Using Kurtosis for Selecting One-Sample T-Test or Wilcoxon Signed-Rank Test

Methods Article

Abstract

Aims / objectives: To introduce a statistical test, which is a mixture of the one-sample t-test and Wilcoxon signed-rank test and depends on the sample kurtosis, using data from a symmetric univariate distribution with finite variance.
Study approach: Computer simulation.
Methodology: Data are generated with differing sample sizes from the Normal, Uniform, student-t with small degrees of freedom, and Laplace distributions. Coverage probabilities and power calculations are compared using the one-sample t-test, the Wilcoxon signed-rank test, and three proposed mixture tests which select the one-sample t-test or the Wilcoxon signed-rank test based on the sample kurtosis being significantly low or significantly high or either. Nonstandard values of α, the probability of a Type I error, are selected to account for the discrete nature of the Wilcoxon signed-rank test, allowing fair comparisons among the Wilcoxon signed-rank test, the t-test, and the three mixture tests.
Results: The false positive rate and power calculations are simulated for these nine distributions for both two-sided and one-sided tests, allowing comparisons among these five testing procedures.
Conclusion: When a small dataset is sampled from a symmetric distribution, then in comparison to the t-test, the Wilcoxon signed-rank test is equal in preference for the Normal distribution and is in fact more preferable for the non-Normal distributions tested herein. For small sample sizes, the mixture test based on high kurtosis is preferred over the t-test, but otherwise the t-test is preferred over all three mixture tests.

Keywords: Kurtosis, T-test, Wilcoxon signed-rank test, Normal distribution, Uniform distribution, T-distribution, Laplace distribution.

2020 Mathematics Subject Classification: 62F03, 62G05

1 Introduction

To determine the appropriate statistical test, researchers must first examine the shape and spread of their data. The most commonly utilized statistical tests require specific assumptions. When employing the Student's t-test, samples should meet certain criteria, such as Normality, equal variances and independence [Kim, 2015]. The Normality assumption can be relaxed if the sample size is large enough, according to the Central Limit Theorem. As sample size increases, the distribution of the sample mean will approximate Normality for finite variance [Kwak and Kim, 2017]. Therefore, the Student's t-test is often used in fields with abundant data and large sample sizes [Livingston, 2004].

In other instances, data might be less abundant and might not meet the assumptions discussed above. For the two-sample case, the Welch's t-test can be substituted for the Student's t-test when the two populations are unequal in variance [West, 2021]. Ruxton posits that the Welch's t-test is underutilized because researchers often mistakenly assume their datasets are of equal variances [Ruxton, 2006]. A previous study found that the Welch's t-test remedied effects of unequal variances, but not effects of non-Normality, which might be assessed using kurtosis [Zimmerman and Zumbo, 1993].

Kurtosis is a comparison between a dataset's peak and tails [DeCarlo, 1997]. Threshold values were determined for both low sample kurtosis (5th percentile based on Normal data) and high sample kurtosis (95th percentile based on Normal data) for sample sizes of 5, 10, 15, and 20, as shown in Table 1 [Garren and Osborne, 2021]. If a dataset meets the above criteria, the Wilcoxon signed-rank test might be used instead. Multiple studies have demonstrated that the Wilcoxon test is more powerful than the t-test when populations are not Normally distributed in some situations [Zimmerman and Zumbo, 1993] [Sawilowsky and Blair, 1992]. This is because the Wilcoxon test is less sensitive to outliers than the Student's t-test and the Welch's t-test [Woolson, 2008]. Zimmerman compared the Student's t-test and Wilcoxon test on non-Normally distributed data of unequal sample sizes. He concluded that the Wilcoxon test was more powerful in detecting differences between population means when the smaller sample had a smaller variance [Zimmerman, 1987].

Table 1: Threshold Values for Low and High Sample Kurtosis

n	5% threshold	95% threshold
5	1.278078	2.876782
10	1.563894	3.940937
15	1.721885	4.118671
20	1.830854	4.149351

However, the Wilcoxon test has limitations. Conover observed that the t-test is more powerful than competing tests, if samples were Normally distributed [Conover, 1971]. Another study found that the Wilcoxon test can give high false negative rates because the test is based on ranks, which are discrete [Xia, 2020]. De Winter found that a Type I error occurs at a high rate in datasets with unequal variances and unequal sample sizes for two-sample tests. In contrast, Type II error rates are smaller with small sample sizes if the effect size is large [de Winter, 2013]. Only one-sample tests are used in the simulations discussed herein.

Past researchers, like Zimmerman and Conover, have compared the t-test to its nonparametric equivalents [Sawilowsky and Blair, 1992; Zimmerman and Zumbo, 1993; Zimmerman, 1987; Conover, 1971]. However, a study is yet to explore kurtosis as a determining factor for selecting between the t-test and Wilcoxon test. This study herein will produce coverage probabilities from nine different distributions and four sample sizes based on kurtosis values calculated in the R-package `moments`, using one-sample testing procedures. This `moments` R-package contains various test functions, but only the kurtosis function is used for this study [Komsta and Novomestky, 2022].

The nine different distributions studied were Uniform, Normal, T with degrees of freedom equal to 5, 6, 7, 8, 9, and 10, and Laplace. The sample sizes tested were n=5, 10, 15, and 20 to assure violation of the assumptions needed for the Central Limit Theorem.

2 Method for Constructing Simulations

In some scenarios, especially involving paired studies, symmetry of a population is a reasonable assumption. In a paired-sample experiment, a drug and a placebo might have identical distributions under a null hypothesis, in which case the difference in response variables would have a symmetric distribution. For example, when testing for the lowering of cholesterol by a drug in comparison to a placebo, the difference between cholesterol levels within the same block (i.e., where one person in the pair uses the drug and the other person in the pair uses the placebo) is just as likely to be in any given range of numbers as in the same but negated range, under the null hypothesis that the drug and placebo are equivalent.

If univariate data are assumed to be from a symmetric distribution with finite variance, then the researcher may select whether to use a t-test on the mean or a Wilcoxon signed-rank test on the median, since the population mean and population median would be equivalent. If the data show signs of high or low kurtosis, then the researcher might choose to use the Wilcoxon signed-rank test, but otherwise might choose to use the t-test. When the decision to select the t-test or the Wilcoxon signed-rank test is made after observing the data, then the false positive rates and powers are affected. Herein, this decision is based on the sample kurtosis.

The Laplace distribution is also called the Double Exponential distribution and is symmetric with density

$$\exp\{|x-\mu|\ (\sqrt{2}\ )/\sigma\}\ /\ (\sigma\ \sqrt{2}\ ),\ \forall x \in \Re,$$

where μ and σ are the population mean and standard deviation, respectively. The symmetric distributions examined herein are the Normal, Uniform, Laplace, and t-distributions.

Pearson's population kurtosis is defined to be $\kappa_4\ /\ \sigma^4$, where κ_4 is the fourth central population moment and σ is the population standard deviation. The population kurtosis for a Normal distribution is 3; for Uniform is 1.8; for T_5 is 9; for T_6 is 6; for T_7 is 5; for T_8 is 4.5; for T_9 is 4.2; for T_{10} is 4; and for Laplace is 6. Although multiple definitions of sample kurtosis exist in the literature, the common definitions differ by merely a constant multiplicative function of the sample size. Therefore, the particular definition of sample kurtosis is irrelevant for our purposes. We used the `kurtosis` function from the R-package `moments`, which defines sample kurtosis to be $m_4\ /\ m_2^2$, where m_2 and m_4 are the second and fourth sample central moments, respectively.

For each of the Tables 2 through 5, a total of 200 million simulations were produced, requiring about twelve days of computing time. Tables 2 and 3 show the false positive rates, which are the probabilities of a Type I error, for a two-sided test and a one-sided test, respectively. Tables 4 and 5 show power calculations, where the null and alternative means differ by 0.3, for a two-sided test and a one-sided test, respectively. The false positive rates and power calculations are listed as percentages to make the tables easier to read. The sample sizes selected were 5, 10, 15, and 20. The distributions in the tables are listed in the order of Uniform, Normal, $T_{10}, \ldots, T_5$, and Laplace, so that kurtosis is roughly decreasing by that ordering, although the Laplace and T_6 distributions have the same kurtosis of 6. In Tables 2 and 3, the standard error for each false positive rate is approximately

$$\sqrt{0.05 * 0.95/200,000,000} \approx 0.0015\%\ .$$

In Tables 4 and 5, the standard error for each power is no more than

$$\sqrt{0.5 * 0.5/200,000,000} \approx 0.0035\%\ .$$

Hence, the probabilities tend to be precise to the digits shown in Tables 2, 3, 4, and 5.

For small sample sizes, setting the significance level α to be 0.05 does not lend itself to a fair comparison between the t-test and the Wilcoxon signed-rank test, since the Wilcoxon signed-rank test statistic is discrete due to the integer values of ranks. In fact, for a two-sided test and a sample size of $n = 5$, the smallest possible p-value is $1/2^4 = 0.0625$. Furthermore, for a one-sided test and a sample size of $n = 5$, the largest possible p-value no larger than 0.05 is only $1/2^5 = 0.03125$.

However, the continuity of the t-test allows the possibility of a p-value matching any value of α for any given number of significant digits. Therefore, the values of α used in the tables are based on the largest possible Wilcoxon p-value no larger than 0.05, with the exception of using $\alpha = 0.0625$ for a two-sided test with $n = 5$.

In addition to the one-sample t-test and the Wilcoxon signed-rank test, the false positive rates and powers are shown for three mixture tests using the results from Table 1. The *low kurtosis* test uses the Wilcoxon signed-rank test when sample kurtosis is too low, but otherwise uses the t-test. The *low or high kurtosis* test uses the Wilcoxon signed-rank test when sample kurtosis is either too low or too high, but otherwise uses the t-test. The *high kurtosis* test uses the Wilcoxon signed-rank test when sample kurtosis is too high, but otherwise uses the t-test.

The colors of red and green are used to make comparisons across rows in Tables 2 and 3. A false positive rate shown in red is the largest probability without exceeding α among the t-test and the three mixture tests; i.e., excluding the Wilcoxon signed-rank test. Hence, a false positive rate in red indicates the best test in terms of being conservative for that distribution and sample size, excluding comparisons to the Wilcoxon signed-rank test. A false positive rate shown in green is the smallest probability among the t-test and the three mixture tests in a row where all four probabilities exceed α. Hence, a false positive rate in green indicates the best test in terms of being the least anti-conservative for that distribution and sample size, again excluding comparisons to the Wilcoxon signed-rank test.

3 Results and Discussion

If a researcher is willing to employ nonstandard values of α, the probability of a Type I error, by selecting α to be a value equal to a possible p-value based on the discrete Wilcoxon test, then Tables 2 and 3 show that the false positive rate for the Wilcoxon signed-rank test actually achieves the value of α, for one-sided and two-sided tests for all nine distributions selected herein. Thus, the Wilcoxon signed-rank test is neither conservative nor anti-conservative for these nonstandard values of α. Therefore, the Wilcoxon signed-rank test is at least as good as the t-test and the three mixture tests for all nine distributions and all four sample sizes tested.

Tables 2 and 3 show that the t-test is anti-conservative for the Uniform distribution but is too conservative for the other non-Normal distributions for the smaller sample sizes. As an example for two-sided tests in Table 2, for the Uniform distribution and $n = 5$ the t-test has a false positive rate of 0.0778, which exceeds $\alpha = 0.0625$. However, for the Laplace distribution and $n = 5$ the t-test has a false positive rate of 0.0453, which is somewhat less than $\alpha = 0.0625$. Similar results hold for the one-sided t-test in Table 3.

However, many researchers who are willing to assume symmetry in the univariate distribution might be unfortunately unwilling to replace the t-test with the Wilcoxon signed-rank test for small sample sizes without examining the data first. Perhaps such researchers can be convinced to calculate the sample kurtosis prior to deciding whether to use the t-test or the Wilcoxon signed-rank test. For two-sided tests with sample sizes of 5 and 10, and also for one-sided tests with a sample size of 5, the high-kurtosis test is preferred over the t-test, except when the data are from a Normal distribution, based on the simulation results found in Tables 2 and 3. For all other scenarios, the t-test is preferred over the three mixture tests. Overall, however, the Wilcoxon signed-rank test is more preferred than the three mixture tests and is at least as preferable as the t-test, using the specific values of α shown in Tables 2 and 3.

Power calculations are shown in Tables 4 and 5, based on a shift in the location parameter of 0.3, for both the two-sided test and one-sided test, respectively. Comparisons of power within any given row reveal just modest differences in power in some situations, but somewhat large differences in power in other situations. For example, in Table 4, for two-sided tests with $n = 5$ and $\alpha = 0.0625$, the Laplace distribution produces powers of 0.1138, 0.1417, 0.1163, 0.1276, and 0.1280, for the t-test, Wilcoxon signed-rank test, the *low kurtosis* test, the *low or high kurtosis* test, and the *high kurtosis* test, respectively. Furthermore, in Table 5, for one-sided tests with $n = 5$ and $\alpha = 0.03125$, the Uniform distribution produces powers of 0.1443, 0.1160, 0.1462, 0.1205, and 0.1398, for the t-test, Wilcoxon signed-rank test, the *low kurtosis* test, the *low or high kurtosis* test, and the *high kurtosis* test, respectively.

4 CONCLUSIONS

a Introduced in section 2 are nonstandard values of α, the probability of the Type I error, to allow fair comparisons of simulated false positive rates and powers between the one-sample t-test and the Wilcoxon signed-rank test, when the distribution can be assumed to be symmetric with finite variance. Three new tests are proposed, based on a mixture of the one-sample t-test and the Wilcoxon signed-rank test, depending on whether the sample kurtosis is too low, too high, or either one. Nine distributions are analyzed, including the Normal, the Uniform, and seven distributions with large population kurtosis values.

b In section 3, the Wilcoxon signed-rank test is shown to be at least as preferred to the one-sample t-test for these values of α and both two-sided tests and one-sided tests. For small sample sizes, the *high kurtosis* test is preferred over the t-test, but otherwise the t-test is preferred, for both two-sided tests and one-sided tests.

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Table 2: False Positive Rate for Two-Sided Test, as a Percentage

Distribution	n	α	T-test	Wilcoxon test	low kurtosis	low or high kurtosis	high kurtosis
Uniform	5	6.25	7.78	6.25	7.88	6.50	7.53
Normal	5	6.25	6.25	6.25	6.36	6.05	6.45
T_{10}	5	6.25	5.86	6.25	5.97	5.91	6.19
T_9	5	6.25	5.81	6.25	5.93	5.90	6.16
T_8	5	6.25	5.75	6.25	5.87	5.87	6.13
T_7	5	6.25	5.68	6.25	5.80	5.84	6.09
T_6	5	6.25	5.58	6.25	5.70	5.80	6.03
T_5	5	6.25	5.43	6.25	5.55	5.73	5.95
Laplace	5	6.25	4.53	6.25	4.71	5.49	5.30
Uniform	10	4.88	5.30	4.88	5.13	5.05	5.13
Normal	10	4.88	4.88	4.88	4.78	4.84	4.93
T_{10}	10	4.88	4.67	4.88	4.57	4.71	4.83
T_9	10	4.88	4.64	4.88	4.54	4.70	4.83
T_8	10	4.88	4.60	4.88	4.51	4.67	4.81
T_7	10	4.88	4.56	4.88	4.46	4.64	4.80
T_6	10	4.88	4.49	4.88	4.40	4.59	4.78
T_5	10	4.88	4.38	4.88	4.30	4.52	4.75
Laplace	10	4.88	4.07	4.88	3.98	4.35	4.61
Uniform	15	4.79	5.01	4.79	4.97	4.77	5.03
Normal	15	4.79	4.79	4.79	4.72	4.53	5.06
T_{10}	15	4.79	4.65	4.79	4.59	4.40	5.04
T_9	15	4.79	4.63	4.79	4.57	4.38	5.04
T_8	15	4.79	4.60	4.79	4.54	4.36	5.03
T_7	15	4.79	4.56	4.79	4.50	4.32	5.03
T_6	15	4.79	4.50	4.79	4.45	4.27	5.02
T_5	15	4.79	4.42	4.79	4.37	4.19	5.01
Laplace	15	4.79	4.28	4.79	4.22	4.04	5.02
Uniform	20	4.84	4.99	4.84	4.99	4.84	4.99
Normal	20	4.84	4.84	4.84	4.78	4.58	5.11
T_{10}	20	4.84	4.74	4.85	4.69	4.43	5.15
T_9	20	4.84	4.72	4.84	4.67	4.41	5.15
T_8	20	4.84	4.70	4.85	4.66	4.39	5.16
T_7	20	4.84	4.67	4.84	4.63	4.35	5.16
T_6	20	4.84	4.62	4.84	4.58	4.30	5.17
T_5	20	4.84	4.55	4.84	4.51	4.22	5.17
Laplace	20	4.84	4.47	4.84	4.45	4.06	5.25

Table 3: False Positive Rate for One-Sided Test, as a Percentage

Distribution	n	α	T-test	Wilcoxon test	low kurtosis	low or high kurtosis	high kurtosis
Uniform	5	3.13	3.89	3.12	3.94	3.25	3.77
Normal	5	3.13	3.13	3.13	3.18	3.03	3.23
T_{10}	5	3.13	2.93	3.12	2.99	2.95	3.10
T_9	5	3.13	2.91	3.13	2.96	2.95	3.08
T_8	5	3.13	2.88	3.12	2.94	2.94	3.07
T_7	5	3.13	2.84	3.12	2.90	2.92	3.04
T_6	5	3.13	2.79	3.13	2.85	2.90	3.02
T_5	5	3.13	2.71	3.13	2.78	2.87	2.97
Laplace	5	3.13	2.27	3.12	2.36	2.74	2.65
Uniform	10	4.20	4.31	4.20	4.25	4.17	4.34
Normal	10	4.20	4.20	4.20	4.14	4.03	4.37
T_{10}	10	4.20	4.11	4.20	4.05	3.96	4.35
T_9	10	4.20	4.09	4.20	4.04	3.95	4.34
T_8	10	4.20	4.08	4.20	4.02	3.94	4.34
T_7	10	4.20	4.05	4.20	4.00	3.92	4.34
T_6	10	4.20	4.02	4.20	3.96	3.89	4.33
T_5	10	4.20	3.96	4.20	3.91	3.84	4.32
Laplace	10	4.20	3.86	4.20	3.80	3.74	4.32
Uniform	15	4.73	4.75	4.73	4.77	4.72	4.76
Normal	15	4.73	4.73	4.73	4.68	4.51	4.95
T_{10}	15	4.73	4.69	4.73	4.65	4.41	5.00
T_9	15	4.73	4.68	4.73	4.64	4.40	5.01
T_8	15	4.73	4.67	4.73	4.63	4.38	5.01
T_7	15	4.73	4.65	4.73	4.62	4.36	5.02
T_6	15	4.73	4.63	4.73	4.60	4.33	5.03
T_5	15	4.73	4.59	4.73	4.56	4.28	5.04
Laplace	15	4.73	4.58	4.73	4.55	4.18	5.14
Uniform	20	4.87	4.87	4.86	4.92	4.86	4.87
Normal	20	4.87	4.87	4.86	4.82	4.66	5.07
T_{10}	20	4.87	4.84	4.87	4.81	4.56	5.16
T_9	20	4.87	4.84	4.86	4.80	4.54	5.16
T_8	20	4.87	4.83	4.87	4.80	4.52	5.17
T_7	20	4.87	4.82	4.86	4.79	4.50	5.19
T_6	20	4.87	4.80	4.86	4.78	4.47	5.20
T_5	20	4.87	4.78	4.87	4.75	4.42	5.22
Laplace	20	4.87	4.79	4.87	4.77	4.31	5.34

Table 4: Power for Two-Sided Test, as a Percentage, with Location Shift of 0.3

Distribution	n	α	T-test	Wilcoxon test	low kurtosis	low or high kurtosis	high kurtosis
Uniform	5	6.25	15.08	12.13	15.28	12.60	14.61
Normal	5	6.25	10.07	9.82	10.23	9.59	10.30
T_{10}	5	6.25	9.35	9.64	9.51	9.23	9.75
T_9	5	6.25	9.26	9.62	9.42	9.19	9.69
T_8	5	6.25	9.16	9.59	9.32	9.14	9.61
T_7	5	6.25	9.02	9.56	9.18	9.06	9.52
T_6	5	6.25	8.84	9.52	9.00	8.96	9.39
T_5	5	6.25	8.58	9.46	8.74	8.82	9.22
Laplace	5	6.25	11.38	14.17	11.63	12.76	12.80
Uniform	10	4.88	27.38	25.28	26.23	25.28	27.38
Normal	10	4.88	13.37	13.15	13.12	13.08	13.44
T_{10}	10	4.88	12.07	12.29	11.87	12.02	12.34
T_9	10	4.88	11.92	12.19	11.73	11.89	12.23
T_8	10	4.88	11.75	12.08	11.56	11.74	12.09
T_7	10	4.88	11.51	11.94	11.33	11.54	11.91
T_6	10	4.88	11.19	11.76	11.02	11.27	11.68
T_5	10	4.88	10.76	11.51	10.60	10.90	11.37
Laplace	10	4.88	15.93	18.18	15.81	16.91	17.20
Uniform	15	4.79	43.15	38.98	41.23	38.97	43.16
Normal	15	4.79	18.62	18.12	18.40	17.63	19.11
T_{10}	15	4.79	16.41	16.61	16.26	15.72	17.31
T_9	15	4.79	16.17	16.46	16.03	15.51	17.13
T_8	15	4.79	15.87	16.27	15.73	15.24	16.89
T_7	15	4.79	15.49	16.03	15.36	14.91	16.61
T_6	15	4.79	14.99	15.72	14.87	14.46	16.24
T_5	15	4.79	14.28	15.30	14.18	13.84	15.75
Laplace	15	4.79	21.59	25.48	21.53	22.08	24.98
Uniform	20	4.84	57.54	51.64	54.10	51.64	57.54
Normal	20	4.84	24.25	23.51	24.04	22.98	24.78
T_{10}	20	4.84	21.01	21.33	20.88	20.18	22.17
T_9	20	4.84	20.67	21.11	20.55	19.87	21.91
T_8	20	4.84	20.24	20.83	20.12	19.49	21.58
T_7	20	4.84	19.69	20.49	19.58	19.00	21.18
T_6	20	4.84	18.96	20.04	18.87	18.34	20.66
T_5	20	4.84	17.97	19.43	17.89	17.44	19.96
Laplace	20	4.84	27.26	32.94	27.23	28.06	32.14

Table 5: Power for One-Sided Test, as a Percentage, with Location Shift of 0.3

Distribution	n	α	T-test	Wilcoxon test	low kurtosis	low or high kurtosis	high kurtosis
Uniform	5	3.13	14.43	11.60	14.62	12.05	13.98
Normal	5	3.13	9.28	9.01	9.43	8.81	9.48
T_{10}	5	3.13	8.58	8.79	8.72	8.44	8.93
T_9	5	3.13	8.50	8.76	8.64	8.40	8.87
T_8	5	3.13	8.40	8.73	8.54	8.34	8.80
T_7	5	3.13	8.27	8.69	8.41	8.27	8.70
T_6	5	3.13	8.10	8.64	8.24	8.17	8.57
T_5	5	3.13	7.85	8.58	7.99	8.03	8.40
Laplace	5	3.13	11.09	13.79	11.33	12.42	12.46
Uniform	10	4.20	37.81	35.26	36.88	35.25	37.83
Normal	10	4.20	19.59	19.17	19.35	18.75	20.00
T_{10}	10	4.20	17.89	17.93	17.70	17.26	18.56
T_9	10	4.20	17.70	17.80	17.51	17.10	18.40
T_8	10	4.20	17.48	17.65	17.29	16.90	18.23
T_7	10	4.20	17.17	17.45	16.99	16.63	17.99
T_6	10	4.20	16.77	17.19	16.60	16.28	17.68
T_5	10	4.20	16.21	16.84	16.05	15.78	17.26
Laplace	10	4.20	23.14	25.84	23.03	23.48	25.50
Uniform	15	4.73	57.61	52.74	55.56	52.74	57.61
Normal	15	4.73	28.52	27.75	28.32	27.19	29.09
T_{10}	15	4.73	25.58	25.68	25.44	24.58	26.68
T_9	15	4.73	25.26	25.46	25.12	24.29	26.43
T_8	15	4.73	24.87	25.19	24.74	23.93	26.13
T_7	15	4.73	24.36	24.86	24.24	23.48	25.74
T_6	15	4.73	23.69	24.42	23.58	22.87	25.24
T_5	15	4.73	22.77	23.83	22.67	22.03	24.57
Laplace	15	4.73	32.04	36.64	32.00	32.57	36.11
Uniform	20	4.87	71.18	65.14	67.94	65.14	71.18
Normal	20	4.87	35.75	34.71	35.55	34.20	36.26
T_{10}	20	4.87	31.66	31.94	31.54	30.66	32.94
T_9	20	4.87	31.22	31.66	31.10	30.27	32.61
T_8	20	4.87	30.67	31.30	30.56	29.78	32.19
T_7	20	4.87	29.96	30.85	29.86	29.15	31.67
T_6	20	4.87	29.04	30.26	28.95	28.31	31.00
T_5	20	4.87	27.76	29.47	27.69	27.14	30.10
Laplace	20	4.87	38.88	45.38	38.86	40.00	44.26