Original Research Article

Equation for Energy Dissipation for Spillways with equal Step Heights and Widths of 0.305m

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ABSTRACT

Stepped spillways are current method of choice for safe discharge of flood water due to their inherent ability to employ their stepping nature to safely dissipate substantial energy. Hence, the authors in this study aim to provide to design engineers new models for estimating energy dissipation in a stepped spillway with an equal step heights and widths of 0.3 m. Many researchers have investigated both the hydraulic and the geometric relationships of stepped spillways of varying sizes that resulted in significant energy dissipation, but quite a few of them has conducted researches on stepped spillways with equal heights and widths of 0.3 m, thereby leading to limited information at the disposal of design engineers involved in the design of stepped spillway with equal heights and widths of 0.3 m. The authors obtained more than 500 measured data from the Engineering Research Center Colorado State University Fort Collins, Colorado work published in April, 2002 and about 300 of them that had complete data were re-analyzed and used to develop energy dissipation model that govern nappe and skimming flows over a wide range of operating conditions. This energy dissipation model was formulated in terms of the number of the steps of the stepped spillway and the flow critical depth using multiple regression analysis and matrix method. This developed model was later calibrated and yielded very high coefficients of correlation that ranged from 0.8808 to 0.9982, which upon verification gave good predictions between the measured and estimated data. Results showed that the rate of energy dissipation along a stepped spillway increases with increasing numbers of steps, but decreases with increasing rate of discharge.

Keywords: [Energy Dissipation, Regression analysis, Stepped spillway, Measured data, Nappe flow}

1. INTRODUCTION

Stepped spillways are current method of choice for safe discharge of flood water due to their inherent ability to employ their stepping nature to safely dissipate substantial energy. Many researchers have investigated both the hydraulic and the geometric relationships of stepped spillways of varying sizes that resulted in significant energy dissipation. These researchers provided design guidelines for stepped spillway channels with various channel geometries and slopes. Most recent research focused on the air-water flow properties and energy dissipation performances, including some more detailed air-water flow properties such as bubble count rate, turbulence, and microscopic air-water properties (Chanson & Toombes, 2002; Gonzalez, 2005; Chanson & Carosi, 2007; Toombes and Chanson, 2008a, b; Felder & Chanson, 2009b, 2011a); Meireles et al., 2009). However, despite the enormous amount of information and guidelines on stepped spillways provided by these scholars to design engineers, huge information gap and guidelines still existed in the area of stepped spillways with equal treads and heights of 0.305 m. It is, therefore, the purpose of this present study to obtain near-prototype scale data on the hydraulic characteristics of stepped spillway flow including air concentration, bulked flow depth, clear water depth, and flow velocity from the Engineering Research Center Colorado State University Fort Collins, Colorado work published in April, 2002. These collected data were analyzed to quantify energy dissipation and to develop design guidelines and models that would aid design engineers involved in the design of stepped spillways with equal heights and widths of 0.3 m.

These Researchers were able to identify three kinds of flows that take place over a stepped spillway such as a) nappe flow regime, b) transional flow regime, and c) skimming flow regime.

In nappe flow regime, sequence of drops from one step to the next step below it with the formation of hydraulic jump at every drop is observed. This type of flow can be likened to a sequence of separates drop structures (Chamani and Rajaratnam 1994; Chanson 1993).

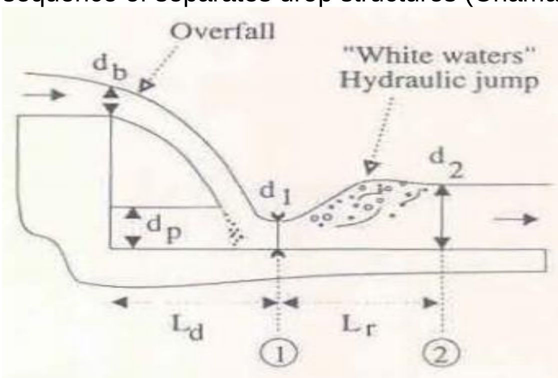

Fig. 1. Nappe flow regime (Flow at a drop structure)

The water flows over one step of the spillway and lands on the next step with energy loss happening from a) the disintegration of the spout in the air, and b) the blending of flow on the steps, with or without the development of hydraulic jump on the step (Chanson, 1994; Rajaratnam, 1990) and these energy losses could be computed using equations 2.2a and 2.2b.

$$\frac{\Delta E}{E_o} = 1 - \frac{\frac{d_1}{y_c} + \frac{1}{2}\left(\frac{y_c}{d_1}\right)^2}{H_{max} + \frac{3}{2}} \qquad (ungated\ spillway) \qquad 1.1a$$

$$\frac{\Delta E}{E_o} = 1 - \frac{\frac{d_1}{y_c} + \frac{1}{2}\left(\frac{y_c}{d_1}\right)^2}{\frac{(H_{max} + H_o)}{d_c}} \qquad (gated\ spillways) \qquad 1.1b$$

And Chanson (1994) later expressed these equations – 1a and 1b – in terms the spillway step height, the critical flow depth, and the dam height as:

$$\frac{\Delta E}{E_o} = 1 - \left[\frac{0.54\left(\frac{y_c}{h}\right)^{0.275} + \frac{3.43}{2}\left(\frac{y_c}{h}\right)^{-0.55}}{\frac{3}{2} + \frac{H_{dam}}{d_c}}\right] \quad ungated\ spillway \quad 1.2a$$

$$\frac{\Delta E}{E_o} = 1 - \left[\frac{0.54\left(\frac{y_c}{h}\right)^{0.275} + \frac{3.43}{2}\left(\frac{y_c}{h}\right)^{-0.55}}{\frac{H_{dam} + H_o}{d_c}}\right] \quad gated\ spillway \quad 1.2b$$

Where ΔE is the energy loss, E_o is the maximum available energy, y_c is critical flow depth, h is the height of the spillway step, H_{max} is the dam height.
Nappe flow with completely established hydraulic jump (Figure 1), usually arises from small discharges with shallow flow depths and flow over the step with formation of supercritical at the edge of the step and returns to subcritical flow downstream of the jump.

In skimming flow regime, the flow occurs with the submergence of the steps with the development of fully aerated uniform flow in the downstream region in a long chute. Along the upstream steps, a non-aerated flow region exists within which a turbulent boundary layer develops. Air entrainment in the flow begins where the boundary layer intersects the free surface, referred to as the point of inception. Downstream from the point of inception, the flow continues to aerate and varies gradually in depth (Figure 2). The flow eventually becomes fully aerated, uniform flow in which the water depth, velocity, and air concentration become constant (Bindo et. al. 1993) (Figure 3).

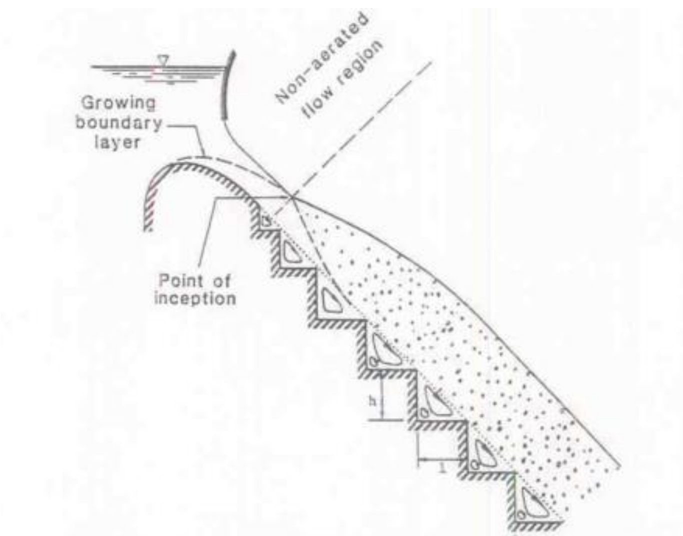

Fig. 2. Skimming flow regime - Sorensen (1985)

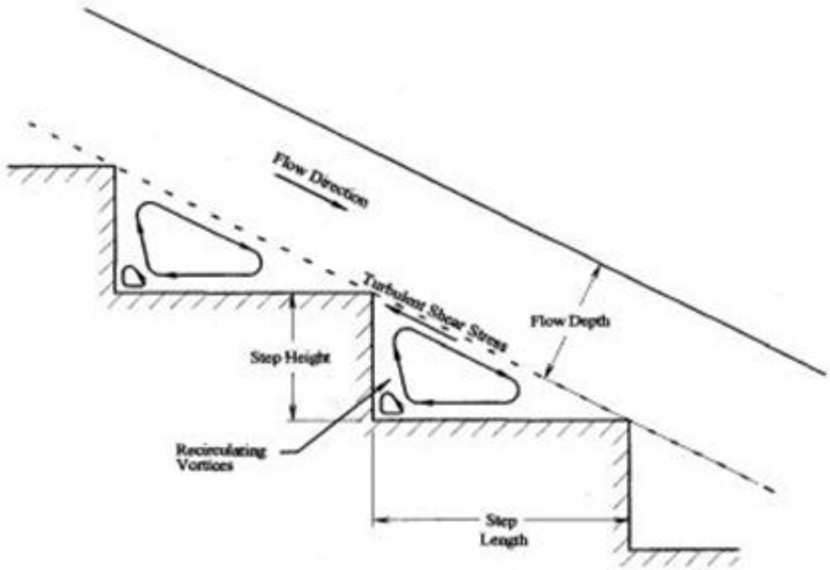

Fig.3. Skimming flow regime with uniform flow conditions

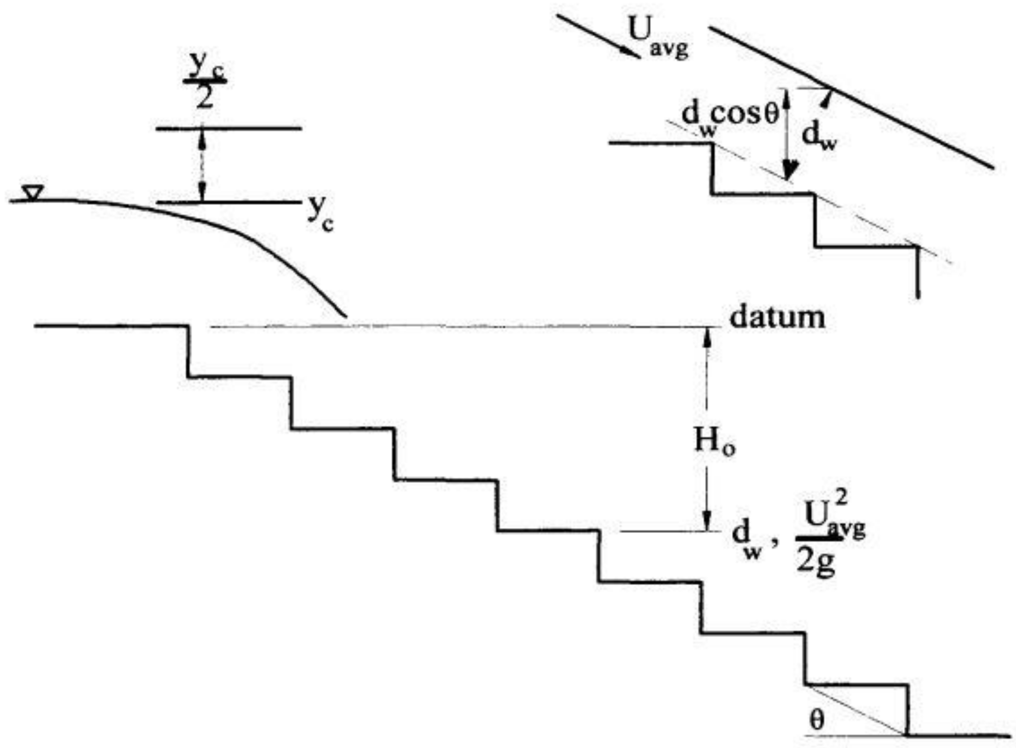

Fig 4. Arrangement of the spillway with the definition of the variables

In skimming flow, most of the energy is dissipated in the maintenance of stable depression vortices. If uniform flow conditions are reached at the downstream end of the spillway, this energy loss could be computed as follows

$$\frac{\Delta E}{E_o} = \frac{d_w \cos\theta + \frac{U_{avg}^2}{2g}}{Nh + \frac{3}{2} y_c} \qquad [1.3\}$$

the total head loss may be rewritten in terms of the friction factor, the spillway slope, the critical depth and the dam height:

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{f}{8 sin\theta}\right)^{1/3} cos\theta + \frac{E}{2}\left(\frac{f}{8 sin\theta}\right)^{-2/3}}{\frac{H_{dam}}{d_c} + \frac{3}{2}} \qquad [1.4]$$

where E is the kinetic energy correction coefficient, ϑ is the dam slope in degrees, friction factor: f = 0.03 and f = 1.30, that represent average flow resistance on smooth spillways and stepped spillways.

2. MATERAL AND METHODS

The authors obtained and re-analyzed the experimental data from the published works of the Final report of the Research Project 99FC800156 HYDRAULIC DESIGN OF STEPPED SPILLWAYS Prepared for: U.S. Bureau of Reclamation Denver, Colorado Prepared by: James F. Ruff and Jason P. Ward Engineering Research Center Colorado State University Fort Collins, Colorado published in April, 2002 and used them to develop an energy dissipation models that governed both nappe and skimming flows over a wide range of operating conditions. These improved models were verified with the measured data fro m these researchers and compared with the results from the existing models. These new models were later verified using part of the experimental data that were not used in the calibration. The extent of fit between measured to predicted ratio of energy dissipation to maximum elevation found by estimation of the Spearman – Pearson coefficient of correlation were compared with the existing model.

2.1 FORMULATION OF THE MODEL

In modeling, it is necessary to determine the values of the parameters that can fit the model to the system it shall describe (Agunwamba, 2007). By least square method, the best fit curve for this study was formulated as a function of the ratio of number of steps to the critical water depth as:

$$\frac{\Delta H}{H_{max}} = \left[\alpha_o \frac{Nh}{y_c}\right]^{\alpha_1} N^{\alpha_2} \qquad [2.1]$$

These improved energy dissipation models were formulated as a function of the ratio of number of steps to the critical water depth using multiple regression analysis.

where

ΔH is head loss,

H_{max} is maximum energy,

N = number of steps,

h = height of step (m),

y_c = critical water depth (m),

α_o, = constant,

α_1 and α_2 = coefficients.

Using this equation and the measured data, the authors used multiple regression analysis and matrix method to solve the resultant equations, which yielded the constant α_o along with the coefficients α_1, and α_2 for the developed models

and the 3 No normal equations of [2.3]

Prediction of new models verified with the experimental data was equated with the outcomes from the current model.

2.2 MODEL VERIFICATION

The equation obtained was verified using a part of the experimental data that were not used in calibration. The extent of fit between measured to predicted ratio of energy dissipation to maximum elevation was found by estimation of the Spearman – Pearson coefficient of correlation expressed by

$$\gamma = \sqrt{\frac{[n\sum xy - (\sum x \sum y)]^2}{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}} \qquad [2.4]$$

2.3 COMPARISON WITH EXISTING MODELS

The developed models (Equations 3.1.1, 3.1.2, and 3.1.3) were compared with the existing Equations (Equations 1.2a and 1.3) using the criteria specified in Equation (2.4).

3. RESULTS AND DISCUSSION

The analyses yielded the values of constant α_o, the coefficients α_1, and α_2 for nappe, transition, skimming flow regimes, respectively, which were then substituted in Equation [2.1] to give the developed models in 3.1.

3.1. Developed models for nappe, transition, and skimming flow regimes.

3.1.1 Developed model for nappe flow regime ($y_{c/}h \leq 0.46$ and h = 0.305 m)

$$\frac{\Delta H}{H_{max}} = \left[0.486\frac{Nh}{y_c}\right]^{13.21} N^{-13.32} \qquad (3.1)$$

3.1.2. Developed model for transition flow regime ($0.46 < y_c/h \leq 0.95$ and h = 0.305 m)

$$\frac{\Delta H}{H_{max}} = \left[1.10\frac{Nh}{y_c}\right]^{79.92} N^{-82.39} \qquad (3.2)$$

3.1.3. Developed model for skimming flow regime ($0.95 < y_c/h \leq 2.69$ and h = 0.305 m)

$$\frac{\Delta H}{H_{max}} = \left[\frac{Nh}{y_c}1.23e^{-10}\right]^{0.11} N^{0.55} \qquad (3.3)$$

3.2 Chart for nappe flow regime for $y_{c/}h \leq 0.46$

Figure 5.1 depicts the energy loss rate and an expression of a dam height divided by the critical depth, and plotted with the measured data from Carosi & Chanson (2006), the developed analytical formulation (Eq 3.1) as well as the existing model for the computation of energy dissipation (Eq 1.2a). The figure showed some traditional concave shape distributions for all the three plotted data for energy dissipation for all the flow rates (Chanson, 2001). As indicated in the chart, energy loss increases with decreasing discharges and again increases with increasing dam height for a particular discharge which is in accordance with the earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a). The data sets from the field work and developed model (Eq 3.1) were in very close agreement with the coefficient of correlation computed as 0.99. However, the case was entirely different when the field data were compared with the predicted data using the existing model (Eq 1.2a): the existing model predicted unrealistically high energy dissipation values compared to the measured data for Nh/y_c within the values of 13 and 48. For instance, the predicted energy dissipation values of 79% and 94% as against the actual energy dissipation values of 22% and 81% were observed when the values of Nh/yc were 13 and 47 respectively. It is, therefore, not advisable to use the existing equation (Eq 1.2a) for values of Nh/yc of between 13 and 47 when h is equal to 0.305 m.

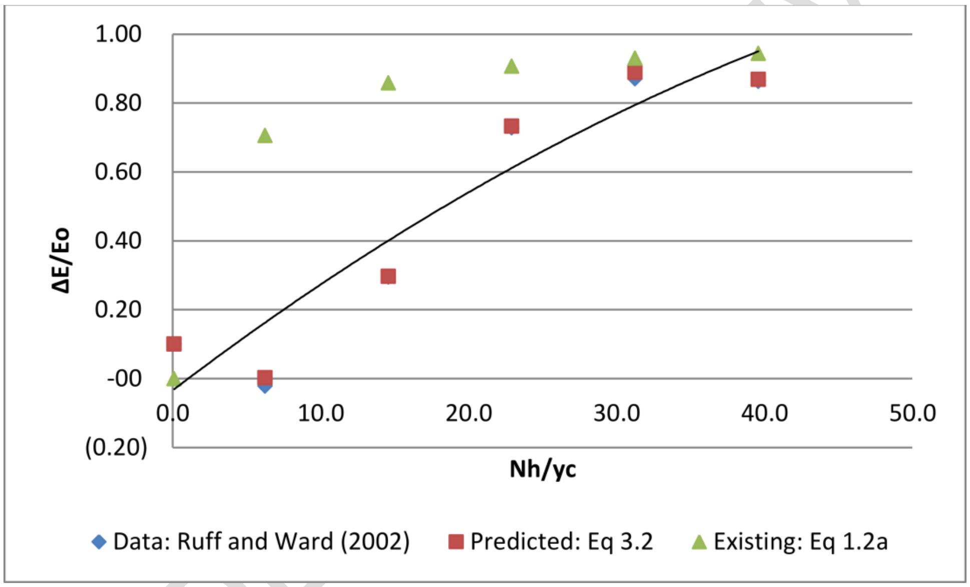

Fig 5.1. Variation of relative head loss $\Delta H/H$ with yc/Nh for $y_c/h \leq 0.46$

3.3 Chart for transition flow regime for $0.46 < y_c/h \leq 0.95$

Figure 5.1 depicts the energy loss rate and an expression of a dam height divided by the critical depth, and plotted with the measured data from Carosi & Chanson (2006), the developed analytical formulation (Eq 3.2) as well as the existing model for the computation of energy dissipation (Eq 1.2a). The figure showed some traditional concave shape distributions for all the three plotted data for energy dissipation for all the flow rates (Chanson, 2001). As indicated in the chart, energy loss increases with decreasing discharges and again increases with increasing dam height for a particular discharge which is in accordance with the earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a). The data sets from the field work and developed model (Eq 3.2) were in very close agreement with the coefficient of correlation computed as 0.99. However, the case was again entirely different when the field data were compared with the predicted data using the existing model (Eq 1.2a): the existing model predicted unrealistically high energy dissipation values compared to the measured data for Nh/y_c within the values of 6 and 23. For instance, the predicted energy dissipation values of 71% and 91% as against the actual energy dissipation values of 20% and 73% were observed when the values of Nh/yc were 6 and 23 respectively. It is, therefore, not advisable to use the existing equation (Eq 1.2a) for values of Nh/yc of between 6 and 23 when h is equal to 0.305 m.

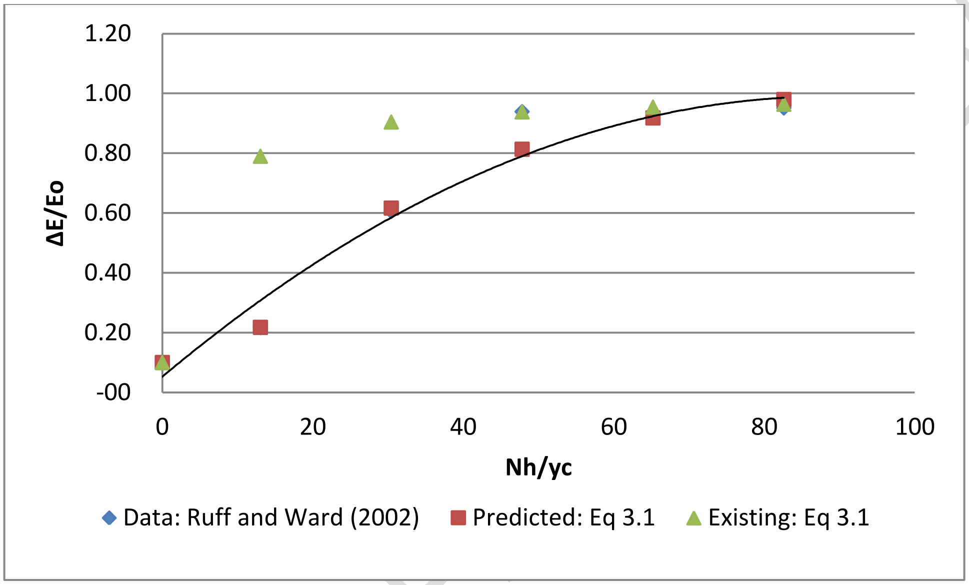

Fig 5.2. Variation of relative head loss $\Delta H/H$ with yc/Nh for 0.46 < y_c/h ≤ 0.95

3.4.1 Charts for skimming flow regime for 0.95 < y_c/h ≤ 1.48, 1.48 < y_c/h ≤ 1.91, 1.91 < y_c/h ≤ 2.32, and 2.32 < y_c/h ≤ 2.69

Figures 5.3, 5.4, 5.5, and 5.6 display patterns that are very closely related to each other and would, hence, be discussed as a group here. These figures depict the energy loss rate and an expression of a dam height divided by the critical depth and plotted with the measured data from Carosi & Chanson (2006), the developed analytical formulation (Eq 3.3) as well as the existing model for the computation of energy dissipation (Eq 1.3). The figures show some traditional concave shape distributions for all the three plotted data for energy dissipation for all the flow rates (Chanson, 2001). As shown in the chart, energy loss increases with decreasing discharges and again increases with increasing dam height for a particular discharge which is in accordance with the earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a). The data sets from the field work, the developed model (Eq 3.3), and the existing model (Eq 1.3) for computing energy dissipation are all in a very close agreement with the coefficients of correlation of 0.99 computed for the measured data and the developed model, and 0.98 computed for the measured data and the existing model for energy dissipation. These figures have, therefore, confirmed the appropriateness for use of the existing model (Eq 1.3) when the spillway step height, h, is 0.305 m. The developed model (Eq 3.3) is, however, simple and easy to use than the existing model (Eq 1.3).

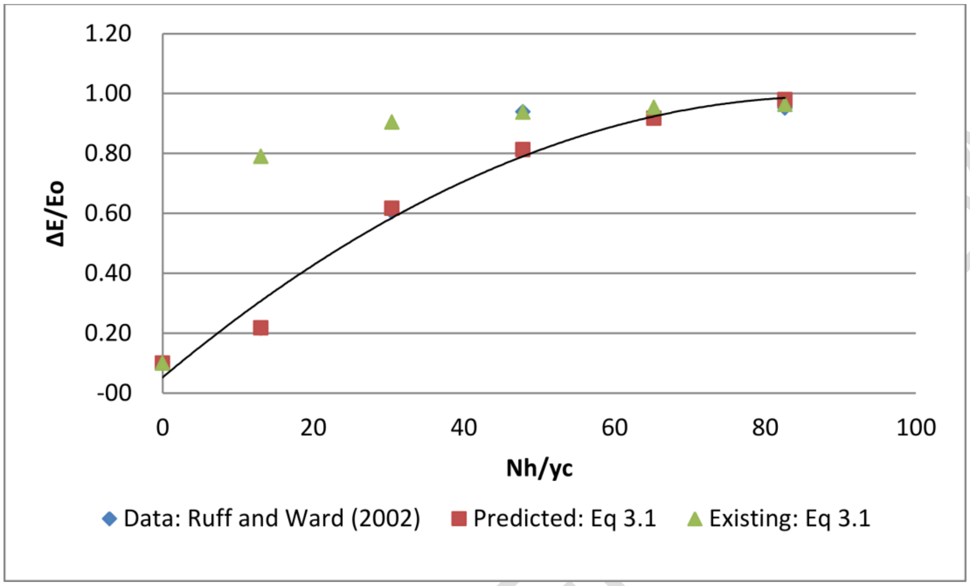

Fig 5.3. Variation of relative head loss $\Delta H/H$ with yc/Nh for 0.95 $<$ y_c/h $\leq$ 1.48

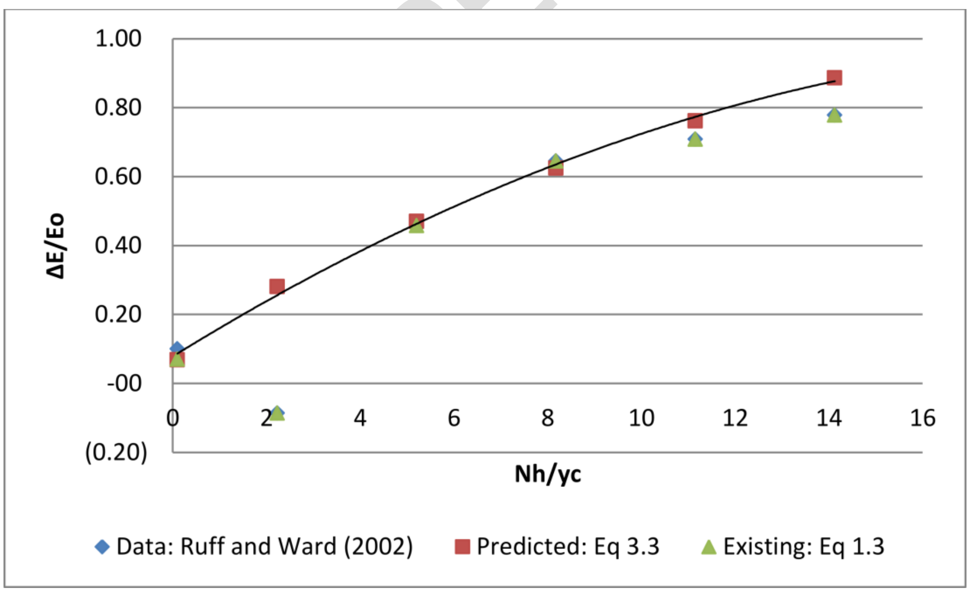

Fig 5.4. Variation of relative head loss $\Delta H/H$ with yc/Nh for 1.48 $<$ y_c/h $\leq$ 1.91

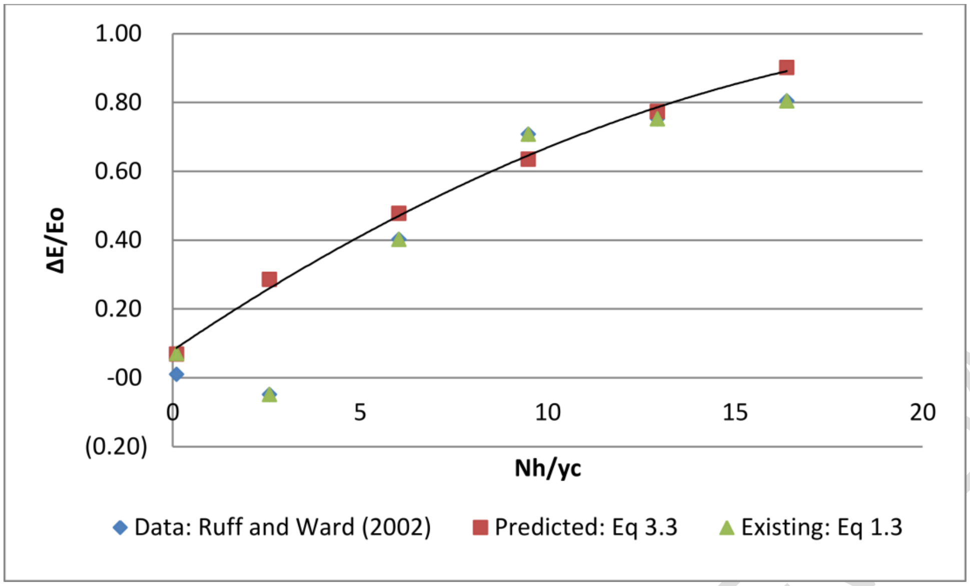

Fig 5.5. Variation of relative head loss $\Delta \mathrm{H}/\mathrm{H}$ with yc/Nh for 1.91 $<$ y_c/h $\leq$ 2.32

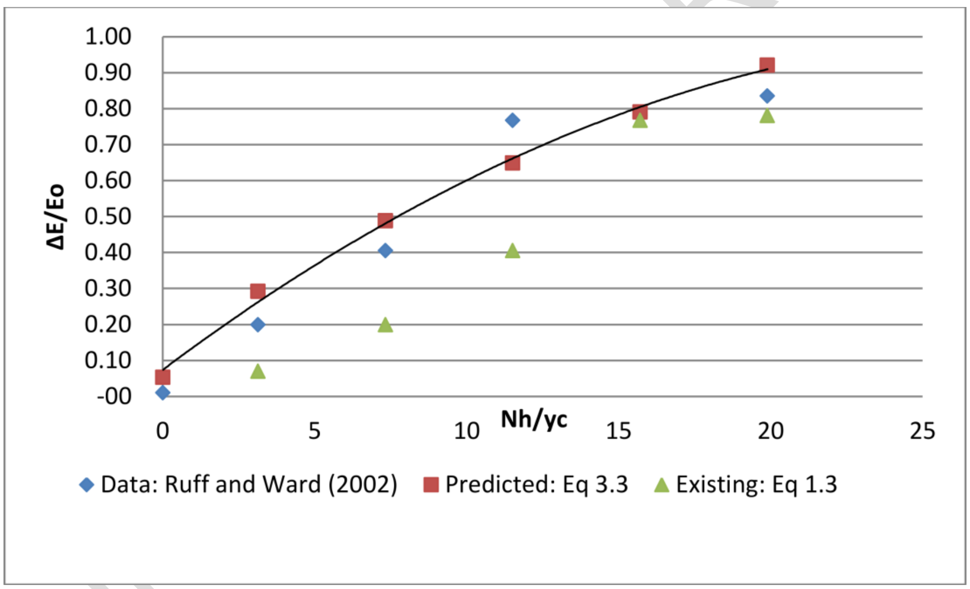

Fig 5.6. Variation of relative head loss $\Delta \mathrm{H}/\mathrm{H}$ with yc/Nh for 2.32 $<$ y_c/h $\leq$ 2.69

4. CONCLUSION

These new models – Eq 3.1, Eq 3.2, and Eq 3.3 - not only predicted the rate of energy dissipation for nappe, transition, and skimming flow regime, respectively that are in a very close agreement when compared with the measured data, but are also simpler and easier to use that the existing models. The coefficients of correlation computed for these developed models are very high and range between 0.98 and 0.99.

It is to be noted that:

a) Eq 3.1 is only valid for use when y_c/h is less than 0.46 and h = 0.305 m, and that the existing model (Eq 1.2a) is not appropriate for use with the Nh/y_c values of between 13 and 48 when h = 0.305 m,
b) Eq 3.2 is only valid for use when y_c/h is between 0.46 and 0.95 and h = 0.305 m, and that the existing (Eq 1.2a) is not appropriate for use with the Nh/yc values of between 6 and 23 when h = 0.305 m,
c) Eq 3.3 is only valid for use when y_c/h is between 0.95 and 2.69 and h = 0.305 m. Although this study confirms the appropriateness for use of the existing model (Eq 1.3), the developed model (Eq 3.3) is, however, simpler and easier to use that the existing model..

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