# QUEING THEORY, A TOOL FOR POLIO ERADICATION IN NIGERIA

Comment [H1]: Queuing not Queing

ABSTRACT

To curb the spread of contagious diseases and the recent polio outbreak in Nigeria, health departments must set up and operate clinics to dispense medications or vaccines. Residents arrive according to an external (not necessarily Poisson) Arrival process to the clinic. When a resident arrives, he goes to the first workstation, based on his or her information, the resident moves from one workstation to another in the clinic.The queuing network is decomposed by estimating the performance of each workstation using a combination of exact and approximate models. A key contribution of this research is to introduce approximations for workstations with batch arrivals and multiple parallel servers, for workstations with batch service processes and multiple parallel servers, and for self service workstations. We validated the models for likely scenarios using data collected from one of the states vaccination clinics in the country during the vaccination exercises.

**Key words:** Polio, Queuing network, Servers, Work station, Processes, Service, Vaccination.

Comment [H2]: Abstract contents should be reported in past tense

## I. INTRODUCTION

The threat of an outbreak of contagious disease caused by untreated water or a natural occurrence, has prompted public health departments to update and enhance their plans for responding to such events. Especially in regions that are densely populated such as the nation's capital, state capitals and local government head quarters. In the worst-case scenario, terrorists could release a lethal virus, such as smallpox, into the general population or as a result of not well treated water supply which may result in the outbreak of "polio" popularly known as cripple disease.

Polio invades the nervous system, and can cause total paralysis in a matter of hours. It affects children under five years through contaminated drinking water. The virus enters the body through the mouth and multiplies in the intestine. Initial symptoms are fever, fatigue, headache and vomiting. Other symptoms are stiffness in the neck and pains in the limbs. One in 200 infections leads to irreversible paralysis 'usually in the legs'. Amongst those paralyzed, 5%-10% die when their breathing muscles become immobilized. Although polio paralysis is the most visible sign of polio infection, less than 1% of polio infections ever result in paralysis. Poliovirus can spread widely before cases of paralysis are seen. Poliovirus can travel from village to village and country to country, through un-immunized children. One unimmunized child can leave tens or hundreds more paralyzed for life. While polio exists anywhere, children everywhere are at risk. Although different responses are available, mass vaccination should be an effective policy *(Kaplan et al. 2002).*

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In the case of Polio, every child below the age of six in the affected nation has to be vaccinated. For example, Okene town in Kogi state of Nigeria would need to vaccinate nearly 361,000 thousand people. To vaccinate so many people in a short period it would have to set up mass dispensing and vaccination clinics. Cities across the federal republic of Nigeria have to create plan for this type of response if polio and other infected diseases has to be reduced to the minimum possible level.

Models of clinics are useful during the planning process. Two key clinic performance measures are the clinic capacity and the average time that a customer spends in the clinic (from arrival to departure), which we call cycle time 'also known as flow time or throughput time'. Clinic capacity is important for verifying that the clinic can treat the affected population in the required time. Estimating cycle time is necessary to determine how much space to allow in the clinic for queues. From the clinic planning perspective, reducing queuing is important to reduce the number of residents in the clinic, since large numbers of people increase crowding, confusion, and the chance of chaos.

While the study of queuing networks has resulted in numerous results, the need to model queuing networks with batch service processes performed by multiple parallel servers and self service stations led us

to propose the model to be developed here.

As we cannot progress without taking a look at the summary of what are the objectives set for achievement in the study, we hereby list them as well as the expected contributions to knowledge.

The objectives of this research are to:

(i) model a formula of the form $\frac{(\overline{K}_{Ai}-m_i+m_iu_i-U_i)(\overline{K}_{Ai}-m_i+m_iu_i-U_i+1)}{2\overline{K}_{Ai}}\frac{t_i}{m_i}$ for wait in batch time ($WIBT_i$) at station i, $u_i^{\sqrt{2m_i+2}-1}$ for steady state probability ($U_i$) that all the servers at station i are busy in a given time, $\overline{K}_{Ai}-m_i+m_iu_i-U_i$ for average number of residents ($X_i$) that wait in the batch at station *i*, To use the formula $\frac{k_i-1}{2\lambda_i}$ for wait to batch time ($WTBT_i$) at station i. where $m_i, t_i, k_i, \lambda_i, \overline{K}_{Ai}$ are respectively the number of staffs, processing time, batch size, processing rate and average batch arrival size at station i, model a self service station as G/G/∞ queuing system

(ii) use the modeled formula to calculate the average cycle time in all the stations from 1 to I before the final station I+1 which is the exit.

(iii) use the result obtained at each station to calculate the average cycle time of patients in the mass dispensing and vaccination clinics.

(iv) use the average cycle time obtained to estimate how many patients can wait in the `clinic at the same time and hence the planning for mass dispensing and vaccination clinic to vaccinate large population within a limited time to aid the reduction of polio and other infectious diseases to the minimum possible level.

## II. LITERATURE REVIEW

Recent polio outbreak and other infectious diseases in Nigeria and other parts of the world have focused increase attention on the ability of state and local public health authorities to provide affected individuals and communities with rapid, reliable access to medications or vaccination.

Fortunately, guidelines and standards provided by National Primary Health Care Development Agency (NPHCDA), National Emergency Management Agency (NEMA) or non-federal health organizations do exist to aid planners of the clinics in their work. Moreover, In order to design the best policy of managing the clinics and give the personnel training under real working conditions, local governments sometimes run full-scale disaster simulations. During these exercises, the performance measures recorded there were used to build a computer simulation model and construct the several pieces of software and spreadsheets. These software packages along with their related tools are basically constructed based on the employment of **s**tatistics and operations research discipline called queuing theory which is mainly used to approximate the performance of the queuing networks like what we did in this paper as dispensing and vaccination clinics for polio reduction or eradication if possible. Since there is plenty of room for improvement in the currently available software tools, particularly with regard to their ability to adapt their models to a particular situation, the role of queuing network theory in updating the existing models as well as introducing the new queue approximations by utilizing more exact approaches is undeniable.

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## QUEUING THEORY IN GENERAL

Queuing theory is generally considered a branch of operations research, and it is simply the science of waiting. Since jobs "stand in line" while waiting to be processed, waiting to move, waiting for parts, and so on. Queuing theory is a powerful tool for studying and modeling any system having a queue inside such as manufacturing, transportation, and telecommunication system.

The theory enables mathematical analysis of several related processes, including arriving at the queue (arrival or input process), waiting in the queue (waiting process) and being served at the workstations (service process). Each workstation consists of units which provide service to the arriving entities such as jobs or customers. These units are usually called servers and can be either people or machines. A queuing system combines the components that have been considered so far: an arrival (input) process, a queue, and a service process. For the arrival process, in most cases, the arrival process is the product of external factors.

Therefore, the best way, one can do is to describe the arrival process in terms of random variables which can represent either the number of arrivals during a time interval or the time interval between successive arrivals. For a queue, the possible queuing discipline can be First-In First-Out (FIFO), Last-In First Out (LIFO), Highest Priority In First Out (HPIFO), Shortest Process Time (SPT), Earliest Due Date (EDD), or any of a host of priority schemes. In many situations customers in some cases get priority in service over others.

## III. METHODOLOGY

As part of developing and testing the queuing models, we will use simulation discrete-event models of queuing systems in various ways such as validation and experimentation. Discrete-event simulation models carried out in this study were all created by Rockwell Software's Arena 7.0 ®.

One of the most important performance characteristics that queuing theory is used to describe is the time a customer or job spends waiting to find an idle server. To cover all of the cases, we have to find an approximation that satisfy the cases with general arrival and process distribution in which we have multiple servers working in parallel to serve several customers at once. *Sakasegawa (1977)* proposed an approximation for this queuing time for *G/G/m*, with $m$ representing the number of servers, given in Equation 1 below. Moreover, $C_a^2$ and $C_e^2$ respectively represent inter-arrival time and service time variability (SCV) when m=1, this equation reduces to G/G/1 approximation. The G/G/m approximation for queuing time is: $\boldsymbol{CT_q = (\frac{C_a^2+C_e^2}{2})\frac{u^{\sqrt{2m+2}-1}}{m(1-u)}t}$ **…… (1)** *(Sakasegawa 1977)*

To analyze batch arrivals, we study queuing systems in which customers arrive at a station in batches but are processed as individuals. There are two ways of handling them. The first method of un-batching is to treat them as individuals arriving in a process with an extremely high SCV; the arrival variability of individuals out of a batch is given below,

$$C_a^2 = KC_{b,a}^2 + (K-1) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$ (Curry, 2002)

Where the processing time SCV is $C_{b,a}^2$ *and* $K$ *is the arriving batch size*:

To explain Wait in Batch Time WIBT more, since there are *k* items in the batch, the items have different delays while awaiting their turn at service. The first item served from a batch has no additional delay due to waiting for others from the same batch, while the second item serviced waits for the first item; the third item waits for the first two selected items, and so on. $CT_q = \left(\frac{c_{b,a}^2+\frac{c_e^2}{K}}{2}\right)\frac{u}{1-u}(kt)$ …… (3)

WIBT = $\frac{(k-1)t}{2}$ …… (4)

These Equations are for a queuing system with a fixed size arrival batch size and a single server with individual service process (*G*/*G*/1).

The approximation for WIBT must be adjusted to accommodate a station with multiple servers, again by scaling the mean service time.

$CT_q = (\frac{C_a^2+C_e^2}{2})\frac{u^{\sqrt{2m+2}-1}}{2(1-u)m}t$ …… (5)

$WIBT = \frac{(k-1)t}{2m}$ …… (6)

**Comment [H7]:** equations 3, 4, 5 and 6 are not referenced

To demonstrate the accuracy of this approximation, it is compared to an equivalent simulation model. The results of the simulation for confidence interval 95% are given in Table 1, along with the values obtained using the new approximation for both portions of the waiting time. The magnitude of error between the two is given as a percentage of the simulation value which Arena calculated for each of the performance measures.

### COMPLETE QUEUING MODELING FRAMEWORK FOR THE CLINICS

**INPUTS**

*P* = Size of population to be treated (residents)
*L* = Time allotted for treatment (days)
*h* = Daily hours of operation (hours per day)
*N* = Number of clinics

$m_i$ = Number of staff at station *i*
$t_i$= Mean process time at station *i* (minutes)
$\sigma_i^2$= Variance of mean service time at station *i* (minutes2)
$k_i$ = Processing batch size at station *i*
$d_{ij}$ = Distance from station *i* to station *j* (feet)
$v$ = Average walking speed (feet per second)
$p_{ij}$ = Routing probability from station *i* to station *j*
$k_0$= Bus arrival size
$\bar{k}_{ai}$ =Aggregate batch arrival size to the station *i*
$c_{a1}^2$ = inter-arrival time SCV at station 1

**OUTPUTS**
*TH'* = Required throughput (residents per minute)
$m_i'$= Minimum staff at station *i*
$WTBT_i$= Wait to batch time at station *i* (minutes)
$WIBT_i$ = Wait in batch time at station *i* (minutes)
$CT_i$ = Cycle time at station *i* (minutes)
*TCT* = Total average time in clinic (minutes)
*WIP* = Average number of residents in clinic
$\lambda_i$= Batch arrival rate at station *i* (batches per minute)
$C_{ai}^2$= Inter-arrival time SCV at station *i*
$C_{ei}^2$ = Process time SCV at station *i*
$C_{di}^2$ = Inter-departure time SCV at station *i* for process batches
$R$= Clinic capacity (residents per minute)
$CTq_i$= Average queue time at station *i* (minutes)
$W_i$ = Average time spent traveling to the next station after station *i* (minutes)
$Q_i$ = Average queue length at station *i*
$u_i$= Utilization at station *i*

**EQUATIONS**
The throughput required to treat the population in the given time is $TH' = \frac{p}{60LhH}$ If residents arrive individually, the user specifies the arrival variability $C_{a1}^2$. Else, the individual resident arrival variability is given as $C_{a1}^2 = k_0 - 1$
All arriving residents go to the first station. We calculate the arrival rates for the other stations based on the routing probabilities:

$$\lambda_1 = \begin{cases} TH' & (i = 1) \\ \sum_{j=1}^{i-1} \lambda_j p_{ji} & (i > 1) \end{cases}$$

At each station after the first, we calculate arrival batch size based on the process batch size of the previous stations:

$$\bar{k}_{ai} = \begin{cases} k_0 & (i = 1) \\ \sum_{j=1}^{i-1} k_j \frac{\lambda_j p_{ji}}{\lambda} & (i > 1) \end{cases}$$

We use station arrival rates to determine the minimum staff at each station:

$$m'_i = \frac{\lambda_i t_i}{\bar{k}_{ai}}$$

We then use user-selected staff levels $m_i$ to calculate station utilization:

$$u_i = \frac{\lambda_i t_i}{m_i k_i}$$

We calculate the variability of arrivals, processes, and departures from each station:

$$C_{ai}^2 = \sum_{j=1}^{i-1} \{(C_{di}^2 - 1)P_{ji} + 1\}. \frac{\lambda_j P_{ji}}{\lambda_i}$$

$$C_{ei}^2 = \frac{\delta_i^2}{t_i^2}$$

$$C_{di}^2 = (K_i - 1 + K_i)\left[1 + (1 - U_i^2)\left\{\frac{C_{ai}^2}{K_i} - 1\right\} + \frac{U_i^2}{\sqrt{m_i}}(C_{ei}^2 - 1)\right]$$

The average time spent waiting at station *i* depends upon the arrival and process batch sizes; denotes time waiting for service, while $WIBT_i$ represents time waiting in arrival batches and $WTBT_i$ represents time waiting to form a process batch.

$$\overline{K}_{ai} = \sum_{j=1}^{i-1} K_j \frac{\lambda_j P_{ji}}{\lambda}$$

$$CT_{qi} = \left(\frac{C_{ai}^2 + C_{ei}^2}{2}\right).\left(\frac{u_i^{\sqrt{2m_i+2}-1}}{m_i(1-u_i)}\right).\frac{t_i}{m_i} \quad K_i = 1,\ \overline{K}_{ai} > 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$CT_{qi} = \left(\frac{\frac{C_{ai}^2}{K_i} + C_{ei}^2}{2}\right)\left(\frac{u_i^{\sqrt{2m_i+2}-1}}{(1-u_i)}\right).\frac{t_i}{m_i} \quad K_i > 1, \overline{K}_{ai} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$WTBT_i = \frac{K_i - 1}{2\lambda_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$WIBT_i = \frac{(\overline{K}_{ai} - 1)t_i}{2m_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The average time spent traveling to the next station after station *i* depend upon the routing probabilities and the average walking speed:

$$W_i = \frac{1}{60v}\sum_{j=i+1}^{I+1} P_{ij} d_{ij}$$

The cycle time at station *i* is

$$CT_i = WTBT_i + CT_{qi} + t_i + W_i$$

We weight the station cycle times by their arrival rates to calculate the total average time in clinic:

$$\text{CT} = \frac{1}{\lambda_1}\sum_{i=1}^{I} \lambda_i CT_i$$

Other statistics we calculate include clinic capacity, the average queue length at each station, and the average clinic WIP:

$$\text{R} = \min_{i=1,\ldots,I}\left(\frac{m_i\lambda_i}{t_i\lambda_i}\right)$$

$$\text{WIP} = \lambda_1 . CT$$

$$Q_i = CT_{qi}\lambda_i$$

**The new Approach to WIBT and the Equations.**

As an important point, we should say that the Equation extracted in this section is applicable for the scenarios in which the arrival batch sizes is larger than the number of servers.

We will use the following notation:

$m_i$ = Number of staff at station *i*

$t_i$ = Mean process time at station *i* (minutes)

$\lambda_{Ai}$ = Batch arrival rate at station *i* (batches per minute)

$\overline{K}_{Ai}$ = Average batch size of all batches that come to station *i*

$u_i$ = Utilization at station *i*

$P_n(i)$ = Steady-state probability of having *n* residents in station *i*.

$U_i$ = Steady-state probability of all of the servers at station *i* being busy

$X_i$ = Average number of residents that wait in the batch at station *i*.

$WIBT_i$ =Average wait in batch time at station *i* (minutes)

We can estimate the wait-in-batch-time for multiple servers by referring to the previous equations as follows:

$$WIBT_i = \frac{(\overline{K}_{Ai} - 1)t_i}{2\text{m}_\text{i}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

As we will see, this is not a good approximation, so we will derive a new Equation for the wait-in-batch-time. To do so, we start by calculating the following terms:

$$u_i = \frac{\lambda_{Ai}\overline{K}_{Ai}t_i}{m_i}$$

$$U_i = \sum_{n=m_i}^{\infty} p_n(i) = 1 - \sum_{n=0}^{n_i-1} p_n(i)$$

**Comment [H8]:**

**Comment [H9]:** Ensure each equation is numbered and referenced accordingly, this should reflect throughout the paper.

It will be useful to note the following

$$\sum_{n=0}^{m_i-1} np_n(i) = m_i u_i - m_i \sum_{n=m_i}^{\infty} p_n(i) = m_i(u_i - U_i)$$

If, when the batch arrives, the number of residents, who are already in the system, is greater than or equal to the number of servers, all of the servers are busy, so the batch waits in the queue. Eventually, the batch is at the head of the queue and one of the servers completes a resident. Then the batch opens, one resident begins service without waiting in batch, and all of the others wait in the batch. If, when the batch arrives, the number of residents, who are already in the system, is less than the number of servers, one or more servers are idle, so the batch opens and one or more residents begin service immediately. From this we estimate $X_i$ as follows:

$X_i = \sum_{n=0}^{m_i-1} p_n(i)(\overline{K}_{Ai} - m_i + n) + \sum_{n=m_i}^{\infty} p_n(i)(\overline{K}_{Ai} - 1)$ $= \overline{K}_{Ai} - m_i(1 - U_i) + m_i(u_i - U_i) - U_i$

$= \overline{K}_{Ai} - m_i + m_i u_i - U_i$

Thus $\overline{K}_{Ai} - X_i$ residents go to server immediately. For them, WIBT is equal to zero. Assuming that the servers, when busy, complete a resident every $\frac{t_i}{m_i}$ minute, the first resident of those remaining must wait-in-batches for $\frac{t_i}{m_i}$ minutes. The second waits for $\frac{2t_i}{m_i}$ minutes, and so forth. The last resident in the batch waits for $\frac{X_i t_i}{m_i}$ minutes. Then we can estimate the average wait-in-batch-time as follows:

$$WIBT_i = \frac{1}{\overline{K}_{Ai}} \sum_{n=1}^{X_i} \frac{nt_i}{m_i} = \frac{X_i(X_i+1)}{2\overline{K}_{Ai}} \frac{t_i}{m_i} = \frac{(\overline{K}_{Ai} - m_i + m_i u_i - U_i)(\overline{K}_{Ai} - m_i + m_i u_i - U_i + 1)}{2\overline{K}_{Ai}} \frac{t_i}{m_i} \quad \text{..... (12)}$$

It is obvious that in addition to batch size $k$, number of servers $m$ and at a given time $t$ as shown by (Hopp and Spearman, 2001). The above Equation considers other parameters like station utilization $u_i$ and the steady state probability that all the servers are busy $U_i$ at a given time and this will give a good result as will be applied to our clinic model.

To obtain the relationship between $U_i$ *and* $u_i$ we estimate $U_i$ Following Shore (1988) and dropping the station subscript for the moment, we let $E(N_c)$ be the mean number of customers in the system and $E(N_1)$ be the mean number of customers in the corresponding GI/G/1 queue having the same traffic intensity.

$$E(N_c) = m_i u_i + \left[\frac{u_i^{\sqrt{2m_i+2}}}{1-u_i}\right]\left[\frac{C_{ai}^2 + C_{ei}^2}{2}\right] \quad \text{..... (13)}$$

$$E(N_1) = m_i u_i + \left[\frac{u_i^2}{1-u_i}\right]\left[\frac{C_{ai}^2 + C_{ei}^2}{2}\right] \quad \text{..... (14)}$$

*Shore (1988)* shows that $U_i = u_i(E(N_c) - m_i u_i)/(E(N_1) - u_i)$

From this, we extract $U_i = u_i^{\sqrt{2m_i+2}-1}$. Since this is not affected by the arrival variability, we will use this result for our batch arrival case. Going back to the original notation, we have $U_i = u_i^{\sqrt{2m_i+2}-1}$ ..... (15)

## IV. RESULTS AND DISCUSSIONS

**Table 1. Experiment with exponential distributed process times.**

| Scenarios | Batch Size | Mean Inter-arrival Time (mins) | Mean Processing Time(mins) | Number Of servers |
|---|---|---|---|---|
| **E-5-1-99** | **5** | **0.1684** | **0.0333** | **1** |
| **E-5-1-95** | **5** | **0.1754** | **0.0333** | **1** |
| **E-5-1-90** | **5** | **0.1852** | **0.0333** | **1** |
| **E-5-1-80** | **5** | **0.2083** | **0.0333** | **1** |
| **E-5-1-50** | **5** | **0.3333** | **0.0333** | **1** |
| **E-5-3-99** | **5** | **0.1684** | **0.1000** | **3** |
| **E-5-3-95** | **5** | **0.1754** | **0.1000** | **3** |
| **E-5-3-90** | **5** | **0.1852** | **0.1000** | **3** |
| **E-5-3-80** | **5** | **0.2083** | **0.1000** | **3** |
| **E-5-3-50** | **5** | **0.3333** | **0.1000** | **3** |

**Comment [H10]:** Try and rewrite your result discussions, and all tables not showing result should be remove.

**Comment [H11]:** Table 1 is not discussed

**Table 2. Result for Experiment with exponentially distributed process times.**

| Scenario | WIBT from simulation (mins) | WIBT from Equation 11 (mins) | Relative error, Equation (11) | WIBT from Equation (12) (mins) | Relative error, Equation (12) |
|---|---|---|---|---|---|
| E-5-1-99 | 0.0667 | 0.0667 | 0.050% | 0.0667 | 0.050% |
| E-5-1-95 | 0.0667 | 0.0667 | 0.050% | 0.0667 | 0.050% |
| E-5-1-90 | 0.0667 | 0.0667 | 0.050% | 0.0667 | 0.050% |
| E-5-1-80 | 0.0667 | 0.0667 | 0.050% | 0.0667 | 0.050% |
| E-5-1-50 | 0.0667 | 0.0667 | 0.050% | 0.0667 | 0.050% |
| E-5-3-99 | 0.0660 | 0.0667 | 1.010% | 0.0663 | 0.475% |
| E-5-3-95 | 0.0660 | 0.0667 | 1.010% | 0.0649 | 1.720% |
| E-5-3-90 | 0.0600 | 0.0667 | 11.111% | 0.0630 | 4.959% |
| E-5-3-80 | 0.0600 | 0.0667 | 11.111% | 0.0590 | 1.748% |
| E-5-3-50 | 0.0480 | 0.0667 | 38.889% | 0.0453 | 5.717% |

**Table 3. Experiment with gamma distribution process times and 1 server.**

| Scenarios | Batch Size | Mean Inter-arrival Time (mins) | Mean Processing Time (mins) | Number Of servers |
|---|---|---|---|---|
| G-5-1-99 | 5 | 0.1684 | 0.3333 | 1 |
| G-5-1-95 | 5 | 0.1754 | 0.3333 | 1 |
| G-5-1-90 | 5 | 0.1852 | 0.3333 | 1 |
| G-5-1-80 | 5 | 0.2083 | 0.3333 | 1 |
| G-5-1-50 | 5 | 0.3333 | 0.3333 | 1 |
| G-20-1-99 | 20 | 0.1684 | 0.0083 | 1 |
| G-20-1-95 | 20 | 0.1754 | 0.0083 | 1 |
| G-20-1-90 | 20 | 0.1852 | 0.0083 | 1 |
| G-20-1-80 | 20 | 0.2083 | 0.0083 | 1 |
| G-20-1-50 | 20 | 0.3333 | 0.0083 | 1 |

**Table 4. Result for Experiment with gamma distribution process times and 1 server.**

| Scenario | WIBT From Simulation (Mins) | WIBT From Equation 11 (Mins) | Relative Error Equation 11 | WIBT From Equation 12 (Mins) | Relative Error Equation 12 |
|---|---|---|---|---|---|
| G-5-1-99 | 0.0665 | 0.0667 | 0.251% | 0.0667 | 0.251% |
| G-5-1-95 | 0.0665 | 0.0667 | 0.251% | 0.0667 | 0.251% |
| G-5-1-90 | 0.0665 | 0.0667 | 0.251% | 0.0667 | 0.251% |
| G-5-1-80 | 0.0665 | 0.0667 | 0.251% | 0.0667 | 0.251% |
| G-5-1-50 | 0.0665 | 0.0667 | 0.251% | 0.0667 | 0.251% |
| G-20-1-99 | 0.0790 | 0.0792 | 0.211% | 0.0792 | 0.211% |
| G-20-1-95 | 0.0790 | 0.0792 | 0.211% | 0.0792 | 0.211% |
| G-20-1-90 | 0.0790 | 0.0792 | 0.211% | 0.0792 | 0.211% |
| G-20-1-80 | 0.0790 | 0.0792 | 0.211% | 0.0792 | 0.211% |
| G-20-1-50 | 0.0790 | 0.0792 | 0.211% | 0.0792 | 0.211% |

**Table 5. Experiment with gamma distribution process times and 3 server.**

| Scenarios | Batch Size | Mean Inter-arrival Time (mins) | Mean Processing Time(mins) | Number Of servers |
|---|---|---|---|---|
| G-5-3-99 | 5 | 0.1684 | 0.1000 | 3 |
| G-5-3-95 | 5 | 0.1754 | 0.1000 | 3 |
| G-5-3-90 | 5 | 0.1852 | 0.1000 | 3 |
| G-5-3-80 | 5 | 0.2083 | 0.1000 | 3 |
| G-5-3-50 | 5 | 0.3333 | 0.1000 | 3 |
| G-20-3-99 | 20 | 0.1684 | 0.0250 | 3 |
| G-20-3-95 | 20 | 0.1754 | 0.0250 | 3 |
| G-20-3-90 | 20 | 0.1852 | 0.0250 | 3 |
| G-20-3-80 | 20 | 0.2083 | 0.0250 | 3 |
| G-20-3-50 | 20 | 0.3333 | 0.0250 | 3 |

**Table 6. Result for Experiment with gamma distribution process times and 3 server.**

| Scenario | WIBT From Simulation (Mins) | WIBT From Equation 11 (Mins) | Relative Error Equation 11 | WIBT From Equation 12 (Mins) | Relative Error Equation 12 |
|---|---|---|---|---|---|
| G-5-3-99 | 0.0660 | 0.0667 | 1.010% | 0.0663 | 0.475% |
| G-5-3-95 | 0.0643 | 0.0667 | 3.681% | 0.0649 | 0.878% |
| G-5-3-90 | 0.0621 | 0.0667 | 7.354% | 0.0630 | 1.409% |
| G-5-3-80 | 0.0576 | 0.0667 | 15.741% | 0.0590 | 2.346% |
| G-5-3-50 | 0.0429 | 0.0667 | 55.400% | 0.0453 | 5.491% |
| G-20-3-99 | 0.0787 | 0.0792 | 0.593% | 0.0729 | 7.373% |
| G-20-3-95 | 0.0779 | 0.0792 | 1.626% | 0.0729 | 6.422% |
| G-20-3-90 | 0.0770 | 0.0792 | 2.814% | 0.0729 | 5.328% |
| G-20-3-80 | 0.0750 | 0.0792 | 5.556% | 0.0729 | 2.803% |
| G-20-3-50 | 0.0688 | 0.0792 | 15.068% | 0.0729 | 5.956% |

From the simulation model we could calculate the average wait-in-batch-time of residents. We also used Equation 11 and Equation 12 to estimate the average wait-in-batch-time. Tables 3, 5, and 7 show the respective results of each scenario in table 2, 4 and 6. The table lists the average wait-in-batch-time from the simulation model, the estimate from Equation 11, and the estimate from Equation 12. Also listed are the relative errors for the estimates. We see that Equation 12 provides a much better estimate than Equation 11 when compared to the simulation result.

## RESULT TABLES FOR CLINIC PERFORMANCE MEASURES

The clinic capacity is determined by bounds set by each station's capacity and the relative arrival rates: $R=\min_{i=1,2,..I}\left\{\frac{k_i m_i r_i}{t_i r_i}\right\}$ Because of the stochastic routing, the clinic's total cycle time is a weighted sum of the station cycle times: $TCT=\frac{1}{r_i}\sum_{i=1}^{I} r_i CT_i$

The average number of residents in the clinic follows from Little's Law: $WIP=r_i TCT$

**Comment [H12]:** It should be the other way round;
Tables 2, 4 and 6 are results, while tables 1, 3, 5 and 7 are scenarios

**Comment [H13]:** Rewrite it and provide more detail discussions on the significance of the results. Provide convincing explanation for each of the tables.

**Comment [H14]:**

**Comment [H15]:** Should be just paragraph not heading

**Table 7. Capacity for Mass Dispensing and Vaccination Clinic Station**

| Work Station | Station Capacity (Residents/min) | Relative throughput | Bound on Clinic Capacity Residents/min) |
|---|---|---|---|
| **Triage** | **19.293** | **1.000** | **19.293** |
| **Symptoms Room** | **2.473** | **0.048** | **51.849** |
| **Holding Room** | **0.789** | **0.032** | **24.905** |
| **Registration** | **65.844** | **0.973** | **67.659** |
| **Education** | **10.000** | **0.973** | **10.276** |
| **Screening** | **5.219** | **0.973** | **5.363** |
| **Consultation** | **1.592** | **0.255** | **6.249** |
| **Vaccination** | **4.908** | **0.958** | **5.123** |

**Table 8 Comparison of total cycle time for the clinic**

| Scenario | Arrival rate to the clinic (residents/min) | Total cycle time from simulation | Total cycle time from clinic mathematical model and formula | Percentage error % |
|---|---|---|---|---|
| **1** | **10.00** | **16.65** | **16.78** | **0.79%** |
| **2** | **9.09** | **11.96** | **12.83** | **7.30%** |
| **3** | **8.00** | **10.26** | **9.80** | **4.48%** |
| **4** | **6.67** | **9.34** | **9.17** | **2.32%** |
| **5** | **5.00** | **8.76** | **8.67** | **1.07%** |
| **6** | **3.33** | **8.50** | **8.42** | **0.95%** |
| **7** | **2.50** | **8.41** | **8.34** | **0.84%** |
| **8** | **2.00** | **8.37** | **8.30** | **0.91%** |

## CONCLUSION

The overall goal of this study has been to provide public health emergency preparedness and response planners with mathematical models that can help them to estimate the important performance measures such as total waiting or cycle time in the mass dispensing and vaccination clinic. With this information, planners become better informed when they have to make decisions regarding staff placement, POD layout, and other relevant concerns. Hence the reduction of polio or complete eradication if possible will cease to be a herculean task.

**Comment [H16]:** Not discussed properly

**Comment [H17]:** Not discussed anywhere

**Comment [H18]:** Briefly highlight pass research and future challenges.

**References**


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**Comment [H19]:** Not cited anywhere

**Comment [H20]:** Not cited anywhere