Application of Autoregressive Integrated Moving Average Model and Weighted Markov Chain on Forecasting Under-Five Mortality Rates in Nigeria

Abstract: The aim of this study is to obtain the best model that will be used to predict Under-Five Mortality Rate (U5MR) between Autoregressive Integrated Moving Average (ARIMA) model and Weighted Markov Chains (WMC). The historical dataset of U5MR in Nigeria for the period 1980-2019 is obtained from the official website of World Bank. ARIMA modeling involved differencing of the data to attain stationarity, while WMC involved classification of the datasets into clusters (using k-means cluster analysis and transition of states). Two performance measures Theil's U Statistic and MAPE are used to evaluate the two models based on in-sample and out-sample. The results shows that ARIMA(0,3,2) is a better model to forecast U5MR in Nigeria.

Keywords: U5MR, ARIMA, Weighted Markov Chain (WMC), MAPE, Theil's U Statistic, K-Mean Cluster

1. Introduction

Under-Five Mortality Rate is the probability of a child dying between birth and exactly 5 years of age, expressed per 1,000 live births in a given year for a particular geographical area [4,13]. In developing countries, childhood mortality rate is affected by socioeconomic, demographic, health variable, and region [9]. Nigeria and other countries in Sub Saharan Africa though experienced a decline in U5MR from 1980 to 2019, still maintain relatively and unacceptable high Mortality compared to many countries in Europe and America [2,3,8]. [1] compared the effect of ARIMA, Artificial Neural Networks, and Exponential Smoothing. [5] studied the U5MR of Malaysia by gender and developed a forecasting model for future prediction; the result showed that the U5MR for both genders decreased slowly. [9] analyzed the Under-5 mortality annual closing rate (CMACR) in Nigeria using Weighted Markov Chain and ARIMA model, the findings showed that ARIMA predicts CMACR better than WMC.

This paper attempts to establish an adequate model using the Autoregressive Integrated Moving Average (ARIMA) and Weighted Markov Chain (WMC) to forecast the Under-five Mortality Rate in Nigeria.

2. Materials and Methods

2.1 Autoregressive Integrated Moving Average (ARIMA) Model

ARIMA is a statistical model which is used to predict future values based on past values. The 'AR' stands for Autoregressive, 'MA' stands for Moving Average, and 'I' stands for Integrated (that is the data values are replaced by difference between the data values and the previous values). ARIMA model is denoted by $ARIMA(p,d,q)$ and it is written as

$$y'_t = c + \phi_1 y'_{t-1} + \cdots + \phi_p y'_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t \quad (1)$$

where $\phi_1, \phi_2, \cdots, \phi_p$ are Autoregressive model's parameters; $\theta_1, \theta_2, \cdots, \theta_q$ are Moving Average model's parameters; c is a constant; ε_t is a white noise, and y'_t is the differenced series which might been differenced more than once.

2.1.1 Autoregressive Moving Average (ARMA) Model

When the time series data is stationary and however does not require differencing, then the resultant model is an Autoregressive Moving Average (ARMA) model. ARMA model is denoted by $ARMA(p,q)$ and it is written as

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t \quad (2)$$

2.1.2 Autoregressive (AR) Model

AR model is the regression of the current observations against one or more past observations. That is the current observation y_t are generated by a weighted averages of past time series data going back p periods, together with a random disturbance in the current period. The AR of order p denoted by $AR(p)$ is defined as

Comment [DO1]: paper

Comment [DO2]: State the recorded time of your historical dataset: daily, weekly, monthly, quarterly or yearly.

Comment [DO3]: (i)The Analysis technique(s) and Method(s) used are missing. (ii) The significance of the paper to the Policymakers, government, stakeholders and citizenry should be explicitly stated.

Comment [DO4]: #The Introduction and Literature Review are too short.
The conclusion of this work is closely related to [9] Obasohan, (2020), you need to make stronger argument.

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \varepsilon_t \qquad (3)$$

Where ε_t is a white noise; $\phi_1, \phi_2, \cdots, \phi_p$ are the parameters of the AR model; y_t is the current observation, $y_{t-1}, y_{t-2}, \cdots, y_{t-p}$ are past observations.

2.1.3 Moving Average (MA) Model

MA is a linear combination of error terms occurring at various times in the past. MA model of order q is denoted as $MA(q)$ and it is written as

$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \qquad (4)$$

2.2 ARIMA Fitting

ARIMA model is fitted to the time series data (historical data) using the Box-Jenkins method. Four (4) steps are employed here, which are: Checking the historical data for stationarity, Estimation of Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF), Model Identification, and Model Adequacy Check.

Comment [DO5]: Box-Jenkins step-by-step modelling method:
- Model Identification using ACF and PACF;
- Estimation
- Model Diagnostic (Adequacy Check)
- Forecasting

2.2.1 Identification of Stationarity Time Series

a. If the Autocorrelation Function (ACF) drops to zero relatively quickly as the number of lags increases, the time series data is stationary
b. If the Autocorrelation Function (ACF) drops very slowly as the number of lags increases, the time series data is not stationary
c. If there is presence of a unit root in the time series data, then the time series is not stationary. This study adopts Augmented Dickey-Fuller (ADF) Test for Unit Root Test

2.2.2 Differencing of Time Series

This is the process of making a non-stationary time series stationary. It stabilizes the mean of time series by removing the changes in the series and eliminating or reducing trend and seasonality.

First Order Differenced series denoted as y_t' is the change between consecutive observations in the original series. It is written as

$$y_t' = y_t - y_{t-1} \qquad (5)$$

If the first differenced series fails to be stationary, there is need to carry out second differencing
Second Order Differenced series denoted as y_t'' is written as

$$y_t'' = y_t' - y_{t-1}' = (y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) = y_t - 2y_{t-1} + y_{t-2} \qquad (6)$$

The general dth order difference using the Backshift Operator B, is written as

$$y_t^d = (1 - B)^d y_t \qquad (7)$$

2.2.3 Model Adequacy Check

Comment [DO6]: (a) Mathematically, state your Estimation approach (like Least Squares Method or Maximum Likelihood Method) for your model ARIMA and show how the parameters are estimated using chosen method.
(b) Mathematically, show steps in Forecasting your ARIMA model
Immediately after your model adequacy check

The Akaike Information Criterion (AIC) and/or Bayesian Information Criterion (BIC) is/are used to check for model adequacy. The AIC is written as

$$AIC = n log(\hat{\sigma}^2) + 2k \qquad (8)$$

k is the number of model parameters; $\hat{\sigma}^2$ is the residual sum of squares, and n is the sample size

Bayesian Information Criterion (BIC) is written as

$$BIC = nlog(\hat{\sigma}^2) + klog(n) \tag{9}$$

The ARIMA or ARMA model with the lowest AIC and/or BIC are/is considered the best model among others.

2.3 Weighted Markov Chain (WMC)

2.3.1 Markov Chain

Markov chain is a stochastic process $X_t, t = 0,1,2,\cdots$ having the property that given the present state of the system, the past and the future are conditionally independent.
Whenever the process is in state i, there is a fixed probability P_{ij} that it will be in state j next, then the property of Markov chains is define as

$$P(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \cdots, X_0 = i_0) = P_{ij} \tag{10}$$

for all states $i_0, i_1, \cdots, i_{n-1}, i, j$ and all $n \geq 0$

where j is the future state, i is the present or current state, $i_{n-1}, i_{n-2}, \cdots i_1, i_0$ are the past states

2.3.2 Weighted Markov Chain

The method of Weighted Markov Chain adopted in this study for forecasting the U5MR is the one expressed by [6,7,10-12], which is categorized into seven (7) steps:

a. Set up a classification standard for the historical data, Under-five Mortality Rate (U5MR) using the K-means Cluster Analysis
b. Determine the m states, that is, the states of the historical data (U5MR) according to the classification standard
c. Obtain the Frequency Matrix (or Transition Matrix) according to step 2

$$F = \begin{pmatrix} f_{11} & f_{12} & \cdots & f_{1m} \\ f_{21} & f_{22} & \cdots & f_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mm} \end{pmatrix} \tag{11}$$

d. Obtain the One-step Transition Probability Matrix $P = (p_{ij})$ and the Marginal Matrix $Q = (q_i)$ using the matrix of step 3, where

$$p_{ij} = \frac{f_{ij}}{\sum_{j=1}^{m} f_{ij}} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{pmatrix} \tag{12}$$

$$q_{ij} = \frac{f_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij}} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mm} \end{pmatrix} \tag{13}$$

Hence the Marginal Matrix

$$q_i = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \end{pmatrix} \tag{14}$$

where $q_1 = q_{11} + q_{12} + \cdots q_{1m}$; $q_2 = q_{21} + q_{22} + \cdots q_{2m}$ up to $q_m = q_{m1} + q_{m2} + \cdots q_{mm}$

e. Test whether the Transition Probability Matrix obtained in step 4 has a Markov Property, using Chi-square test, defined as

$$\chi^2 = 2\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij}\left|ln\frac{p_{ij}}{q_i}\right| \quad (15)$$

with degree of freedom $(m-1)^2$. The stochastic process (Transition Probability Matrix) has a Markov Property if χ^2 is greater than $\chi^2_{\alpha,(m-1)^2}$

f. Obtain the weight of the various steps Markov Chain, w_k transition probabilities matrices, which is defined as

$$w_k = \frac{|r_k|}{\sum_{k=1}^{K} r_k} \quad (16)$$

where r_k is the autocorrelation coefficient of the historical data with $k\epsilon\{1,2,\cdots,K\}$, and is computed as

$$r_k = \frac{\sum_{t=1}^{n-k}(y_t-\bar{y})(y_{t+k}-\bar{y})}{\sum_{t=1}^{n}(y_t-\bar{y})^2} \quad (17)$$

where y_t is the time series at time t, $\bar{y}$ is the average of time series y_t, and n is the number of time series y_t

g. Take the weighted average of various predicting probabilities of the same state as predicting probabilities of the U5MR, defined as

$$\hat{p}_{ij} = \sum_{k=1}^{K} w_k p_{ij}^{(k)} \quad (18)$$

for every $j\epsilon\{1,2,\cdots,m\}$, $\hat{p}_{ij}$ is the probability for a time series y_t to be in the state j in the future.

The forecast result is in the form of a state, state j obtained by $arg\ max\{\hat{p}_{ij}, j = 1,2,\cdots,m\}$

2.4 Forecast Adequacy Check

The measures of forecast accuracy adopted in this study is Theil's U Statistic and Mean Absolute Percentage Error (MAPE).

2.4.1 Theil's U Forecast Accuracy

The Theil's U shows how the forecast conforms to the values of the future periods. It is written as

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t-\hat{y}_t)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}y_t^2}+\sqrt{\frac{1}{n}\sum_{t=1}^{n}\hat{y}_t^2}} \quad (19)$$

where Y_t is the actual value of a point for a given time period t, $\hat{Y}_t$ is the forecast value, n is the number of the data points.

If U falls within the range $0 \leq$U< 1, the proposed model is a good fit
If U$= 0$, the proposed model is a perfect fit
If U≥ 1, the proposed model is not a good fit

2.4.2 Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error (MAPE) is used to measure the error of both methods (ARIMA and WMC). The model with the smallest MAPE is considered the appropriate model. It is defined as

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right| \times 100\% \qquad (20)$$

3. Results/Findings

Figure 1 shows the timeplot for Under-five Mortality Rate in Nigeria for the period 1980-2019. U5MR in Nigeria shows a partial decrease over the years. In Table 1, the average Under-Five Mortality Rate (U5MR) is 174.315 deaths per 1000 live births, and standard deviation of 34.6777 deaths per 1000 live births.

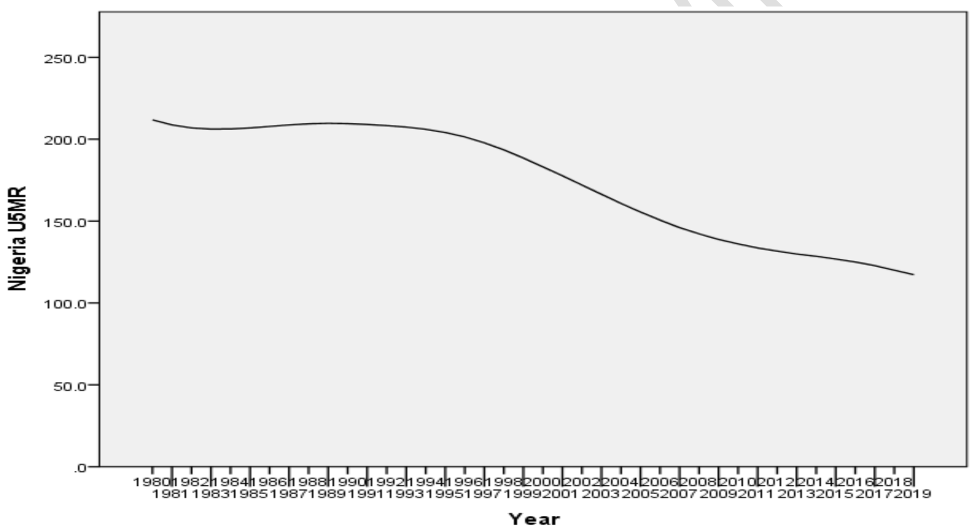


Fig. 1. Timeplot of U5MR in Nigeria for the Period 1980-2019

Table 1. Descriptive Statistics for U5MR

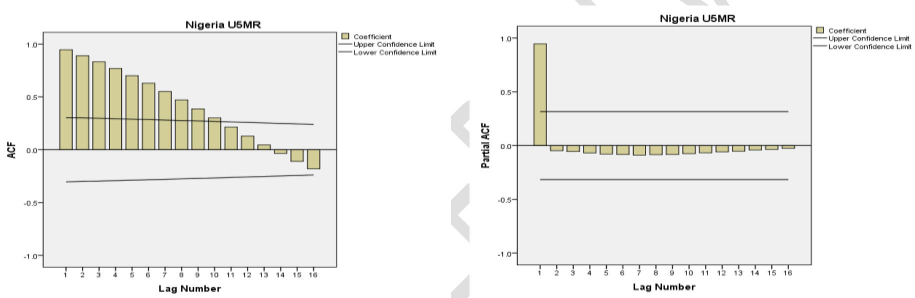
Descriptive Statistics						
	N	Minimum	Maximum	Mean		Std. Deviation
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
U5MR	40	117.2	211.8	174.315	5.4830	34.6777

Table 2. Unit Root Test in Under-Five Mortality Rate (U5MR)

Null Hypothesis: U5MR has a unit root			
Exogenous: Constant			
Lag Length: 3 (Automatic - based on SIC, maxlag=9)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.111199	0.9623
Test critical values:	1% level	-3.626784	
	5% level	-2.945842	
	10% level	-2.611531	
*MacKinnon (1996) one-sided p-values.			

In Table 2, the Augmented Dickey-Fuller (ADF) test statistic is 0.111199 with p-value of 0.9623 less than 0.05, implying that there is presence of unit root in the Under-Five Mortality Rate (U5MR), which indicates that there is need to difference at least once to get to stationarity.

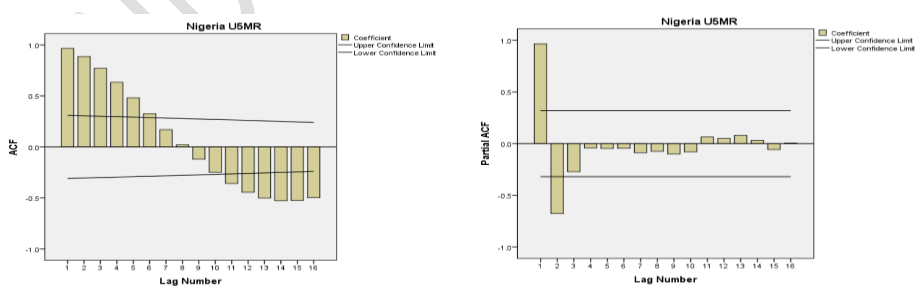
Comment [DO7]: Your P-value is greater than the 5%(0.05) so CHECK



A B

Fig. 2. (A) ACF Plot for Nigeria's U5MR (B) PACF Plot for Nigeria's U5MR

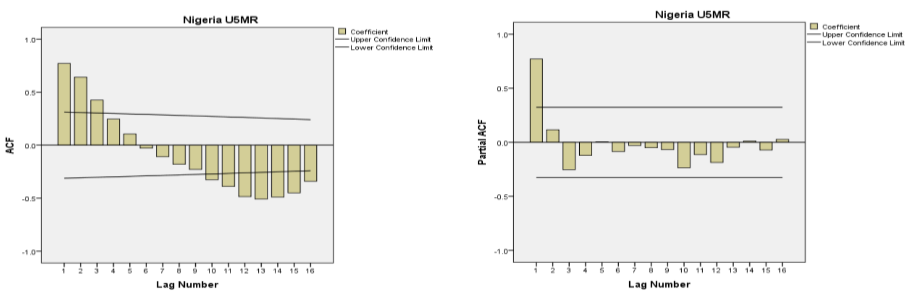
Figure 2A shows a slow fall of the lags as the lag number increases, thereby indicating a non-stationarity of Nigeria's U5MR. However, a first differenced ACF and PACF of Nigeria's U5MR is obtained as shown in Figure 3.



C D

Fig. 3. (C) First Differenced ACF Plot for Nigeria's U5MR (D) First Differenced PACF Plot for Nigeria's U5MR

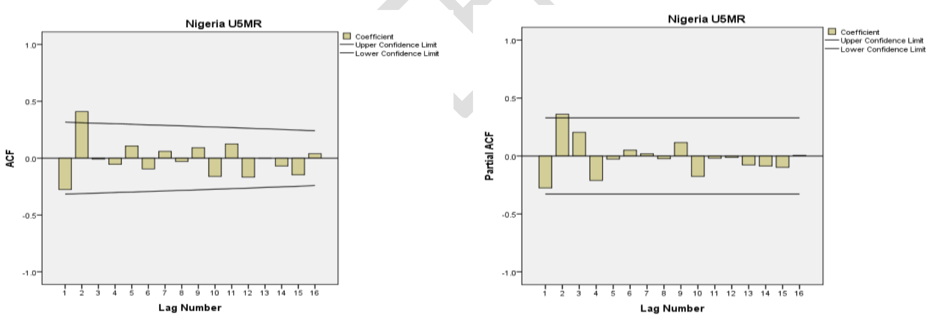
There is still a very slow fall of the lags as the lag number increases in Figure 3C which indicates that the first differenced Nigeria's U5MR is non-stationary, and will however require to be differenced the second time. Second differenced ACF and PACF of Nigeria's U5MR is obtained as shown in Figure 4



E F

Fig. 4. (E) Second Differenced ACF Plot for Nigeria's U5MR (F) Second Differenced PACF Plot for Nigeria's U5MR

There is still a very slow fall of the lags as the lag number increases in Figure 4E which indicates that the second differenced Nigeria's U5MR is non-stationary, and will however requires to be differenced the third time. Third differenced ACF and PACF of Nigeria's U5MR is obtained as shown in Figure 5.



G H

Fig. 5. (G) Third Differenced ACF Plot for Nigeria's U5MR (H) Third Differenced PACF Plot for Nigeria's U5MR

Figure 5G shows a quick fall at lag 1, thereby implying that the third differenced Nigeria's U5MR is now stationary. Lag 2 is significant as it cuts through the upper bound. And in Figure 5H, lag 2 is the only significant lag, which implies that the required ARIMA model is of order 2.

Comment [DO8]: Give more explanation on how you arrive at your Autoregressive model and Moving Average model lags using your ACF and PACF.

In Table 3, ARIMA(0,3,2) has the smallest Bayesian Information Criteria (BIC) of -2.679 which indicates that the required ARIMA model that will be used to forecast Nigeria's U5MR is the ARIMA(0,3,2). The ARIMA (0,3,2) model that will be used to forecast the Nigeria's U5MR is written as

$$y'_t = -0.581\varepsilon_{t-1} + \varepsilon_t \quad (21)$$

Table 3. ARIMA Model Adequacy Check

ARIMA(p, d, q)	BIC
ARIMA(0,3,2)	-2.679
ARIMA(2,3,0)	-2.372
ARIMA(2,3,2)	-2.260
ARIMA(1,3,2)	-2.375
ARIMA(2,3,1)	-2.281
ARIMA(1,3,0)	-2.339
ARIMA(0,3,1)	-2.295

Table 4 shows the classification of Under-Five Mortality Rate (U5MR) in Nigeria into six (6) blocks (class intervals) with their respective states.

Table 4. Classification of Under-Five Mortality Rate (U5MR)

State	Block of the Nigeria's U5MRs
1	$y \leq 129.4$
2	$129.4 < y \leq 155.9$
3	$155.9 < y \leq 180.3$
4	$180.3 < y \leq 197.6$
5	$197.6 < y \leq 207.9$
6	$y > 207.9$

Table 5. U5MR and State of Transition

Year	U5MR	State	State Transition
1980	211.8	6	
1981	208.7	6	66
1982	206.9	5	65
⋮	⋮	⋮	⋮
2000	183.1	4	44
2001	177.7	3	43
2002	172.0	3	33
⋮	⋮	⋮	⋮
2018	120.0	1	11
2019	117.2	1	11

In Table 5, taking year 1980, it is placed in state 6 but did not transit from another state due to no state preceding it. In the case of 1982, the U5MR transited from state 6 to state 5 in which it is being classified. And again, for 2019, the U5MR is transited from state 1 (U5MR of 2018) which comes before it.

The system in equation (22) is the Transition Matrix of the state transitions in Table 5; the system in equations (23), and (24) are One-Step Transition Probability Matrix, One-Step Marginal Probability Matrix and Marginal Matrix respectively.

$$F = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 1 & 8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 8 & 1 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{pmatrix} \end{array} \begin{array}{l} \\ = 5 \\ = 9 \\ = 4 \\ = 3 \\ = 10 \\ = 8 \end{array} \tag{22}$$

$$P = (p_{ij}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.11 & 0.89 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0.75 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0.10 & 0.80 & 0.10 \\ 0 & 0 & 0 & 0 & 0.25 & 0.75 \end{pmatrix} \tag{23}$$

$$q_{ij} = \begin{pmatrix} 0.128 & 0 & 0 & 0 & 0 & 0 \\ 0.026 & 0.205 & 0 & 0 & 0 & 0 \\ 0 & 0.026 & 0.077 & 0 & 0 & 0 \\ 0 & 0 & 0.026 & 0.051 & 0 & 0 \\ 0 & 0 & 0 & 0.026 & 0.205 & 0.026 \\ 0 & 0 & 0 & 0 & 0.051 & 0.153 \end{pmatrix}; \quad Q = (q_i) = \begin{pmatrix} 0.128 \\ 0.231 \\ 0.103 \\ 0.077 \\ 0.257 \\ 0.204 \end{pmatrix} \tag{24}$$

Table 6 shows the computation of the Chi-square for the verification of Markov using the information in the systems in equation (23) and (24).

Table 6. Chi-square Test for Markov Property

State, i	f_{ij}	p_{ij}	q_i	$ln\frac{p_{ij}}{q_i}$	$f_{ij}\left(ln\frac{p_{ij}}{q_i}\right)$
1	5	1.00	0.128	2.0557	10.2786
2	1	0.11	0.231	-0.7419	-0.7419
2	8	0.89	0.231	1.3488	10.7904
3	1	0.25	0.103	0.8867	0.8867
3	3	0.75	0.103	1.9853	5.9560
4	1	0.33	0.077	1.4553	1.4553
4	2	0.67	0.077	2.1635	4.3269
5	1	0.10	0.257	-0.9439	-0.9439
5	8	0.80	0.257	1.1355	9.0843
5	1	0.10	0.257	-0.9439	-0.9439
6	2	0.25	0.204	0.2033	0.4067
6	6	0.75	0.204	1.3020	7.8117
			Total		48.3669

In Table 6, the Chi-square computed is 48.3669 and the Chi-square tabulated is 37.652 ($\chi^2_{\alpha,(m-1)^2} = \chi^2_{0.05,25} = 37.652$), this implies that the stochastic process obtained has a Markov Property. Table 7 shows the weights w_k and the autocorrelation coefficients r_k for previous five-time series and the second, third, fourth, and fifth step transition probability matrices are shown in the system in Equation (25).

Table 7. Estimated Autocorrelation Coefficients and Weights of Markov Chain

	k					
	1	2	3	4	5	Total
r_k	0.94610	0.89009	0.83132	0.76859	0.70121	4.13731
w_k	0.22868	0.21514	0.20093	0.18577	0.16948	

$$P^{(2)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.44 & 0.56 & 0 & 0 & 0 & 0 \\ 0.03 & 0.41 & 0.56 & 0 & 0 & 0 \\ 0 & 0.08 & 0.47 & 0.45 & 0 & 0 \\ 0 & 0 & 0.03 & 0.15 & 0.67 & 0.15 \\ 0 & 0 & 0 & 0.03 & 0.39 & 0.58 \end{pmatrix} \quad P^{(3)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.58 & 0.42 & 0 & 0 & 0 & 0 \\ 0.07 & 0.51 & 0.42 & 0 & 0 & 0 \\ 0 & 0.20 & 0.50 & 0.30 & 0 & 0 \\ 0 & 0 & 0.07 & 0.17 & 0.57 & 0.19 \\ 0 & 0 & 0 & 0.06 & 0.46 & 0.48 \end{pmatrix}$$

$$P^{(4)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.68 & 0.32 & 0 & 0 & 0 & 0 \\ 0.13 & 0.55 & 0.32 & 0 & 0 & 0 \\ 0.03 & 0.30 & 0.47 & 0.20 & 0 & 0 \\ 0 & 0.03 & 0.11 & 0.17 & 0.50 & 0.19 \\ 0 & 0 & 0.02 & 0.08 & 0.49 & 0.41 \end{pmatrix} \quad P^{(5)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.76 & 0.24 & 0 & 0 & 0 & 0 \\ 0.19 & 0.57 & 0.24 & 0 & 0 & 0 \\ 0.06 & 0.28 & 0.42 & 0.24 & 0 & 0 \\ 0 & 0.05 & 0.12 & 0.16 & 0.46 & 0.20 \\ 0 & 0 & 0.05 & 0.10 & 0.50 & 0.35 \end{pmatrix} \quad (25)$$

Table 8. Forecast for 2015 Under-Five Mortality Rate (U5MR)

Initial Year	State	Step, k	Weight, w_k	1	2	3	4	5	6	Probability Source
2014	1	1	0.22868	1	0	0	0	0	0	$P^{(1)}$
2013	2	2	0.21514	0.44	0.56	0	0	0	0	$P^{(2)}$
2012	2	3	0.20093	0.58	0.42	0	0	0	0	$P^{(3)}$
2011	2	4	0.18577	0.68	0.32	0	0	0	0	$P^{(4)}$
2010	2	5	0.16948	0.76	0.24	0	0	0	0	$P^{(5)}$
$\hat{p}_{ij}$ (Weighted Average)				0.695	0.305	0	0	0	0	

Table 8 shows $max\{\hat{p}_{ij}\} = 0.695$, indicating that the U5MR in 2014 is in state 1 with the highest probability of 0.695, and it satisfies the interval $y \leq 129.4$, where the actual U5MR in 2015 which is 126.8 falls within the interval. Taking the average of the interval, the forecast of U5MR is 64.7. Similarly, forecasting the U5MR for 2016 using 2011-2015 as initial states as given in **Table 9**, U5MR is in state 1 with probability 0.815, and it falls within the interval $y \leq 129.4$. The actual U5MR in 2016 which is 125 shows that the prediction is also true. The same procedure is used to forecast for the remaining years. **Table 10** shows the In-sample for the period 2000-2019 and Out-of-Sample forecast of U5MR for the period 2020-2030 using ARIMA(0,3,2) and WMC and performance measures (Theil's U statistic and MAPE)

Table 9. Forecast for 2016 Under-Five Mortality Rate (U5MR)

Initial Year	State	Step, k	Weight, w_k	1	2	3	4	5	6	Probability Source
2015	1	1	0.22868	1	0	0	0	0	0	$P^{(1)}$
2014	1	2	0.21514	1	0	0	0	0	0	$P^{(2)}$
2013	2	3	0.20093	0.58	0.42	0	0	0	0	$P^{(3)}$

2012	2	4	0.18577	0.68	0.32	0	0	0	0	$P^{(4)}$
2011	2	5	0.16948	0.76	0.24	0	0	0	0	$P^{(5)}$
$\hat{p}_{ij}$ (Weighted Average)				0.815	0.185	0	0	0	0	

Table 10. In-sample and Out-of-Sample Forecast of U5MR for 2000-2030 and Performance Measures

Year	2000	2001	⋯⋯	2029	2030	MAPE	Theil's U Statistics
ARIMA(0,3,2)	182.9	177.3	⋯⋯	97.4	96.6	0.174 3 36%	0.000014
WMC	182.0	168.1	⋯⋯	64.7	64.7	17.5101 30%	0.001702

ARIMA(0,3,2) in Table 10 has a Theil's U Statistic of 0.000014 and MAPE of 0.174336% which are all lower than that of the WMC having a Theil's U Statistic of 0.001702 and MAPE of 17.510130%. This implying that ARIMA(0,3,2) is selected as the best model to forecast U5MR in Nigeria. However, by 2030, the U5MR will drop to 96.6 deaths per 1000 live births, which shows a drop of 20.6%. Figure 6 shows the timeplot for the historical data for the period 1980-2019 and out-of-sample forecast of U5MR for the period 2020-2030.

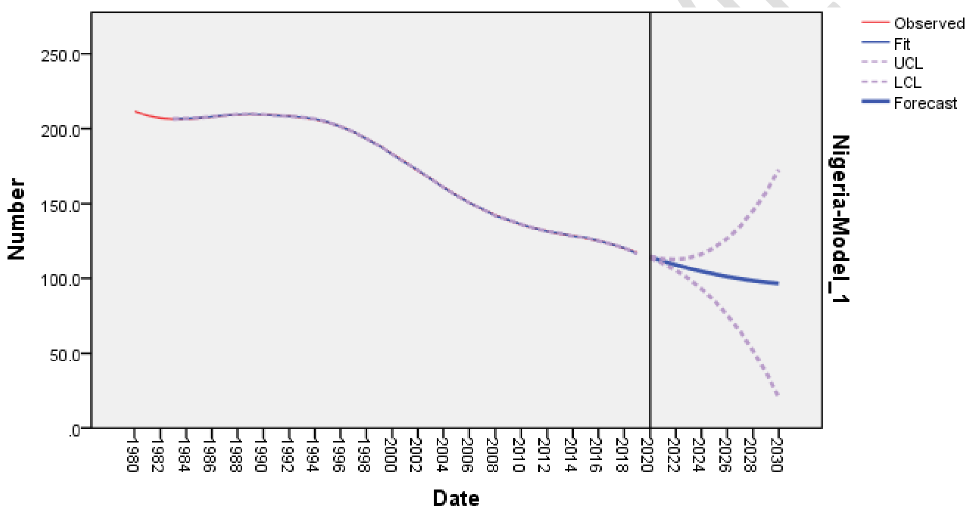


Fig. 6. Timeplot of the Actual U5MR and Out-Sample Forecast of U5MR

4. Conclusion

The purpose of this paper is to identify the best model that will be used to forecast U5MR in Nigeria. ARIMA(0,3,2) predicts U5MR better than WMC, and based on the modeling and forecasting, the U5MR is showing an instrinsic decrease from year to year. The findings of this study can help promote health policies in order to address and to reduce U5MR in the future, as well as to establish a basis for implementing optimal strategies that can be used to overcome U5MR in order to meet up with the target of SDGs.

Comment [DO9]: Include factors that must have caused persistence decrease in U5MR in your data as also seen your Forecast plot

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