Original Research Article

A NUMERICAL INVESTIGATION OF PRESSURE-VELOCITY COUPLING SCHEMES FOR BOUYANCY DRIVEN FLOW IN A DIFFERENTIALLY HEATED CAVITY

Abstract

Numerical analysis of fluid flow is anchored on the laws of conservation. A challenge in solving the momentum equation arises due to the unavailability of an explicit pressure equation. To circumvent this problem most researchers, use the vorticity-stream function method to eliminate the pressure term in the momentum equation. This approach introduces more variables to be solved than those in the original equations. In order to overcome this challenge, the pressure equation is obtained in terms of the primitive variables using various forms of pressure-velocity coupling schemes. In this study, pressure equation is obtained from the continuity equation using the SIMPLE, SIMPLER and SIMPLEC pressure-velocity schemes. The schemes are used in analysing flow characteristics for a laminar buoyancy driven flow in order to establish the scheme that gives results consistent with bench mark data. The domain of the flow is a rectangular enclosure differently heated on two horizontal opposite sides. The cavity has two insulating baffles attached to the vertical walls and is full of air of Prandtl number of 0.71. The equations governing the flow are solved iteratively using finite volume method together with the central difference interpolating scheme. The solutions are presented for Rayleigh numbers of 10^3, 10^4, and 10^5. This resulted in the velocity profiles for the SIMPLE, SIMPLER, and SIMPLEC algorithm for a Rayleigh number of 10^4 and 10^5 converging to the same path. At a Rayleigh number of 10^3 however, SIMPLER behaves as the SIMPLE and SIMPLEC in the baffle free regions but undergoes a degradation in convergence with grid refinement at the baffle region. Results predicted by using the SIMPLEC algorithm are thus able to effectively compute the velocity of fluid flow in a differentially heated square enclosure with baffles for both low and higher Rayleigh numbers irrespective of the grid size. The results are consistent with M. Zeng et al., (2002) study on the convergence comparison of four algorithm namely; the SIMPLE, SIMPLER, SIMPLEC and SIMPLEX algorithm.

Nomenclature

g	Gravitational acceleration, (m/s^2)
p	Pressure of the fluid, (N/m^2)
p^*	Pressure Dimensionless pressure
Ra	Rayleigh number
u,v	Velocity in the x and y directions
u^*, v^*	Dimensionless velocity component
v_{nb}, u_{nb}	Neighbouring finite volumes

Acronyms

SIMPLE	Semi Implicit Method for Pressure Linked Equations
SIMPLER	SIMPLE Revised
SIMPLEC	SIMPLE Consistent
FVM	Finite Volume Method

1.0 Background of the Study

In computational fluid dynamic (CFD), temperature and velocity flow profiles are obtained from the equations governing the flow. Explicit equations for all the flow variables in the governing equations of flow are available except for pressure. To solve the momentum equation the pressure fields must be therefore determined first. To circumvent this problem most researchers, use the vorticity-stream function method to eliminate the pressure term in the momentum equation. This approach introduces more variables to be solved than those in the original equations and is applicable only to 2-dimensional (2D) flows as vorticity does not exist in 3-dimensional flows (3D).

To overcome this challenge, this study has obtained the pressure equation from the continuity equation using the SIMPLE, SIMPLER and SIMPLEC pressure-velocity schemes resulting in physically meaningful results in solving both 2-dimensional (2D) and 3-dimensional (3D) flows. The coupling between the momentum and the continuity equation is the key to the pressure- velocity coupling schemes. These schemes are based on the finite volume discretization of the momentum equation.

Two major assumptions are made in SIMPLE algorithm (Tao, 2001), an initial estimate of the pressure fields and the initial estimate of the velocity fields are made with no interconnection between them. The second assumption involves omission of the summation $\sum a_{nb} u'_{nb}$ and $\sum a_{nb} v'_{nb}$ in order to simplify the solution. SIMPLER algorithm overcomes the first approximation of the SIMPLE algorithm by computing a good pressure field from good velocity guess instead of having to guess it. It uses the continuity equation to create a separate equation for pressure computation instead of the pressure correction equation in SIMPLE. In the SIMPLEC algorithm instead of dropping the velocity neighbour correction terms like in SIMPLE it is retained and instead an approximation $\sum a_{nb} u'_{nb} \approx u'_{i,J} \sum a_{nb}$ made.

1.1 Review of Previous Related Studies

Most researchers have used a certain pressure velocity schemes to derive an equation for pressure. Saeid et al, (1) studied fluid flow and natural convection heat transfer in a differentially heated square cavity with a fin attached to its cold wall. The governing equations were written in terms of the primitive variables and solved numerically using the finite volume method and the SIMPLER algorithm. Himsar et al, (2) carried out a numerical study on laminar natural convection heat transfer in a differentially heated air filled cavity with two insulated baffles attached to its horizontal walls. The SIMPLE algorithm was used to couple the velocity, pressure and temperature fields. Other researchers have compared the performance of various pressure velocity schemes, Zeng et al, (3) carried out a comparative study on the convergence characteristics of the SIMPLE, SIMPLER, SIMPLEC and the SIMPLEX at a varying grid system for natural convection in a square enclosure. The SIMPLEC algorithm was then recommended especially for computation at fine meshes. However no study has compared the performance of SIMPLE, SIMPLER and SIMPLEC pressure-velocity schemes for laminar buoyancy driven flow. In this study, the SIMPLE, SIMPLER and SIMPLEC schemes will be used with the aim of identifying the most appropriate pressure velocity scheme for laminar buoyancy flow.

2.0 Governing Equations

The non- dimensional equations for incompressible, steady, buoyancy driven flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \Pr\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

(2)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \Pr\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + Ra\Pr\theta$$

(3)

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right)$$

(4)

2.1 Discretised Equations

2.1.1 Continuity equation

Integrating the continuity equation over the control volume

$$\int_s^n \int_w^e \frac{\partial u}{\partial x} dxdy + \int_w^e \int_s^n \frac{\partial u}{\partial y} dydx$$

(5)

Gives

$$\left[(A_e u_e - A_w u_w) + (A_n v_n - A_s v_s) \right] = 0$$

(6)

2.1.2 Momentum equation in x direction

Integrating the momentum equation the x direction over the control volume

$$\int_s^n \int_w^e u \frac{\partial u}{\partial x} dxdy + \int_w^e \int_s^n v \frac{\partial u}{\partial y} dydx = \int_s^n \int_w^e \frac{\partial p}{\partial x} dxdy + \Pr \int_s^n \int_w^e \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) dxdy + \Pr \int_w^e \int_s^n \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) dydx$$

(7)

Results to,

$$a_P u_{i,J} = a_W u_{i-1,J} + a_E u_{i+1,J} + a_N u_{i,J+1} + a_S u_{i,J-1} + (P_{I-1,J} - P_{I,J}) A_{i,J}$$

(8)

In standard form

$$a_P u_{i,J} = \sum a_{nb} u_{nb} + (P_{I-1,J} - P_{I,J}) A_{i,J} \tag{9}$$

2.1.3 Momentum equation in y direction

Integrating the y momentum equation over the control volume

$$\int_s^n \int_w^e u \frac{\partial v}{\partial x} dxdy + \int_w^e \int_s^n v \frac{\partial v}{\partial y} dydx = -\int_w^e \int_s^n \frac{\partial p}{\partial y} dydx + \Pr \int_s^n \int_w^e \frac{\partial^2 v}{\partial x^2} dxdy + \Pr \int_w^e \int_s^n \frac{\partial^2 v}{\partial y^2} dydx$$
$$+ Ra \Pr \int_w^e \int_s^n \theta dydx \tag{10}$$

Results in

$$a_P v_{I,j} = \sum a_{nb} v_{nb} + b_{I,j} + (P_{I,J-1} - P_{I,J}) A_{I,j} \tag{11}$$

The discretised momentum equations can now be solved iteratively using the SIMPLE, SIMPLER and SIMPLEC pressure-velocity scheme.

2.1.4 Discretised energy equation

Integration of the terms in the energy equation

$$\int_{s}^{n}\int_{w}^{e} u\,\frac{\partial\theta}{\partial x}dxdy+\int_{w}^{e}\int_{s}^{n} v\,\frac{\partial\theta}{\partial y}dydx=\int_{s}^{n}\int_{w}^{e}\frac{\partial^{2}\theta}{\partial x^{2}}dxdy+\int_{w}^{e}\int_{s}^{n}\frac{\partial^{2}\theta}{\partial y^{2}}$$

(12)

Yields

$$a_{P}\theta_{I,J} = a_{E}\theta_{I+1,J} + a_{W}\theta_{I-1,J} + a_{N}\theta_{I,J+1} + a_{S}\theta_{I,J-1}$$

(13)

2.2. Numerical Methodology

2.2.1 SIMPLE algorithm

Using an estimated pressure and velocity field momentum equations is solved and the

Correction formulae applied which results in

$$a_{i,J}u'_{i,J} = \sum a_{nb}u'_{nb} + (P'_{I-1,J} - P'_{I,J})A_{i,J} + b_{i,J}$$

(14)

$$a_{I,j}v'_{I,j} = \sum a_{nb}v'_{nb} + \left(P'_{I,J-1} - P'_{I,J}\right)A_{I,j} + b_{I,j}$$

(15)

Omitting the summation and further Substitution of the equations into the discretised continuity equation gives

$$\begin{aligned}&(A)_{i+1,J}\left(u^{*}_{i+1,J} + d_{i+1,J}\left(P'_{I,J} - P'_{I+1,J}\right)\right) - (A)_{i,J}\left(u^{*}_{i,J} + d_{i,J}\left(P'_{I-1,J} - P'_{I,J}\right)\right)\\&+(A)_{I,j+1}\left(v^{*}_{I,j+1} + d_{I,j+1}\left(P'_{I,J} - P'_{I,J+1}\right)\right) - (A)_{I,j}\left(v^{*}_{I,j} + d_{I,j}\left(P'_{I,J-1} - P'_{I,J}\right)\right)\end{aligned}$$

(16)

Which is now expressed as

$$a_{I,J}P'_{I,J} = a_{I+1,J}P'_{I+1,J} + a_{I-1,J}P'_{I-1,J} + a_{I,J+1}P'_{I,J+1} + a_{I,J-1}P'_{I,J-1} + b'_{I,J}$$

(17)

The equation above is solved to obtain the pressure correction field at all points. The correct pressure field at all points P is calculated by adding P′ to P*. Velocities are calculated from their starred values using the velocity-correction equations. The corrected pressure P obtained is treated as a new guessed pressure P* and the procedure repeated until a converged solution is obtained. The discretization equation for other dependent variables such as temperature are then solved.

2.2.2 SIMPLER algorithm

The steps implemented in the SIMPLER algorithm are similar to those of the SIMPLE except that

i) The pressure is not guessed but is instead obtained by substitution of pseudal velocities of the discretised momentum equation into the discretised continuity equation which results to

$$A_{i+1,J}\left(\hat{u}_{i+1,J}+d_{i+1,J}(P_{I,J}-P_{I+1,J})\right)-A_{i,J}\left(\hat{u}_{i,J}+d_{i,J}(P_{I-1,J}-P_{I,J})\right)$$
$$+A_{I,j+1}(\hat{v}_{I,j+1}+d_{I,j+1}(P_{I,J}-P_{I,J+1}))-A_{I,j}\left(\hat{v}_{I.j}+d_{I,j}(P_{I,J-1}-P_{I,J})\right)$$

(18)

Which is now rearranged to give the discretised pressure equation below

$$a_{I,J}P_{I,J}=a_{I+1,J}P_{I+1,J}+a_{I-1,J}P_{I-1,J}+a_{I,J+1}P_{I,J+1}+a_{I,J-I}P_{I,J-1}+b_{I,J}$$

(19)

ii) The summation in the discretised momentum equation is not omitted.

2.2.3 SIMPLEC algorithm

The steps implemented in the SIMPLEC are similar to those of the SIMPLE except that the summation term $\sum a_{nb}u'_{nb}$ and $\sum a_{nb}v'_{nb}$ are not omitted but retained by instead making an approximation that,

$$\sum a_{nb}u'_{nb}\approx u'_{i,J}\sum a_{nb}$$

(20)

$$\sum a_{nb}v'_{nb}\approx v'_{I,j}\sum a_{nb}$$

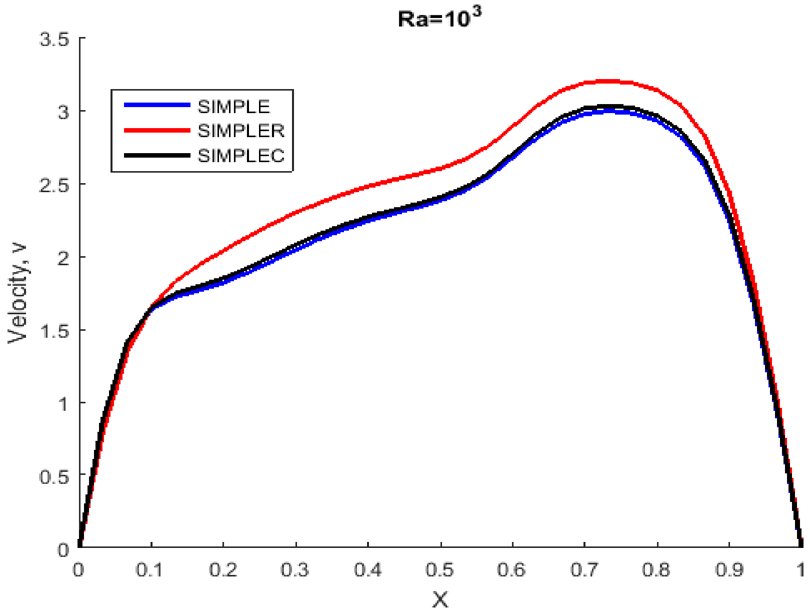
(21)

3. **Results and Discussions**.

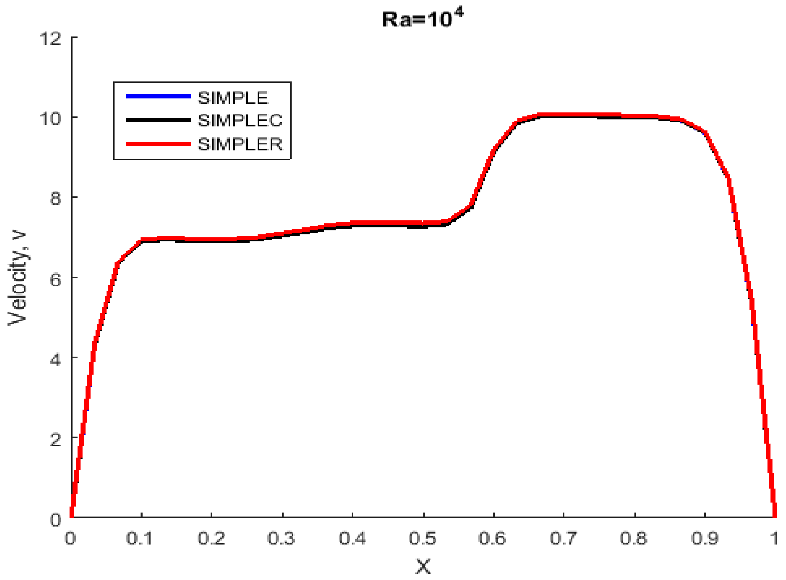
Results on the effect of varying the pressure velocity schemes and the Rayleigh number on velocity have been presented by use of velocity profiles.

3.1 Effect of varying the Rayleigh number and pressure- velocity schemes on velocity profiles

The results of the primary velocity were negligible because buoyancy which is reason for flow of air in the cavity is assumed to be in the y-momentum equation. Fig 3.1 shows secondary velocities obtained for different Rayleigh numbers for the SIMPLE, SIMPLER and SIMPLEC algorithms.



Ra=10^3
SIMPLE
SIMPLER
SIMPLEC
Velocity, v
X



Ra=10^4
SIMPLE
SIMPLEC
SIMPLER
Velocity, v
X

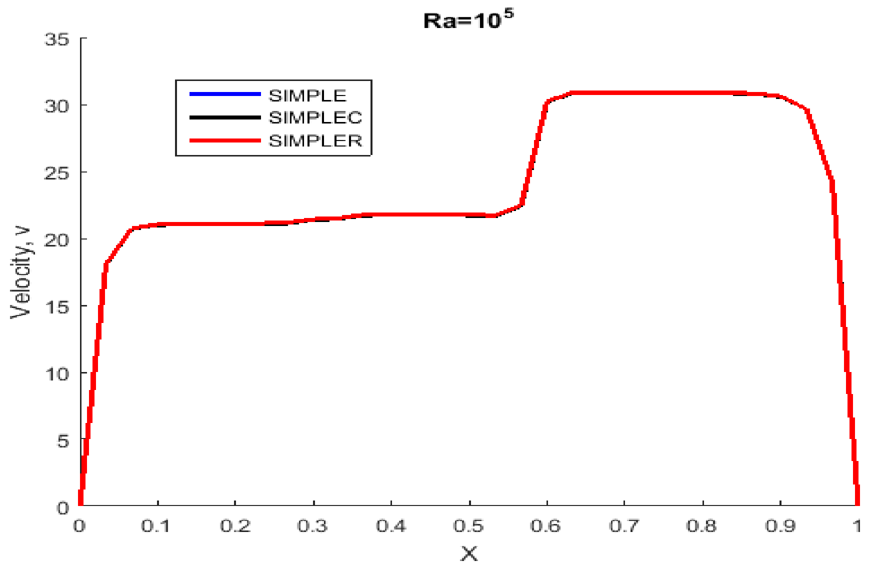
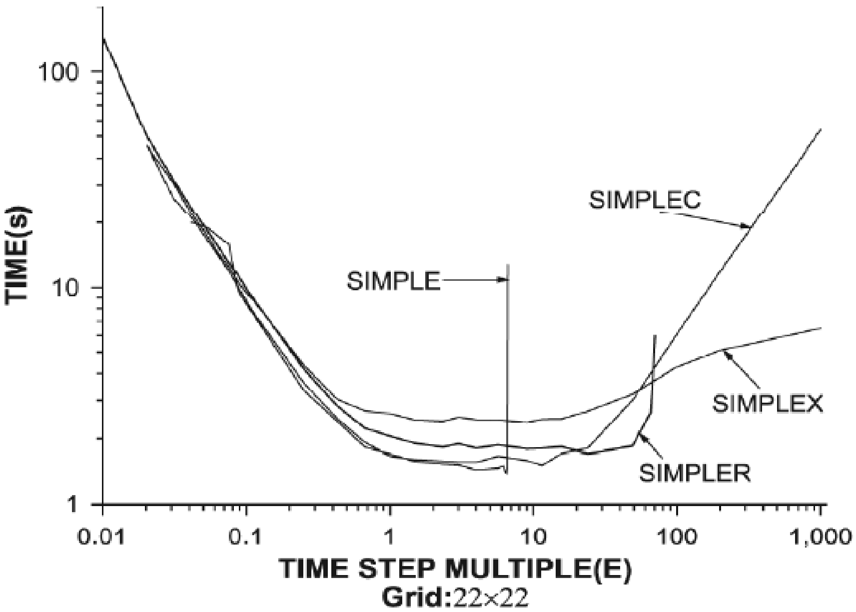


Figure 1; Secondary velocities at varying Rayleigh numbers for the SIMPLE, SIMPLER and SIMPLEC algorithm

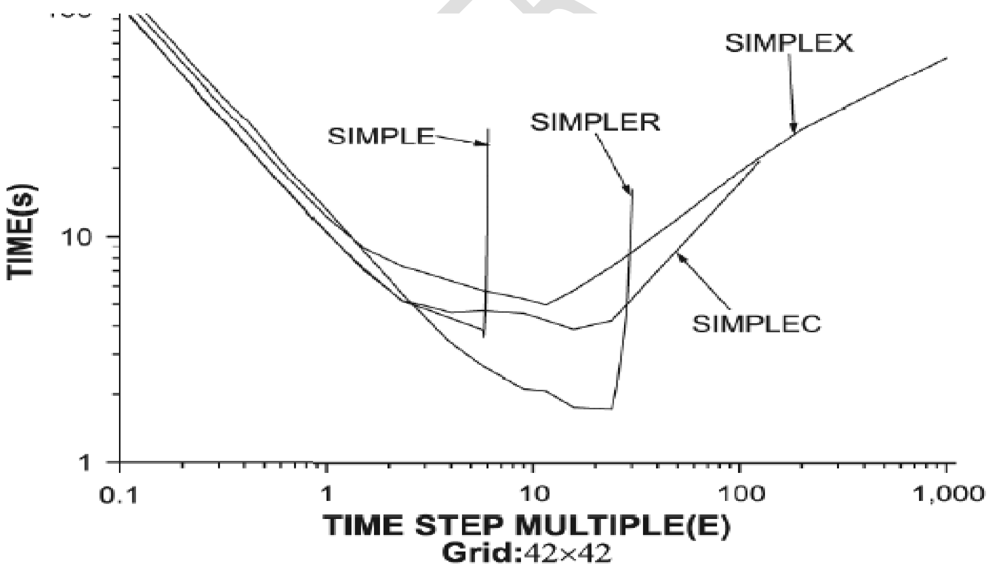
At a Rayleigh number of 10^3 the SIMPLER behaves as the SIMPLE and SIMPLEC in the baffle free region but experiences a degradation in convergence at the baffle region hence the reason higher values of velocity within the baffle region as compared to that of the SIMPLE and SIMPLEC. At higher Rayleigh numbers of 10^4 and 10^5 the secondary velocities for the SIMPLE, SIMPLER and SIMPLEC converge to the same path.

3.2 Comparison with the benchmark results

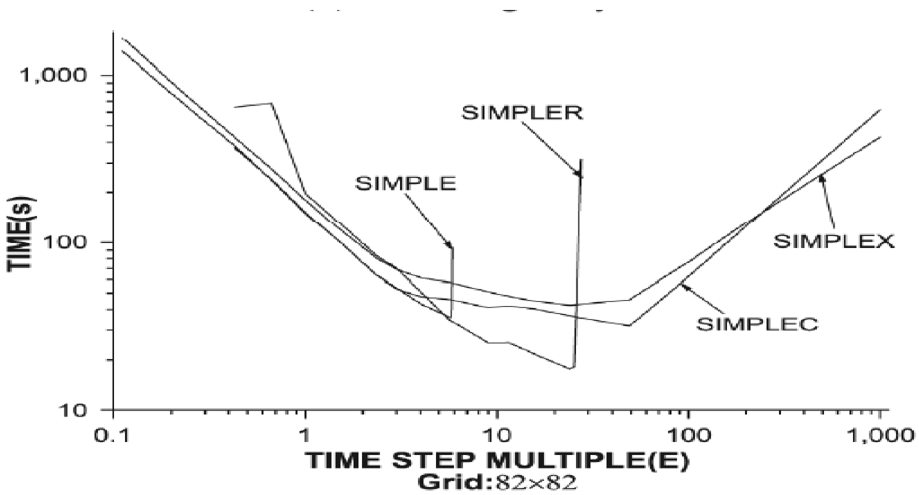
M. Zeng (2002), carried out a study on the convergence characteristics of the SIMPLE, SIMPLER, SIMPLEX and SIMPLEC algorithm for natural convection in a square enclosure at varying grid sizes. The figure below shows the SIMPLE, SIMPLER, SIMPLEX and SIMPLEC algorithm profiles at varying grid sizes for a Rayleigh number of 10^4



(a) 22 × 22 grid system



(b) 42 × 42 grid system



(c) 82 × 82 grid system

Figure 2; Robustness comparison of the SIMPLE, SIMPLER, SIMPLEX and SIMPLEC algorithm at a Rayleigh number of 10^4 for natural convection in a square differentially heated cavity.

SIMPLE and SIMPLER algorithm is not able to compute the velocity effectively for both course and fine grid. SIMPLEC algorithm on the other hand is able to effectively compute velocity both at course and fine grids.

4. Conclusion

At a low Rayleigh numbers of 10^3 the SIMPLE and SIMPLER behaves as the SIMPLEC algorithm in a coarse grid but experiences a degradation in convergence with grade refinement. SIMPLEC is thus more effectively able to compute velocity for laminar buoyancy flow in a differentially heated enclosure with baffles at both the baffle and the baffle free regions.

5. REFERENCES

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