CROSS SPECTRUM ANALYSIS OF LEAD-LAG RELATIONSHIP BETWEEN BCG AND DPT VACCINATIONS IN ANAMBRA STATE

Abstract

This study examined the lead-lag relationship between between Bacille Calmette-Guerin (BCG) and Diphtheria, Pertussis and Tetanus (DPT) vaccinations in Anambra State using periodogram of cross spectral analysis with BCG as the input series. The monthly data on the number of vaccinations for BCG and DPT were collected from Expanded Programmes on Immunization office in Awka, Anambra State for the period of January, 2012 to December, 2020. The Fisher's Kappa and Bartlett Komolgorov-Smirnov white noise tests showed that both series are white noise. The series were differenced stationary using Augumented Dickey-fuller test. The coherence squared presented a strong relationship between BCG and DPT vaccinations, though it did not show consistent pattern of relationship, however identified pattern of relationship existed at both lower and higher frequencies. In addition, the phase showed that at lower frequencies BCG led DPT, while at higher frequencies BCG lagged DPT. The analysis showed that there is a high awareness for both vaccinations in the State and it should be sustained by the state government.

Keywords : Vaccination, Periodogram, Cross spectrum, Coherence

1.0 Introduction

Vaccination can be defined as an act to introduce specific antigenic substances into the body to produce immunity to a specific disease. It constitutes an important public health management function globally as this strategically reduces morbidity and mortality that maybe associated with identified infectious diseases. Therefore, among these vaccines are BCG (Bacille Calmette Guerin) vaccine which has shown to be very effective against tuberculosis, and DPT vaccine used against three infectious diseases in human: diphtheria, pertussis (whooping cough), and tetanus [1]. Consequently, the administration of these vaccines is carried out with BCG given at birth and DPT administered at 6, 10 and 14 weeks of age [2].

In Nigeria, Anambra State is one of the states in south eastern part of the country with strong interest to promoting vaccinations of BCG and DPT with a vision to improve the health of children in the state by eradicating diseases preventable by these vaccines. Nevertheless, there are still challenges to achieving the vision such as immunization fatigue, political problems, poor storage system and poor accessibility to rural dwellers [3]. Therefore, it becomes imperative, to assess the comovement relationship between BCG and DPT vaccination in Anambra State since BCG is administered first at birth and it is expected that other vaccinations should follow up. Thus, does administration of BCG at birth encourage a follow up for DPT vaccination in the state?

Hence, to answer the above question a frequency domain of time series analysis shall be adopted in this study. This method is periodogram of cross spectral analysis using the Fourier sinusoidal transformation. Consequently, the periodogram analysis of lead-lag relationship between BCG and DPT vaccinations in Anambra State shall be evaluated with BCG as the input series.

2.0 Method

The method that was employed to establish a lead-lag relationship between BCG (Bacille Calmette-Guerin) and DPT (Diphtheria, Pertussis and Tetanus) with BCG as the input series is periodogram of cross spectral analysis. Nevertheless, before estimating the cross spectrum, the properties of the variables were substantiated in terms stationarity and white noise. The statistical tools that were used for these verifications are the Augmented Dickey-Fuller test for stationarity and, Fisher's Kappa and Bartlett Kolmogorov-Smirnov tests for white noise. In addition, the degree of relationship among these variables was ascertained using the computed squared coherence while, the negative and positive signs of the phase showed the lead-lag between BCG and DPT.

The data for individual variable included in this work was obtained from the Expanded Programmes on Immunization office in Awka, Anambra State. The scope is a monthly data from January, 2012 to December, 2020, while the variables are:

BCG (Bacille Calmette-Guerin) as the input series (q)

DPT (Diphtheria, Pertussis and Tetanus) as the output series (k)

2.1 Model Specifications

In order to establish the relationship between BCG and DPT comovement, the individual frequency spectrum models for the two series were developed using the Fourier transformation for sinusoidal movements. Therefore, BCG (q) model is:

$$q_t = \hat{\mu} + \sum_{j=1}^{M}\left\{\hat{\alpha}_j.\cos[\omega_j(t-1)] + \hat{\delta}_j.\sin[\omega_j(t-1)]\right\} \text{ with } \hat{\mu} = \bar{q} \text{ (sample mean BCG)…… (1)}$$

$$\hat{\alpha}_j = (2/T)\sum_{t=1}^{T} q_t.\cos[\omega_j(t-1)] \text{ For } j = 1, 2…M \text{ …….. (2)}$$

$$\hat{\delta}_j = (2/T)\sum_{t=1}^{T} q_t.\sin[\omega_j(t-1)] \text{ For } j = 1, 2…M \text{ ………. (3)}$$

The sample variance of q_t can be expressed as

$$(1/T)\sum_{t=1}^{T}(q_t - \bar{q})^2 = (1/2)\sum_{j=1}^{M}\left(\hat{\alpha}_j^2 + \hat{\delta}_j^2\right) \text{………………. (4)}$$

And the portion of the sample variance of q that can be attributed to cycles of frequency ω_j is given by

$$\frac{1}{2}\left(\hat{\alpha}_j^2 + \hat{\delta}_j^2\right) \text{……………………. (5)}$$

The portion of the sample variance of q that can be attributed to cycles of frequency ω_j can equivalently be expressed as

$$\frac{1}{2}\left(\hat{\alpha}_j^2 + \hat{\delta}_j^2\right) = (4\pi/T).\hat{s}_q(\omega_j) \text{…………. (6)}$$ where $\hat{s}_q(\omega_j)$ is the periodogram at frequency ω_j

Also, the DPT (k) model can be expressed as:

$$k_t = \hat{\mu} + \sum_{j=1}^{M}\left\{\hat{a}_j.\cos[\omega_j(t-1)] + \hat{d}_j.\sin[\omega_j(t-1)]\right\} \text{ with } \hat{\mu} = \bar{k} \text{ (sample mean DPT)…… (7)}$$

$$\hat{a}_j = (2/T)\sum_{t=1}^{T} k_t.\cos[\omega_j(t-1)] \text{ For } j = 1, 2…M \text{ …….. (8)}$$

$$\hat{d}_j = (2/T)\sum_{t=1}^{T} k_t.\sin[\omega_j(t-1)] \text{ For } j = 1, 2 \ldots M \text{ .... (9)}$$

The sample variance of k_t can be expressed as

$$(1/T)\sum_{t=1}^{T}(k_t - \bar{k})^2 = (1/2)\sum_{j=1}^{M}(\hat{a}_j^2 + \hat{d}_j^2) \text{.................. (10)}$$

And the portion of the sample variance of k that can be attributed to cycles of frequency ω_j is given by

$$\frac{1}{2}(\hat{a}_j^2 + \hat{d}_j^2) \text{........................ (11)}$$

The portion of the sample variance of k that can be attributed to cycles of frequency ω_j can equivalently be expressed as

$$\frac{1}{2}(\hat{a}_j^2 + \hat{d}_j^2) = (4\pi/T).\hat{s}_k(\omega_j) \text{............. (12)}$$ where $\hat{s}_k(\omega_j)$ is the periodogram at frequency ω_j

2.2 The Cross Spectrum Model

The cross spectrum q and k can be expressed through their covariance as:

$$\left(\frac{1}{T}\right)\sum_{t=1}^{T}(q_t - \bar{q})(k_t - \bar{k}) = \left(\frac{1}{2}\right)\sum_{j=1}^{M}(\hat{\alpha}_j \hat{a}_j - \hat{\delta}_j \hat{d}_j) \text{................. (13)}$$

Hence the portion of the sample covariance between q and k that is due to common dependence on cycles of frequency ω_j is given by:

$$\left(\frac{1}{2}\right)(\hat{\alpha}_j \hat{a}_j - \hat{\delta}_j \hat{d}_j) \text{........... (14)}$$ and this is called the cospectrum.

The sample cross periodogram from q to k at frequency ω_j can be expressed as:

$$\hat{s}_{kq}(\omega_j) = \left[T/(8\pi)\right].(\hat{a}_j + \hat{d}_j)(\hat{\alpha}_j - i.\hat{\delta}_j) = \left[\frac{T}{(8\pi)}\right].(\hat{a}_j\hat{\alpha}_j + \hat{d}_j\hat{\delta}_j) + i.\left[\frac{T}{(8\pi)}\right].(\hat{d}_j\hat{\alpha}_j - \hat{a}_j\hat{\delta}_j) \text{...... (15)}$$

The real part of (15) is called the cospectrum, while the imaginary part is called the quadrature spectrum and represented in (16) as:

$$\hat{s}_{kq}(\omega_j) = \hat{g}_{kq}(\omega_j) + i.\hat{h}_{kq}(\omega_j) \text{.......... (16)}$$

Where

$$\hat{g}_{kq}(\omega_j) = \left[\frac{T}{(8\pi)}\right].(\hat{a}_j\hat{\alpha}_j + \hat{d}_j\hat{\delta}_j)$$

$$\hat{h}_{kq}(\omega_j) = \left[\frac{T}{(8\pi)}\right].(\hat{d}_j\hat{\alpha}_j - \hat{a}_j\hat{\delta}_j)$$

However in all cases above, T = total number of observations, $M \equiv (T-1)/2$ when T is odd, $\hat{\alpha}_j, \hat{a}_j, \hat{\delta}_j, \hat{d}_j$ are coefficients.

2.3 The Squared Coherence Model

The coherence squared is defined below as:

$$z_{kq}(\omega_j) = \frac{[g_{kq}(\omega_j)]^2 + [h_{kq}(\omega_j)]^2}{s_{qq}(\omega_j)s_{kk}(\omega_j)} \quad .... (17)$$

where $s_{qq}(\omega_j)$ and $s_{kk}(\omega_j)$ are nonzero. If $s_{qq}(\omega_j)$ or $s_{kk}(\omega_j)$ is zero, the coherence squared is defined as zero. $0 \le z_{kq}(\omega_j) \le 1$ for all ω_j. If $z_{kq}(\omega_j)$ is large, this indicates that q and k have important cycles of frequency [4].

2.3 The Squared Coherence Significance Test

$$C^2 = 1 - \alpha^{\frac{1}{(n-1)}} \quad .... (18) \ [5]$$

$$\alpha = 1 - p$$

P = significance level

N = degrees of freedom

The null hypothesis of zero coherency is rejected if the value of the computed coherence squared exceeds the tabulated value at a chosen probability level and degrees of freedom [6]. In this study p = 0.95.

2.4 The Phase Model

The phase gives phase difference between two series that yields the greatest correlation for the given frequency and signs (positive or negative) of the phase determine the lead-lag. Positive sign shows that the input series is leading while; negative sign shows that the input series is lagging.

$$\frac{\sin[\theta(\omega_j)]}{\cos[\theta(\omega_j)]} = \frac{h_{kq}(\omega_j)}{g_{kq}(\omega_j)} \quad \text{......... (19)} \quad [4]$$

2.5 The Stationarity Test

A lot of tests are available to test for the stationarity of these variables, but in this work, the Augmented Dickey-Fuller test was applied and is given by the equation

$$\Delta Y_t = \beta_0 + \beta_1 t + \phi Y_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta Y_{t-j} + \varepsilon_t \quad , t = p+1 \ldots\ldots T \quad \text{........... (20)}$$

Where p lags of ΔY_{t-j} are added to remove serial correlation

Hypothesis

$H_0: \phi = 0$ (there is a unit root in the series)

$H_1: \phi < 0$ (there is no unit root in the series)

The hypothesis is tested on the basis of t-statistic of the coefficient of ϕ, with $\alpha = 0.05$

Decision rule: Reject H_0 if test statistic is less than critical values, otherwise do not reject [7].

2.6 The Fisher's Kappa White Noise Test

This tests the null hypothesis that the values in the series are white noise against the alternative hypothesis that the series has a component. Kappa is the ratio of the maximum value of the periodogram, I(f), and its average value

$$\Pr(K > k) = 1 - \sum_{j=0}^{q} (-1)^j \binom{q}{j} \left[\max\left(1 - \frac{jk}{q}, 0\right) \right]^{q-1} \quad \text{.................. (21)}$$

$q = \frac{N}{2}$ if N is even, $q = \frac{(N-1)}{2}$ if N is odd. K is the observed value of Kappa. The null hypothesis is rejected if this probability is less than the significance level.

2.6 Data

The number of monthly vaccinations of BCG and DPT in Anambra State were detrended using monthly growth rates of these vaccinations as:

$$q_t = 100.\left[\ln\left(BCG_t\right) - \ln\left(BCG_{t-1}\right)\right]$$
$$k_t = 100.\left[\ln\left(DPT_t\right) - \ln\left(DPT_{t-1}\right)\right] \quad \text{.............. (22)}$$

3.0 Resul

3.1 Descriptive Statistics

Table 1: Descriptive Statistics on Number of Monthly BCG and DPT Vaccinations in Anambra State

Statistic	B.C.G	D.P.T
No. of observations	108	108
Minimum	0.00	0.00
Maximum	8900.00	29320.00
Mean	2550.94	6640.94
Variance	22048.74	292298.84
Standard deviation	148.49	540.65

Source: Authors Computation

Table 1 shows the descriptive statistics of BCG and DPT monthly vaccinations of Anambra State within the period under consideration in this study.

3.2 Stationarity Test

Table 2.0 Dickey-Fuller Test (ADF) for q Series

Tau (Observed value)	-5.2268
Tau (Critical value)	-0.7888
p-value (one-tailed)	0.0001
alpha	0.05

Source: Authors Computation

Table 2.1 Dickey-Fuller test (ADF) for k Series

Tau (Observed value)	-5.9298
Tau (Critical value)	-0.7888
p-value (one-tailed)	< 0.0001
alpha	0.05

Source: Authors Computation

Tables 2.0 and 2.1 show that the computed p-values are less than the alpha level of 0.05, therefore there is no unit root in both series q and k. This means that both series are stationary.

3.3 White Noise Test

Table 3.0 Fisher's Kappa Test for q Series

White noise tests (q):		
Statistic	Value	p-value
Fisher's kappa	3.7644	0.7545

Source: Authors Computation

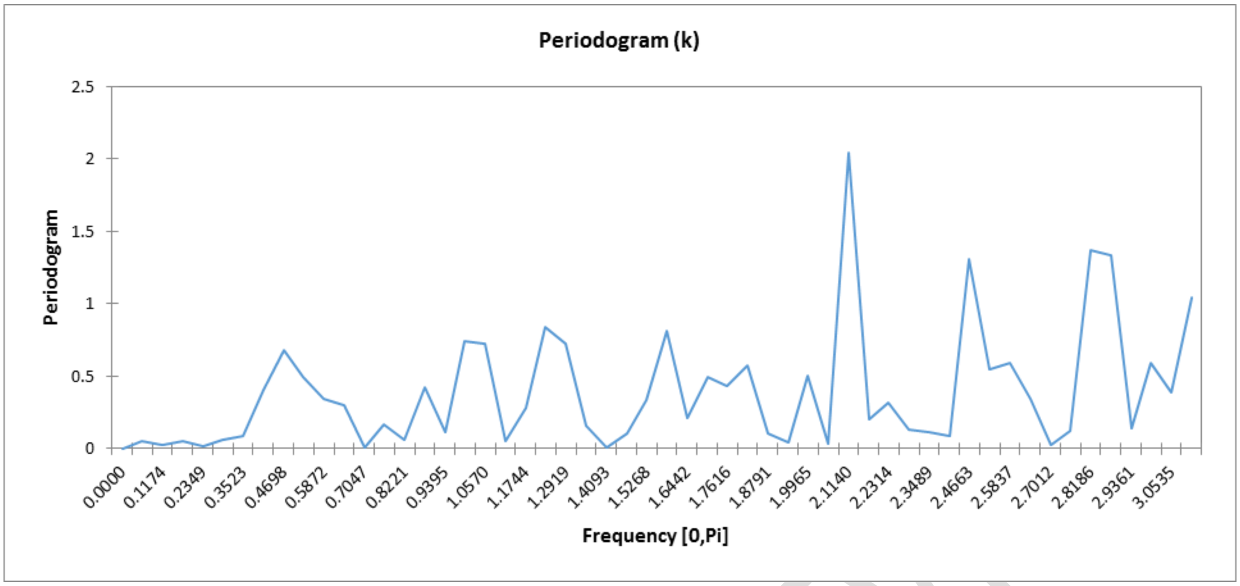
Table 3.1 Fisher's Kappa Test for k Series

White noise tests (k):		
Statistic	Value	p-value
Fisher's kappa	5.1141	0.2515

Source: Authors Computation

Tables 3.0 and 3.1 show that the computed p-values are greater than the alpha level of 0.05, this indicates that both series q and k are white noise and do not contain any component.

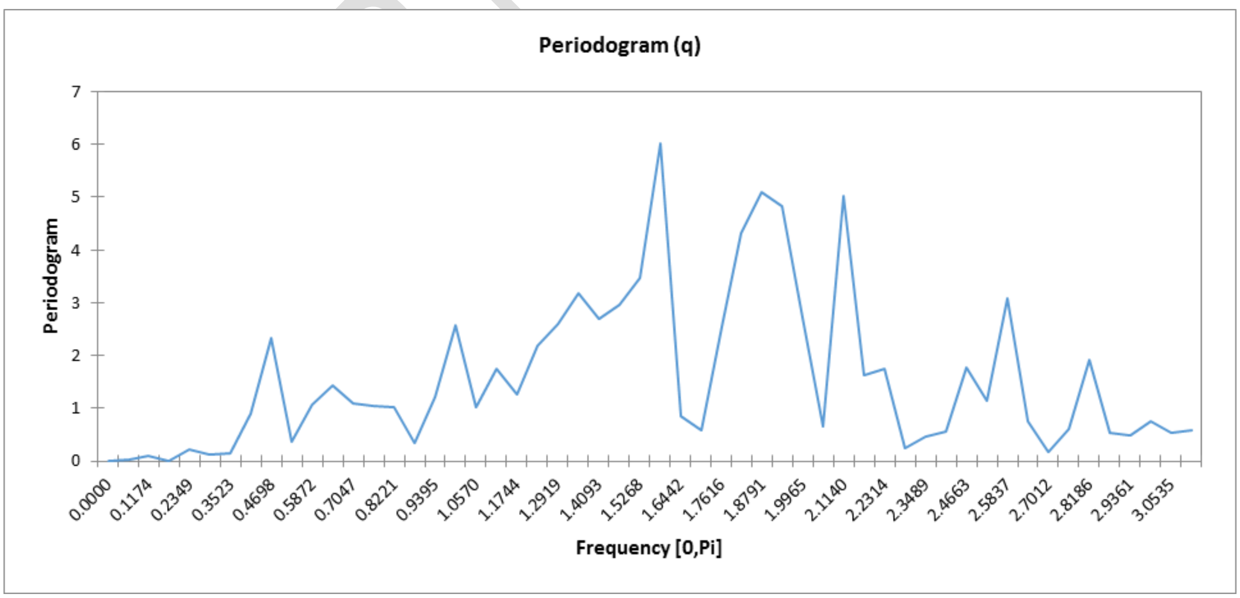
3.4 Periodogram of monthly growth rate of DPT and BCG Vaccination series



Source: Authors Computation

Fig. 1: The Periodogram of Monthly Growth Rate of DPT Vaccination in Anambra State

Fig.1 shows that within the power spectrum of the monthly growth rate of DPT vaccinations in Anambra State, frequency 2.1140 contributed the highest proportion of 9.65% to the total variance of DPT, followed by frequency 2.8186 which add 6.50% proportion to the total variation. However, frequency 1.4093 contributed the lowest proportion of 0.03% of the total variance.

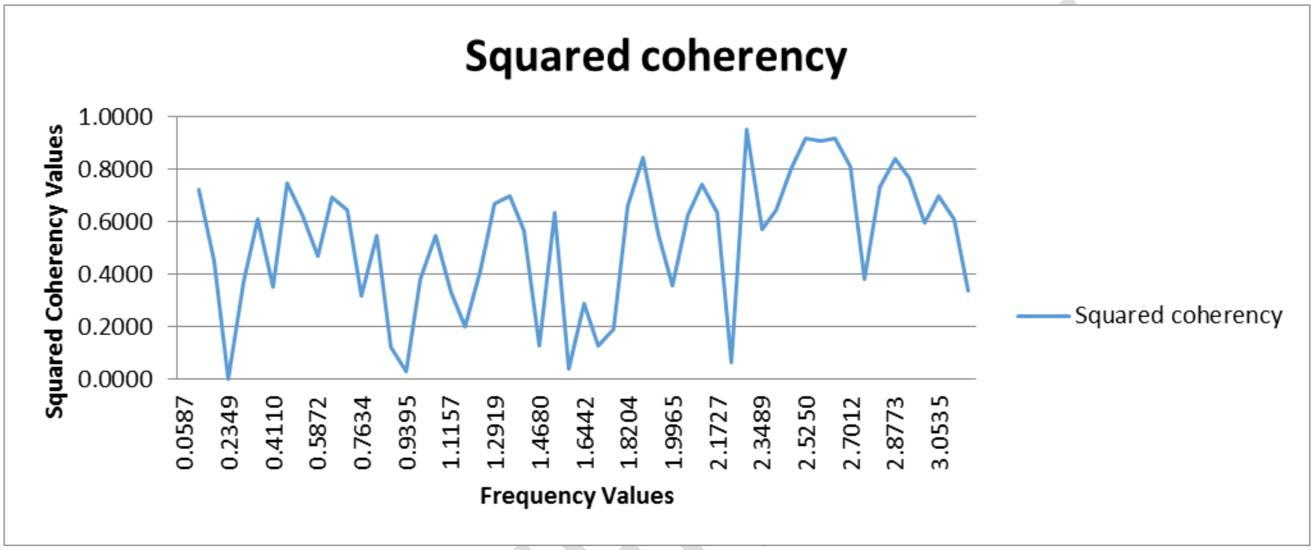


Source: Authors Computation

Fig. 2: The Periodogram of Monthly Growth Rate of BCG Vaccination in Anambra State

In Fig.2, the power spectrum of the monthly growth rate of BCG vaccinations in Anambra State shows that frequency 1.5855 contributed the highest proportion of 7.10% to the total variance of BCG series, while frequency 1.8791 followed with a contribution of 6.03%. Frequency 0.1762 added the lowest proportion of 0.01%.

3.5 The Squared Coherence

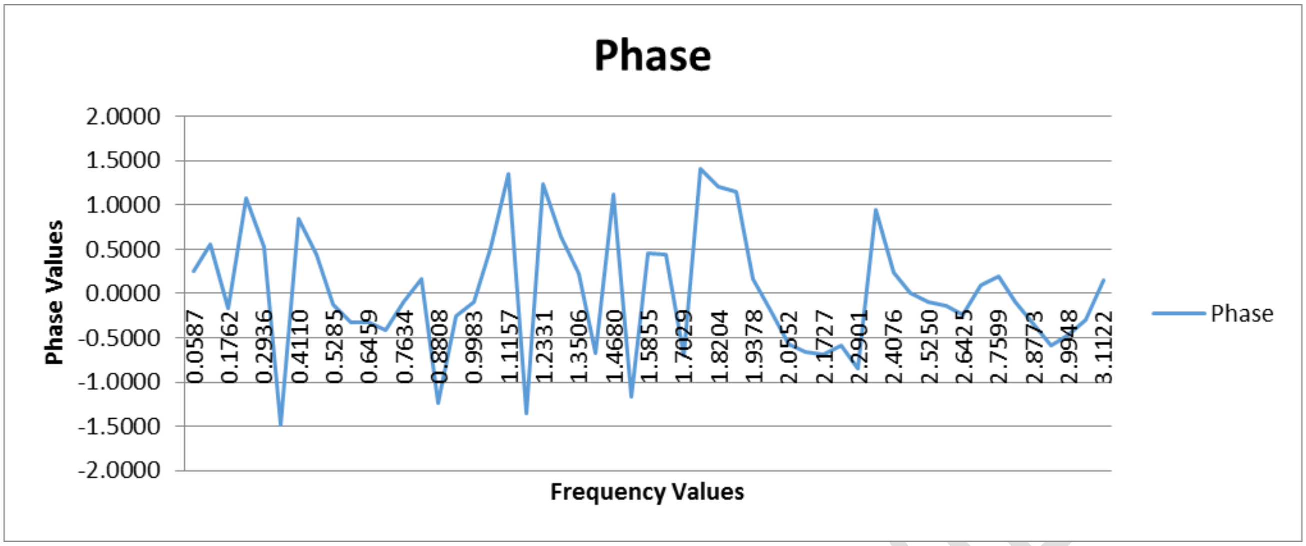


Source: Authors Computation

Fig. 3: The Squared Coherence of Monthly Growth Rate of BCG and DPT Vaccinations in Anambra State

The tabulated squared coherency value at 106 degrees of freedom and probability of 0.95 is 0.0279. Therefore, fig.3 shows that majority of the square coherency values of monthly growth rate of BCG and DPT vaccinations are significant at different frequencies since these values are greater than the tabulated squared coherency value. The highest coherency value of 0.9520 occurred at frequency 2.2314 and it is significant while the least coherency value of 0.0041 happened at frequency 0.1762.

3.6 The Phase



Source: Authors Computation

Fig. 4: The Phase of Monthly Growth Rate of BCG and DPT Vaccinations in Anambra State

Fig. 4 shows that that the phase between monthly growth rate of BCG and DPT vaccinations in Anambra State did not show consistent pattern.

4.0 Discussion

A cross-spectral analysis was conducted with number of BCG vaccinations as the input series and number of DPT vaccinations as the output series. Fig.3 and Fig. 4 show respectively the coherence and phase plots of monthly growth rate of BCG and DPT vaccinations in Anambra State. The horizontal axis of both Fig.3 and Fig.4 shows the frequency plots and vertical axis shows indicates coherence and phase levels respectively. A positive phase value means that the input series is leading ahead the output series, while negative value of phase means that the input series is lagging behind the output series. However in Fig.3, majority the coherence values between monthly growth rate of BCG and DPT in the State were all significant at probability of 0.95 which indicates that there is a strong relationship and comovement between the series. The highest coherency value of 0.9520 occurred at frequency 2.2314; this shows that both series are highly linear at frequency 2.2314.

In addition, in Fig.4 the phase did not show a consistent pattern, nevertheless in terms of lead-lag relationship between the two series with BCG as the input series, at lower frequencies the phase showed more of positive sign indicating that BCG vaccinations lead DPT vaccinations confirming that BCG is administered to children first before DPT in Anambra State, therefore suggesting high awareness of these vaccination in the State. Also, at higher frequencies BCG vaccinations lagged behind DPT vaccinations showing that more DPT vaccinations are administered to children in Anambra State after taking BCG vaccine.

5.0 Conclusion

This study used periodogram of cross-spectral method to analyze the lead-lag relationship between monthly BCG and DPT vaccination in Anambra State. The analysis of the coherence showed a significant strong relationship between BCG and DPT at almost all the frequencies. From the phase diagram in Fig.4, the monthly BCG vaccinations led ahead DPT vaccination at lower frequencies, in the middle frequencies, there was no clear pattern of leads and lags observed since the diagram crossed the horizontal line severally. However, at higher frequencies BCG vaccination lagged behind DPT vaccinations. Nevertheless, shows that there is a high awareness and administration of these vaccines in the State.

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