Dark matter in the planetary system

Abstract *In this analysis, we use the data from the eight planets in our planetary system to show that it is stress-free, and then by radial (R) extension show that is consistent with an inner spherical Universe of radius,* R_o *=1.2×10*^{16} m, *where* R_o *is the radius at which the azimuthal orbital velocity(U) attains the velocity of light. The orbital data are fitted by the extended Newtonian gravitational model,* $U^2 = G\,M_o/R(1 + (R/R_o)^3\ ) + c^2\,(R/R_o)^2$, *where G is the universal gravitational constant,* M_o *is the mass of the Sun, and the second term arises from the mass of dark matter* $(M = 1/3\,\rho_2\,{R_o}^3)$ *where* $\rho_2 = 3\,c^2/(R_o^2 G)$. *This model applied to the complete inner Universe predicts that the ratio of the mass of ordinary matter to the mass of dark matter,* $1/(1+3\pi/2)$, *is 17.5 %, in close agreement with the Central Density data [1]. The nature of dark matter is also investigated, guided by the planetary data, in which the ratio of ordinary matter to dark matter for Earth (and also Mercury and Jupiter) is close to one, whereas for the outer planets it is much less than unity. This has led us to believe that the key property of dark matter is the presence of an organising principle, which is not present in ordinary matter. The ratio of dark matter to ordinary matter in the complete inner Universe attests to a creative universe. We also show that the inner Universe is bounded by a macro-constant stress layer in which galaxies are created in partnership with the micro-constant layer in which photons are created*

Keywords: Universe, the extended law of Newtonian gravitation, the creative nature of dark matter

1. INTRODUCTION

Dark matter is a controversial topic, which is extensively referenced in [1]. This paper, however, offers a new approach to its understanding. We show that it has a gravitational origin, as was suggested in [2], where it was shown that the planetary system (Table 1) may have originated from a stress-free continuum system, before it evolved into a discrete system in which mass and stress had separated. We suggest that the stress-free continuum system is composed of dark matter

In [2] we focussed on the discrete system which consists of ordinary matter which is subject to stress. Here we focus on the stress-free continuum system and contrast its properties with the discrete system. Gravitational theory enables this aim to be addressed.

2. THE GRAVITATIONAL MODEL

The basic relation in Newton's gravitational model is

$$U^2 = G\,(M_o + M_1)/R \qquad (1)$$

where in a cylindrical co-ordinate system, (R,ϕ) in which R is the radial co-ordinate and ϕ is the azimuthal co-ordinate, and M_o is the mass of the primary body, and $M_1(R)$ is the external mass, and G is the universal gravitational constant. $U = 2\,\pi\,R/T$ in which T is the orbital period is the azimuthal velocity of the spinning disk (M_1). Let us suppose that M_1 consists of two components,

$$M_1 = M_P + M \qquad (2)$$

where M_P is due to ordinary matter, i.e. planets, and M is due to dark matter, which share the common property of mass.

In order to solve (1), we require to evaluate both M_P and . M_P is well known from observations, which indicate that in our planetary sytem $M_P << M_o$ where M_o is the mass of the Sun, i.e. $M_o = 2\times10^{30}$ kg, and the mass of the largest planet, Jupiter, $M_P = 1.9\times10^{27}$ kg, and hence $M_P/M_o \approx 10^{-3}$

2.1 The dark matter mass

An important question is how significant is the dark matter mass, , compared with the planetary mass M_P. Each planet exists in an annulus between its neighbouring planets, of radial extent, ½ $(R_2 + R)$ – ½ $(R_1 + R) =$ ½ $(R_2 - R_1)$ where R_1 and R_2 are respectively the orbital radii of the interior and exterior planets. Thus, in order to compare the ordinary matter mass (M_P) with the dark matter mass (M), we need to know the dark matter mass in the annulus of width ½ $(R_2 - R_1)$. The dark matter mass relation is,

$$dM = m\,dR \qquad (3)$$

where the mass density, $m \equiv m(R) = \rho\, 2\pi\, R\, W$ in which W is the thickness of the spinning disk, and ρ is the azimuthal mean density. Hence the mass of the dark matter in the planetary annulus is,

$$M_D = \tfrac{1}{2}\, m(R_2 - R_1) \qquad (4)$$

The mass of the planetary body which has formed in the annulus [2], is,

$$M_P = \tfrac{1}{2}\, \pi^2\, \rho\, D^2\, R \qquad (5)$$

where $D = W$ is its diameter. Hence, on eliminating ρ using the mass density relation, $M_P = ¼\, m\, D$, and the ratio,

$$M_D/M_P = 2(R_2 - R_1)/\pi D \qquad (6)$$

Table 2 shows the ratio of the dark matter to the ordinary matter (M_D/M_P) for the planetary system. It is clear that the mass of the dark matter is much greater than the mass of the ordinary matter for each planet, that is throughout the planetary system. Of particular interest is that the ratio is very similar for Venus, Earth and Jupiter at about 5000, and is larger for the outer planets and for Mars. This suggests that this region of the planetary system may have been a conversion zone for planetary formation, see Section 2.4. At all events, dark matter mass convincingly dominates ordinary matter mass throughout the planetary system.

2.2 Dark matter dynamics: The transverse radius

On neglecting the ordinary planetary masses (M_P), (1) reduces to the relation,

$$U^2 = G\,(M_o + M)/R \qquad (7)$$

in which from (3), $M = \int_o^R m\, dR$, and for $M >> M_o$, we have $m = m_o$ where $m_o = c^2/G$ and c is the velocity of light. We have investigated the solution of (7) for the power law relation for $m = C\, R^b$ where C and b are constants. A particularly interesting relation in the tradition of Newton's original theory, is,

$$m_2 = \rho_2\, R^2 \qquad (8)$$

in which $C = \rho_2$ has the dimensions of density, and $b = 2$ is an integer. Eq. (8) will be used as a model for dark matter in the planetary system in the theoretical analysis of Sections 2.3 – 2.5.

Before commencing however we need to generalize the results in Section 3.3 of [2], which assumed that m is a constant, for $m = m(R)$. This yields after some algebra, the expression for the friction velocity,

$$u_* = \kappa / W \; [\, ½ \, R^{½} \, m \, d(W/m)/dR \,] \, [G(M_o + M)]^{½} \qquad (9)$$

which reduces to Eq. (9) in [2] for m = constant, and in the stress-free state yields

$$R^{½}/W \; m \; d \; (W/m) \; / \; dR \; = 0 \qquad (10)$$

The stress-free solution of which is,

$$W \; / \; m = \text{constant} \qquad (11)$$

and on substituting (11) in the expression for the mass density relation we obtain,

$$R\rho \; = \text{constant} \qquad (12)$$

as in [2]. Eq. (11) is an exceedingly significant relation, since for $m = m_o$ where $m_o = c^2 / G$, the thickness of the spinning disk,

$$W_o = m_o \, (W \; / \; m\,) \qquad (13)$$

which is a measure of the size of the Universe as determined from the orbital parameters (W and m) of the planetary system. Table 2 indicates that $W \; / \; m$ is almost a constant over the planetary system consistent with a stress-free origin, except for Mercury and Mars, for which we attribute the departures of W/m from the mean value as being due to post- formation interplanetary processes. The mean value of $W \; / \; m$ excluding these two planets is 1.92×10^{-11} m^2 kg^{-1}. Hence for m_o = 1.35×10^{27} kg m^{-1} (c = 2.998×10^8 ms^{-1} and G = 6.673×10^{-11} m^3 kg^{-1} s^{-2}), W_o = 2.59×10^{16} m, and the transverse radius of the (inner) Universe, ½ W_o = 1.3×10^{16} m.

2.3 Dark matter dynamics: The longitudinal radius

We will now consider the longitudinal properties as represented by our planetary system. On assuming the b = 2 model for the mass density (m), and substituting (8) in (7), we obtain,

$$U^2 = G\,(M_o + 1/3\,\rho_2\,R^3)/R \qquad (14)$$

which as R → R_o, which is the radial extent of the planetary system, yields,

$$c^2 = G\,(M_o + 1/3\,\rho_2\,{R_o}^3)/R_o \qquad (15)$$

On substituting (15) in (14), we obtain,

$$U^2 = G\,M_o/R(1 + (R/R_o)^3) + c^2\,(R/R_o)^2 \qquad (16)$$

Eq. (16) is the gravitational expression for the azimuthal velocity of the dark matter, which shows that U tends to the Newtonian azimuthal velocity as $R/R_o \rightarrow 0$, and to the velocity of light as $R/R_o \rightarrow 1$. .

Further, since in the planetary system,

$$M_o << 1/3\,\rho_2\,{R_o}^3 \qquad (17)$$

on substituting (8) in (15), we obtain,

$$c^2 = 1/3\,G\,m_2\,(R_o/R)^2 \qquad (18)$$

which on substituting $m_o = c^2/G$, yields,

$${R_o}^2 = 3\,m_o/m_2\,R^2 \qquad (19)$$

Eq. (19) is an expression for the longitudinal radius (R_o) of the planetary system. The results for $R_o = (3\,m_o/m_2)^{1/2}R$ for the planets in our planetary system are shown in Table 2, which indicates that, $R/\sqrt{m}$ is almost constant over the planetary system in agreement with the prediction of the $b = 2$ model, except for Mars, and for Saturn, Uranus and Neptune. The reason for the departures of Saturn, Uranus and Neptune is given in Section 2.4.

The mean value of $R/\sqrt{m}$, excluding these four planets, is 183 kg$^{-1/2}$ m$^{-3/2}$ Hence for m_o = 1.35×10^{27} kg m^{-1} , the mean value for the longitudinal radius of the planetary system, R_o = 1.2×10^{16} m, which is almost identical with the transverse radius ($W_o/2$) of 1.3×10^{16} m. . This result indicates that in terms of the dark matter, the inner Universe ($0 < R < R_o$) as measured by our planetary system is spherical. Thus the archetypical model for the planet is the inner Universe itself, see Section 2.4

2.4 Properties of the inner Universe

The basic property of the dark matter Universe which emerges from Sections 2.2 and 2.3 is that it is stress-free and isotropic. We now explore some further properties of the inner Universe including that of the relation between ordinary and dark matter. In all instances, the theoretical expressions allow quantitative results to be obtained.

From (15) , since in the planetary system, $M_o/R_o \ll c^2/G$, on substituting $m_o = c^2/G$, we obtain,

$$\rho_2 = 3\, m_o \,/\, {R_o}^2 \qquad (20)$$

Hence for m_o = 1.35×10^{27} kg m^{-1} , and R_o = 1.25×10^{16} m, which is the mean value of the longitudinal and transverse values from the planetary system data, we obtain, ρ_2 = 2.6×10^{-5} kg m^{-3} , and on evaluating $M_D = \int_o^{R_o} m\, dR$ using (8), we obtain $M_D = 1/3\, \rho_2\, {R_o}^3$ which from (20) yields,

$$M_D = m_o R_o \qquad (21)$$

or inversely, $R_o = G M_D / c^2$. Hence M_D = 1.7×10^{43} kg is the mass of the dark matter contained within the radial disk (0 < $R < R_o$) which may be called the inner Universe. The mean density of the inner Universe, $\rho_D = \rho_2 / 4\pi$, is 2.1×10^{-6} kg m^{-3}. In comparison, the mass of the ordinary matter in the observable planetary system is 3×10^{27} kg [1].

Eq. (16) indicates that the azimuthal velocity, U(R), has a minimum, (dU^2/dR = 0). This occurs at $R = R_c$ where,

$$R_c = (1/2\, G\, M_o \,/\, [\, c^2 / R_o{}^2 + G\, M_o \,/\, R_o{}^3]\,)^{1/3} \qquad (22)$$

which in the planetary system where $M_o / R_o << c^2 / G$ reduces to,

$$R_c = (M_o R_o{}^2 / 2\, m_o)^{1/3} \qquad (23)$$

The orbital period (T) from (16) is,

$$T = 2\,\pi \,/\, [G\, M_o\, (1/R^3 - 1/R_o{}^3) + (c/R_o)^2\,]^{1/2} \qquad (24)$$

which reduces to the Newtonian formula, $T = 2\,\pi\, R_c{}^{3/2} \,/\, (G\, M_o)^{1/2}$ for $R_o \rightarrow \infty$, and for $R/R_o \rightarrow 1$, the period tends to,

$$T_o = 2\pi\, R_o \,/\, c \qquad (25)$$

On evaluating (25) for R_o = 1.25×10^{16} m, we find that T_o = 8.30 yrs.

R_c is an iconic radius for the planetary system. In analogy with the point on a stream at a bend where it locally slows and a part of its sedimentary load is expelled and deposited, we suggest that, this is the radius at which dark matter may be converted into ordinary matter. On evaluating R_c , we obtain R_c = 4.9×10^{11} m, which lies between Mars and Jupiter, and is the radius of transition between the terrestrial and the major planets (Table 1). There is ‘fossilized’ evidence for this process in Table 2, which shows that the ratio of the planetary mass to the dark matter mass has a maximum over the three planets, Venus, Earth and Jupiter.

The evidence for dark matter in the planetary system can now be examined in more detail, by considering the ratio of dark matter to ordinary matter. For each planet, the mass density, $m = 2\pi\, R\, W\, \rho$ where $\rho = D/3\pi R\, \rho_P$ in which ρ_P is the planetary density, and the mass density due to dark matter, $m_2 = \rho_2\, R^2$, where the mass density is the rate of increase in mass with orbital radius. These quantities and their ratio, $m\, /\, m_2$ are shown in Tables 1 and 2.

We immediately see that the outer planets are very inefficient users of dark matter, which is due to the physical limits on ρ_P which are attainable, however the ratios for Mercury, Jupiter and Earth are close to unity. Earth, in particular, has an almost perfect intake of dark matter into ordinary matter. Our two sister planets display an interesting pattern in which Venus has a more than expected intake from the dark matter ($m/m_2 > 1$), and Mars has a much less intake than expected ($m/m_2 << 1$). Hence, in terms of the density, for ordinary matter, $m = 2/3\, D^2\, \rho_P$ and for dark mater, $m_2 = 2/3\, D^2\, \rho_D$ where ρ_D is the density of dark matter, from which the ratios of the mass densities and of the densities in the planets are equal, which is shown in Table 2, where $\rho_D = 3/2\, \rho_2\, (R/D)^2$ round-off errors excepted. The requirement for the complete assimilation of dark matter into ordinary matter, is a planetary density, $\rho_P = \rho_D$, which is just met by Earth, and by Jupiter, for which $\rho_P = 1250$ kg m^{-3} and $\rho_D = 1160$ kg m^{-3} (Tables 1 and 2).

The orbital period (T) is also of interest. Eq. (24) shows that it increases with radius (Table 1). In the Earth orbit, the dark matter only travels a little faster than the planet.

At the radius of Neptune, however, dark matter travels much faster than its accompanying planet, and T is 8.29 yr, which is almost equal to T_o (8.30 yr). Thus essentially, the orbital periods at radii greater than the extent of the planets are constant, and a solid body dark matter universe occurs. Here, in the annulus, $dR,\ dM = \rho_2 R^2\, dR$, and in terms of the dark matter density, ρ_D , $dM = 4\pi\, R^2\, \rho_D$, and hence $\rho_D = \rho_2/4\pi$, as also found above.

2.5 Properties of the outer Universe

We will call the region beyond $R = R_o$, the outer Universe. Here it would be expected that for $R \geq R_o$, the mass density would gradually reduce, and as a working relation, we assume the exponential decay,

$$m = m_o\, exp\, (- \lambda\, (R - R_o)\,) \qquad (26)$$

where λ^{-1} is a decay length. On substituting (26) in (3), we obtain,

$$dM/dR = m_o\, exp\, (\, -\lambda\, (R - R_o\,)) \qquad (27)$$

which, on integration yields,

$$M(R_1) = m_o/\, \lambda\, \ (1 - exp\, (- \lambda(R_1 - R_o\,)) \qquad (28)$$

where $R_1 >> R_o$, is the radius of the outer Universe, and the ratio of the mass in the outer Universe to its radius.

$$M(R_1)/R_1 = m_o\, (1 - exp\, (\, - \lambda\, R_1))\, /\, (\lambda R_1) \qquad (29)$$

Observations [3] indicate that $M(R_1) = 1.5\times10^{53}$ kg and $R_1 = 4.4\times10^{26}$ m from which $\lambda\, R_1 = 3.92$, $m_o = 0.27\times10^{27}$ kg m^{-1} , and $\theta = R_1 / R_o$ is 3.5×10^{10}. An important result follows from evaluating the density (ρ_U) of matter in the annulus (R_1, R_o), due to the presence of the inner Universe, which is,

$$\rho_U \;=\; M(R_1)/\,4\pi\,{R_1}^3 \qquad (30)$$

We find that ρ_U = 4.2×10^{-28} kg m^{-3}, whereas the density of dark matter, ρ_D = 2.1×10^{-6} kg m^{-3} and thus ρ_U << ρ_D. In other words the presence of the inner Universe hardly raises a ripple on the fabric of dark matter in which it sits. Some further comments are made in Section 4.

An analogy to this model of the Universe is the fried egg, in which the well-defined central yolk is the inner Universe and the surrounding white albumen, which varies from cooking to cooking, is the outer Universe.

3. THE ORIGIN OF GALAXIES

In [4] we introduced a basic model for a black hole in which at the juncture between the stress-free Newton dynamics of the outer region (the firmament) and the stress-free dynamics of the inner region (the black hole) there exists a constant stress layer ($\tau_{R\phi}$ = constant) in which turbulent velocity fluctuations occur. In this micro-environment, the speed of these fluctuations may attain the velocity of light as evidenced by the bright ring occurring around the black hole [4]. In effect there is a conversion from dark matter to ordinary matter by the production of a photon

The same juncture between the Newtonian and the Einsteinian dynamics, occurs on a macro-scale, at the radius (R_o). Here a constant stress layer occurs between the inner and the outer Universe. Thus a constant stress layer is a boundary for the inner Universe of ordinary matter at both the micro-scale limit inside of which photons are formed, and the macro-scale limit outside of which galaxies are formed. Both are essentially particles which arise from instabilities in the constant stress layer.

The analysis for the micro-scale constant stress layer is given in Section 5.4 of [4]. The steps in the macro-scale analysis are similar, and in both cases we assume that the constant stress layer exists in the annulus $(R_o\ ,\ )$ where $R\ > R_o$, i.e. it impinges on the ordinary matter in the micro-scale analysis and it impinges on the dark matter in the macro-scale analysis.

Following closely the analysis of Section 5.4 of [4] which is based on the Prandtl model for a turbulent boundary layer, we obtain the azimuthal stress due to the discontinuity between the Newtonian and the Einsteinian dynamics,

$$\tau_{R\phi} = \kappa^2\,\rho\,U^2 \qquad (31)$$

where the azimuthal velocity,

$$U\ =\ ½\,c,\ f = 1 \qquad (32)$$

which is applied in the constant stress layer. With the assumption that the mean speed of the turbulent particles is fc, where $f\ <\ 1$, we obtain, as for Eq, (12) in [2], that,

$$1/8\ (\kappa/f)^2\ (\theta\ +\ 1)^2\ =\ (\theta - 1)^2 \qquad (33)$$

where $=\ 0.4$ [6], $\theta\ = R\ /\ R_o$. The solution of (33) is,

$$\theta\ =\ (1\ +\ \kappa\ /\ f\sqrt{8})\ /\ (1\ -\ \kappa\ /\ f\sqrt{8}) \qquad (34)$$

in which for, $\theta\ \rightarrow\ \infty$, i.e. the annulus in which the constant stress occurs extends out to infinity, $f\ =\ \kappa/\sqrt{8}\ =\ 0.14$. This limit is very closely matched by the data in Section 2.5 for which $\theta = 3.5\times10^{10}$ The inference is that turbulent particles, i.e. galaxies, with a speed *greater* than 14% of the speed of light, populate both the inner and the outer Universe.

Even those particles travelling into the outer Universe at speeds in the range of 14 to 100 % of the speed of light, are observable from Earth, as is well known. This distinguishes the outer Universe from the black hole in which the particles, i.e. photons, entering at the speed of light are undetectable, whereas the galaxies can be tracked.

4. SOME GENERAL CONCLUSIONS

Some general conclusions can be drawn by considering the ratio of the mass of ordinary matter in the planets to the mass of dark matter over the full extent of the inner Universe.

For the complete inner Universe, $R = R_o$ and $= 2R_o$, and the mean annular density of the ordinary matter, $\rho_P = 2\ M_P/(\pi^2\ D^2\ R_o\ )$ where $M_P\ =\ 4/3\ \pi\ {R_o}^3\ (\rho_D\ \text{-}\rho_P)$, in which ρ_D is the density of dark matter, and since for a sphere the mean annular density is identical with the mean density, ρ_P is the mean density of ordinary matter. Hence,

$$\rho_P = \ 2/3\pi\ (\rho_D\ -\ \rho_P\ ) \qquad (35)$$

Eq. (35) is a geometrical relation which predicts that the ratio of the mass of ordinary matter to the mass of dark matter,

$$\rho_P\ /\ \rho_D\ =\ 2/3\pi\ /\ (1\ +\ 2/3\pi) \qquad (36)$$

is 17.5 %. This fundamental result is in substantial agreement with observational estimates based on critical density [3] which indicate that the ratio of ordinary matter to dark matter in the Universe is 18% The only other solution of (35) is ρ_D = ρ_P = 0.

The results of this paper show how the basic properties of the Universe may be derived when the concept of stress is embraced in the analysis. and dark matter is included.

It is apparent that a steady-state Universe of which we are a part, can be countenanced, on the assumption of the universal quantities of the velocity of light and the gravitational constant, and also that the inner Universe is energized by the constant stress layers which exist on the micro-scale and the macro-scale, and respectively provide lights and galaxies for our enjoyment. In this important sense, von Karman's constant is also a universal quantity [4].

In the analysis, an essential element is dark matter, which we show in Section 2.4 to have a mass approximately five times that of ordinary matter. The deeper question is, what is dark matter? Essentially, ordinary matter and dark matter are NOT two forms of matter. The remarkable fact is that on Earth, our personal environment, the densities of ordinary matter and dark matter are approximately equal (Table 2).

This is a vital clue to the nature of dark matter. All complex systems exhibit a randomness, for example, climate in which the random walks always bring surprises, see for example [7]. This is the principle behind dark matter as the creative agent. The ratio of ordinary matter to dark matter in the planetary system (Table 2) is a measure of the bounded random walks which are occurring. The higher the ratio, the greater is the creativity until the ratio achieves unity. Earth provides an insight into how this process works. The density of dark matter in the inner Universe, $\rho_D = 2.1\times10^{-6}$ kg m^{-3} (Section 2.4), and meteorological data from the atmosphere surrounding Earth show that this density is attained at an altitude of about 95 km [8]. Hence, the dark matter can be regarded as seeking a home in a body of greater density through the gaseous envelope with which it is surrounded. Beyond this envelope, the dark matter has no home and although present, its existence is undetected. Observations only detect the properties of the ordinary matter [1]. This scenario recalls the search for the origin of life on Earth as coming from space, but this paper puts it at a more fundamental level in terms of dark matter and ordinary matter.

Mercury, Earth and Jupiter, in their own ways have creativity at the centre of their beings. Mars in which the proportion of ordinary matter to dark matter is much less than unity has not achieved its creative potential, as is also abundantly clear for the outer planets in which the proportion of ordinary matter to dark matter is very small (Table 2). Venus however is an outlier in which the mass of ordinary matter has outgrown that of dark matter. This is reflected, possibly coincidentally, in a run-away greenhouse shrouded hostile environment, which is a stark warning for Earth people.

The outcome of the discussion in Section 2.4 is that the inner Universe is supported by a fabric of dark matter, which is interspersed by points of ordinary matter which occur on all scales and demonstrate its creative ability. There is however, according to the geometrical relation (35), an overall constant proportion between dark matter and ordinary matter as the creative process evolves from the 'big bang '

In practical terms, the conclusion in Section 2.6 is felicitous. The outer Universe is quintessentially the province of dark matter, our focus is directed decisively to the inner Universe of which our planetary system provides ample clues to its properties.

At all events, the basic outcome of this analysis is that dark matter and ordinary matter differ only in the presence or absence of an organizing principle. The ratios of ordinary matter to dark matter reflect the efficiency of the creative mechanism which is occurring at a given location.

REFERENCES

1. Dark Matter – Wikipedia. https://en.wikipedia.org/wiki/Dark_matter

2. Bye JAT The Adjunct Force of Gravity. Intl. Astron. and Astrophys. Res. J. 2021; 3(1): 1-7.

3. Observable Universe. – Wikipedia. https://en-wikipedia.org/wiki/Observable_universe

4. Bye JAT Stress: The forgotten Gravitational Force. Intl. Astron. and Astrophys. Res. J. 2021; 3(3): 27-32.

5. Baumert HZ. Primitive turbulence: kinetics, Prandtl's mixing length, and von Karman's constant. 2009;arXiv: 0907.0223v2.

6. Jones BW. The solar system. Pergamon Press. 1984;336

7. Bye JAT, Fraedrich K.,Kirk E.,Schubert S. and Zhu X. Random walk lengths of about 30 years in global climate. Geophys. Res. Letters. 2011; 38; L05806. doi:10.1029/2010GL046333.2011

8 NRLSISE Standard Atmosphere Model

List of Tables

Table 1. Orbital parameters for the Planetary System adopted from James [6]

M_P x 10^{23}	R x 10^{11}	ρ	D x 10^{6}	ρR x 10^{11}	m x 10^{17}	ρ_P	T

	kg	m	kg m^{-3}	m	kg m^{-1}	kg m^{-1}	kg m^{-3}	yr
Mercury	3.2	0.579	0.0485	4.88	0.0281	0.86	5260	0.24
Venus	48.7	1.082	0.0623	12.1	0.0674	5.12	5250	0.61
Earth	59.8	1.496	0.0498	12.78	0.0745	5.97	5470	0.99
Mars	6.44	2.28	0.0124	6.8	0.0282	1.2	3900	1.83
Jupiter	19000	7.783	0.0242	142.8	0.188	168.7	1245	6.80
Saturn	5690	14.27	0.00561	120	0.08	60.3	630	7.99
Uranus	876	28.69	0.00231	51.8	0.0662	21.5	1200	8.26
Neptune	1030	44.97	0.00192	49.2	0.0863	26.7	1650	8.29

Table 2. Important quantities for the Planetary System

	$M_D/M_P \times 10^5$	$W / m \times 10^{-11}$ $kg^{-1} m^2$	$R / \sqrt{m} \times 10^2$ $kg^{1/2} m^{3/2}$	$m_2 \times 10^{17}$ $kg\ m^{-1}$	m/m_2	ρ_D $kg\ m^{-3}$	ρ_P/ρ_D
Mercury	0.142	5.67	1.98	0.87	0.99	5460	0.96
Venus	0.048	2.36	1.51	3.04	1.68	3120	1.71
Earth	0.060	2.14	1.94	5.82	1.03	5340	1.02
Mars	0.589	5.67	6.58	13.5	0.089	44000	0.089
Jupiter	0.053	0.85	1.89	157.5	1.07	1160	1.08
Saturn	0.111	1.99	5.81	529	0.113	5500	0.115
Uranus	0.377	2.41	19.56	2140	0.010	$1.2\ 10^5$	0.010
Neptune	0.421*	1.84	27.52	5258	0.005	$3.3\ 10^5$	0.005

* $R_2 - R_1 = 2 (R - R_1)$