Original Research Article

Comparative Analysis of the Molodensky and Kotsakis Ellipsoidal Heights Transformation between Geocentric and Non-Geocentric Datums Models

ABSTRACT

The non-availability of ellipsoidal heights of local geodetic Datums has made it necessary for the application of ellipsoidal heights transformation models to the available global ellipsoidal heights to obtain their respective theoretical heights in local Datums. It is required to know the accuracy, as well as reliability of any model of interest before its application. For that reason, this study comparatively analyses the Molodensky and Kotsakis models for the transformation of ellipsoidal heights between geocentric and non-geocentric Datums to determine the reliability of the Kotsakis model. The GNSS data of the used stations were processed in WGS84 datum to obtain their global geographic coordinates and ellipsoidal heights. The coordinates, ellipsoidal heights and the transformation parameters between WGS84 and Minna Datums were applied to the Molodensky and Kotsakis models to compute the Clarke 1880 theoretical heights of the stations. The Molodensky model was used as a reference to which the Kotsakis model ellipsoidal heights were compared to obtain the Kotsakis model ellipsoidal heights discrepancies, as well as residuals. The residuals were used to compute the RMSE of the Kotsakis model. The computed RMSE, as well as reliability of the model is 1.244m. The study concluded that the low reliability, as well as accuracy of the Kotsakis model might be as a result of the two rotation datum shift parameters in it as they are the main differences between the two models.

Keywords: Datum, ellipsoidal, geocentric, Height, Kotsakis, model, Molodensky, transformation

1. INTRODUCTION

Practical height computation from processed observed GNSS data requires the application of geoid height to the ellipsoidal height obtained from the processed observations. Ellipsoidal heights are theoretical heights from GNSS observations, which are measured from the surface of the reference ellipsoid to the observed point on the earth surface [1]. Non-geocentric datum ellipsoidal heights are not readily available in most GNSS observation areas and regions. Most GNSS height adjustments, as well as fitting during observations processing are done using the orthometric heights of existing control, used as reference station in the observation. However, ellipsoidal heights are applied to GNSS observation for theoretical height adjustment likewise orthometric height to spirit levelling for practical heights reduction. Orthometric heights are measured along the gravity vector direction and referenced to the geoid, as well as the mean seal level [2]. Ellipsoidal and orthometric heights have their respective reference surfaces. The erroneous use of the orthometric height for GNNS observations processing to obtain local ellipsoidal heights of points is as a result of the unavailability of ellipsoidal heights in the observation area or region. The Clarke 1880 ellipsoid adopted for geodetic computation in Nigeria is flatter and bigger than WGS 84 ellipsoid (see Figure 1). Nigeria is located between latitudes 4^{o}N and 14^{o}N, which is closer to the equator than the North Pole. So, it is expected that the ellipsoidal heights computed on the Clarke 1880 are smaller in value than those computed on the WGS84 ellipsoid.

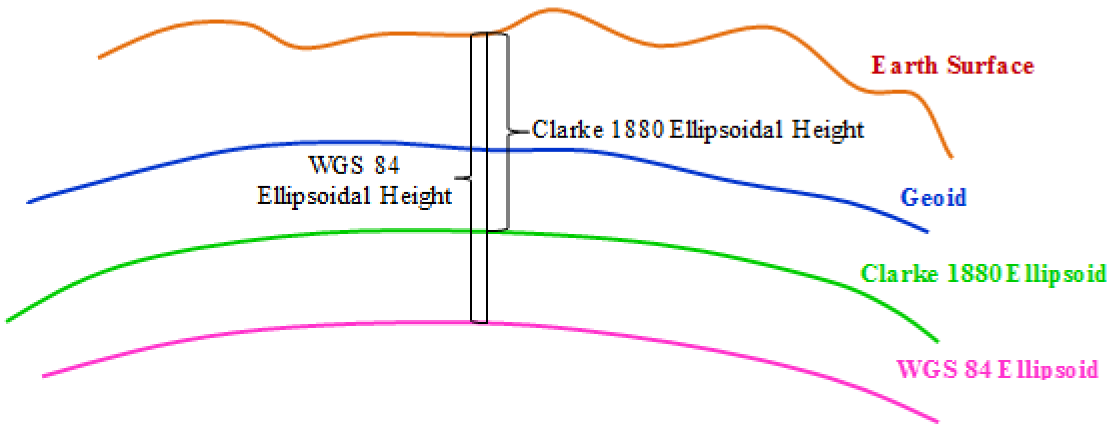


Fig.1. Relationship between WGS84 and Clarke 1880 ellipsoids

The local, as well as non-geocentric datum ellipsoidal heights can be obtained from the conversion of ellipsoidal heights computed on the global, as well as geocentric (WGS84) ellipsoid. The conversion can be achieved through the application of the 5-parameters Molodensky's model [3, 4, 5] and the 8-parameters Kotsakis model [6] for ellipsoidal heights transformation between geocentric and non-geocentric Datums, as well as reference frames. The Molodensky model involves the use of the 3 translation datum shift parameters, change in semi-major axis and difference in flattening between the two reference frames, as well as ellipsoids while the Kotsakis model comprises 3 translation and 2 rotation datum shift parameters, change in scale, change in semi-major axis and difference in flattening between the two reference ellipsoids. The Molodensky method was recently used by [7] for the determination of the ellipsoidal height of the Nigerian geodetic/Minna datum and RMSE of 0.00m was achieved. The method was compared with other two methods and was recommended as the best among the three methods. The Kotsakis method has not really been applied to Nigeria. Here, the Molodensky method is used as a reference to which the Kotsakis method is compared to determine its reliability. Consequently, this study comparatively analyses the Molodensky and Kotsakis ellipsoidal heights transformation between geocentric and non-geocentric Datums models to determine the reliability of the Kotsakis model.

2. METHODOLOGY

The adopted methodology involves the transformation of geocentric datum (WGS84) ellipsoidal heights obtained from GNSS observations to local ellipsoidal height in the Nigeria Minna datum using the Molodensky and Kotsakis methods and comparing their results. The application of the two methods requires the use of the 5-parameters Molodensky change in ellipsoidal height computation model, 8-parameters Kotsakis model, datum shift, as well as transformation parameters between WGS84 and Minna Datums, and the two Datums, as well as ellipsoids properties (semi-major axis and flattening).

2.1 The 5-Parameters Molodensky Model

The 5-parameters Molodensky model used for the transformation of ellipsoidal heights between geocentric and non-geocentric reference frames is [3, 4, 5, 7, 8]

$$\Delta h_{WGS84} = T_X \cos\varphi \cos\lambda + T_Y \cos\varphi \sin\lambda + T_Z \sin\varphi - \Delta a\left(\frac{a}{R_N}\right) + \Delta f\left(\frac{b}{a}\right) R_N \sin^2\varphi \quad (1)$$

Where,

$T_X, T_Y\ T_Z$ =Translation parameters between WGS84 and Minna Datum.

φ, λ = Geographic coordinates (Latitude and Longitude) of points.

a = Equatorial radius of the Clarke 1880 ellipsoid.
b = Polar radius of the Clarke 1880 ellipsoid.
f = Flattening of the Clarke 1880 ellipsoid.
Δa = Change in equatorial radius between the two ellipsoids (Minna minus WGS84)
Δf = Change in flattening between the two ellipsoids (Minna minus WGS84)

$$R_N = \text{Radius of curvature in prime vertical} = \frac{a}{\left(1-e^2\sin^2\varphi\right)^{1/2}} \tag{2}$$

$$e^2 = \text{Eccentricity squared} = 2f - f^2 = \frac{a^2-b^2}{a^2} \tag{3}$$

$$b = a(1-f) \tag{4}$$

Having computed Δh_{WGS84}, the non-geocentric (Clarke 1880) ellipsoidal heights ($h_{Clarke1880}$) are obtained using [5, 7]

$$h_{Clarke1880} = h_{WGS84} + \Delta h_{WGS84} \tag{5}$$

2.2 The 8-Parameters Kotsakis Model

The 8-parameters Kotsakis model used for the transformation of ellipsoidal heights between geocentric and non-geocentric reference frames is [6]

$$h' - h = \delta h(t_x) + \delta h(t_y) + \delta h(t_z) + \delta h(\varepsilon_x) + \delta h(\varepsilon_y) + \delta h(\delta s) + \\ + \delta h(\delta a) + \delta h(\delta f) \tag{6}$$

Where,

$$\delta h(t_x) = t_x \cos\varphi \cos\lambda \tag{7}$$

$$\delta h(t_y) = t_y \cos\varphi \sin\lambda \tag{8}$$

$$\delta h(t_z) = t_z \sin\varphi \tag{9}$$

$$\delta h(\varepsilon_x) = -\varepsilon_x N e^2 \sin\varphi \cos\varphi \sin\lambda \tag{10}$$

$$\delta h(\varepsilon_y) = \varepsilon_y N e^2 \sin\varphi \cos\varphi \cos\lambda \tag{11}$$

$$\delta h(\delta s) = (aW + h)\delta s \tag{12}$$

$$\delta h(\delta a) = -W\delta a \tag{13}$$

$$\delta h(\delta f) = \frac{a(1-f)}{W}\sin^2\varphi\,\delta f \tag{14}$$

$$W = \sqrt{1-e^2\sin^2\varphi} \tag{10}$$

N in equations (10) and (11) is the radius of curvature in prime vertical as given in equation (2)
The quantities $\delta a = \Delta a = a' - a$ and $\delta f = \Delta f = f' - f$ correspond to the difference in the numerical values for the semi-major axis and the flattening of the reference ellipsoid, as these are used in the respective reference frames, GRF1 and GRF2 [6].

2.3 Transformation Parameters between WGS84 and Minna Datum (Clarke 1880 Ellipsoid)

The transformation parameters from WGS84 to Minna datum are [9]

$$
\left.\begin{array}{l}
T_X = 93.809786\text{m} \pm 0.375857310\text{m} \\
T_Y = 89.748672\text{m} \pm 0.375857310\text{m} \\
T_Z = -118.83766\text{m} \pm 0.375857310\text{m} \\
R_X = 0.000010827829 \pm 0.000010311322 \\
R_Y = 0.0000018504213 \pm 0.0000015709539 \\
R_Z = 0.0000021194542 \pm 0.0000013005997 \\
\text{S} = 0.99999393 \pm 0.0000010048219
\end{array}\right\} \quad (11)
$$

2.4 The Nigerian Geodetic and WGS 84 Datums

The Nigerian geodetic datum (Clarke 1880 ellipsoid) and WGS84 ellipsoid semi-major axes (*a*) and flattening (*f*) are respectively [10] 6378249.145m and 1/293.465, and 6378137.000m and 1/298.257223563. $\delta a = \Delta a = a' - a = 112.145$
$\delta f = \Delta f = f' - f = 0.0000547507139518535$

2.5 Root Mean Square Error (RMSE)

The Root Mean Square Error (RMSE) of a model is computed to indicate its accuracy, as well as reliability. Here, the RMSE is computed by comparing the transformed ellipsoidal heights obtained from the two models using the Molodensky model as a reference. The computation of the RMSE of the transformation model is done using [11]

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2} \quad (12)$$

Where,

$e^2 = (h_{Molodensky} - h_{Kotsakis})^2$

$h_{Molodensky} =$ Molodensky model ellipsoidal height

$h_{Kotsakis} =$ Kotsakis model ellipsoidal height.

n = Number of points.

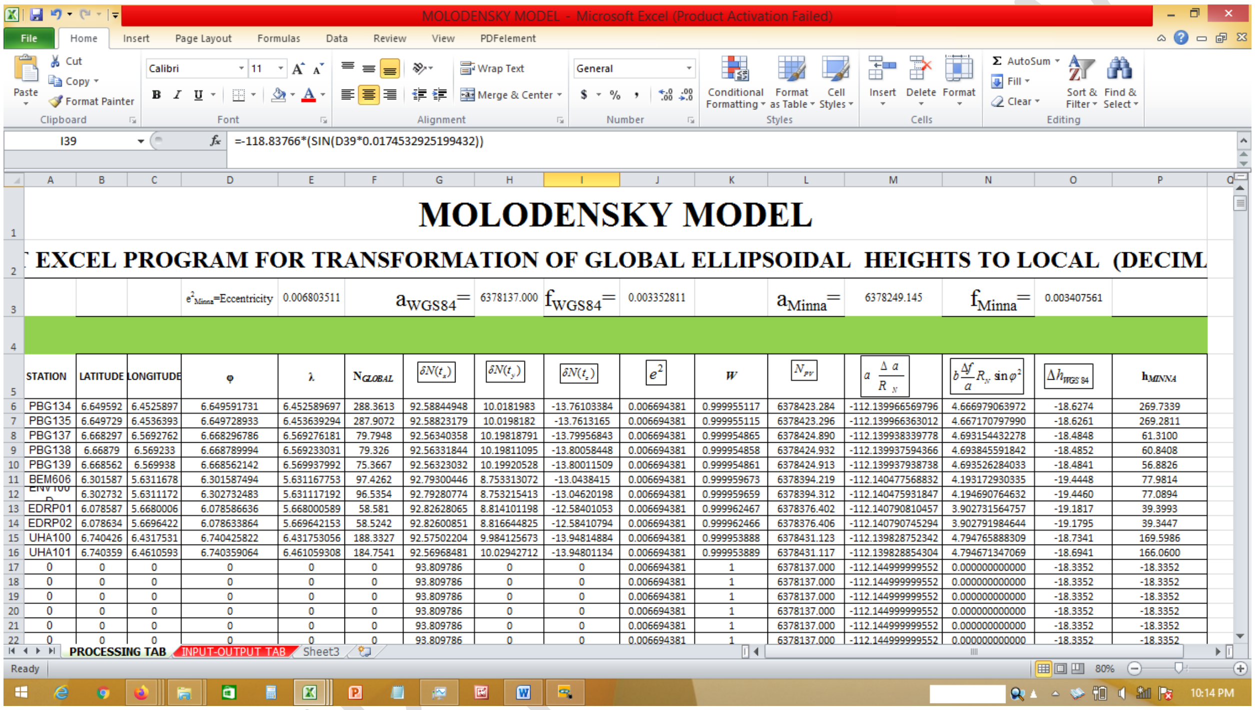
A total of 11 GNSS points located within Edo State were used in the study. The observation of the points was carried out with 5 dual-frequencies GNSS receivers. The geographic coordinates and ellipsoidal heights of the points were processed on the WGS84 ellipsoid using the Compass Post-processing software as the study involves the transformation of global dataset to local. Table 1 shows the geocentric (WGS84) datum geographic coordinates and ellipsoidal heights of the used stations.

Table 1: Geographic latitudes, longitudes and ellipsoidal heights of stations

Station	WGS 84 Datum		
	Latitude (Decimal. Degree)	**Longitude (Decimal. Degree)**	**Ellipsoidal Height (m)**
PBG134	6.649591731	6.452589697	288.3613
PBG135	6.649728933	6.453639294	287.9072
PBG137	6.668296786	6.569276181	79.7948
PBG138	6.668789994	6.569233031	79.3260
PBG139	6.668562142	6.569937992	75.3667
BEM606	6.301587494	5.631167753	97.4262
ENV100D	6.302732483	5.631117192	96.5354

EDRP01	6.078586636	5.668000589	58.5810
EDRP02	6.078633864	5.669642153	58.5242
UHA100	6.740425822	6.431753056	188.3327
UHA101	6.740359064	6.461059308	184.7541

The changes in ellipsoidal heights between the WGS84 and Clarke 1880 spheroids and the Clarke 1880 ellipsoidal heights regarding the Molodensky model were respectively computed using equations (1) and (5) (see Figure 2) while those of the Kotsakis model were computed using equation (6) (see Figure 3). The reliability, as well as the root mean square error of the Kotsakis model, was computed using equation (12).


MOLODENSKY MODEL

EXCEL PROGRAM FOR TRANSFORMATION OF GLOBAL ELLIPSOIDAL HEIGHTS TO LOCAL (DECIM

e^2_{Minna}=Eccentricity 0.006803511 a_{WGS84}= 6378137.000 f_{WGS84}= 0.003352811 a_{Minna}= 6378249.145 f_{Minna}= 0.003407561

STATION	LATITUDE	LONGITUDE	φ	λ	N_{GLOBAL}	$\delta N(t_x)$	$\delta N(t_y)$	$\delta N(t_z)$	e^2	W	N_{PV}	$a\frac{\Delta a}{R_N}$	$b\frac{\Delta f}{a}R_N \sin\varphi^2$	Δh_{WGS84}	h_{MINNA}
PBG134	6.649592	6.4525897	6.649591731	6.452589697	288.3613	92.58844948	10.0181983	-13.76103384	0.006694381	0.999955117	6378423.284	-112.139966569796	4.666979063972	-18.6274	269.7339
PBG135	6.649729	6.4536393	6.649728933	6.453639294	287.9072	92.58823179	10.0198182	-13.7613165	0.006694381	0.999955115	6378423.296	-112.139966363012	4.667170797990	-18.6261	269.2811
PBG137	6.668297	6.5692762	6.668296786	6.569276181	79.7948	92.56340358	10.19818791	-13.79956843	0.006694381	0.999954865	6378424.890	-112.139938339778	4.693154432278	-18.4848	61.3100
PBG138	6.66879	6.569233	6.668789994	6.569233031	79.326	92.56331844	10.19811095	-13.80058448	0.006694381	0.999954858	6378424.932	-112.139937594366	4.693845591842	-18.4852	60.8408
PBG139	6.668562	6.569938	6.668562142	6.569937992	75.3667	92.56323032	10.19920528	-13.80011509	0.006694381	0.999954861	6378424.913	-112.139937938738	4.693526284033	-18.4841	56.8826
BEM606	6.301587	5.6311678	6.301587494	5.631167753	97.4262	92.79300446	8.753313072	-13.0438415	0.006694381	0.999959673	6378394.219	-112.140477568832	4.193172930335	-19.4448	77.9814
ENV100D	6.302732	5.6311172	6.302732483	5.631117192	96.5354	92.79280774	8.753215413	-13.04620198	0.006694381	0.999959659	6378394.312	-112.140475931847	4.194690764632	-19.4460	77.0894
EDRP01	6.078587	5.6680006	6.078586636	5.668000589	58.581	92.82628065	8.814101198	-12.58401053	0.006694381	0.999962467	6378376.402	-112.140790810457	3.902731564757	-19.1817	39.3993
EDRP02	6.078634	5.6696422	6.078633864	5.669642153	58.5242	92.82600851	8.816644825	-12.58410794	0.006694381	0.999962466	6378376.406	-112.140790745294	3.902791984644	-19.1795	39.3447
UHA100	6.740426	6.4317531	6.740425822	6.431753056	188.3327	92.57502204	9.984125673	-13.94814884	0.006694381	0.999953888	6378431.123	-112.139828752342	4.794765888309	-18.7341	169.5986
UHA101	6.740359	6.4610593	6.740359064	6.461059308	184.7541	92.56968481	10.02942712	-13.94801134	0.006694381	0.999953889	6378431.117	-112.139828854304	4.794671347069	-18.6941	166.0600
0	0	0	0	0	0	93.809786	0	0	0.006694381	1	6378137.000	-112.144999999552	0.000000000000	-18.3352	-18.3352
0	0	0	0	0	0	93.809786	0	0	0.006694381	1	6378137.000	-112.144999999552	0.000000000000	-18.3352	-18.3352
0	0	0	0	0	0	93.809786	0	0	0.006694381	1	6378137.000	-112.144999999552	0.000000000000	-18.3352	-18.3352
0	0	0	0	0	0	93.809786	0	0	0.006694381	1	6378137.000	-112.144999999552	0.000000000000	-18.3352	-18.3352
0	0	0	0	0	0	93.809786	0	0	0.006694381	1	6378137.000	-112.144999999552	0.000000000000	-18.3352	-18.3352
0	0	0	0	0	0	93.809786	0	0	0.006694381	1	6378137.000	-112.144999999552	0.000000000000	-18.3352	-18.3352

Fig. 2. Molodensky model change in ellipsoidal heights computation

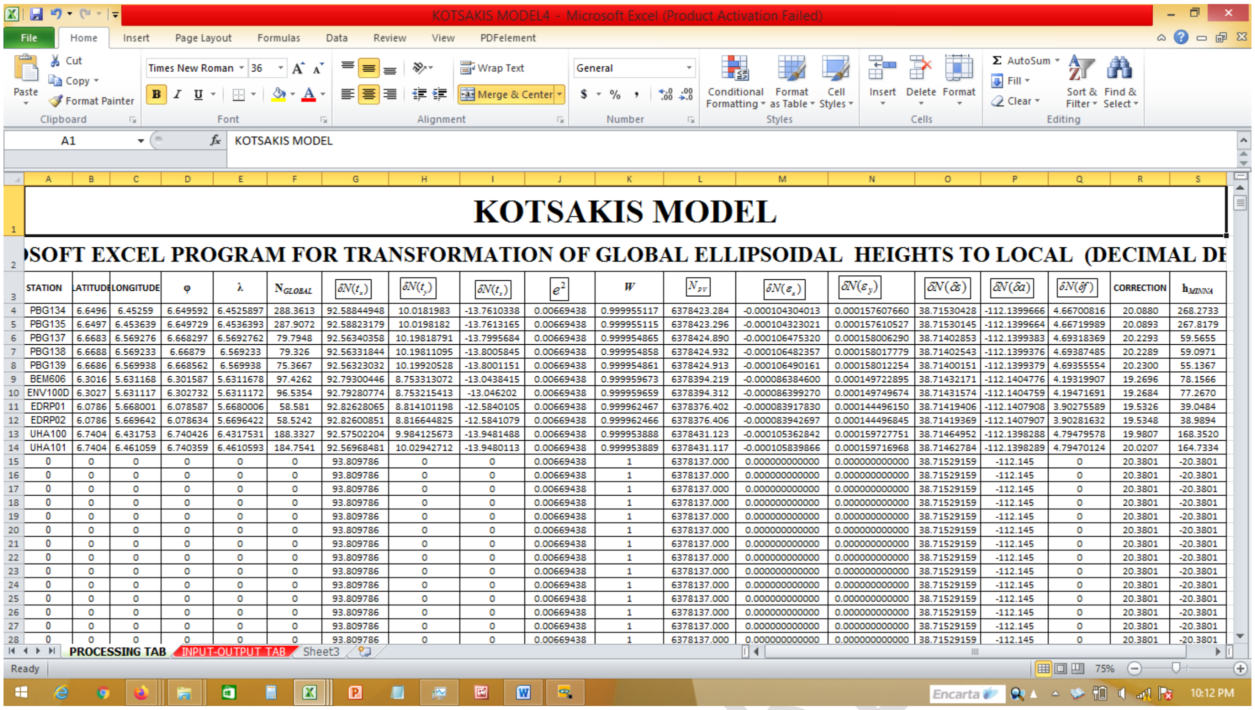


Fig. 3. Kotsakis model Clarke 1880 ellipsoidal heights computation

3. RESULTS AND DISCUSSION

Table 2 presents the discrepancies in the ellipsoidal heights and Root Mean Square Error, RMSE of the Kotsakis model. They were computed to show the range of the discrepancies, as well as the differences in ellipsoidal heights between the two models and the accuracy of the Kotsakis model relative to the Molodensky model. It can be seen from Table 2 that the minimum and maximum discrepancies of the Kotsakis model ellipsoidal heights are -0.1775m and 1.7459m respectively. It implies that the Kotsakis model ellipsoidal heights discrepancies range from -0.1775 to 1.7459m. The computed range is limited to the used stations, as well as the location of the points (Edo State). I can also be seen in Table 2 that the RMSE of the Kotsakis model is 1.244m which implies that the model has a reliability, as well as accuracy of 1.244m. The low accuracy of the model might be as a result of the two rotation datum shift parameters terms in it since they are the main differences between the two models.

Table 2. Transformed ellipsoidal heights discrepancies and RMSE of Kotsakis model

STATION	h_{GLOBAL}	$h_{LOCAL/MINNA}$ (m)		DIFFERENCE/ DISCREPANCY (m)	DIFFERENCE SQUARED (m^2)
		MOLODENSKY MODEL	KOTSAKIS MODEL		
PBG134	288.3613	269.7339	268.2733	1.4606	2.1335
PBG135	287.9072	269.2811	267.8179	1.4633	2.1411
PBG137	79.7948	61.3100	59.5655	1.7446	3.0436
PBG138	79.326	60.8408	59.0971	1.7436	3.0402
PBG139	75.3667	56.8826	55.1367	1.7459	3.0482
BEM606	97.4262	77.9814	78.1566	-0.1752	0.0307
ENV100D	96.5354	77.0894	77.2670	-0.1775	0.0315
EDRP01	58.581	39.3993	39.0484	0.3509	0.1231
EDRP02	58.5242	39.3447	38.9894	0.3554	0.1263
UHA100	188.3327	169.5986	168.3520	1.2466	1.5540

UHA101	184.7541	166.0600	164.7334	1.3266	1.7599
				Kotsakis Model RMSE =	**1.2443m**

4. CONCLUSION

The study has comparatively analyzed the Molodensky and Kotsakis ellipsoidal heights transformation between geocentric and non-geocentric Datums models and determined the accuracy, as well as reliability of the Kotsakis model. The study has determined the range of the discrepancies of the Kotsakis model limited to the used stations to be -0.1775 to 1.7459m. It has also determined the accuracy of the Kotsakis model to be 1.244m. It again stated that the low accuracy of the model might result from the two rotation datum shift parameters terms in it.

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