
A numerical method for pulse propagation in nonlinear media

Abstract

In this work, the pulse propagation in a nonlinear dispersive optical medium is numerically investigated. The finite difference time-domain scheme of third order with periodic boundary conditions is used to solve generalized nonlinear Schrödinger equation governing the propagation of the pulse. As a result a discrete system of ordinary differential equations is obtained and solved numerically by fourth order Runge-Kutta algorithm. Varied input ultrashort laser pulses are used. Accurate results of the solutions are obtained and the comparison with other results is excellent.

Keywords: finite difference time-domain method; generalized nonlinear Schrödinger equation; periodic boundary conditions.

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1 Introduction

Theoretical and experimental research for the propagation process of ultrashort laser pulses in a medium have been the subject of intensive research within the last few years [1-8]. Wave propagation in an optical fiber is governed by the nonlinear Schrödinger equation (NLSE). This particular form of the Schrödinger equation is obtained from the general set of Maxwell equations taking advantage of a certain number of assumptions made possible from the very specific characteristics

of (quasi-)monochromatic wave propagation in a medium such as a fiber [2]. The nonlinear Schrödinger-type equations are essential to describe the optical soliton propagation in a variety of branches of fiber communication sciences, e.g., nonlinear optics. Additionally, these models have successfully addressed the ultrashort pulses of the wave dynamics, which will increase the power of high-bit-rate transmission systems. Consequently, understanding the dynamics of the soliton can lead to an extensive improvement in technology and industry. Therefore, there has been significant progress in the development of diverse schemes for treating NLSE and nonlinear partial differential equations (NPDEs) in the general case. For approximate schemes, we cite the Adomian decomposition method, collocation method, homotopy perturbation method, homotopy analysis method, reduced differential transform method, q-homotopy analysis method, variational iteration method, reproducing kernel Hilbert space method, iterative Shehu transform method, and residual power series method [3] (see also the references that appear therein). While constructing an exact analytic solution is of more importance since this can provide the best understanding of the model's nature to be processed in an efficient way, researchers have developed various powerful tools to analyze NPDEs. Furthermore, these can be utilized to estimate the boundary data that are used in numeric and semi-analytic methods. Such techniques include the Kudryashov method and its modifications, functional variable method, generalized Riccati equation mapping method, Jacobi elliptic function method, sine-Gordon expansion method, Hirota method, subequation method, soliton ansatz method, G/G-expansion method, new extended direct algebraic method, extended trial function method, new generalized exponential rational function method, integral dispersion equation method, modified extended tanh-function method, simple equation method, and modified simple equation methods [3] (see also the references that appear therein).

Even if the resolution of the generalized nonlinear Schrödinger equation has been intense research object the subject remains a matter of great concern. Due to nonlinearity, the NLSE requires some numerical solution and some of the most popular numerical methods are: split-step method, the spectral methods, the finite difference time-domain method. One of the most prevalent challenges in developing these numerical schemes is that they satisfy the conservation laws[4]

Based on different techniques of discretization of own equations, several differential numerical time domain methods have efficiently simulated nonlinear dispersive media. These approaches include the linear and nonlinear properties and take into account physical complexity of media such as Kerr effects, Raman diffusion and/or photonic absorption[5]The FDTD method is one of the most widely used methods for this purpose, due to the simplicity of its implementation, its applicability to arbitrary scattering media (e.g., media exhibiting anisotropic and/or nonlinear responses), and the high accuracy of the generated results [6]As far as the finite difference method is concerned, accurate and stable schemes are often required to solve a large-scale linear system. We solved generalized nonlinear Schrödinger equation by using finite difference time-domain scheme of third order with periodic boundary conditions. The advantage of using boundary periodic conditions is to avoid the problems of discontinuities which could arise during numerical treatment. The FDTD method solves a discretized Schrödinger equation in an explicitly iterative process. The fourth order Runge-Kutta algorithm is applied to the obtained system of ordinary differential equations.

The present article is arranged as ensued. In Sect. 2, we describe the governing model of the propagation of the pulse. In Sect. 3, we use the FDTD method with the boundary periodic conditions to get the system of ordinary differential equations. In Sect. 4, we accomplish the integration of the system of differential equations by the fourth order Runge-Kutta method. In Sect. 5, we present the results and discussions. For Sect. 6, a brief conclusion is given.

2 Governing model

We consider the generalized nonlinear Schrödinger equation with time $z \in \mathbb{R}^+$, $t \in \Omega =] - T, T[$ the space variable in the case of one dimensional problem and $A = A(z, t)$ the unknown function that

represents the slowly varying electric pulse envelope, in the form [7]:

$$\begin{aligned} i\frac{\partial A}{\partial z} + i\delta_3\frac{\partial^3 A}{\partial t^3} + \frac{1}{2}\frac{\partial^2 A}{\partial t^2} \\ + \beta_0 n_2 L_D \left[(A^2 A) + i s \frac{\partial(A^2 A)}{\partial t} \right] = 0 \end{aligned} \quad (2.1)$$

$\forall t z \in \Omega \times \mathbb{R}_+^*$

To this equation, we add the following initial conditions :

$$A t 0 = g(t), \quad t \in \Omega$$

The function g is a given function in the Hilbert space $\mathbb{L}^2(\mathbb{R}, \mathbb{C})$. The periodic boundary conditions used are:

$$\begin{cases} A - Tz = ATz & z > 0 \\ \frac{\partial A}{\partial z} - Tz = \frac{\partial A}{\partial z} Tz & z > 0 \\ \frac{\partial^2 A}{\partial z^2} - Tz = \frac{\partial^2 A}{\partial z^2} Tz & z > 0 \end{cases}$$

The parameter $\delta_3 = \beta_3 / (6T_0 |\beta_2|)$ takes into account the third order dispersion effects governed by β_3 , $s = 1 / (\omega_0 T_0)$ is the parameter responsible for self-steepening. Let's introduce $\gamma = \beta_0 n_2 L_D$ and $N^2 = \beta_0 n_2 I_0 L_D / 6$, I_0 is maximum pulse intensity and N governs the soliton order; L_D is the dispersion length. As pointed out in [8], intrapulse Raman scattering is ignored in this work.

3 The finite-difference time-domain scheme

We consider a uniform grid $t_{j1 \leq j \leq N}$ step h in t . Let $A_j(z)$ an approximation of $A t_j z$ for all $z \in]0, L[$ and $j = 1, 2, \dots, J$.

Let's use approximation with six points $\{t_k z\}_{k=j-3}^{j+2}$ to express the partial derivatives $\frac{\partial A}{\partial t} t_j z$, $\frac{\partial^2 A}{\partial t^2} t_j z$ and $\frac{\partial^3 A}{\partial t^3} t_j z$

Let's expand the partial derivatives in the following form:

$$K_{j,r} A z = a A_{j-3}(z) + b A_{j-2}(z) + c A_{j-1}(z) + d A_j(z) + e A_{j+1}(z) + f A_{j+2}(z)$$

such that for fixed $r \in \{1, 2, 3\}$, we have

$$K_{j,r} A z = \frac{\partial^r A}{\partial t^r} t_j z + O(h^{6-r}) \quad (3.1)$$

This accuracy is desired heuristically that the leading term of the truncation error be $\frac{\partial^6 A}{\partial t^6} t_j z$ and does not bring a parasite dispersion. Assuming that A is regular and by the Taylor expansion up to 6th order of $K_{j,r} A z$ at the $t_j z$, we have

$$\begin{aligned} K_{j,r} A z &= (a + b + c + d + e + f) A t_j z \\ &+ h(-3a - 2b - c + e + 2f) \frac{\partial A}{\partial t} t_j z \\ &+ \frac{h^2}{2!}(9a + 4b + c + e + 4f) \frac{\partial^2 A}{\partial t^2} t_j z \\ &+ \frac{h^3}{3!}(-27a - 8b - c + e + 8f) \frac{\partial^3 A}{\partial t^3} t_j z \\ &+ \frac{h^4}{4!}(81a + 16b + c + e + 16f) \frac{\partial^4 A}{\partial t^4} t_j z \\ &+ \frac{h^5}{5!}(-243a - 32b - c + e + 32f) \frac{\partial^5 A}{\partial t^5} t_j z \\ &+ \frac{h^6}{6!}(729a + 64b + c + e + 64f) \frac{\partial^6 A}{\partial t^6} t_j z + \dots \end{aligned}$$

Equation 3.1 becomes

$$\begin{cases} a + b + c + d + e + f & = 000 \\ -3a - 2b - c + e + 2f & = \frac{1}{h}00 \\ 9a + 4b + c + e + 4f & = 0\frac{2}{h^2}0 \\ -27a - 8b - c + e + 8f & = 00\frac{6}{h^3} \\ 81a + 16b + c + e + 16f & = 000 \\ -243a - 32b - c + e + 32f & = 000 \end{cases}$$

Solving this system of equations we obtain:

$$\begin{cases} a & = -\frac{2}{60h} - \frac{0}{12h^2} - \frac{1}{4h^3} \\ b & = \frac{15}{60h} - \frac{1}{12h^2} - \frac{7}{4h^3} \\ c & = -\frac{60h}{20} - \frac{12h^2}{30} - \frac{4h^3}{10} \\ d & = \frac{60h}{30} - \frac{12h^2}{16} - \frac{4h^3}{1} \\ e & = \frac{60h}{3} - \frac{12h^2}{1} - \frac{4h^3}{1} \\ f & = -\frac{1}{60h} - \frac{1}{12h^2} - \frac{1}{4h^3} \end{cases}$$

One can deduce that

$$729a + 64b + c + e + 64f = -\frac{720}{60h} - \frac{96}{12h^2} - \frac{360}{4h^3}$$

then the leading term of the truncation error is:

$$E_p = -\frac{h^5}{60} - \frac{h^4}{90} - \frac{h^3}{8} \frac{\partial^6 A}{\partial t^6} x_j z.$$

We have

$$\begin{cases} \frac{\partial A}{\partial t} t_j z & = \frac{1}{60h} (-2A_{j-3}(z) + 15A_{j-2}(z) - 60A_{j-1}(z) \\ & + 20A_j(z) + 30A_{j+1}(z) - 3A_{j+2}(z)) + \frac{h^5}{60} \frac{\partial^6 A}{\partial t^6} t_j z + \dots \\ \frac{\partial^2 A}{\partial t^2} t_j z & = \frac{1}{12h^2} (-A_{j-2}(z) + 16A_{j-1}(z) \\ & - 30A_j(z) + 16A_{j+1}(z) - A_{j+2}(z)) + \frac{h^4}{90} \frac{\partial^6 A}{\partial t^6} t_j z + \dots \\ \frac{\partial^3 A}{\partial t^3} t_j z & = \frac{1}{4h^3} (A_{j-3}(z) - 7A_{j-2}(z) + 14A_{j-1}(z) \\ & - 10A_j(z) + A_{j+1}(z) + A_{j+2}(z)) - \frac{h^3}{8} \frac{\partial^6 A}{\partial t^6} t_j z + \dots \end{cases}$$

In condensed form the previous system is equivalent to the following system:

$$\begin{cases} \frac{\partial A}{\partial t} t_j z & = K_{j,1} A z + \frac{h^5}{60} \frac{\partial^6 A}{\partial t^6} t_j z + \dots \\ \frac{\partial^2 A}{\partial t^2} t_j z & = K_{j,2} A z + \frac{h^4}{90} \frac{\partial^6 A}{\partial t^6} t_j z + \dots \\ \frac{\partial^3 A}{\partial t^3} x_j z & = K_{j,3} A z - \frac{h^3}{8} \frac{\partial^6 A}{\partial t^6} x_j z + \dots \end{cases}$$

it follows that the system of ordinary differential equations:

$$\frac{dA_j}{dz} = \delta_3 K_{j,3} A z + \frac{1}{2} K_{j,2} A z - s\gamma K_{j,1} A^2 A z + \nu\gamma A_j^2 A_j$$

must be cast into the form :

$$\begin{aligned}
\frac{dA_j}{dz} &= \frac{\delta_3}{4h^3} [A_{j-3}(z) - 7A_{j-2}(z) + 14A_{j-1}(z) \\
&\quad - 10A_j(z) + A_{j+1}(z) + A_{j+2}(z)] \\
&\quad + \frac{\iota}{24h^2} [-A_{j-2}(z) + 16A_{j-1}(z) \\
&\quad - 30A_j(z) + 16A_{j+1}(z) - A_{j+2}(z)] \\
&\quad - \frac{s\gamma}{60h} [-2A_{j-3}(z)^2 A_{j-3}(z) + 15A_{j-2}(z)^2 A_{j-2}(z) \\
&\quad - 60A_{j-1}(z)^2 A_{j-1}(z) + 20A_j(z)^2 A_j(z) \\
&\quad + 30A_{j+1}(z)^2 A_{j+1}(z) - 3A_{j+2}(z)^2 A_{j+2}(z)] \\
&\quad + \nu\gamma [A_j(z)^2 A_j(z)] \\
&\quad \forall z \in]0, T[\text{ et } j = 4, \dots, J-2
\end{aligned} \tag{3.2}$$

Through periodic boundary conditions, the system completed by the following equations 3.3, 3.4, 3.5 et 3.6:

$$\begin{aligned}
\frac{dA_1}{dz} &= \frac{\delta_3}{4h^3} [A_{J-3}(z) - 7A_{J-2}(z) + 14A_{J-1}(z) \\
&\quad - 10A_1(z) + A_2(z) + A_3(z)] \\
&\quad + \frac{\iota}{24h^2} [-A_{J-2}(z) + 16A_{J-1}(z) \\
&\quad - 30A_1(z) + 16A_2(z) - A_3(z)] \\
&\quad - \frac{s\gamma}{60h} [-2A_{J-3}(z)^2 A_{J-3}(z) + 15A_{J-2}(z)^2 A_{J-2}(z) \\
&\quad - 60A_{J-1}(z)^2 A_{J-1}(z) + 20A_1(z)^2 A_1(z) \\
&\quad + 30A_2(z)^2 A_2(z) - 3A_3(z)^2 A_3(z)] \\
&\quad + \nu\gamma [A_1(z)^2 A_1(z)] \\
&\quad \forall z \in]0, T[
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
\frac{dA_2}{dz} &= \frac{\delta_3}{4h^3} [A_{J-2}(z) - 7A_{J-1}(z) + 14A_1(z) \\
&\quad - 10A_2(z) + A_3(z) + A_4(z)] \\
&\quad + \frac{\iota}{24h^2} [-A_{J-1}(z) + 16A_1(z) \\
&\quad - 30A_2(z) + 16A_3(z) - A_4(z)] \\
&\quad - \frac{s\gamma}{60h} [-2A_{J-2}(z)^2 A_{J-2}(z) + 15A_{J-1}(z)^2 A_{J-1}(z) \\
&\quad - 60A_1(z)^2 A_1(z) + 20A_2(z)^2 A_2(z) \\
&\quad + 30A_3(z)^2 A_3(z) - 3A_4(z)^2 A_4(z)] \\
&\quad + \nu\gamma [A_2(z)^2 A_2(z)] \\
&\quad \forall z \in]0, T[
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
\frac{dA_3}{dz} &= \frac{\delta_3}{4h^3} [A_{J-1}(z) - 7A_1(z) + 14A_2(z) \\
&\quad - 10A_3(z) + A_4(z) + A_5(z)] \\
&\quad + \frac{\iota}{24h^2} [-A_1(z) + 16A_2(z) \\
&\quad - 30A_3(z) + 16A_4(z) - A_5(z)] \\
&\quad - \frac{s\gamma}{60h} [-2A_{J-1}(z)^2 A_{J-1}(z) + 15A_1(z)^2 A_1(z) \\
&\quad - 60A_2(z)^2 A_2(z) + 20A_3(z)^2 A_3(z) \\
&\quad + 30A_4(z)^2 A_4(z) - 3A_5(z)^2 A_5(z)] \\
&\quad + \nu\gamma [A_3(z)^2 A_3(z)] \\
&\quad \forall z \in]0, T[
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\frac{dA_{J-1}}{dz} &= \frac{\delta_3}{4h^3} [A_{J-4}(z) - 7A_{J-3}(z) + 14A_{J-2}(z) \\
&\quad - 10A_{J-1}(z) + A_1(z) + A_2(z)] \\
&\quad + \frac{\nu}{24h^2} [-A_{J-3}(z) + 16A_{J-2}(z) \\
&\quad - 30A_{J-1}(z) + 16A_1(z) - A_2(z)] \\
&\quad - \frac{s\gamma}{60h} [-2A_{J-4}(z)^2 A_{J-4}(z) + 15A_{J-3}(z)^2 A_{J-3}(z) \\
&\quad - 60A_{J-2}(z)^2 A_{J-2}(z) + 20A_{J-1}(z)^2 A_{J-1}(z) \\
&\quad + 30A_1(z)^2 A_1(z) - 3A_2(z)^2 A_2(z)] \\
&\quad + \nu\gamma [A_{J-1}(z)^2 A_{J-1}(z)] \\
&\quad \forall z \in]0, T[
\end{aligned} \tag{3.6}$$

4 Integration over space using fourth order Runge-Kutta method

This integration will be done by fourth order of classical Runge-Kutta method that will give us anyway a precision in $O(\Delta t^4)$.

The system of ordinary differential equations is:

$$\frac{dA_j}{dz} = \delta_3 K_{j,3} A z + \frac{\nu}{2} K_{j,2} A z - s\gamma K_{j,1} A^2 A z + \nu\gamma A_j^2 A_j \text{ pour } j = 1, \dots, J$$

Let's denote $A(z) = A_j(z)_{j=1}^J$ and $\mathcal{L} = L_j_{j=1}^J$ where

$$L_j(A(z)) = \delta_3 K_{j,3} A z + \frac{\nu}{2} K_{j,2} A z - s\gamma K_{j,1} A^2 A z + \nu\gamma A_j(z)^2 A_j(z)$$

We'll deal with Cauchy problem:

$$\begin{cases} \frac{dA}{dt} = \mathcal{L}(A) \\ A(0) = g(t_j)_{j=1}^J \end{cases}$$

The classic fourth order Runge-Kutta method applied to the obtained system of ordinary equations can be put in the form:

$$A(z + \Delta z) = A(z) + \frac{1}{6} (\mathcal{K}_1 + 2\mathcal{K}_2 + 2\mathcal{K}_3 + \mathcal{K}_4)$$

avec

$$\begin{cases} \mathcal{K}_1 = \Delta z \mathcal{L}(A(z)) \\ \mathcal{K}_2 = \Delta z \mathcal{L}(A(z) + \frac{1}{2}\mathcal{K}_1) \\ \mathcal{K}_3 = \Delta z \mathcal{L}(A(z) + \frac{1}{2}\mathcal{K}_2) \\ \mathcal{K}_4 = \Delta z \mathcal{L}(A(z) + \mathcal{K}_3) \end{cases}$$

The coupling of the fourth order Runge Kutta method and the finite difference method adopted above leads to a discrete scheme globally consistent with the Schrödinger equation and accurate at fourth order for the parameters s and δ taken strictly positive. This scheme is conditionally stable under the constraint:

$$\begin{cases} \gamma s \max_{1 \leq j \leq J} A_j(z)^2 \frac{\Delta z}{\Delta t} & 1.7 \\ \frac{\Delta z}{\Delta t^2} & 1 \\ \delta \frac{\Delta z}{\Delta t^3} & 0.34 \end{cases}$$

5 Results and discussion

The results are presented in three parts: the first part presents a comparison between our method and another method. The agreement between these methods shows the accuracy of our method. The second part is devoted to the shape of the pulse during propagation in the medium. First of all, as pointed out by Van Cao et al [9], the higher-order dispersion and nonlinearity effects cannot be neglected. Under influence of these parameters, the pulses changes in a more complicated way as we can see in Fig. 1, Fig. 2, Fig. 3, Fig. 4, and Fig. 5. All the studied input pulses have symmetry with periodic boundary conditions, but during the propagation, the symmetry is broken and the pulse become more asymmetric in a complicated way. Let's denote that $\delta_3 < 1$ and $s < 1$ so it convenient to treat the higher order effects as perturbation.

In Fig. 1, we choose the following parameters: $\beta_2 = -1[ps^2/km]$, $T_0 = 10[fs]$, which imply that $L_D = \frac{T_0^2}{|\beta_2|}$ is equal to $0.1[mm]$. The propagation distance is equal to $10L_D$ equivalent to the normalized distance equal to 10. The input pulse envelope $A = A_0 sech(t)$ with A_0 chosen so much so that $N = 1$. The other parameters are: $\delta_3 = 0.02[ps^3/km]$, the steepening parameter is $s = 0.08$. We compare the intensity profile at the propagation distance equal to $10L_D$ corresponding to normalized propagation distance $z = 10$ with the sech input pulse profile. The calculated energy of input and output pulse shows that energy is conserved. The peak moves towards the trailing part and the shape of the pulse remains virtually unchanged with negligible temporal broadening [2]. It is also seen that the shape of the output pulse is slightly unchanged and peak pulse moves as a soliton. As pointed out in [7] the third order dispersive effects are not strong enough to break the balance between the self-phase modulation and group velocity dispersion. In this figure our results agree with those obtained by Hile et al.[10-11], shown by GNLS equation and the time transformation approach developed in [2]. The time-transformation method directly maps the electric field in the time-domain and does not require the use of the slowly varying envelope approximation. The authors use the finite-difference time-domain method integrating Maxwell's equations directly.

Fig. 2 corresponds to the gaussian input pulse which is the most widely used, $g(t) = \exp\left(-\frac{t^2}{\sigma^2}\right)$; where σ is a constant.

The normalized propagation distances are ($z = 0.2$) and ($z = 0.7$). The other parameters used are $s = 0.8$, $T_0 = 10[fs]$, $\delta_3 = 0.02[ps^3/km]$. As the propagation distance increases, the shape of the pulse is deformed.

In Fig. 3 the input pulse is given by: $g(t) = A_0 sech(t) \cos(t/s)$; where s is the self-steepening parameter. As noted in Fig.2 as the propagation distance is increasing the pulse shape is deformed significantly.

Fig. 4 presents the input sech pulse given by: $A = A_0 sech(t)$ whereas Fig.5 we consider two solitons hyperbolic secant used in the collision models and is written : $g(t) = A_0 sech(t-t_1) + r sech(t+t_2) \exp(i\theta)$; where r is the relative amplitude of the two solitons, θ is the relative phase between them. The parameters are: $r = 1$, $\theta = 0$ (equal amplitude and in phase case) and $t_1 = t_2 = 2$ (initial spacing) (8).

6 Conclusion

We have performed a numerical study of ultrashort optical pulses in nonlinear dispersive optical media. We used FDTD scheme of third order with boundary conditions to solve NLSE. We have shown that comparisons with time-transformation method and FDTD for Maxwell equations are excellent. This can be interpreted as the scheme used in the paper is accurate and makes the method appropriate. The results obtained in this paper exhibit the impact that the higher-order dispersive and nonlinear effects have on the ultrashort pulses propagation in its whole importance. The role of the higher-order terms in the stability and dynamics of the solitons thus needs to be understood. Since the ultrashort

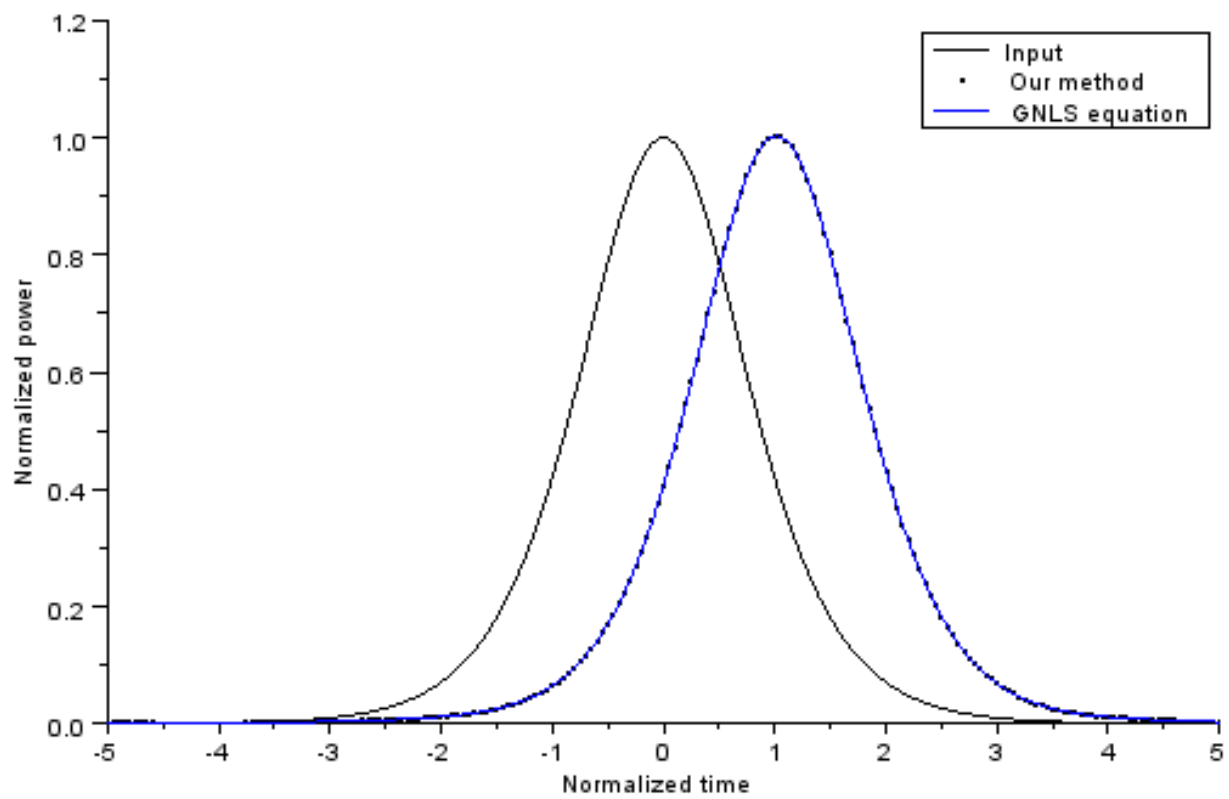


Figure 1: Intensity profile of a secant hyperbolic input pulse at normalized distances $z = 0$ (solid black) and $z = 10$ (dotted black). The parameters used are: $T_0 = 10[fs]$, $s = 0.08$, $\delta_3 = 0.02[ps^3/km]$. Plot of our method, plot of GNLS equation are shown

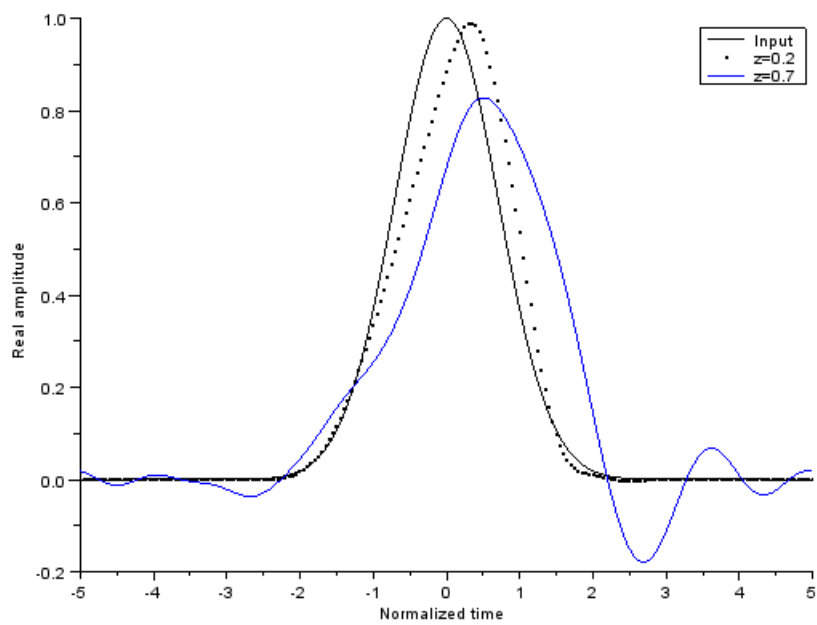


Figure 2: The input gaussian electric profile (solid black line) and the dash ($z = 0.2$) and solid blue line ($z = 0.7$). The parameters used are $s = 0.8$, $T_0 = 10[fs]$, $\delta_3 = 0.02[ps^3/km]$.

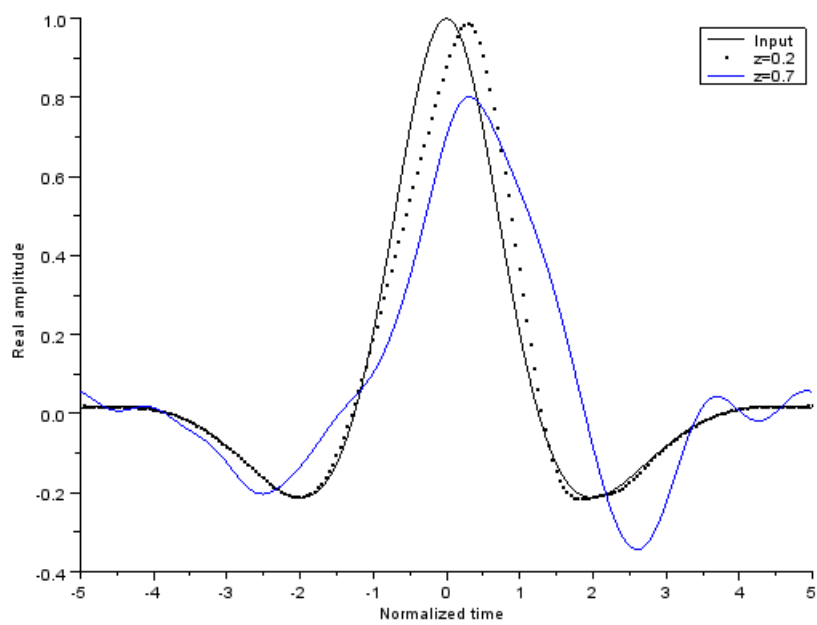


Figure 3: The cosine modulated by sech function (solid black) and output electric profile depicted the dash line ($z = 0.2$) and solid black line ($z = 0.7$). The pulse width $T_0 = 10[fs]$

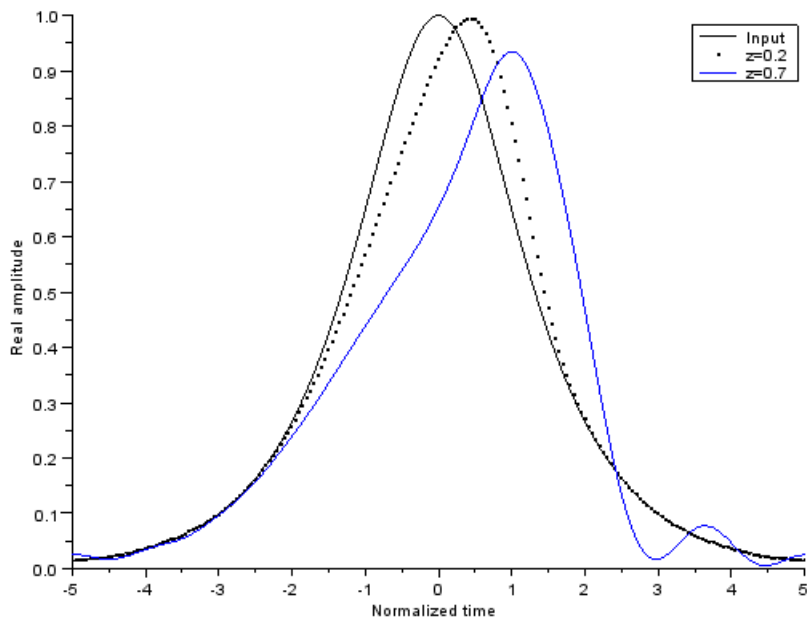


Figure 4: The sech function (solid black) and output electric profile depicted the dash line ($z = 0.2$) and solid black line ($z = 0.7$). The pulse width $T_0 = 10[fs]$

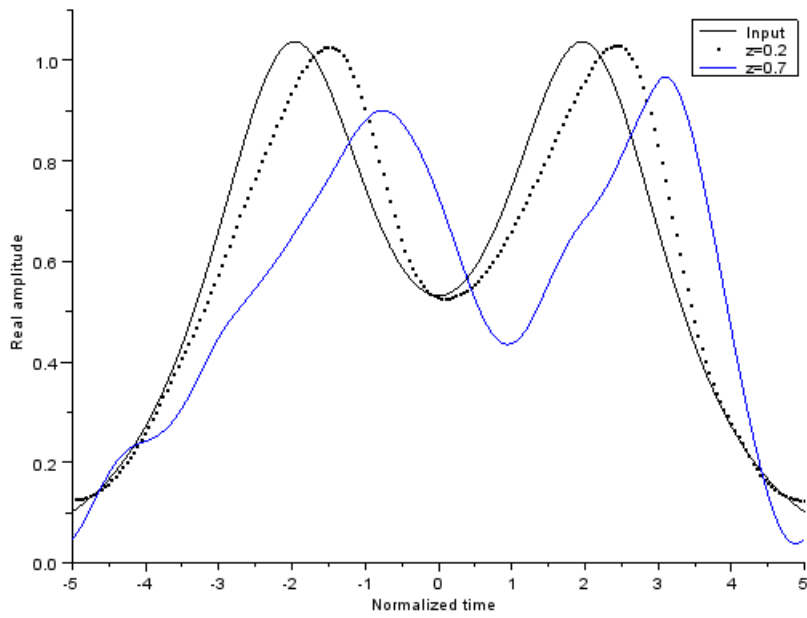


Figure 5: The input pulse is a soliton pair entering the medium. The output pulse for propagation distances $z = 0.2$ (dashed line) and $z = 0.7$ (solid blue line)

pulses are very used in optical telecommunications, the interest of such pulses will always attract. In our future work, we'll use the FDTD approach to solve partial derivative equation with higher order and focus on the stability of finite difference scheme.

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