y''' = f(x, y, y', y''),\ y(a) = y_0,\ y'(a) = y_1,\ y''(a) = y_2$ move it to literature review (1.1)

are solved by a reduction to a system of first order ordinary differential equation of the form

$y' = f(x, y),\ y(a) = y_0,\ a \le x \le b,\ x, y \in \Re$ move it to literature review (1.2)

Then any appropriate numerical methods would be used to solve the resulting equation. This approach is extensively discussed by scholars such as [1- 5] input all citation [1, 5]. It was noticed that this reduction process has a lots of setbacks such as difficulties in writing computer program for the method, computational burden which affects the accuracy of the method in terms of error and time consuming. In order to overcome these challenges or difficulties in reduction method, we will proposed the direct method.

In predictor-corrector method, an explicit method is usually meant for predictor step while an implicit method for the corrector step. The development of Linear Multistep Method (LMM) through the predictor-corrector mode has been carefully considered by scholars such as in [6-8] among others. This method can only computes the numerical solution at one point at a time.

In order to overcome the difficulties mentioned in predictor-corrector method, block method was developed [9]. This method computes the discrete method at more than one grid point simultaneously. According to [10, 11], the block method was originally proposed by Milne [12] who advocated the use of block as a means of getting a starting value for predictor-corrector algorithm and later adopted as a full method in [13].

Therefore, we will develope a direct method using a block algorithms for solving (1.1) without reduction to (1.2) as suggested in by [14-16].Much and considerable attention have been dedicated to solving higher order ordinary differential equations of the form (1.1) directly without being reduced to system of first order ordinary differential equation. For instance, [17-21] etc. proposed block methods for direct solution of third order ordinary differential equation, the outcome is better when reduced to first order ordinary differential equation.

2 Mathematical Formulation and Methodology (or Method and Materials)

2.1 Mathematical Formulation

The mathematical formulation of the method with six partition shall be described in this section for treating third order initial value problems of the form (1.1). The one-step linear block approach with six partition are obtain from the expression

$$y_{n+\varphi} = \sum_{i=0}^{2} \frac{(\varphi h)^i}{i!} y_n^{(i)} + \sum_{i=0}^{6} \left(\eta_{i\varphi} f_{n+i} + \tau_{i\varphi} g_{n+i} \right), \quad \varphi = \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, 1 \qquad (2.1)$$

Obtaining the first and second derivative schemes of one step block method from

$$y_{n+\varphi}^{(\kappa)} = \sum_{i=0}^{2-\kappa} \frac{(\varphi h)^i}{i!} y_n^{(i+\kappa)} + \sum_{i=0}^{7} \psi_{\varphi i \kappa} f_{n+i}, \ \kappa = 1_{\left(\varphi = \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, 1\right)}, \ \kappa = 2_{\left(\varphi = \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, 1\right)} \qquad (2.2)$$

$\eta_{\varphi i} = Q^{-1} X$ and $\psi_{\varphi i \kappa} = Q^{-1} G$ where that do mean here k=2 here?

$$Q=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \frac{h}{6} & \frac{2h}{6} & \frac{3h}{6} & \frac{4h}{6} & \frac{5h}{6} & h \\ 0 & \frac{\left(\frac{h}{6}\right)^2}{2!} & \frac{\left(\frac{2h}{6}\right)^2}{2!} & \frac{\left(\frac{3h}{6}\right)^2}{2!} & \frac{\left(\frac{4h}{6}\right)^2}{2!} & \frac{\left(\frac{5h}{6}\right)^2}{2!} & \frac{(h)^2}{2!} \\ 0 & \frac{\left(\frac{h}{6}\right)^3}{3!} & \frac{\left(\frac{2h}{6}\right)^3}{3!} & \frac{\left(\frac{3h}{6}\right)^3}{3!} & \frac{\left(\frac{4h}{6}\right)^3}{3!} & \frac{\left(\frac{5h}{6}\right)^3}{3!} & \frac{(h)^3}{3!} \\ 0 & \frac{\left(\frac{h}{6}\right)^4}{4!} & \frac{\left(\frac{2h}{6}\right)^4}{4!} & \frac{\left(\frac{3h}{6}\right)^4}{4!} & \frac{\left(\frac{4h}{6}\right)^4}{4!} & \frac{\left(\frac{5h}{6}\right)^4}{4!} & \frac{(h)^4}{4!} \\ 0 & \frac{\left(\frac{h}{6}\right)^5}{5!} & \frac{\left(\frac{2h}{6}\right)^5}{5!} & \frac{\left(\frac{3h}{6}\right)^5}{5!} & \frac{\left(\frac{4h}{6}\right)^5}{5!} & \frac{\left(\frac{5h}{6}\right)^5}{5!} & \frac{(h)^5}{5!} \\ 0 & \frac{\left(\frac{h}{6}\right)^6}{6!} & \frac{\left(\frac{2h}{6}\right)^6}{6!} & \frac{\left(\frac{3h}{6}\right)^6}{6!} & \frac{\left(\frac{4h}{6}\right)^6}{6!} & \frac{\left(\frac{5h}{6}\right)^6}{6!} & \frac{(h)^6}{6!} \end{pmatrix} \tag{2.3}$$

$$X=\left(\frac{(\varphi h)^3}{3!}\quad \frac{(\varphi h)^4}{4!}\quad \frac{(\varphi h)^5}{5!}\quad \frac{(\varphi h)^6}{6!}\quad \frac{(\varphi h)^7}{7!}\quad \frac{(\varphi h)^8}{8!}\quad \frac{(\varphi h)^9}{9!}\right)^T \tag{2.4}$$

$$G=\left(\frac{(\varphi h)^{3-\kappa}}{(3-\kappa)!}\quad \frac{(\varphi h)^{4-\kappa}}{(4-\kappa)!}\quad \frac{(\varphi h)^{5-\kappa}}{(5-\kappa)!}\quad \frac{(\varphi h)^{6-\kappa}}{(6-\kappa)!}\quad \frac{(\varphi h)^{7-\kappa}}{(7-\kappa)!}\quad \frac{(\varphi h)^{8-\kappa}}{(8-\kappa)!}\quad \frac{(\varphi h)^{9-\kappa}}{(9-\kappa)!}\right)^T \tag{2.5}$$

equation (2.1) and (2.2) can also be written in the following form

$$\left.\begin{aligned}
y_{n+\frac{1}{6}} &= y_n+\frac{h}{6}y'_n+\frac{\left(\frac{h}{6}\right)^2}{2!}y''_n+h^3\left[\eta_{01}f_n+\eta_{11}f_{n+\frac{1}{6}}+\eta_{21}f_{n+\frac{1}{3}}+\eta_{31}f_{n+\frac{1}{2}}+\eta_{41}f_{n+\frac{2}{3}}+\eta_{51}f_{n+\frac{5}{6}}+\eta_{61}f_{n+1}\right]\\
y_{n+\frac{1}{3}} &= y_n+\frac{h}{3}y'_n+\frac{\left(\frac{h}{3}\right)^2}{2!}y''_n+h^3\left[\eta_{02}f_n+\eta_{12}f_{n+\frac{1}{6}}+\eta_{22}f_{n+\frac{1}{3}}+\eta_{32}f_{n+\frac{1}{2}}+\eta_{42}f_{n+\frac{2}{3}}+\eta_{52}f_{n+\frac{5}{6}}+\eta_{62}f_{n+1}\right]\\
y_{n+\frac{1}{2}} &= y_n+\frac{h}{2}y'_n+\frac{\left(\frac{h}{2}\right)^2}{2!}y''_n+h^3\left[\eta_{03}f_n+\eta_{13}f_{n+\frac{1}{6}}+\eta_{23}f_{n+\frac{1}{3}}+\eta_{33}f_{n+\frac{1}{2}}+\eta_{43}f_{n+\frac{2}{3}}+\eta_{53}f_{n+\frac{5}{6}}+\eta_{63}f_{n+1}\right]\\
y_{n+\frac{2}{3}} &= y_n+\frac{2h}{3}y'_n+\frac{\left(\frac{2h}{3}\right)^2}{2!}y''_n+h^3\left[\eta_{04}f_n+\eta_{14}f_{n+\frac{1}{6}}+\eta_{24}f_{n+\frac{1}{3}}+\eta_{34}f_{n+\frac{1}{2}}+\eta_{44}f_{n+\frac{2}{3}}+\eta_{54}f_{n+\frac{5}{6}}+\eta_{64}f_{n+1}\right]\\
y_{n+\frac{5}{6}} &= y_n+\frac{5h}{6}y'_n+\frac{\left(\frac{5h}{6}\right)^2}{2!}y''_n+h^3\left[\eta_{05}f_n+\eta_{15}f_{n+\frac{1}{6}}+\eta_{25}f_{n+\frac{1}{3}}+\eta_{35}f_{n+\frac{1}{2}}+\eta_{45}f_{n+\frac{2}{3}}+\eta_{55}f_{n+\frac{5}{6}}+\eta_{65}f_{n+1}\right]\\
y_{n+1} &= y_n+hy'_n+\frac{(h)^2}{2!}y''_n+h^3\left[\eta_{06}f_n+\eta_{16}f_{n+\frac{1}{6}}+\eta_{26}f_{n+\frac{1}{3}}+\eta_{36}f_{n+\frac{1}{2}}+\eta_{46}f_{n+\frac{2}{3}}+\eta_{56}f_{n+\frac{5}{6}}+\eta_{66}f_{n+1}\right]
\end{aligned}\right\} \tag{2.6}$$

$$\left.\begin{aligned}
y_{n+\frac{1}{6}} &= y_n + \frac{h}{6}y'_n + h^2\left[\psi_{101}f_n + \psi_{111}f_{n+\frac{1}{6}} + \psi_{121}f_{n+\frac{1}{3}} + \psi_{131}f_{n+\frac{1}{2}} + \psi_{141}f_{n+\frac{2}{3}} + \psi_{151}f_{n+\frac{5}{6}} + \psi_{161}f_{n+1}\right] \\
y_{n+\frac{1}{3}} &= y_n + \frac{h}{3}y'_n + h^2\left[\psi_{202}f_n + \psi_{212}f_{n+\frac{1}{6}} + \psi_{222}f_{n+\frac{1}{3}} + \psi_{232}f_{n+\frac{1}{2}} + \psi_{242}f_{n+\frac{2}{3}} + \psi_{252}f_{n+\frac{5}{6}} + \psi_{262}f_{n+1}\right] \\
y_{n+\frac{1}{2}} &= y_n + \frac{h}{2}y'_n + h^2\left[\psi_{303}f_n + \psi_{313}f_{n+\frac{1}{6}} + \psi_{323}f_{n+\frac{1}{3}} + \psi_{333}f_{n+\frac{1}{2}} + \psi_{343}f_{n+\frac{2}{3}} + \psi_{353}f_{n+\frac{5}{6}} + \psi_{363}f_{n+1}\right] \\
y_{n+\frac{2}{3}} &= y_n + \frac{2h}{3}y'_n + h^2\left[\psi_{404}f_n + \psi_{414}f_{n+\frac{1}{6}} + \psi_{424}f_{n+\frac{1}{3}} + \psi_{434}f_{n+\frac{1}{2}} + \psi_{444}f_{n+\frac{2}{3}} + \psi_{454}f_{n+\frac{5}{6}} + \psi_{464}f_{n+1}\right] \\
y_{n+\frac{5}{6}} &= y_n + \frac{5h}{6}y'_n + h^2\left[\psi_{505}f_n + \psi_{515}f_{n+\frac{1}{6}} + \psi_{525}f_{n+\frac{1}{3}} + \psi_{535}f_{n+\frac{1}{2}} + \psi_{545}f_{n+\frac{2}{3}} + \psi_{555}f_{n+\frac{5}{6}} + \psi_{565}f_{n+1}\right] \\
y_{n+1} &= y_n + hy'_n + h^2\left[\psi_{606}f_n + \psi_{616}f_{n+\frac{1}{6}} + \psi_{626}f_{n+\frac{1}{3}} + \psi_{636}f_{n+\frac{1}{2}} + \psi_{646}f_{n+\frac{2}{3}} + \psi_{656}f_{n+\frac{5}{6}} + \psi_{666}f_{n+1}\right]
\end{aligned}\right\} \quad (2.7)$$

and

$$\left.\begin{aligned}
y'_{n+\frac{1}{6}} &= y'_n + h\left[\psi_{1101}f_n + \psi_{1111}f_{n+\frac{1}{6}} + \psi_{1121}f_{n+\frac{1}{3}} + \psi_{1131}f_{n+\frac{1}{2}} + \psi_{1141}f_{n+\frac{2}{3}} + \psi_{1151}f_{n+\frac{5}{6}} + \psi_{1161}f_{n+1}\right] \\
y'_{n+\frac{1}{3}} &= y'_n + h\left[\psi_{2202}f_n + \psi_{2212}f_{n+\frac{1}{6}} + \psi_{2222}f_{n+\frac{1}{3}} + \psi_{2232}f_{n+\frac{1}{2}} + \psi_{2242}f_{n+\frac{2}{3}} + \psi_{2252}f_{n+\frac{5}{6}} + \psi_{2262}f_{n+1}\right] \\
y'_{n+\frac{1}{2}} &= y'_n + h\left[\psi_{3303}f_n + \psi_{3313}f_{n+\frac{1}{6}} + \psi_{3323}f_{n+\frac{1}{3}} + \psi_{3333}f_{n+\frac{1}{2}} + \psi_{3343}f_{n+\frac{2}{3}} + \psi_{3353}f_{n+\frac{5}{6}} + \psi_{3363}f_{n+1}\right] \\
y'_{n+\frac{2}{3}} &= y'_n + h\left[\psi_{4404}f_n + \psi_{4414}f_{n+\frac{1}{6}} + \psi_{4424}f_{n+\frac{1}{3}} + \psi_{4434}f_{n+\frac{1}{2}} + \psi_{4444}f_{n+\frac{2}{3}} + \psi_{4454}f_{n+\frac{5}{6}} + \psi_{4464}f_{n+1}\right] \\
y'_{n+\frac{5}{6}} &= y'_n + h\left[\psi_{5505}f_n + \psi_{5515}f_{n+\frac{1}{6}} + \psi_{5525}f_{n+\frac{1}{3}} + \psi_{5535}f_{n+\frac{1}{2}} + \psi_{5545}f_{n+\frac{2}{3}} + \psi_{5555}f_{n+\frac{5}{6}} + \psi_{5565}f_{n+1}\right] \\
y'_{n+1} &= y'_n + h\left[\psi_{6606}f_n + \psi_{6616}f_{n+\frac{1}{6}} + \psi_{6626}f_{n+\frac{1}{3}} + \psi_{6636}f_{n+\frac{1}{2}} + \psi_{6646}f_{n+\frac{2}{3}} + \psi_{6656}f_{n+\frac{5}{6}} + \psi_{6666}f_{n+1}\right]
\end{aligned}\right\} \quad (2.8)$$

To obtain the unknown coefficients η, it is defined that $\eta_{ki} = Q^{-1}X$ where Q and X are given in (2.3) and (2.4). The coefficients are

$$(\eta_{01},\eta_{11},\eta_{21},\eta_{31},\eta_{41},\eta_{51},\eta_{61})=\left(\frac{343801}{78382080},\frac{6031}{9331200},-\frac{32981}{52254720},\frac{5177}{9797760},-\frac{15107}{52254720},\frac{5947}{6531840},-\frac{9809}{78382080}\right)$$

$$(\eta_{02},\eta_{12},\eta_{22},\eta_{32},\eta_{42},\eta_{52},\eta_{62})=\left(\frac{6887}{3061800},\frac{1499}{255150},-\frac{233}{58320},\frac{52}{15309},-\frac{379}{204120},\frac{149}{255150},-\frac{491}{6123600}\right)$$

$$(\eta_{03},\eta_{13},\eta_{23},\eta_{33},\eta_{43},\eta_{53},\eta_{63})=\left(\frac{1959}{358400},\frac{1599}{89600},-\frac{537}{71680},\frac{1}{120},-\frac{327}{71680},\frac{129}{89600},-\frac{71}{358400}\right)$$

$$(\eta_{04},\eta_{14},\eta_{24},\eta_{34},\eta_{44},\eta_{54},\eta_{64})=\left(\frac{3863}{382725},\frac{4664}{127575},-\frac{226}{25515},\frac{272}{15309},-\frac{31}{3645},\frac{344}{127575},-\frac{142}{382725}\right)$$

$$(\eta_{05},\eta_{15},\eta_{25},\eta_{35},\eta_{45},\eta_{55},\eta_{65})=\left(\frac{505625}{31352832},\frac{162125}{2612736},-\frac{85625}{10450944},\frac{66875}{373248},-\frac{119375}{10450944},\frac{1625}{373248},-\frac{18625}{31352832}\right)$$

$$(\eta_{06},\eta_{16},\eta_{26},\eta_{36},\eta_{46},\eta_{56},\eta_{66})=\left(\frac{33}{1400},\frac{33}{350},-\frac{3}{560},\frac{2}{35},-\frac{3}{280},\frac{3}{350},-\frac{1}{1200}\right)$$

$$(\psi_{101},\psi_{111},\psi_{121},\psi_{131},\psi_{141},\psi_{151},\psi_{161})=\left(\frac{28549}{4354560},\frac{275}{20736},-\frac{5717}{483840},\frac{10621}{1088640},-\frac{7703}{1451520},\frac{403}{241920},-\frac{199}{870912}\right)$$

$$(\psi_{202},\psi_{212},\psi_{222},\psi_{232},\psi_{242},\psi_{252},\psi_{262})=\left(\frac{1027}{68040},\frac{97}{1890},-\frac{2}{81},\frac{197}{8505},-\frac{97}{7560},\frac{23}{5670},-\frac{19}{34020}\right)$$

$$(\psi_{303},\psi_{313},\psi_{323},\psi_{333},\psi_{343},\psi_{353},\psi_{363})=\left(\frac{253}{10752},\frac{165}{1792},-\frac{267}{17920},\frac{5}{128},-\frac{363}{17920},\frac{57}{8960},-\frac{47}{53760}\right)$$

$$(\psi_{404},\psi_{414},\psi_{424},\psi_{434},\psi_{444},\psi_{454},\psi_{464})=\left(\frac{272}{8505},\frac{376}{2835},-\frac{2}{945},\frac{656}{8505},-\frac{2}{81},\frac{8}{945},-\frac{2}{1701}\right)$$

$$(\psi_{505},\psi_{515},\psi_{525},\psi_{535},\psi_{545},\psi_{555},\psi_{565})=\left(\frac{35225}{870912},\frac{8375}{48384},\frac{3125}{290304},\frac{25625}{217728},-\frac{625}{96768},\frac{275}{20736},-\frac{1375}{870912}\right)$$

$$(\psi_{606},\psi_{616},\psi_{626},\psi_{636},\psi_{646},\psi_{656},\psi_{666})=\left(\frac{41}{840},\frac{3}{14},\frac{3}{140},\frac{17}{105},\frac{3}{280},\frac{3}{70},0\right)$$

$$(\psi_{110},\psi_{111},\psi_{112},\psi_{113},\psi_{114},\psi_{115},\psi_{116})=\left(\frac{19087}{362880},\frac{2713}{15120},-\frac{15487}{120960},\frac{293}{2835},-\frac{6737}{120960},\frac{263}{15120},-\frac{863}{362880}\right)$$

$$(\psi_{220},\psi_{221},\psi_{222},\psi_{223},\psi_{224},\psi_{225},\psi_{226})=\left(\frac{1139}{22680},\frac{47}{189},\frac{11}{7560},\frac{166}{2835},-\frac{269}{7560},\frac{11}{945},-\frac{37}{22680}\right)$$

$$(\psi_{330},\psi_{331},\psi_{332},\psi_{333},\psi_{334},\psi_{335},\psi_{336})=\left(\frac{137}{2688},\frac{27}{112},\frac{387}{4480},\frac{17}{105},-\frac{243}{4480},\frac{9}{560},-\frac{29}{13440}\right)$$

$$(\psi_{440},\psi_{441},\psi_{442},\psi_{443},\psi_{444},\psi_{445},\psi_{446})=\left(\frac{143}{2835},\frac{232}{945},\frac{64}{945},\frac{752}{2835},\frac{29}{945},\frac{8}{945},-\frac{4}{2835}\right)$$

$$(\psi_{550},\psi_{551},\psi_{552},\psi_{53},\psi_{554},\psi_{555},\psi_{556})=\left(\frac{3715}{72576},\frac{725}{3024},\frac{2125}{24192},\frac{125}{567},\frac{3875}{24192},\frac{235}{3024},-\frac{275}{72576}\right)$$

$$(\psi_{660},\psi_{661},\psi_{662},\psi_{663},\psi_{664},\psi_{665},\psi_{666})=\left(\frac{41}{840},\frac{9}{35},\frac{9}{280},\frac{34}{105},\frac{9}{280},\frac{9}{35},\frac{41}{840}\right)$$

3 Investigating the Properties of the Block Method

The properties examined for the new block method are the properties that are mandatory to ensure convergence of the block method when modified to solve initial value problems of higher order ordinary differential equations.

3.1 Order and error constant of the block method

To accomplish the order of the block method, by Taylor series expansions about x_n defined as

$$y^{(m)}(x_n+ah)=y^{(m)}(x_n)+ahy^{(m+1)}(x_n)+\frac{(ah)^2}{2!}y^{(m+2)}(x_n)+\frac{(ah)^3}{3!}y^{(m+3)}(x_n)+\cdots \qquad (3.1)$$

Where $y^{(m)}x_n=\left.\frac{d^my}{dx^m}\right|_{x=x_n},m=1,2,\cdots$

Using the linear operator

$$L[y(x);h]=\sum_{j=0}^{1}\alpha_j y_{n+j}-\sum_{j=0}^{1}\beta_j f_{n+j}+\sum_{j=0}^{1}\gamma_j f'_{n+j} \qquad (3.2)$$

expanding (3.2) using Taylor series expansions about x_n and comparing the coefficient h and the method is said to be of order p if $C_0=C_1=\cdots=C_{p+1}=0, C_{p+2}=0, C_{p+3}\neq 0$ and C_{p+3} is the error constant, [4]. Therefore the order and error constant of new method are $p=[5,5,5,5,5,5]^T$ with error constant

$$C_8=\left[6.7679\times10^{-09}, 5.0402\times10^{-09}, 5.9803\times10^{-09}, 5.0420\times10^{-09}, 6.7679\times10^{-09}, 4.4653\times10^{-09}\right]^T$$

3.2 Consistency

The block method is said to be consistent if the order is greater than or equal to one i.e. $p\geq 1$. Therefore the new method is consistent in [19].

3.3 Zero Stability

The block method is said to be zero-stable, if the roots $z_s, s=1,2,\cdots;k$ of the first characteristics polynomial $\vartheta(z)$ defined by $\vartheta(z)=\det(zA^{(0)}-E)$ satisfies $|z_s|\leq 1$ and every root satisfies $|z_s|=1$ have multiplicity not exceeding the order of the differential equation [9]. To analyze the block method for zero stability, the roots of the first characteristic polynomial

$$\vartheta(z)=\left|zI_6-\begin{pmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{pmatrix}\right| \qquad (3.3)$$

Solving for Z in (3.3) gives $\vartheta(z)=z^6-z^5$.

3.4 Convergence

The block method is to be convergent if it is consistent and zero-stable. Hence the block method is convergent in [14]. At what time block method converge, means at k= what time block method converge. State the convergent series and what if is not converge, what happen.

More evidence on convergence of a method

4 Numerical Implementation of the Problems

The subsequent mathematical problems are measured for determination of viewing the accuracy of the new block method when compared with previously existing methods. The accuracy and convergence of the block method will be considered using some highly stiff third order linear problems, textual form and vividly shown.

Problem one: Consider the third order highly non-stiff linear problem

$$\frac{d^3y}{dx^3}=3\cos(x), \quad y(0)=1, \frac{dy}{dx}(0)=0, \frac{d^2y}{dx^2}(0)=2,$$

with the exact solution: $y(x)=x^2-3\sin(x)+3x+1$

Source: [21, 22, 18].

Problem two: Consider the third order highly stiff linear problem

$$\frac{d^3y}{dx^3}+4\frac{dy}{dx}-x=0, y(0)=\frac{dy}{dx}(0)=0, \frac{d^2y}{dx^2}(0)=1, h=0.1$$ (h: stand for what?, please define h)

with the exact solution given by

$$y(x)=\frac{3}{16}(1-\cos 2x)+\frac{x^2}{8}$$

Source: [20, 5, 19]

Problem three: Consider the third order highly stiff linear problem

Please provide CPU time repot and software used to give the result

$$\frac{d^3y}{dx^3}+\frac{dy}{dx}=0, y(0)=0, \frac{dy}{dx}(0)=1, \frac{d^2y}{dx^2}(0)=2, h=0.1$$

With the exact solution given by

$$y(x)=2(1-\cos x)+\sin x$$

Source [23-26].

Please where is the result **of this work? and show formulation of this paper**

Table 1: Showing the comparison of error for problem one

x	Exact Result	approximate Result	Error in our Method	Error in [22]	Error in [21]	Error in [18]
0.1	1.01049975005951554310	1.01049975005951554300	1.0000e-19	2.4800e-07	1.9700e-16	0.0000e-00
0.2	1.04399200761481635360	1.04399200761481635330	3.0000e-19	7.3740e-06	1.2639e-15	2.2205e-16
0.3	1.10343938001598127470	1.10343938001598127400	7.0000e-19	6.0542e-05	4.0627e-15	8.8818e-16
0.4	1.19174497307404852500	1.19174497307404852360	1.4000e-18	2.5479e-04	9.4370e-15	1.5543e-15
0.5	1.31172338418739099920	1.31172338418739099680	2.4000e-18	7.7602e-04	1.8205e-14	2.8866e-15
0.6	1.46607257981489392840	1.46607257981489392490	3.5000e-18	1.9261e-03	3.1152e-14	5.3291e-15
0.7	1.65734693828692683900	1.65734693828692683410	4.9000e-18	4.1505e-03	4.9021e-14	7.5495e-15
0.8	1.88793172730143171510	1.88793172730143170860	6.5000e-18	8.3637e-03	7.2504e-14	1.0436e-14
0.9	2.16001927111754983460	2.16001927111754982620	8.4000e-18	1.0224e-13	1.0224e-13	1.4211e-14
1.0	2.47558704557631048000	2.47558704557631047000	1.0000e-17	1.3880e-13	1.3880e-13	1.8208e-14

Source: [21, 22, 18].

Table 2: Showing the comparison of error for problem two

x	Exact Solution	Commutated Solution	Error in our Method	Error in [20]	Error in [5]	Error in [19]	
						Case one	Case two
0.1	0.00498751665476719416	0.00498751665476714555	4.8610e-17	0.2304e-14	2.8818e-09	7.9512e-14	3.1484e-14
0.2	0.01980106362445904698	0.01980106362445885599	1.9099e-16	0.1658e-13	3.2893e-08	8.6717e-13	1.5843e-13
0.3	0.04399957220443531927	0.04399957220443490276	4.1651e-16	0.4850e-13	1.1954e-07	3.1385e-12	4.2347e-13
0.4	0.07686749199740648358	0.07686749199740577542	7.0816e-16	0.1147e-12	2.8709e-07	7.5504e-12	8.5820e-13
0.5	0.11744331764972380299	0.11744331764972275952	1.0435e-15	0.2425e-12	5.5398e-07	1.4585e-11	1.4746e-12
0.6	0.16455792103562370419	0.16455792103562230833	1.3959e-15	0.4436e-12	9.2975e-07	2.4504e-11	2.2752e-12
0.7	0.21688116070620482401	0.21688116070620308777	1.7362e-15	0.7467e-12	1.4149e-06	3.7317e-11	3.2313e-12
0.8	0.27297491043149163616	0.27297491043148960136	2.0348e-15	0.1183e-11	1.9995e-06	5.2765e-1	4.3022e-12
0.9	0.33135039275495382287	0.33135039275495156010	2.2628e-15	0.1753e-11	2.6636e-06	7.0321e-11	5.4266e-12
1.0	0.39052753185258919756	0.39052753185258680323	2.3943e-15	0.2481e-11	3.3776e-06	8.8206e-11	6.5277e-12

Source: [20, 5, 19]

Table 3. Showing the result for problem three

x	Exact Solution	Commutated Solution	Error in new Method	Error in [23]	Error in [24]	Error in [25]	Error in [26]
0.1	0.10982508609077662011	0.10982508609077661962	4.9000e-19	1.6613e-12	1.1177e-10	3.7470e-16	2.4980e-16
0.2	0.23853617511257795326	0.23853617511257795125	2.0100e-18	7.5411e-12	9.3348e-10	8.3267e-16	4.1633e-16
0.3	0.38484722841012753581	0.38484722841012753150	4.3100e-18	1.3843e-09	3.2775e-09	1.3878e-15	8.3267e-16
0.4	0.54729635430288032607	0.54729635430288031857	7.5000e-18	4.5006e-09	8.0524e-09	1.4433e-15	1.4433e-16
0.5	0.72426041482345756807	0.72426041482345755666	1.1410e-17	1.0520e-08	1.6249e-08	1.5543e-15	4.4409e-16
0.6	0.91397124357567876270	0.91397124357567874687	1.5830e-17	1.9715e-08	2.8912e-08	1.9986e-15	1.1102e-16
0.7	1.11453331266871420120	1.11453331266871418040	2.0800e-17	3.2968e-08	4.7125e-08	2.8866e-15	4.4409e-19
0.8	1.32394267220519191980	1.32394267220519189390	2.5900e-17	5.0419e-08	7.1985e-08	4.4409e-15	1.3323e-15
0.9	1.54010697308615447550	1.54010697308615444420	3.1300e-17	7.2608e-08	1.0458e-07	3.5527e-15	4.4409e-15
1.0	1.76086637307161707180	1.76086637307161703510	3.6700e-17	9.9511e-08	1.4596e-07	5.3291e-15	2.2204e-15

Source [23-26].

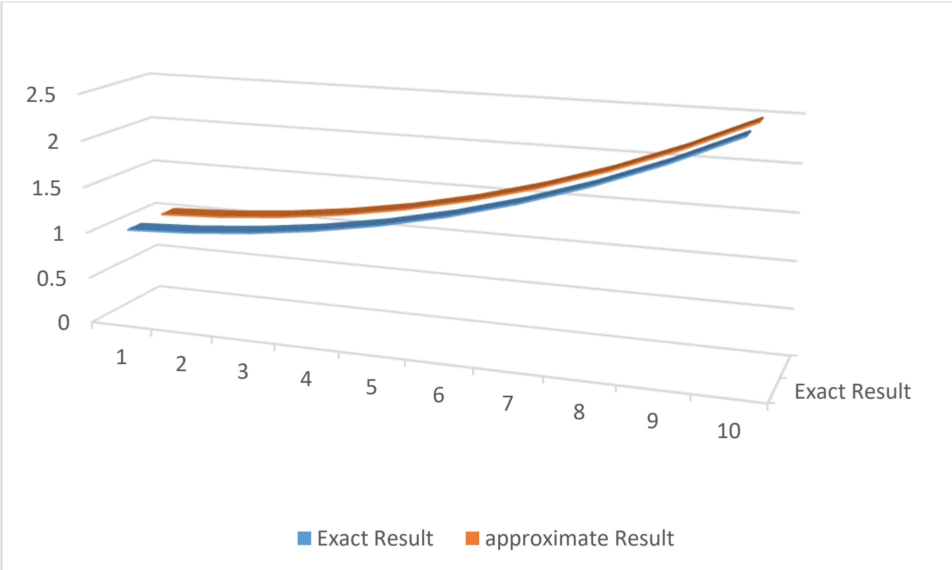

Figure 1. The graphical solution of problem one.

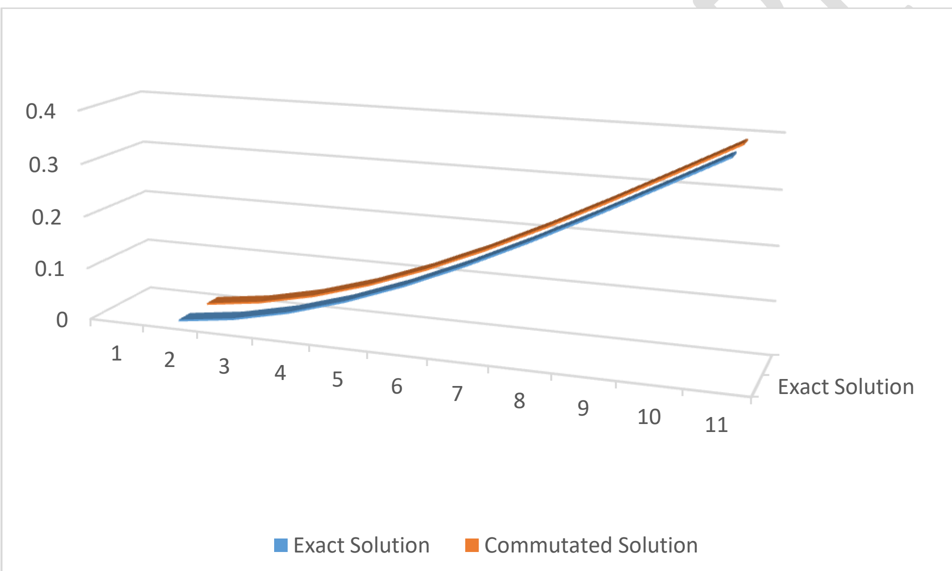

Figure 2. The graphical solution of problem two.

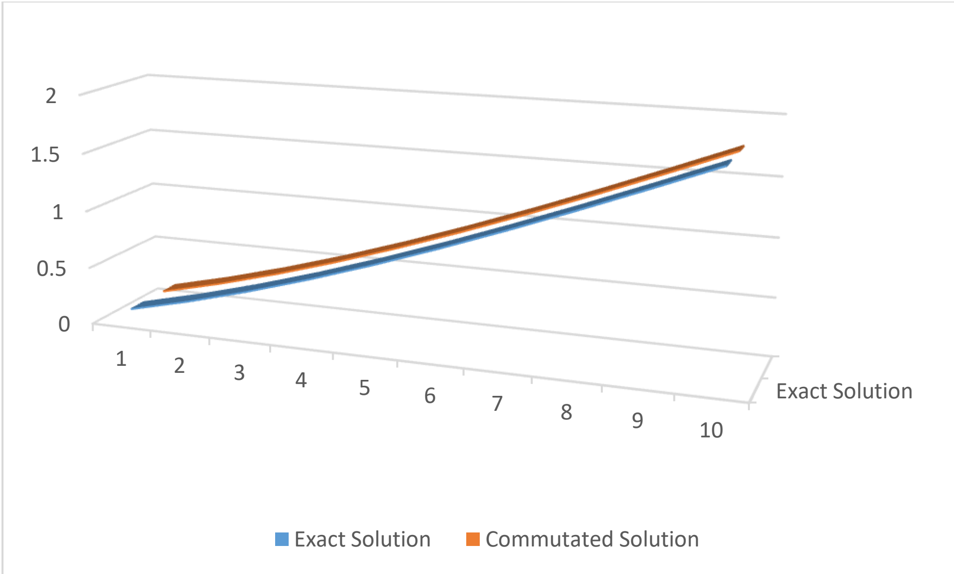

Figure 3. Showing the solution graph of problem three.

5 Summary, Conclusion and Recommendations

In this research, we have proposed the block method for direct solution of third order initial value problems without reduction to a system of first order ordinary differential equation, to address the weaknesses in reduction method. The method is derived using the linear block method through interpolation and collocation. The basic properties of the block method were recovered and was found to be consistent, convergent and zero stability. The new block method is been applied to solve third order initial value problems of ordinary differential equations without reducing the equations to their equivalent systems of first order ordinary differential equations. The result obtained on the process on some sampled modeled third order linear problems give better approximation than the existing methods. The method developed using interpolation and collocation procedure has been recommended for scholars, students and researchers.

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