

Detection of shortfall in fertilizers and applying remedial fertilizers at any time during cultivation period for optimum yield using a mathematical model.

ABSTRACT

At times, it is found that the yield from a commercial plantation is affected adversely because of inadequate intake of fertilizers by the plants. The possible reasons behind decrease in fertilizer intake may be due to both natural causes and man-made. For example, fertilizers applied in the field may be of inferior quality or the fertilizers may be washed away by natural causes like heavy rain or flood resulting in depletion of fertilizer applied at sowing stage. In order to ensure the optimal yield in the event of above mentioned circumstances, remedial supplementary fertilizer doses can be provided to the plant at intermediate stage of cultivation. Such additional fertilizer doses can be applied after detecting that the intake of fertilizers in plants is not adequate for optimal yield. Such a detection process can be carried out at any time within the cultivation period. Here a mathematical model has been proposed for detecting inadequacy of fertilizers and recommends additional fertilizer doses that can be followed by the farmers for optimum possible yield. A garlic plantation has been considered to illustrate the applicability of the proposed mathematical model.

Keywords: *Fertilizer response _ Regression models _garlic plant_ Fertilizer nitrogen _ Fertilizer phosphorous _Newton's Method, _Marquardt's Method*

1. INTRODUCTION

Although the country has increased its agricultural productivity a lot by availing the benefit of hybrid seed and proper utilization of fertilizers, yet there are instances when sufficient nutrients is not available to the plants in some fields hampering the full potential of the crop yield. For instance, unreasonable heavy rain can cause significant nitrogen loss due to leaching and surface runoff[1], Nitrogen can be lost from the farm fields in the form of environmentally harmful gaseous compounds[2], Nitrogen and phosphorous are lost from paddy fields due to bad drainage system[3], cultivation may suffer due to low quality of fertilisers used[4]. Due to depletion of fertilisers in the field, the crop may not be able to collect adequate fertilisers from soil in order to produce anticipated yield.

High yielding garlic crop demands large amount of nutrients out of these nitrogen (N) and phosphorous (P) are major components[5]. Imbalance nutrients N and P may cause a huge loss in garlic[6].

Comment [DKS1]: Revised the abstract, it is not a introduction, add only two starting lines than add location where experiment was conducted than short result and than objective of the study

Comment [DKS2]: Arrange it alphabetically, and maximum five and remove underscore sign

The mathematical model suggested by us can remedy the above mentioned problems to a great extent. The model uses the data generated from experimental results in a trial plantation in the vicinity of a commercial plantation. The model has the following advantages

- (i) It determines the amount of fertilisers that need to be applied in the field at the beginning of the cultivation in order to produce optimum yield.
- (ii) At any intermediate stage of cultivation, the model determines the amount of nutrient uptake required by the plant for producing optimum yield
- (iii) At any intermediate stage of cultivation, the model detects whether there is a deficit in fertilisers available to the plant for producing optimum yield.
- (iv) If there is a deficit in fertilisers, the model determines the additional remedial fertiliser doses to be applied in the field so that yield is not affected adversely.

We have illustrated the applicability of the mathematical model by considering experimental data of garlic[7]Data from 4^2 factorial fertiliser rate trial with four rates of N and four rates P and a total of 16 treatments (i.e. all combinations of four rates of nitrogen and four rates of phosphorous) have been taken into account to develop response surfaces of yield and N and Plant uptakes of garlic crop necessary for the mathematical model.

Several experiments have been undertaken in the past to study the effect of N and P fertilizers on yield of garlic. Soil test based fertilizer recommendation has been used for targeted yield [8]. several experiments also undertaken on different crops for maximizing yield using optimization. Supply of NPK fertilizers to optimize yield of potato has been studied by *Kolbe et al*[9].in their work, the objective function representing the yield, subject to some linear constraints, was optimized by an unconstrained sequential optimization technique. In the work of *Koeh et al.*[10], the 3D response surfaces were employed to evaluate the effect of NPK fertilizers on yield of potato by varying two variables within the experimental range under investigation and holding the other variable at its central level. *Piekutowska et al* [11] developed a linear model based on multiple linear regression analysis and a nonlinear model based on artificial neural network to predict potato cultivation before harvest. In most of the works in the past, the yield and nutrient uptake response surfaces have been assumed to be quadratic or linear. It is easier to optimize a quadratic function of several variables. However, it is not true that the response surfaces will be quadratic functions always, since quadratic surfaces may not fit the data well. Here we have allowed the response surfaces to be of higher degree in order that they can be statistically significant. If the response surfaces are not quadratic, the mathematical techniques used in earlier works to maximize function of several variables are not applicable and we need to apply more advanced techniques.

The experimental data required by the mathematical model and a step by step computational procedure of the proposed mathematical framework has been presented in section-2. In section-3,

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applicability of the mathematical model has been illustrated taking into account the data on garlic plantation presented in the Section-2.

2. Material and Method

2.1. Description of Trial

The current study is developed using the data generated from an investigation on garlic plantation under field condition during *rabi* season [7] at the student's farm, University of Agricultural sciences, Rajendranagar, Hyderabad, Andhra Pradesh. It is geographically situated at 79°23' East longitude and 17°19' north latitude at an altitude of 542.3 meters above the sea level. The soil of the experimental area was sandy loam with good drainage and moderate water holding capacity.

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The whole experimental plot was brought to fine tilth by repeated ploughings, followed by harrowing. It was perfectly levelled. The trial used a randomized complete block design of two replicates with 16 treatments. The 16 treatments are the factorial combination of 4 rates of nitrogen (0, 50, 100 and 150 kg/ha) and four rates of phosphorous (0, 30, 60 and 90 kg/ha). Cultivar used was Jamnagar Laddu which produces medium to large size bulbs with fairly big cloves. It matures between 120 -130 days after planting.

Comment [DKS7]: mention the recommended dose of fertilizers for the region

The entire phosphorous in the form of single super phosphate and a common dose of potassium at 250 kg/ha in the form of sulphate of potash were applied along with the basal dose of nitrogen at planting. The nitrogen was applied at 7.5 cm away from the plants along the row opening a small furrow to a depth of 5 cm in bands and covered with soil immediately and irrigated. Irrigation was given twice a week in the initial stage and subsequently as and when necessary depending upon soil moisture and weather condition.

Comment [DKS8]: mention exact time of irrigation

Nitrogen in plant samples was estimated by Microkjeldahl method [12]. Phosphorous in whole plant samples was determined by Vanadomolybdophosphoric Acid. Calorimetric method [12].

2.2 METHODOLOGY

For our mathematical framework, we need the following data collected at the experimental plantation.

- (i) Yield of garlic corresponding to different level of application of N and P fertilisers at the sowing stage
- (ii) Uptake of P fertiliser by the whole plant measured at different time of cultivation and due to different level of applications of N and P fertilisers at the sowing stage due .
- (iii) Uptake of N fertiliser by the whole plant measured at different time of cultivation and due to different level of applications of N and P fertilisers at the sowing stage due .

The following response surfaces are obtained from the above data

- (a) Yield response surface of garlic using data mentioned in (i) depicting the effect of different levels of N and P fertilisers in trial plantation on yield of garlic. Let the response surface, thus obtained, be the following function of variables N and P

$$Y = FU_Y(N, P) \quad (2.2.1)$$

- (b) Phosphorus uptake response surfaces of garlic plant using data mentioned in (iii) depicting the plant uptake of P fertiliser at time t due to application of N and P fertilisers at the sowing stage. Let the response surface, thus obtained, be the following function of variables N and P and t

$$FU_P(N, P, t) . \quad (2.2.2)$$

- (c) Nitrogen uptake response surfaces of garlic plant using data mentioned in (ii) depicting the plant uptake of N fertiliser at time t due to application of N and P fertilisers at the sowing stage. Let the response surface, thus obtained, be the following function of variables N and P and t

$$FU_N(N, P, t) \quad (2.2.3)$$

The execution of the model starts by maximizing the yield response surface equation $FU_Y(N, P)$ by an unconstrained nonlinear optimization technique. Let the solution of the optimization problem be N_{opt} and P_{opt} . Thus quantity of fertilizers that needs to be applied at the beginning of cultivation to maximize yield are N_{opt} and P_{opt} .

The values N_{opt} and P_{opt} are substituted in uptake response surface $FU_N(N, P, t)$. The substitution results in an equation involving the variable t. Let it be

$$U_N(t) \quad (2.2.4)$$

$U_N(t)$ is the level of N fertilizer nutrient uptake in the plant at time t which is necessary for the plant to produce optimum yield.

Similarly, the parameter values N_{opt} and P_{opt} are substituted in uptake response surface $FU_P(N, P, t)$. The substitution results in an equation involving the variable t . Let it be

$$U_P(t) \tag{2.2.5}$$

$U_P(t)$ is the level of P fertilizer nutrient uptake in the plant at time t which is necessary for the plant to produce optimum yield.

Let t_0 be an arbitrary time within the period of cultivation when it is decided to test commercial plantation to see whether the commercial plantation will produce optimum yield.

To this end, we need to visit the site of the commercial plantation at a time t_0 and identify the N and P fertilizer uptake in the plantation at time t_0 . Suppose $[N_{t_0}]$ and $[P_{t_0}]$ are respectively the levels of N and P fertilizer nutrient uptake of the commercial plantation at time t_0 .

We now determine the quantities $U_N(t_0)$ and $U_P(t_0)$ by substituting t by t_0 in equation (2.2.4) and (2.2.5). The quantities $U_N(t_0)$ and $U_P(t_0)$ are the levels of N and P nutrients the plant should have in form of uptake at time t_0 in order that it can produce optimum yield.

From the quantities $[N_{t_0}]$, $[P_{t_0}]$, $U_N(t_0)$ and $U_P(t_0)$ we can determine whether the fertilizers available to the plant at time t_0 are adequate to produce optimum yield or not. If both the inequalities

$$[N_{t_0}] \geq U_N(t_0) , \tag{2.2.6}$$

$$[P_{t_0}] \geq U_P(t_0) , \tag{2.2.7}$$

are satisfied, we can conclude that adequate fertilizers are available to the plant to produce optimum yield. However, if one of the inequalities (2.2.6) or (2.2.7) is violated, we can conclude that enough fertilizers are not available to the commercial plantation at time t_0 and additional doses of fertilizers need to be applied at time t_0 to make up the shortfall in fertilizer and produce optimum yield.

If the inequalities (2.2.6)-(2.2.7) are not satisfied, we need to determine the amount of fertilizers applied to the commercial plantation at the time of sowing stage before we can recommend additional fertilizer doses. Let N_{app} and P_{app} be respectively the amount of N fertilizer and P fertilizers applied at the

time of sowing in commercial plantation. $N = N_{app}$ and $P = P_{app}$ are the solutions of the following system of equations

$$FU_N(N, P, t_0) = [N_{t_0}] \quad (2.2.8)$$

$$FU_P(N, P, t_0) = [P_{t_0}] \quad (2.2.9)$$

After having determined N_{app} and P_{app} , we can obtain the remedial fertilizer doses necessary for optimal yield. Let N_{rem} and P_{rem} respectively be the remedial fertilizer doses of N and P . Then

$$N_{rem} = N_{opt} - N_{app}, \quad P_{rem} = P_{opt} - P_{app}$$

depending on whether $N_{opt} > N_{app}$ or $P_{opt} > P_{app}$.

There are several nonlinear optimization techniques [13] that can be used to maximize the nonlinear function of several variables present in (2.2.1). Since exact solution of such an optimization problem is hard to attain, we have to use a numeric optimization method to obtain an approximate solution. Marquardt's Method [13] is preferred here because of its wider scope of applicability. To address crop productivity through soil test based plant nutrient management, the ICAR project on *Soil Test Crop Response* has used a targeted yield approach to develop relationship between crop yields on the one hand, and soil test estimates and fertilizer inputs, on the other. In this targeted yield approach, it is assumed that there is a linear relationship between grain yield and nutrient uptake by the crop and for obtaining a targeted yield. A linear relationship may not exist between yield and nutrient uptake. Marquardt's Method is capable of handling implicit nonlinearity between grain yield and nutrient uptake.

The equations (2.3.8) and (2.3.9) constitute a system of nonlinear equations of several variables. It is not possible to solve such system of equations without using numerical analytic techniques. We have used Newton's Method of solution of system of nonlinear equations here.

2.3. Statistical Analysis

The effect of different level of N and P on yield of garlic is provided in table-1. The response surface for yield, after application of multiple linear regressions [14] using data in table-1, is given by

$$FU_y(N, P) = a + bN + cP + dN^2 + eNP + fN^2P + gN^3 + hP^4, \quad (2.3.1)$$

Where, $FU_y(N, P)$ is the response surface of yield of garlic and a, b, c, d, e, f, g and h are regression coefficients having values

$a = 282.8192(P < 0.001)$, $b=7.025663(P < 0.001)$, $c =1.173542(P < 0.001)$, $d=-0.05845(P < 0.001)$,
 $e=0.022476(P = 0.003)$, $f=-0.00013 (P = 0.006)$, $g=0.000294(P < 0.001)$ and $h= -5.9e-07(P = 0.02)$
(2.3.1) has coefficient of determination ($R^2 = 0.99$) and($adjustedR^2 = 0.99$)which shows that response model is highly significant.

Effect of different level of N and P on plant uptake of P in the trial garlic plantation is provided in table-2.

The response surface for uptake of phosphorous, after application of multiple linear regressions using data in table-2, is given by

$$FU_p(N, P, t) = a + btN + ctP + dt^2 + eP^2 + fN^2 + gtN^2 + hN^2P + iP^3 + jt^3 \quad (2.3.2)$$

Where, a, b, c, d, e, f, g, h, I and j are regression coefficients having values:

$a=1.98228(P = 0.01)$, $b=0.000535(P < 0.001)$, $c=0.001109(P < 0.001)$, $d=-0.00196(P = 0.001)$,
 $e=-0.00108(P = 0.002)$, $f=-0.0003(P < 0.001)$, $g=3.98e-06(P < 0.001)$, $h=7.88e-07(P = 0.01)$,
 $i=6.13e-06(P = 0.03)$, $j=2.44e-05(P < 0.001)$.(2.3.2) has coefficient of determination ($R^2 = 0.97$)
and($adjustedR^2 = 0.96$)which shows that response model is highly significant.

Effect of different level of N and P on plant uptake of N in the trial garlic plantation is provided in table-3.

The response surface for uptake of nitrogen, after application of multiple linear regressions using data in table-3, is given by

$$FU_N(N, P, t) = a + btP + ctN + dN^2 + eN^2P + f t^3 + gN^3 + hP^3 \quad (2.3.3)$$

Where, a, b, c, d, e, f, g and h are regression coefficients having values:

$a=1.661157(P = 0.12)$, $b=0.001802(P < 0.001)$, $c=0.006149(P < 0.001)$, $d=-0.00291(P < 0.001)$,
 $e=2.17e-06(P = 0.08)$, $f=1.26e-05(P < 0.001)$, $g=1.25e-05(P < 0.001)$, $h=-5.7e-06(P = 0.03)$. (2.3.3) has
coefficient of determination ($R^2 = 0.98$) and($adjustedR^2 = 0.97$)which shows that response model is
highly significant.

(Table – 1)(Effects of different levels of **N** and **P** on bulb yield (q/ha) in garlic)

| Nitrogen levels(<i>N</i>)(kg/ha) | Potassium levels(<i>P</i>)(kg/ha) | Yield(<i>Y</i>) (q/ha) |
|------------------------------------|-------------------------------------|--------------------------|
| 0 | 0 | 27.84 |
| 50 | 0 | 53.38 |
| 100 | 0 | 68.03 |
| 150 | 0 | 100.72 |
| 0 | 30 | 32.10 |
| 50 | 30 | 58.94 |
| 100 | 30 | 76.40 |
| 150 | 30 | 108.00 |
| 0 | 60 | 34.19 |
| 50 | 60 | 62.62 |
| 100 | 60 | 81.52 |
| 150 | 60 | 109.22 |
| 0 | 90 | 35.42 |
| 50 | 90 | 65.77 |
| 100 | 90 | 85.49 |
| 150 | 90 | 111.90 |

(Table – 2)(Effects of different levels of **N**, **P** and **t** on uptake of phosphorous in garlic (kg/ha))

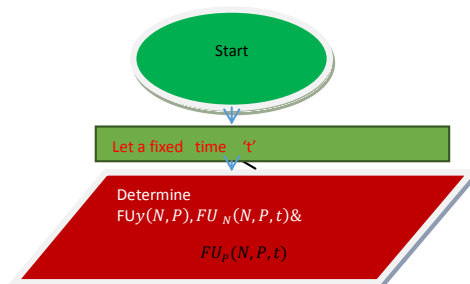
| Sl.no | N(kg/ha) | P (kg/ha) | t (days) | U(P)(uptake of P)(kg/ha) |
|-------|----------|-----------|----------|--------------------------|
| 1 | 0 | 0 | 40 | 0.52 |
| 2 | 50 | 0 | 40 | 0.71 |
| 3 | 100 | 0 | 40 | 0.89 |
| 4 | 150 | 0 | 40 | 1.11 |
| 5 | 0 | 30 | 40 | 0.66 |
| 6 | 50 | 30 | 40 | 0.9 |

| | | | | |
|----|-----|----|----|-------|
| 7 | 100 | 30 | 40 | 1.13 |
| 8 | 150 | 30 | 40 | 1.69 |
| 9 | 0 | 60 | 40 | 0.84 |
| 10 | 50 | 60 | 40 | 1.13 |
| 11 | 100 | 60 | 40 | 1.45 |
| 12 | 150 | 60 | 40 | 1.74 |
| 13 | 0 | 90 | 40 | 0.88 |
| 14 | 50 | 90 | 40 | 1.22 |
| 15 | 100 | 90 | 40 | 1.55 |
| 16 | 150 | 90 | 40 | 1.95 |
| 17 | 0 | 0 | 60 | 1.12 |
| 18 | 50 | 0 | 60 | 2.03 |
| 19 | 100 | 0 | 60 | 3.24 |
| 20 | 150 | 0 | 60 | 3.99 |
| 21 | 0 | 30 | 60 | 1.55 |
| 22 | 50 | 30 | 60 | 2.63 |
| 23 | 100 | 30 | 60 | 4.04 |
| 24 | 150 | 30 | 60 | 4.91 |
| 25 | 0 | 60 | 60 | 1.79 |
| 26 | 50 | 60 | 60 | 3.07 |
| 27 | 100 | 60 | 60 | 4.44 |
| 28 | 150 | 60 | 60 | 5.44 |
| 29 | 0 | 90 | 60 | 2.22 |
| 30 | 50 | 90 | 60 | 3.4 |
| 31 | 100 | 90 | 60 | 4.93 |
| 32 | 150 | 90 | 60 | 6.01 |
| 33 | 0 | 0 | 80 | 2.19 |
| 34 | 50 | 0 | 80 | 3.81 |
| 35 | 100 | 0 | 80 | 5.11 |
| 36 | 150 | 0 | 80 | 7.46 |
| 37 | 0 | 30 | 80 | 3.82 |
| 38 | 50 | 30 | 80 | 5.98 |
| 39 | 100 | 30 | 80 | 9.83 |
| 40 | 150 | 30 | 80 | 11.75 |
| 41 | 0 | 60 | 80 | 4.3 |
| 42 | 50 | 60 | 80 | 6.43 |
| 43 | 100 | 60 | 80 | 10.55 |
| 44 | 150 | 60 | 80 | 13.11 |
| 45 | 0 | 90 | 80 | 4.44 |
| 46 | 50 | 90 | 80 | 7.9 |
| 47 | 100 | 90 | 80 | 11.84 |
| 48 | 150 | 90 | 80 | 13.79 |

(Table – 3)(Effects of different levels of N, P and t on uptake of nitrogen in garlic (kg/ha))

| Sl.no | N(kg/ha) | P (kg/ha) | t(days) | U(N)(uptake of N)(kg/ha) |
|-------|----------|-----------|---------|--------------------------|
| 1 | 0 | 0 | 40 | 5.24 |
| 2 | 50 | 0 | 40 | 8.89 |
| 3 | 100 | 0 | 40 | 12.12 |
| 4 | 150 | 0 | 40 | 16.87 |
| 5 | 0 | 30 | 40 | 5.86 |

| | | | | |
|----|-----|----|----|-------|
| 6 | 50 | 30 | 40 | 9.62 |
| 7 | 100 | 30 | 40 | 13.04 |
| 8 | 150 | 30 | 40 | 17.78 |
| 9 | 0 | 60 | 40 | 6.45 |
| 10 | 50 | 60 | 40 | 10.05 |
| 11 | 100 | 60 | 40 | 13.98 |
| 12 | 150 | 60 | 40 | 18.87 |
| 13 | 0 | 90 | 40 | 6.95 |
| 14 | 50 | 90 | 40 | 10.75 |
| 15 | 100 | 90 | 40 | 14.44 |
| 16 | 150 | 90 | 40 | 20.76 |
| 17 | 0 | 0 | 60 | 7.13 |
| 18 | 50 | 0 | 60 | 16.24 |
| 19 | 100 | 0 | 60 | 26.52 |
| 20 | 150 | 0 | 60 | 38.09 |
| 21 | 0 | 30 | 60 | 8.82 |
| 22 | 50 | 30 | 60 | 17.53 |
| 23 | 100 | 30 | 60 | 30.43 |
| 24 | 150 | 30 | 60 | 40.39 |
| 25 | 0 | 60 | 60 | 10.18 |
| 26 | 50 | 60 | 60 | 20.04 |
| 27 | 100 | 60 | 60 | 31.67 |
| 28 | 150 | 60 | 60 | 43.32 |
| 29 | 0 | 90 | 60 | 12.5 |
| 30 | 50 | 90 | 60 | 21.53 |
| 31 | 100 | 90 | 60 | 32.94 |
| 32 | 150 | 90 | 60 | 47.21 |
| 33 | 0 | 0 | 80 | 10.81 |
| 34 | 50 | 0 | 80 | 23.54 |
| 35 | 100 | 0 | 80 | 35.93 |
| 36 | 150 | 0 | 80 | 56.7 |
| 37 | 0 | 30 | 80 | 14.04 |
| 38 | 50 | 30 | 80 | 28.69 |
| 39 | 100 | 30 | 80 | 50.63 |
| 40 | 150 | 30 | 80 | 65.82 |
| 41 | 0 | 60 | 80 | 15.51 |
| 42 | 50 | 60 | 80 | 28.06 |
| 43 | 100 | 60 | 80 | 53.31 |
| 44 | 150 | 60 | 80 | 69.89 |
| 45 | 0 | 90 | 80 | 15.91 |
| 46 | 50 | 90 | 80 | 33.45 |
| 47 | 100 | 90 | 80 | 55.98 |
| 48 | 150 | 90 | 80 | 72.64 |



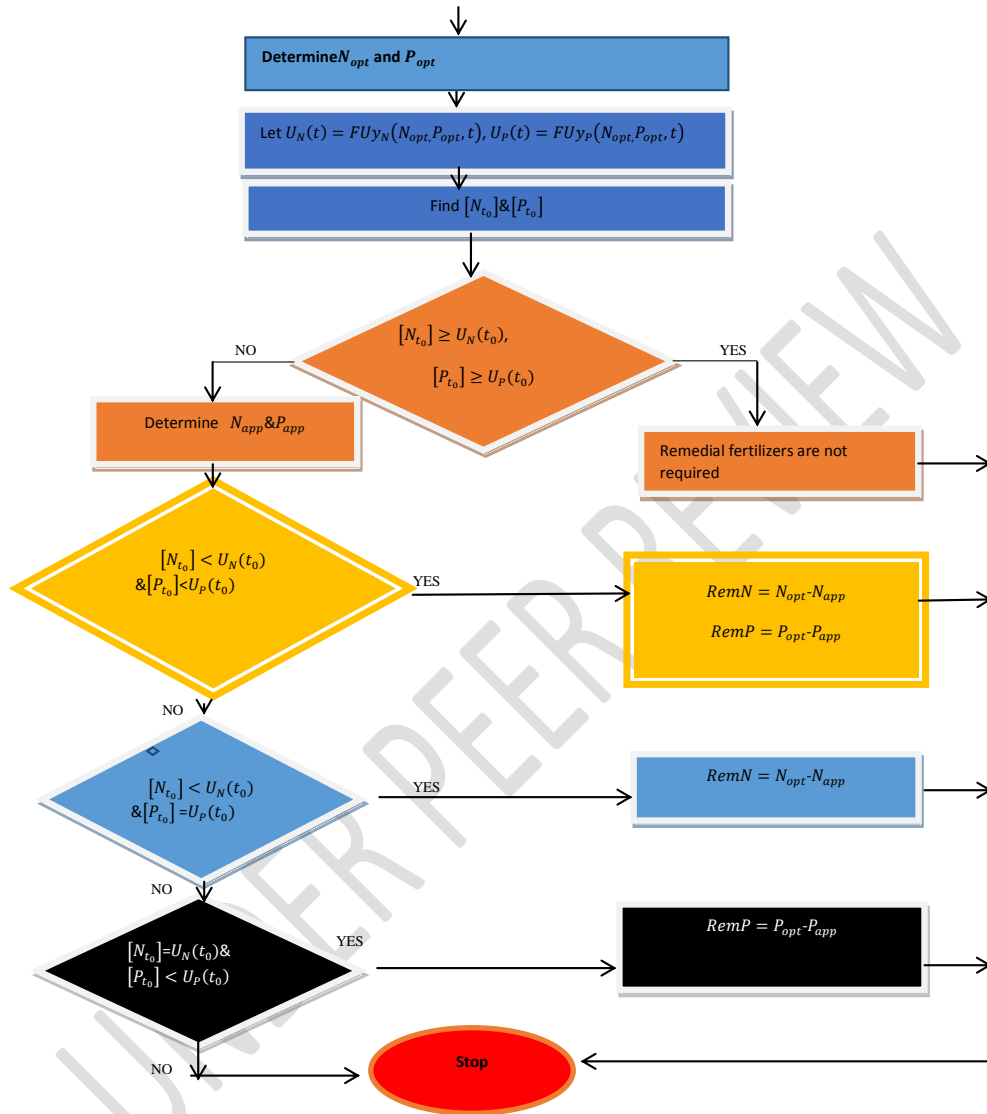
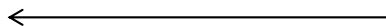
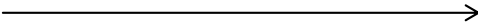


Fig.1. Algorithm of computational steps ($RemN$: Remedial fertilizer of N , $RemP$: Remedial fertilizer of P)

3. RESULT AND DISCUSSION

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We now discuss how we can compute the remedial doses of fertilizers required in a garlic plantation which is cultivated in the neighborhood of the trial plantation.

First the yield function (2.3.1) is optimized by Marquardt's method [13]. After optimization we find that

$$N_{opt} = 90(kg/ha) \text{ and } P_{opt} = 90(kg/ha).$$

The values of N_{opt} and P_{opt} are substituted in uptake response surface (2.3.2). The substitution results in an equation involving the variable 't' and is

$$U_p(t) = 2.44e-05t^3 - 0.00196t^2 + 0.1801t - 4.1524 \quad (3.1)$$

Similarly, the parameter values of N_{opt} and P_{opt} are substituted in uptake response surface (2.3.3). The substitution results in an equation involving the variable 't' and is

$$U_N(t) = 1.26e-05t^3 + 0.7156t - 15.3707 \quad (3.2)$$

Suppose we decide that we shall test the commercial plantation after 45 days from start of cultivation then $t_0=45$. Now we need to visit the site of the commercial plantation at a time $t_0=45$ (days) and identify the N and P fertilizer nutrient uptake in the plantation. Suppose $[N_{45}] = 18(kg/ha)$ and $[P_{45}] = 2(kg/ha)$ are respectively the levels of N and P fertilizer nutrient uptake of the commercial plantation at time 45 (days).

By substituting $t = 45$ in equation (3.1) and (3.2), we find that $U_N(45) = 17.9795(kg/ha)$ and $U_p(45) = 2.2066(kg/ha)$. These are the levels of N and P nutrients the plant should have in form of uptake at time 45 days in order that it can produce optimum yield.

By knowing the quantities $[N_{45}]$, $[P_{45}]$, $U_N(45)$ and $U_p(45)$ we can determine whether the fertilizers available at time $t_0=45$ days are adequate to produce optimum yield or not.

$$\text{Here } [N_{45}] > U_N(45) \text{ but } [P_{45}] < U_p(45).$$

Therefore, we conclude that enough fertilizers are not available to the commercial plantation of garlic at time $t_0=45$ days.

Additional doses of fertilizers need to be applied on 45 day to make up the shortfall in fertilizer and produce optimum yield.

To determine N_{app} and P_{app} we solve the following system of nonlinear equations by Newton's method [15].

$$1.25e-05N^3 + 2.17e-06N^2P - 5.7e-06P^3 - 0.00291N^2 + 0.2767N + 0.0811P + 2.8094 = 18 \quad (3.3)$$

$$6.13e-06P^3+7.88e-07N^2P-0.00108P^2-1.2090e-04N^2+0.0241N+0.0499P+0.2368=2 \quad (3.4)$$

We obtain that $N_{app} = 86.8785(Kg/ha)$ and $P_{app} = 77.7036(kg/ha)$

The remedial fertilizer N_{rem} and P_{rem} can now be found out as

$$N_{rem} = N_{opt} - N_{app} = 3.12(kg/ha) \text{ of fertilizer } N,$$

$$P_{rem} = P_{opt} - P_{app} = 12.29(kg/ha) \text{ of fertilizer } P$$

4. CONCLUSION

Crop yield may be affected by inadequate uptake of fertilizers by plant during the course of cultivation. There are various reasons for below par uptake of fertilizers. A mathematical framework has been presented here to help scientists, farmers and management to rectify any shortfall of fertilizer uptake in plant. The said model relies on experimental results from a trial fertilization in the vicinity of commercial plantation. In a small neighborhood of a trial plantation, the soil and environment condition does not differ much. Therefore, the results obtained from trial plantation can be applied to commercial plantation. The model generates two functions of independent variable time. One function determines the amount of N uptake at time t a plant should have for maximum yield. The other function determines the amount of P uptake at time t for maximum yield. With the help of the two functions, it is possible to identify at any time t the amount of N and P uptake that will enable the plant for maximum yield. If any shortfall in uptake is detected at a time t, necessary remedial fertilizer doses should be supplied to the plant to make up for the deficiency. The model is capable of prescribing the additional doses of fertilizer that should be applied at time t if the plant is found to have shortfall. The proposed mathematical framework can be applied in collaboration with KVKs (KrishiVigyanKendras) where factorial fertilizer trials can be conducted and farmers in the neighborhoods of KVKs can be advised regarding additional doses of fertilizers required in their fields in an intermediate stage of cultivation. Although we have considered garlic cultivation as an example, the model presented by us can be easily extended to any variety of crop. Here we have considered effect of N and P fertilizer on yield. The model can be extended easily to cases where other fertilizers such as potassium are used. We have not taken the economical aspect of the fertilizer use. It may so happen that the objective of optimization of yield will conflict with the objective of economic use of fertilizer. In that case we shall have to solve a multi objective optimization problem instead of a single objective optimization problem that we have solved here.

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Comment [DKS10]: add only latest reviews after 2015

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