Original Research Article

THE ENERGY OF CONJUGATE GRAPH OF A DIHEDRAL GROUP

Abstract

Let $\Gamma^{C}_{D_{2n}}$ and $E(\Gamma)$ denote the conjugate graph of a dihedral group of order $2n$ ($n \in \aleph$) and the energy of a graph respectively. The sum of the absolute values of the eigenvalues of an adjacency matrix's eigenvalues is the energy of a graph. In this paper, we use group representation of a dihedral group of order $2n$ with its conjugacy classes to explicitly design admissible conjugate graphs. We further introduced the general formula for the energy of conjugate graphs of dihedral groups in various circumstances. Also, we deduced the general formula for the conjugate graph of generalized dihedral groups of order $2n$ depending on the nature of *n*.

Keywords: Dihedral group, adjacency matrix, conjugacy class, conjugate graph, energy of a graph.

1. INTRODUCTION

The application of group theory to graph theory has been the subject of numerous studies over the years. The aspect of conjugate graph was first introduced by Erfanian and Tolue (2013) and researches in this aspect commenced since then. These researches ultimately extended to calculating the various energies of the conjugate graphs. It was remarked in Woods (2013) that Gutman was the first to interpret the energy of a generic simple graphs in 1978 after getting inspired by Huckel's Molecular Orbital Theory introduced in 1930. This discovery by Huckel has found application in Chemistry in the aspect of calculating the energy associated with π−electron orbital in conjugated hydrocarbons. A further analysis on

Huckel's work was undertaken in Gunthard and Primas (1956) where they discovered that Huckel's technique is actually based on the first-degree polynomial of a graph matrix.

In one development, Pirzada and Gutman (2008) displayed that a graph's energy cannot be the square root of an odd integer. Further, Mahmoud et al. (2017) came up with a formula for the energy of a non- commutating graph of a dihedral group and recently, Mahmud et al. (2021) computed the energy of the conjugate graph of some finite Metabelian groups of order less than 24. In this paper, we focus our attention on the dihedral group of order $2n$.

This paper is subdivided into three (3) sections. In the second section we present existing definitions and results which will ultimately give light for understanding the results of this work. We present in the third section the findings of this work.

2. PRELIMINARIES

We present in this section preliminary definitions and results. We shall find these definitions and results instrumental in the next section. Some of these definitions can be found in any textbook on Algebra.

Definition 2.1

Let G be a finite group and H be its normal subgroup. The factor group of G modulo H is a group defined by $G/H = \{gH: g \in G\}$.

Definition 2.2

The group of all symmetries of a regular polygon with n vertices is known as the dihedral group, D_{2n}. The group is of order $2n$ ($n \in \mathbb{N}$). D_{2n} is defined in the following way

$$D_{2n} \cong < a, b : a^3 = b^2 = 1, bab = a^{-1} >$$

Its elements is presented below

$$\{1, a, a^2, \dots, a^{n-1}, b, ab, a^2b, \dots, a^{n-1}b$$

Definition 2.3

Let G be a group and $a, b \in G$. The commutator of a and b denoted by $[a, b]$ is the element

$$[a, b] = aba^{-1}b^{-1} \in G.$$

Definition 2.4

Let G be a finite group and let x_1,x_2 be elements in G. The elements x_1,x_2 are said to be conjugate if $x_2 = hx_1h^{-1}$ for some h in G. The set of all conjugate elements of x_1 is called the conjugacy class of x_1.

Definition 2.5

Let G be a finite group. The centre of G denoted Z(G) is defined by:

$$\mathrm{Z}(G) = \{a \in G : ax = xa \ \forall x \in G\}.$$

The centre of G is a subset of elements in G that commute with every element of G.

Definition 2.6

Suppose G is a group, two elements a and b of G are conjugate if there exists an element g in G with $gag^{-1} = b$.

Definition 2.7

Let G be a finite group. Then the conjugacy class of the element a in G is given as:

$$\mathrm{cl}(a) = \{ g \in G / xax^{-1} = g, x \in G \}.$$

Definition 2.8

A pair consisting of a set V of vertices and a set E of edges labelled as $\Gamma = (V, E)$ is called a graph. The elements of E are the lines that combine two elements of V.

Definition 2.9

A complete graph is a simple graph in which every pairs of distinct vertices are adjacent. The complete graph with *n* vertices is denoted as K_n.

Definition 2.10

Let G be a finite non-abelian group with centre $Z(G)$. The conjugate graph is a graph whose vertices are non-central elements of G in which two vertices are adjacent if they are conjugate.

Preposition 2.1

Let *G* be finite group and let *a* and *b* be elements of *G*. *a* and *b* are said to be conjugate if they belong to precisely one conjugacy class. That is *Cl(a)* and *Cl(b)* are equal.

Theorem 2.2 (Samaila et al (2013))

Depending on the parity of n, the conjugacy classes in the dihedral group D_{2n} are as follows:

1. $\{1\}, \{a, a^{-1}\}, \ldots\ldots, \left\{a^{\frac{n-1}{2}}, a^{-\left(\frac{n-1}{2}\right)}\right\}, \left\{a^i b, 0 \leq i \leq n-1\right\}$, if n is odd;

2. $\{1\}, \left\{a^{\frac{n}{2}}\right\}, \{a, a^{-1}\}, \{a^2, a^{-1}\}, \ldots\ldots, \left\{a^{\frac{n-2}{2}}, a^{-\left(\frac{n-2}{2}\right)}\right\}, \left\{a^{2i} b,\ 0 \leq i \leq \frac{n-2}{2}\right\}$

   $\left\{a^{2i+1} b,\ 0 \leq i \leq \frac{n-2}{2}\right\}$, if n is even.

3. MAIN RESULT

We present in this section the findings of this paper. In the result that follows, we characterize the conjugate graph of a dihedral group.

Theorem 3.1

Let G be a dihedral group of order 2*n* where $n \geq 3, n \epsilon N$. Then, the conjugate graph of D_{2n} is,

$$\Gamma^c_{D_{2n}} = \begin{cases} \{Ki_2\}_{i=1}^{\frac{n-1}{2}} \cup Kn & if\ n\ is\ odd \\ \{Ki_2\}_{i=1}^{\frac{n-2}{2}} \cup \{Ki_{\frac{n}{2}}\}_{i=1}^{2} & if\ n\ is\ even \end{cases}$$

Proof

Suppose D_{2n} is a dihedral group of order $2n$ and $\Gamma^{c}_{D_{2n}}$ its conjugate graph. Now,

Case 1: if n odd,

By Theorem 2.2, the set of vertex of the conjugate graph is $V\left(\Gamma^{c}_{D_{2n}}\right) = 2n - 1$, by the vertex adjacency of conjugate graph and by proposition 2.1, the elements are conjugate if they belong to one conjugate class i.e. cl(a) = cl(b). Thus, $\Gamma^{c}_{D_{2n}} = \{Ki_2\}_{i=1}^{\frac{n-1}{2}} \cup Kn$.

Case 2: if n even,

By Theorem 2.2, the set of vertex of conjugate graph is $V\left(\Gamma^{c}_{D_{2n}}\right) = 2(n-1)$ by the vertex of adjacency of conjugate graph and by proposition 2.1, the element are conjugate if they belong to precisely one conjugate class i.e. cl(a) = cl(b). Thus, $\Gamma^{c}_{D_{2n}} = \{Ki_2\}_{i=1}^{\frac{n-2}{2}} \cup \{Ki_{\frac{n}{2}}\}_{i=1}^{2}$.

Example 3.1

Let G be a dihedral group of order 6, $D_6 = \langle a, b : a^3 = b^2 = 1, bab = a^{-1} \rangle$ and let $\Gamma^{G}_{D_6}$ be a conjugate graph of D_6. Then $\Gamma^{C}_{D_6} = K_2 \cup K_3$

Solution

Following from theorem 2.2, there are five non-central elements in D_6 and thus $\left|V(\Gamma^{C}_{D_6})\right| = 5$. By vertices adjacency of conjugate graph and proposition 2.1, the element are conjugate if they belong to one conjugacy class $\{cl(a), cl(a^2)\}$, $\{cl(b), cl(ab), cl(a^2b)\}$, the related vertices are joined by an edge and form a single graph K_2. In the same way, the other three points make up a full graph of K_3. Hence $\Gamma^{C}_{D_6} = K_2 \cup K_3$. The conjugate graph of $\Gamma^{C}_{D_6}$ is shown below:

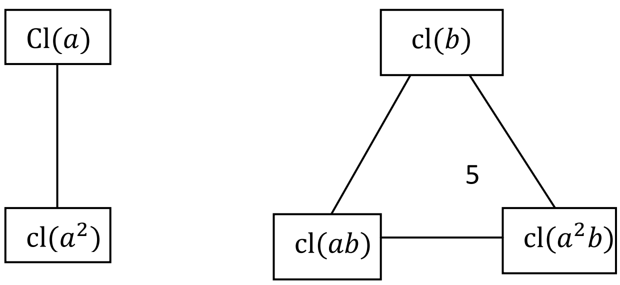

Fig 1: The complete graph of $K_2 \cup K_3$ ■

Example 3.2

Let G be a dihedral group of order 8, $D_8 = \langle a, b \colon a^4 = b^2 = 1, bab = a^{-1} \rangle$ and let $\Gamma^{c}_{D_8}$ be a conjugate graph of D_8. Then $\Gamma^{C}_{D_8} = \{Ki_2\}_{i=1}^{3}$

Solution

Considering theorem 2.2, there will be five non-central elements in D_8, thus $\left|V(\Gamma^{C}_{D_8})\right| = 6$. By vertices adjacency of conjugate graph and proposition 1, the element are conjugate if they belong to one conjugacy class $\{cl(a), cl(a^3)\}$, $\{cl(b), cl(a^2b)\}$, $\{cl(ab), cl(a^3b)\}$. The related vertices are joined by an edge and formed three complete graph of K_2. Hence $\Gamma^{C}_{D_8} = \{Ki_2\}_{i=1}^{3}$the conjugate graph of $\Gamma^{C}_{D_8}$ is shown below:

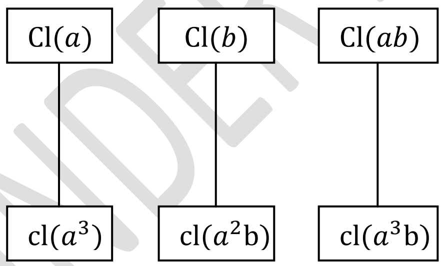

Fig 2: The complete graph of $\{Ki_2\}_{i=1}^{3}$ ■

Example 3.3

Let G be a dihedral group of order 22, $D_{22} = \langle a, b \colon a^8 = b^2 = 1, bab = a^{-1} \rangle$ and let $\Gamma^{G}_{D_{22}}$ be a conjugate graph of D_{22}. Then $\Gamma^{C}_{D_{22}} = \{Ki_2\}_{i=1}^{5} \cup K_{11}$

Solution

It follows from theorem 2.2 that there are five non-central elements in D_{22}, thus $\left|V(\Gamma_{D_{22}}^{C})\right| = 21$. And by vertices adjacency of conjugate graph and proposition 1, the element are conjugate if they belong to one conjugacy class $\{cl(a), \mathrm{cl}(a^{10})\}, \{cl(a^3), cl(a^8)\}, \{cl(a^4), cl(a^7)\}, \{cl(a^4), cl(a^6)\}, \{cl(b), cl(ab), cl(a^2b), cl(a^3b), cl(a^4b), cl(a^5b), cl(a^6b), cl(a^7b), cl(a^8b), cl(a^9b), cl(a^{10}b),\}$, the related vertices are joined by an edge and form five single graph of K_2. In the same way, the other eleven points make up a full graph of K_{11}. Hence $\Gamma_{D_{22}}^{C} = \{Ki_2\}_{i=1}^{5} \cup K_{11}$. The conjugate graph of $\Gamma_{D_{22}}^{C}$ is shown below:

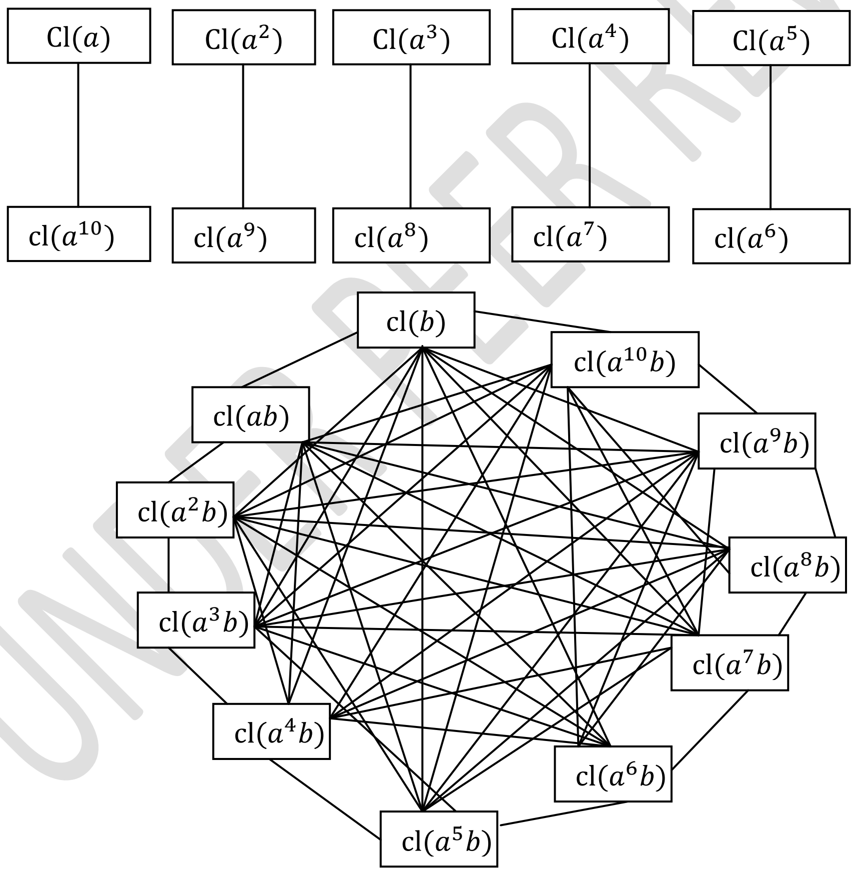

Figure 3: The complete graph of $\{Ki_2\}_{i=1}^{5} \cup K_{11}$

Remark: We remark here that it is not hard to see from the examples above that as we display the conjugate graph of the dihedral groups of order $2n$, the size of the graph gets bigger in proportion to the order.

In what follows, we determine the eigenvalues of the conjugate graph of the dihedral group of order $2n$ as well as the general formulas for its energy. We consider this in cases depending on the parity of n.

Theorem 3.2

Let G be a dihedral group of order $2n$, where n is an odd integer such that $n \geq 3, n \in Z^{+}$ and let $\Gamma_{D_{2n}}^{c}$ be its conjugate graph as usual. Then the energy of the graph $\Gamma_{D_{2n}}^{c}$ is $\varepsilon\left(\Gamma_{D_{2n}}^{c}\right) = \frac{5n-3+2i}{2}$ for $i = 0, 1, 2, 3, \ldots$

Proof

Suppose G is a dihedral group of order $2n$ and $\Gamma_{D_{2n}}^{c}$ be its conjugate graph, then from theorem 3.1, case 1, the conjugate graph of the group G can be expressed as a complete graphs given by $\Gamma_{D_{2n}}^{c} = \{Ki_2\}_{i=1}^{\frac{n-1}{2}} \cup K_n$. Hence, the eigenvalues of the complete graph are: $\lambda_1 = n-1$, $\lambda_2 = 1$ (with $\frac{n-1}{2}$ repeats) and $\lambda_3 = -1$ (with $(n+i)$ repeats where $(i = 0, 1, 2, \ldots)$ and so the energy is $\boldsymbol{\varepsilon}\left(\boldsymbol{\Gamma}_{D_{2n}}^{c}\right) = \frac{\mathbf{5n-3+2i}}{\mathbf{2}}$.

Example 3.4

Let $G = D_6$. Then, the energy of the conjugate graph of G, $\varepsilon(\Gamma_{D_6}) = 6$.

Then, the adjacency matrix A for the conjugate graph

$\Gamma_{D_6}^{C} = K_2 \cup K_3$ is given in the following:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Thus, the characteristic polynomial of A is given as in the following:

$$2+\lambda^5-4\lambda^3-2\lambda^2+3\lambda$$

Hence, It is discovered that the eigenvalues are $\lambda=2$, $\lambda=1$ and $\lambda=-1$ with 3 repeats. Therefore, the energy of the conjugate graph for D_6 is $\varepsilon(\Gamma_{D6})=\sum_{\mathrm{i}=1}^{\mathrm{n}}|\lambda_{\mathrm{i}}|=6$.

Meanwhile by theorem 3.2, $\varepsilon\left(\Gamma_{D_{2n}}^{c}\right)=\frac{5n-3+2i}{2}=6$

Example 3.5

$G = D_{10}$. Then, the energy of the conjugate graph of G, $\varepsilon(\Gamma_{D_{10}}) = 12$.

The adjacency matrix A for the conjugate graph $\Gamma_{D_{10}}^{C}=\{Ki_2\}_{i=1}^{2}\cup \mathrm{K}_5$ is given in the following:

$$A=\begin{bmatrix} 0&1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&1&1&1\\ 0&0&0&0&1&0&1&1&1\\ 0&0&0&0&1&1&0&1&1\\ 0&0&0&0&1&1&1&0&1\\ 0&0&0&0&1&1&1&1&0 \end{bmatrix}$$

Thus, the characteristic polynomial of A is given as in the following:

$$-4+\lambda^9-12\lambda^7-20\lambda^6-6\lambda^5+36\lambda^4+20\lambda^3-12\lambda^2-15\lambda$$

Hence, It is discovered that the eigenvalues are $\lambda=4$, $\lambda=1$ with 2 repeats and $\lambda=-1$ with 6 repeat. Therefore, the energy of the conjugate graph for D_{10} is $\varepsilon(\Gamma_{D5})=\sum_{\mathrm{i}=1}^{n}|\lambda_{\mathrm{i}}|=12$.

Meanwhile by using theorem 3.1 $\varepsilon\left(\Gamma_{D_{2n}}^{c}\right)=\frac{5n-3+2i}{2}=12$

Example 3.6

Let $G = D_{14}$. Then, the energy of the conjugate graph of G, $\varepsilon(\Gamma_{D14}) = 18$.

The adjacency matrix A for the conjugate graph $\Gamma_{D_{14}}^{C}=\{Ki_2\}_{i=1}^{3}\cup \mathrm{K}_5$ is given in the following:

$$
A = \begin{bmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&1&1&1&1\\
0&0&0&0&0&0&1&0&1&1&1&1&1\\
0&0&0&0&0&0&1&1&0&1&1&1&1\\
0&0&0&0&0&0&1&1&1&0&1&1&1\\
0&0&0&0&0&0&1&1&1&1&0&1&1\\
0&0&0&0&0&0&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&0
\end{bmatrix}
$$

Thus, the characteristic polynomial of A is given as in the following:

$(-1+\lambda^2)^3(-6+\lambda^7-21\lambda^5-70\lambda^4-105\lambda^3-84\lambda^2-35\lambda)$

Hence, It is discovered that the eigenvalues are $\lambda = 6$, $\lambda = 1$ with 3 repeats and $\lambda = -1$ with 9 repeats. Therefore, the energy of the conjugate graph for D_{14} is $\varepsilon(\Gamma_{D14}) = \sum_{i=1}^{n}|\lambda_i| = 18$.

And so, $\varepsilon\left(\Gamma^c_{D_{2n}}\right) = \frac{5n-3+2i}{2} = 18$

Theorem 3.3

Let G be a dihedral group of order $2n$ where n is even, $n \geq 6$ $n \in N$ and let $\Gamma^c_{D_{2n}}$ be its conjugate graph. Then the energy of the graph $\Gamma^c_{D_{2n}}$ is $\varepsilon\left(\Gamma^c_{D_{2n}}\right) = 3(n-2)$.

Proof

Suppose G is a dihedral group of order $2n$ and $\Gamma^c_{D_{2n}}$ be its conjugate graph from theorem 3.1, case 2, the conjugate graph of the group G is complete given by $\Gamma^c_{D_{2n}} = \{Ki_2\}_{i=1}^{\frac{n-2}{2}} \cup \{Ki_{\frac{n}{2}}\}_{i=1}^{2}$. Then the eigenvalues of the complete graph are $\lambda_1 = n-2$, $\lambda_2 = n-4-i$ and $\lambda_3 = -1$ with multiplicity $n+i$ where $i = 0,1,2,3,4,5,6, \ldots\ldots$ Hence, the energy is $\left(\Gamma^c_{D_{2n}}\right) = 3(n-2)$.

Example 3.7

Let G = D_{12}. Then, the energy of the conjugate graph of G, $\varepsilon(\Gamma_{D12}) = 12$.

Then, the adjacency matrix A for the conjugate graph $\Gamma^{C}_{D12} = \{Ki_2\}_{i=1}^{2} \cup \{Ki_3\}_{i=1}^{2}$ is given in the following:

$$A = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&0&0&0&0&0 \\ 0&0&1&0&1&0&0&0&0&0 \\ 0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&0&1&1&0 \end{bmatrix}$$

Thus, the characteristic polynomial of A is given as in the following:

$$4 + \lambda^{10} - 8\lambda^{5} - 4\lambda^{7} + 22\lambda^{6} + 20\lambda^{5} - 20\lambda^{4} - 28\lambda^{3} + \lambda^{2} + 12\lambda$$

Hence, It is discovered that the eigenvalues are $\lambda = 2$ with 2 repeats, $\lambda = 1$ with 2 repeats and $\lambda = -1$ with 6 repeats. Therefore, the energy of the conjugate graph for D_{12} is

$$\varepsilon(\Gamma_{D12}) = \sum_{i=1}^{n} |\lambda_i| = 12.$$

Meanwhile by using theorem 3.2. $\varepsilon\left(\Gamma^{c}_{D_{2n}}\right) = \mathbf{3}(\boldsymbol{n} - \mathbf{2})$= 12

Example 3.8

Let $G = D_{16}$. Then, the energy of the conjugate graph of G, $\varepsilon(\Gamma_{D16}) = 18$.

Then, the adjacency matrix A for the conjugate graph $\Gamma^{C}_{D16} = \{Ki_2\}_{i=1}^{3} \cup \{Ki_4\}_{i=1}^{2}$ is given in the following:

$$A = \begin{bmatrix} 0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&1&1&0&0 \\ 0&0&0&1&0&0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&1&1 \\ 0&0&0&0&0&1&0&0&0&0&0&0&1&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&0&0&0&0&0&0&1&0&0 \\ 0&0&0&1&1&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&1&0&0&0&0&0&0&1 \\ 0&0&0&0&0&1&1&0&0&0&0&0&1&0 \end{bmatrix}$$

Thus, the characteristic polynomial of A is given as in the following:

$(-1+\lambda^2)^3\ (-3+\lambda^4-3\ 6\lambda^7-6\lambda^2-8\lambda)^2$

Hence, It is discovered that the eigenvalues are $\lambda = 3$ with 2 repeats, $\lambda = 1$ with 4 repeats and $\lambda = -1$ with 9 repeats. Therefore, the energy of the conjugate graph for D_{16},

is $\varepsilon(\Gamma_{D16}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} |\lambda_{\mathrm{i}}| = 18$

Meanwhile by using theorem 3.3, $\varepsilon\left(\Gamma_{D_{2n}}^{c}\right) = 3(n-2) = 18$

Example 3.9

Let $G = D_{20}$. Then, the energy of the conjugate graph of G, $\varepsilon(\Gamma_{D20}) = 24$.

Then, the adjacency matrix A for the conjugate graph $\Gamma_{D_{20}}^{C} = \{Ki_2\}_{i=1}^{4} \cup \{Ki_5\}_{i=1}^{2}$ is given in the following:

$$A = \begin{bmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&1&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&0&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&1&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0
\end{bmatrix}$$

Thus, the characteristic polynomial of A is given as in the following:

$(-1+\lambda^2)^4(-4+\lambda^5-10\lambda^3-20\lambda^2-15\lambda)^2$

Hence, It is discovered that the eigenvalues are $\lambda = 4$ with 2 repeats, $\lambda = 1$ with 4 repeats and $\lambda = -1$ with 12 repeats. Therefore, the energy of the conjugate graph for D_{20} is

$\varepsilon(\Gamma_{\mathrm{G}}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} |\lambda_{\mathrm{i}}| = 24.$

Meanwhile by using theorem 3.3. $\varepsilon\left(\Gamma_{D_{2n}}^{c}\right) = 3(n-2) = 24$

CONCLUSION

In this paper, we deduced the general formulas for the energy of a conjugate graph of dihedral groups. This formula was found to be $\varepsilon\left(\Gamma_{D_{2n}}^{c}\right) = \frac{5n-3+2i}{2}$ for an odd integer n and $\varepsilon\left(\Gamma_{D_{2n}}^{c}\right) = 3(n-2)$ for an even integer n.

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