Original Research Article

The Role of Dust charging and asymmetric Ion flow on the Dust Lattice Mode in Complex Plasma

Abstract

Interaction mechanism among dust grains immersed in plasma has been reported theoretically in presence of asymmetric ion flow and dynamical charging of dust grains. The ion flow induced particle-wake interaction in addition to repulsive Yukawa type of interaction found to play a significant role in tuning the coupling strength between charged grains and hence the vibrational mode. The transition in coupling strength is obtained as a characteristic of ion flow speed and grain size. The results show a damping in strength of interaction in presence of dynamical charging of dust. The type of interaction shows a mixed phase of Wake-Yukawa in subsonic regime and highly Yukawa dominating phase in supersonic regime of ion flow. The results are significant for understanding the thermodynamic properties and the phase behavior of colloidal systems and nano-crystals.

1. Introduction

The study of inter grain interaction is of fundamental interest in statistical thermodynamics. The phase behavior and the dynamics of a system depend solely on the type and strength of inter grain interaction. Realization of correct and appropriate interaction potential is one of the biggest challenges in many-body systems. The complex plasma or dusty plasma is an immerging branch of research for last few decades because of its strong potentiality to replicate the strongly correlated systems at larger scale. The complex plasma consists of micron-submicron sized dust grains immersed in ambient plasma and radiative background. The dust grains are charged up to a high value of 10^3 to 10^4 electronic charges by various processes [1-2]. In laboratory dusty plasma experiments, the dust grains are commonly negatively charged due to higher electron mobility towards the grains. Because of comparatively larger size of the grains, they can be observed individually and this makes it possible to investigate various phenomena such as phase transition [3], diffusion, self organization[4] etc. at macroscopic kinetic level.

The interaction mechanism around a test dust charge in an isotropic system is caused mainly due to spherically symmetric Yukawa type of potential [5]. Under suitable conditions, when the intergrain interaction is much higher as compared to their thermal energy, the grains show some regular oriented structures, the so called plasma crystals [4,6]. Near the plasma sheath, where laboratory plasma crystals are formed by balancing the gravity and sheath electric field, there is ion streaming towards the lower electrode. This bombardment of ions overshield the spherical dust Debye cloud and elongate it in lower vertical direction. Thus, an asymmetry in charge distribution arises around the grain along the direction of ion flow. This access accumulation of positive charge in the lower side of the grain attracts other dust grains just below it to align along vertical ion flow direction [7] and shows vertical ordering of dust grains (chain formation) often observed in laboratory experiments [8-10].

This leads to an attractive type of interaction potential along vertical direction named as wake potential [11-12]. Most of the theoretical results aim to study the intergrain interaction is based on stationary charge on dust surface. However, in reality the charge on the dust grains exhibit self-consistent fluctuation in response to the oscillation in plasma currents flowing into them. The charge fluctuation on dust surface leads to some new observations [13-14] based on the damping of electrostatic wave [15].

The study of inter grain interaction is primarily based on the local plasma enviroment surrounding the dust. This also helps to reduce the multi-component scenario of dusty plasma to one component plasma (OCP) model of dust subsystem. In this letter, we have reported analytical expressions of intergrain interaction potential having background asymmetric ion flow towards the sheath and self-consistent fluctuation of dust charge, for two vertically placed dust subsystem. The potentials thus obtained shows a damping in strength in presence of dust charging dynamics. Moreover, the strength of interaction along the direction of ion flow is found as a characteristic ion flow speed and grain size. The type of interaction in subsonic regime of plasma flow is found in a mixed phase of repulsive Yukawa and attractive wake potential, whereas the supersonic regime is found in Yukawa dominating phase. The presence of this mixed phase in subsonic regime shows an interesting anomalous phase transition of plasma crystal reported recently by Saurav et.al. [16], which is commonly observed in solid state crystals undergo ion irradiation [17]. Since, the interaction potential is the key to understand the phase behavior of a system, the results reflect a tuned transition in crystal phase based on ion flow speed and particle size, which is of fundamental scientific interest in metal nano crystals [18-19] because phase control is the only step to improve the functionalities for various applications.

2. Theoretical Formulation

Our theoretical model composed of a uniform dusty plasma having background Boltzmanian electrons, cold streaming ions and fluctuating negatively charged dust grains, in a collisionless scenario, where the collision mean free path is greater then the dust Debye length λ_{De} near the plasma sheath [5, 20]. Dust grains near the plasma sheath encounter a down stream ion flow with uniform velocity u_{io} along z-direction towards the plasma sheath. Under the condition that dust particle radius $a < \lambda_{De}$, one can apply OML(orbital motion limited) theory [21] to describe the charging currents. The dielectric response function in presence of dust charging dynamics is given by [22]

$$\epsilon(\omega,k) = 1 + \frac{1}{k^2{\lambda_{De}}^2}(1+P) - \frac{{k_\parallel}^2}{k^2}\frac{{\omega_{pi}}^2}{(\omega - u_{io}k_\parallel)^2}(1+Q) \tag{1}$$

Where propagation vector $k^2 = {k_\parallel}^2 + {k_\perp}^2$, P and Q are normalized electron and ion attachment frequency to dust grain and are given as

P=$\frac{|I_{eo}|}{e(\eta - i\omega)}\frac{n_{do}}{n_{eo}}$,

Q=$\frac{|I_{eo}|}{e(\eta - i\omega)}\frac{n_{do}}{n_{io}}M\sqrt{\frac{\pi m_e}{8m_i}}(n_{io}/n_{eo})^{3/2}(1 - \frac{2eQ_{do}}{m_i {u_{io}}^2 r_d})exp(-\frac{Q_{do}e}{k_B T_e r_d})$.

where $|I_{eo}|$ is the equilibrium elctron current and is given by I_{eo}=$\pi a^2 e\sqrt{\frac{8k_BT_e}{\pi m_e}}n_e exp(e\phi_s/k_BT_e)$, for ϕ_s <0.

Here ϕ_s= $\frac{Q_{do}}{r_d}$ is the dust surface potential or floating potential with respect to surrounding plasma.

$\eta = \frac{1}{\tau_c} = \frac{eI_{eo}}{ak_BT_e}[1 + \frac{1}{M}\sqrt{\frac{\pi m_e n_{io}}{2m_i n_{eo}}}exp(\frac{-eQ_{do}}{k_BT_e a})]$, τ_c is the dust charging time. ω is the frequency of the ion acoustic wave and $k_{||}$ is the propagation parallel to ion flow (along z-direction) towards the plasma sheath.

Using the integral form of electrostatic poisson equation and dielectric response function given in equation (1), we can write the interaction potential due to a moving test dust charge Q_{dt} based on the model described above as follows

$$\Phi(r,t) = \frac{Q_{dt}}{(2\pi)^3 2\pi\epsilon_0}\int d^3k d\omega \frac{\delta(\omega - v_t k)}{k^2\epsilon(\omega,k)} e^{ik.\acute{r}} \tag{2}$$

Where $\acute{r} = r - v_t t$.

For cold stationary dust grains, $v_t = 0$, as compatible to our physical model, the interaction potential is modified as

$$\Phi(r) = \frac{Q_{dt}}{(2\pi)^3\epsilon_0}\int d^3k \frac{1}{k^2\epsilon(0,k)} e^{ik.r} \tag{3}$$

In steady state and normalized scale of electron debye length, the dielectric response function given in equation (1) can be written as

$$\epsilon(0,k) = 1 + \frac{1+P}{k^2} - \frac{1+Q}{M^2 {k_{||}}^2} \tag{4}$$

Where it is assumed that $k_{||} \gg k_{\perp}$.

Above equation can be simplified as follows

$$\frac{1}{k^2\epsilon(0,k)} = \frac{{k_{||}}^2}{({k_{||}}^2 - {k_R}^2)({k_{||}}^2 - {k_{Im}}^2)} \tag{5}$$

Where all k are normalized with λ_{De}. k_R and k_{Im} corresponds to real and imaginary screening modes as a function of $k_{\perp}$, in a complex $k_{||}$ plane.

$$k_R = \sqrt{-A + B} \tag{6}$$

$$k_{Im} = \sqrt{-A-B} \tag{7}$$

Where A and B are given as

$$A = \frac{1}{2}({k_\perp}^2 + 1 + P - \frac{1+Q}{M^2}) \tag{8}$$

$$B = \sqrt{A^2 + \frac{{k_\perp}^2}{M^2}(1+Q)} \tag{9}$$

Using the relation in equation (5) we can write the interaction potential due to test dust particulate in YZ plane in presence of dust charging dynamics as

$$\Phi(r) = \frac{Q_{dt}}{(2\pi)^2 \lambda_{De} \epsilon_0} \int dk_\perp exp(ik_\perp y) \int dk_\parallel \frac{{k_\parallel}^2 exp(ik_\parallel z)}{({k_\parallel}^2 - {k_R}^2)({k_\parallel}^2 - {k_{Im}}^2)} \tag{10}$$

Where r=$\sqrt{y^2+z^2}$, all y and z are normalized with electron Debye length λ_{De}

The second part of the above integral can be solved by performing closed contour integration in a complex $k_\parallel$ plane for two poles corresponding to $k_\parallel$=k_R, (real one) gives oscillatory wake potential $\Phi_w(y,z)$ and $k_\parallel$=k_{Im}, (imaginary one) gives repulsive Yukawa type of potential $\phi_D(y,z)$. Finally using the saddle point integration methode one can get the potentials as follows

$$\phi_w(0,z) = \frac{-Q_{dt}}{4\pi\epsilon_0\lambda_{De}} \sqrt{\frac{2\pi}{6zM}} \frac{(1+Q)^{1/4}}{\sqrt{1+P}} sin[\frac{z}{M}(1+Q)^{1/2}] \tag{11}$$

$$\phi_D(y,z) = \frac{Q_{dt}\sqrt{\pi}}{4\pi\epsilon_0\lambda_{De}} \exp(-\sqrt{1+P}\sqrt{y^2+z^2}) \frac{(1+P)^{7/4}}{(y^2+z^2)^{1/4}} (z/y)^{1/2} \tag{12}$$

The expressions for corresponding interaction potential in absence of dust charge fluctuation can be obtained by putting P=Q=0.

The normalized equation of motion for the mode of vibration of two vertically (z-direction) placed dust grains above the plasma sheath in presence of asymmetric ion flow towards the sheath is given as

$$\frac{d^2 z_{n1}}{dt^2} = [K_z + K_{zw}](z_{n1} - z_{m1}) \tag{13}$$

Here, we have considered the vibrational mode propagates along y-direction. $(n.m)$ represents the particle pair and $z_{(n,m)1}$ stands for their displacements from equilibrium positions. The length and time are normalized by λ_{De} and $\sqrt{m_d \lambda_{De}{}^2 / K_B T_d}$ respectively having K_B as the Boltzmann constant and T_d is the dust temparature. K_z and K_{zw} are the spring constant or coupling strength correspond to repulsive Yukawa and attractive wake potential respectively.

Using equation (11), we can write the expression for the normalized Yukawa potential as

$$K_z = \frac{d^2\Phi_D}{d(y,z)^2}|_{y,z=a} = \frac{\Gamma\sqrt{\pi}(1+P)^{7/4}}{2^{9/4}\kappa^{3/2}} exp\left(-\sqrt{2(1+P)}\kappa\right)\left[5/4 + 2^{3/2}\sqrt{1+P}\kappa + 2(1+P)\kappa^2\right] \tag{14}$$

Here, a is the average interparticle distance, Γ is the Coulomb coupling parameter [lin i] and $\kappa = \frac{a}{\lambda_{De}}$ is the screening constant.

Similarly, using equation (12), we get the spring constant for wake potential as

$$K_{zw} = -\sqrt{\frac{2\pi}{6M}}\frac{(1+Q)^{1/4}}{\sqrt{1+P}}\frac{1}{\kappa^{5/2}}\left[\left(\frac{3}{4} - \frac{\kappa^2(1+Q)}{M^2}\right) sin\left(\frac{\kappa}{M}\sqrt{1+Q}\right) - \frac{z\sqrt{1+Q}}{M} cos\left(\frac{z}{M}\sqrt{1+Q}\right)\right] \tag{15}$$

Thus, the effective spring constant along vertical z-direction is

$$K_{eff} = K_z(Yukawa) + K_{zw}(Wake) \tag{16}$$

Considering a small oscillation in wave amplitude z_{n1}=$A_z$exp[i($k_y a$-ωt)] and substituting this in equation (13), we get the dispersion relation for the transverse vibrational mode as

$$\omega^2 = 2\left(K_z + K_{zw}\right) sin^2\left(\frac{k_y a}{2}\right) \tag{17}$$

3. Results and Discussion

The ion flow induced elongation of spherical dust Debye cloud and the formation of attractive wake potential in addition to repulsive Yukawa type of potential pulls the dust grain just below it to displace along the direction of ion flow. Thus, the effective strength of attraction along vertical direction is mainly depends upon the amount of distortion in Debye cloud and relative strength of both repulsive Yukawa and attractive wake potential respectively. From equation (1) it is clearly visible that the dust Debye length λ_{De} around the grain is modified in two different ways: first, by a factor $\frac{1}{\sqrt{1+P}}$, and is attributed to shift in Boltzmanian electron distribution having normalized electron attachment frequency P and second, by a factor $\frac{M}{\sqrt{1+Q}}$, interms of ion flow speed M and normalized ion attachment frequency Q. Both the changes contribute to two different interaction potentials given in equation (12) and (11) respectively. This shows that the modification in dust Debye cloud is caused mainly due to ion flow speed M and background plasma response towards the grain through P and Q. From Fig. 1, it is seen that P value increases with increase in ion flow speed from subsonic regime and becomes saturated approaching the supersonic regime. This makes the repulsive Yukawa potential independent of M in supersonic regime and can be realized from equation (12). Whereas, the Q increases linearly with ion flow speed M. As a result, the strength of both repulsive Yukawa and attractive wake potential given in equation (12) and (11) are reduced in presence of dynamical charging of grains as compared to that of constant charged dust grains having P=Q=0 and is shown in Fig.2(a) for wake potential, with typical dust plasma parameters having electron temperature T_e=1.7eV, ion temperature T_i=0.02T_e, ion to electron density ratio $\frac{n_i}{n_e}$ ≈1 and normalized dust radius g=r_d/λ_{De} <1. It also clears the fact that for P=0, case having stationary charge on dust surface, Yukawa potential is completely independent of ion flow speed M. The strength of wake potential from equation (11) shows an enhancement with decrease in flow speed M in Fig. 2(b)

It is also shown in Fig. 1 that both the electron(ion) attachment frequency P(Q) increases with increase in grain size g. This says larger grain will have smaller effective Debye length and weaker effective strength of interaction for a given ion flow speed M and is shown in Fig. 2(c) for a given Γ and κ, using equation (16). The corresponding transverse mode of vibration also shows a decrease in frequency of vibration with increase in grain size and is visible from Fig. 2(d). This transition in coupling strength with grain size finally assures a transition in crystal phase, which is of recent scientific interest in metal nano crystals [18].

To get an estimation of the relative strength of Wake and Yukawa potential in both sub and supersonic regime, we defined a dimension less ratio R=$\frac{K_{eff}}{K_z}$, where K_{eff} is the combined strenth of Yukawa and Wake potential from equation (16) and K_z is the coupling constant for Yukawa potential from equation (14). The behavior of relative strength of interaction R with subsonic ion flow speed M is ploted in Fig. 3(a) and is as folllows

Subsonic regime (0.1< M < 0.7): The ratio R shows a fluctuation in strength with decrease in M value and is shown in Fig. 3(a). It is clear from Fig. 2(b), that particle-wake interaction is dominant in subsonic regime of plasma flow and is also reported in recent experiment [23]. However, the effective confinement strength of particle along vertical direction in presence of both Yukawa and Wake undergo a transtion from stronger to weaker and again back to stronger confinement as a characteristic of oscillatory confinement frequency of wake potential with decrease in ion flow speed M. At M=0.249, the ratio R, shows an enhancement. It implies that the effective interaction is repulsive in nature. On decrease of M at 0.18, the effective coupling is relatively weaker and the intercation becomes attractive in nature. The corresponding mode of vibration also decreases and is shown in Fig. 3(b). On further decrease, at M=0.141, the ratio R again shows an increment. As a result, the mode increases once again. The behavior may be realized as such that on gradual decrease in flow speed M, the attractive wake potential of upstream particle starts to dominate and align the lower dust particle along vertical direction of ion flow. On further decrease in M, when the two particle becomes very close to each other, the repulsive Yukawa dominates over there and the particle gets back to its original configuration. The presence of this mixed phase of wake and Yukawa in subsonic regime show an anomalous transition [16] in vibrational mode for given ion flow speeds and is shown in Fig. 3(b).

Supersonic regime (1.0< M <5.0): In this regime the strength of wake potential is significantly reduced and it makes this region highly Yukawa dominating phase of interaction. It can be seen from Fig. 3(c), that the ratio R asymptotically approach 1, with increase in ion flow speed M. This is due to the fact that electron attachment frequency P becomes inpendent of M in supersonic regime and is discussed above. This leads to the strength of

Yukawa potential independent of M, for higher ion flow speed.

4. Conclusion

In this letter, we have presented an elaborate discussion on the formation of wake potential along with Yukawa potential in complex plasma and discussed the strength of interaction for a vertically placed dust subsystem, as a characteristic of ion flow speed and grain size. The observations can be summarized as follows:

(a) The effective plasma debye length surrounding the dust gets distorted and reduced in presence of dust charge fluctuation interms of ion flow speed and particle size.

(b) The strength of both Yukawa and wake potential is reduced with increase in grain size, interms of electron(ion) attachment frequency P(Q). The corresponding mode of vibration also decreases with increase in grain size. This implies an size dependent transition in crystal phase often observed in nano crystal.

(c) The type of interaction is found to be in a mixed phase of Wake and Yukawa in subsonic regime of plasma flow. In this regime the frequency of vibrational mode shows an anomaly with gradual decraese in ion flow speed M. This implies an ion flow induced anomalous transition in crystal phase, a well known phenomena in solid state physics [17]. The supersonic regime is found to be in Yukawa dominating phase.

(d) The study is expected to play a significant role in controlling the phase behavior and diffusion phenomena in complex plasma in absence of magnetic field.

5. References

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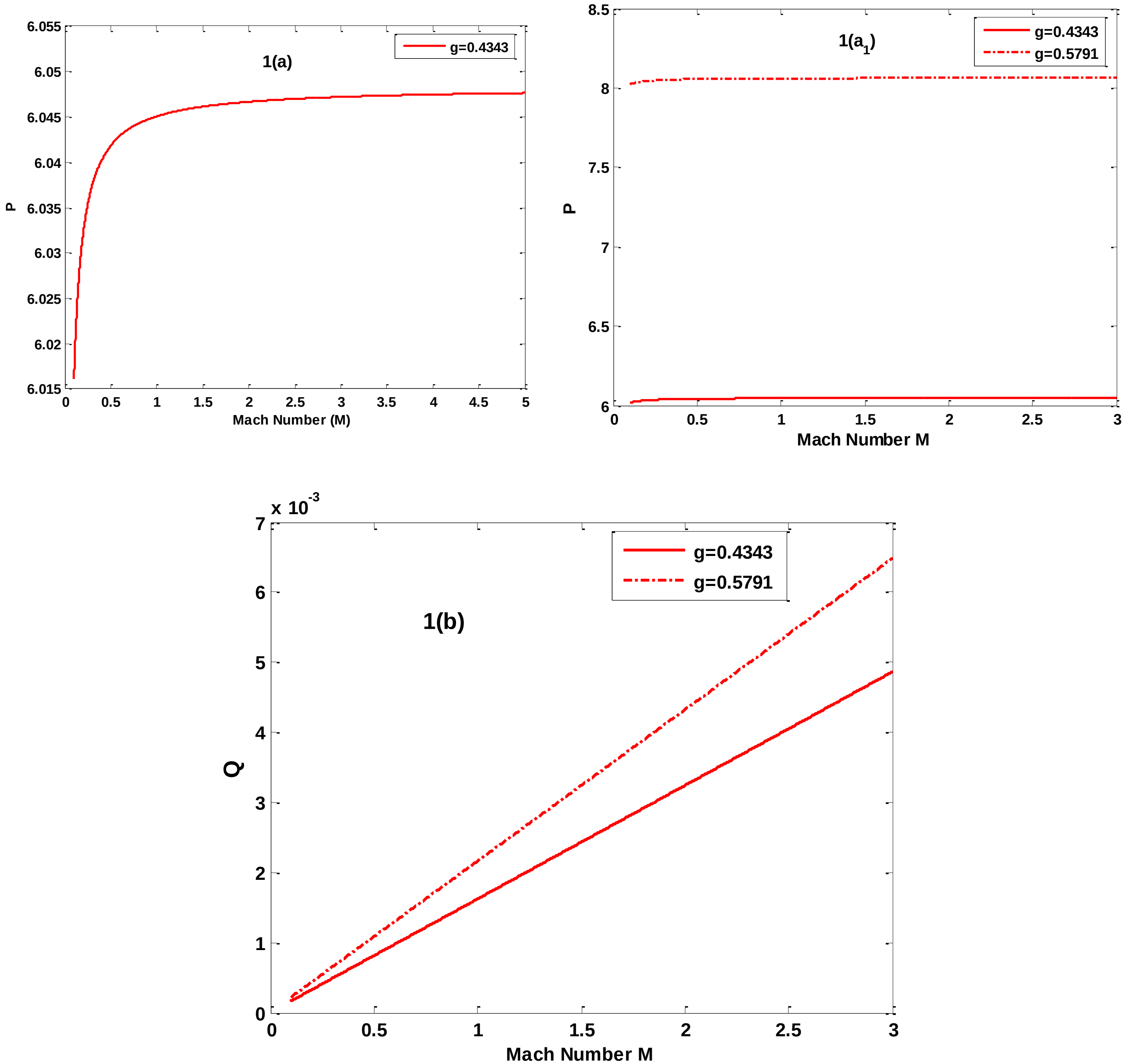
Figure Captions:

FIG. 1: The plot shows (a) the variation of electron attachment frequency P vs. M (Mach number), for a given normalized dust radius g=0.4343. (a_1) P vs. M, for g=0.4343 (red solid) and g=0.5791 (red dash). (c) Ion attachment frequency Q vs. M, for g=0.4343 (red solid), g=0.5791(red dotted).

FIG. 2: The profiles show (a) Normalized wake potential in presence (red solid) and absence (red dotted) of dust charge fluctuation. (b) Normalized wake potential, for Mach number M=0.5 (red solid) and 1.5 (red dotted). (c) The effective vertical spring constant vs. Mach number M, having normalized grain size g=0.4343 (red solid) and g=0.7239 (red

dotted), for given plasma parameters and showing a transition from M=0.241 to M=0.18 to M=0.141. (d) Dispersion relation for transverse vibrational modes having g=0.4343 (red solid), g=0.7239 (red dash) and Mach number M=0.5

FIG. 3: (a) the ratio R vs. Mach Number M, (subsonic regime) for the given set of parameters. (b) The transition in vibrational mode from M=0.241(red solid) to M=0.18 (red dashed) to M=0.141 (red dotted). (c) The ratio R vs. Mach number, (supersonic regime) for the given set of parameters.



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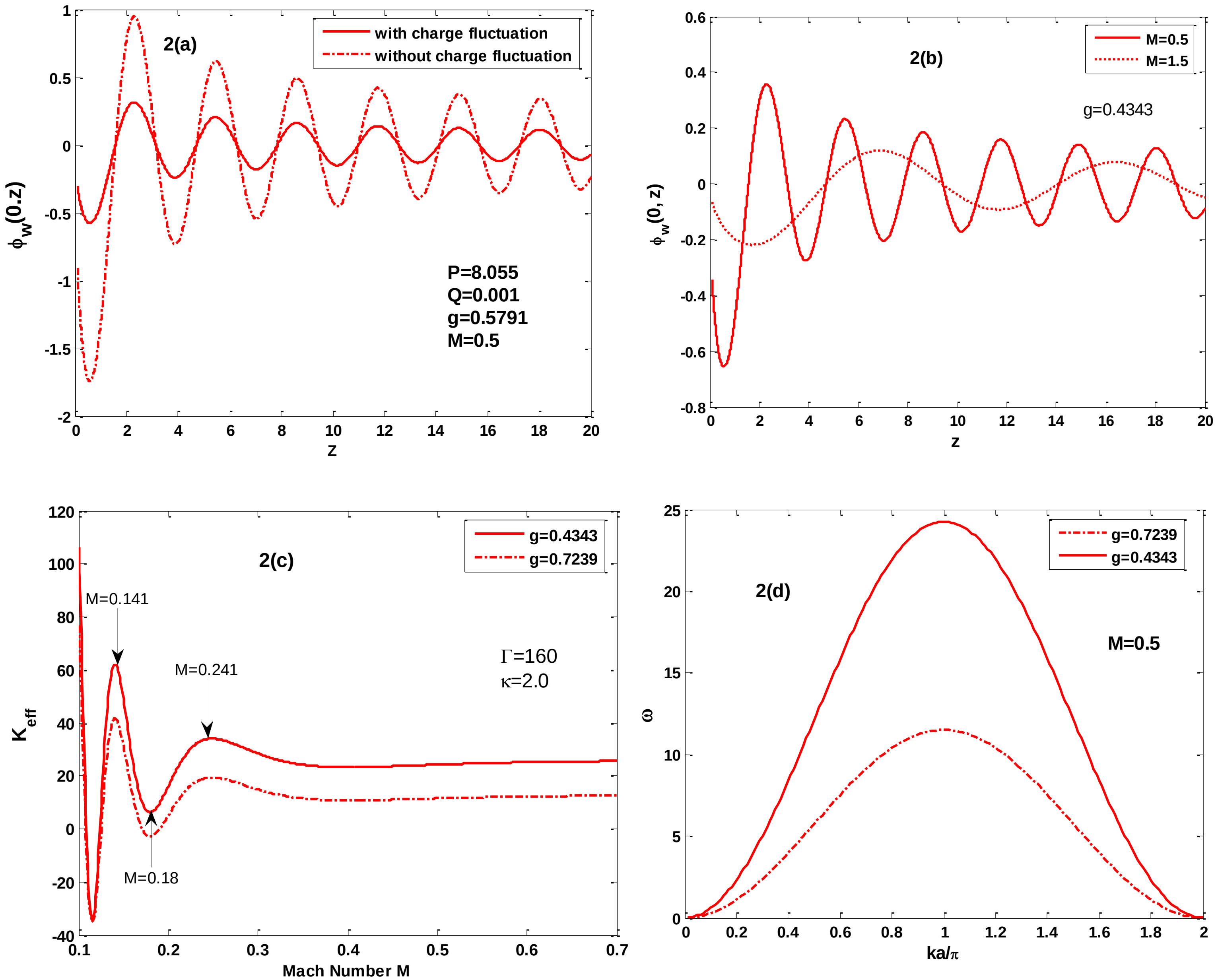

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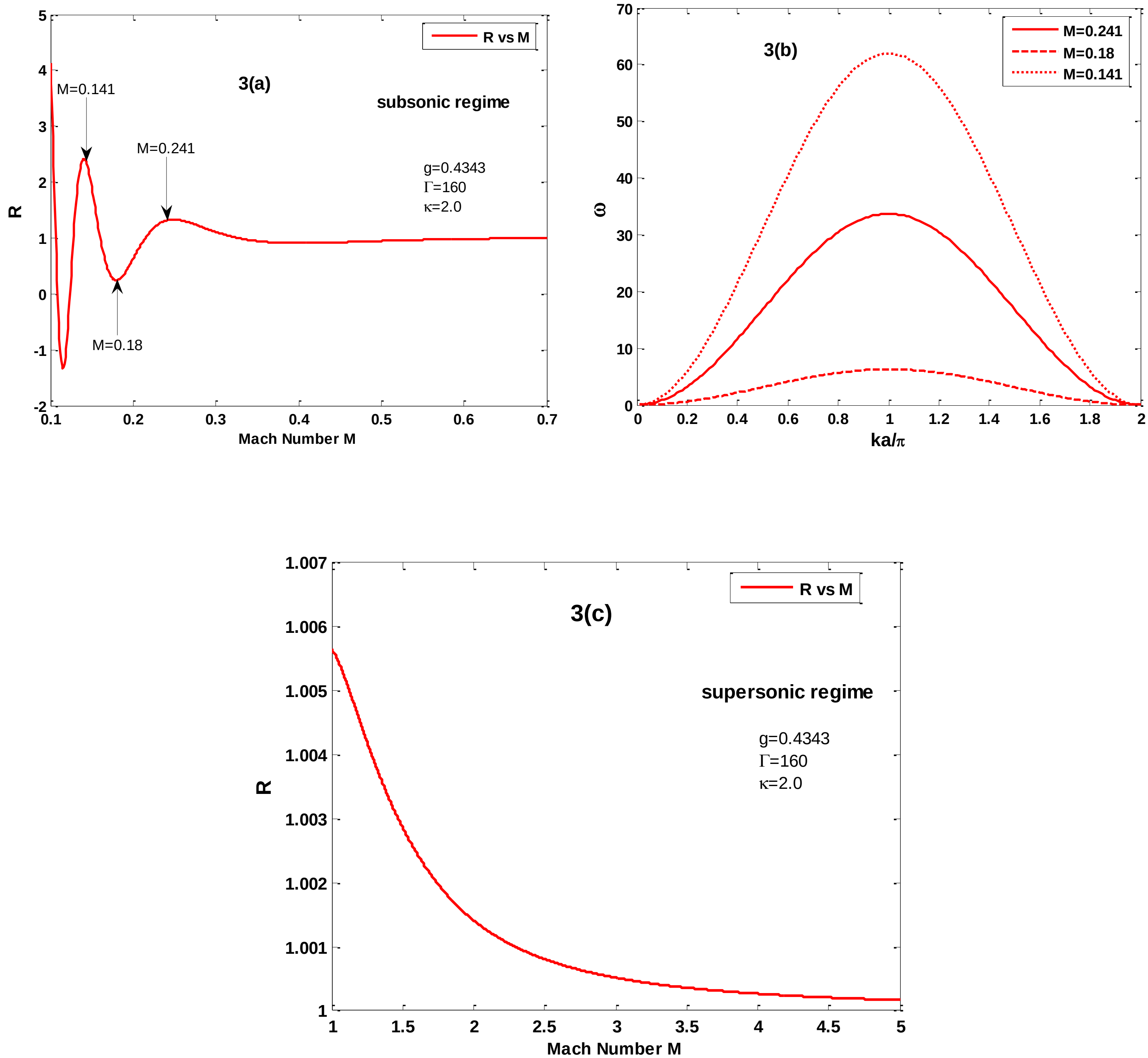

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