*<u>**Original Research Article**</u>*

# Effects of the hydro anisotropy and the magnetic field on the dynamic thermo-bi-diffusive flow in a horizontal cavity confining a porous medium saturated by a binary fluid.

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**Abstract**

We analyze analytically the effects of anisotropy in permeability and that of a transverse magnetic field on thermal convection in a porous medium saturated with a binary fluid and confined in a horizontal cavity. The porous medium, of great extension, is subjected to various conditions at the thermal and solutal boundaries. The axes of the permeability tensor are oriented obliquely with respect to the gravitational field. Based on a scale analysis, the velocity, temperature, and heat and mass transfer rate fields were determined. These results were validated by the study of borderline cases which are: pure porous media and pure fluid media discussed in the literature. It emerges from this study that the anisotropy parameters influence the convective flow. The application of a transverse magnetic field significantly reduces the speed of the flow and thereby affects the temperature field and the rate of heat and mass transfer.

**Keywords:** Porous medium, convection, anisotropy, magnetic field.

**Nomenclatures**

**a, b, c** : Constants

$\boldsymbol{A}$: Aspect ratio of the cavity (L’/H’)

$\vec{\boldsymbol{B}}$: Vector magnetic field

$\vec{\boldsymbol{g}}$ : Vector gravity field

$\boldsymbol{H'}$ : Height of the cavity

$\boldsymbol{j'}$: Current density

$\overline{\overline{\boldsymbol{K}}}'$ : Second-order permeability tensor

$\boldsymbol{K_1}, \boldsymbol{K_2}$ : Flow permeability along the principal axes

$\boldsymbol{K}^*$ : Permeability ratio

$\boldsymbol{q'}$:constant heat flow(per unit of surface), w.m$^{-2}$

$\boldsymbol{L'}$ : Thickness of the cavity

$\boldsymbol{L_x}$ :length of the central region of the cavity

$\boldsymbol{Le}$: Lewis number, $\alpha_p$/D

$\boldsymbol{Sh}$ : Overall Nusselt number

$\boldsymbol{Nu}$ : Overall Sherwood number

$\boldsymbol{P'}$ : Pressure of the saturating vapor

$\boldsymbol{Ra}$ : Rayleigh number

$\boldsymbol{N}$: Ratio of volume force ,($\beta_c \triangle C^* / \beta_T \triangle T^*$)

$\boldsymbol{T'}$ : Dimensional fluid temperature

$\boldsymbol{C'}$ : Dimensional fluid concentration

$\boldsymbol{T}$ : Dimensionless fluid temperature

$C$ : Dimensionless fluid concentration

$\Delta \boldsymbol{T}^* = \mathrm{q'H'/kp}$ :Temperature difference scale

$\Delta \boldsymbol{C}^* = \mathrm{j'H'/Dp}$ : Temperature difference scale

$\boldsymbol{t}$ : Dimensionless time,($t' \alpha_p / H'^2$)

$\boldsymbol{u'}, \boldsymbol{v'}$ : Dimensional velocities in x and y directions

$\boldsymbol{u}, \boldsymbol{v}$ : Dimensionless velocities in $x$ and $y$ directions

$\vec{\boldsymbol{V}}'$ : Vector velocity of filtration of the fluid in porous medium

$\boldsymbol{x'}$ : Dimensional Cartesian coordinate measured along the vertical wall of the cavity

$\boldsymbol{y'}$ : Dimensional Cartesian coordinate measured along the bottom wall of the cavity

$\boldsymbol{x}, \boldsymbol{y}$ : Dimensionless Cartesian coordinates

$\boldsymbol{\beta_T}$ : Coefficient of thermal expansion

$\boldsymbol{\alpha_p}$ : thermal diffusivity of the medium porous saturated $k_p/(\rho C)_f$

$\boldsymbol{\varepsilon}$: Dimensionless porosity, $\varepsilon'/\sigma$

$\boldsymbol{\sigma}$: Heat capacity ratio $(\rho C)_p/(\rho C)_f$

$\boldsymbol{\beta_C}$ : solutal expansion coefficient, kg.mol$\cdot$L$^{-1}$

$\boldsymbol{\varrho}$ : Density of the fluid

$\boldsymbol{\Psi}$: dimensionless current function ($\Psi'/\alpha_p$ )

$(\boldsymbol{\varrho C_p})_{\boldsymbol{f}}$ : Heat capacity of the fluid

$\boldsymbol{\psi}$ : Dimensionless stream function

$\boldsymbol{\vartheta}$: kinematic viscosity of the fluid, m$^2$.s$^{-1}$

## 1. Introduction

The existence of a temperature and concentration gradient in a saturated porous

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medium causes the appearance of a natural convection flow with heat and mass transport. This is because the differences in temperature and concentration cause a non-uniform distribution of the density of the medium, which gives rise to the movement of the fluid under the effect of gravity. This phenomenon is called thermosolutal convection, the fundamental importance of which is no longer to be demonstrated given its practical implications and applications including the storage of radioactive waste and especially the transport and diffusion of pollutants through the soil.

***AMAHMID*** et ***al*** [5] studied analytically and numerically the natural convection in a porous layer of Brinkmann doubly diffusive in a confined anisotropic porous medium taking into account the particular situation where the thermal and solutal volume forces are opposite and of the same intensity. From their studies it appears that the increase in Da induces a decrease in the flow intensity and heat and mass transfers; likewise as RT increases, the intensity of the flow increases monotonously; however the numbers of Nusselt and Sherwood tend asymptotically towards the same value independent of the number of Lewis Le which decreases with Da. Based on the Darcy model and the Boussinesq approximation, ***ATTIA*** et ***al*** [6] studied the Soret and Dufour effects on thermosolutal convection in a porous medium of rectangular cavity saturated by a binary fluid. The horizontal walls of the cavity are subjected to uniform heat fluxes q "and species j", while the vertical walls are considered adiabatic and impermeable. As a result, the Soret and Dufour effects can dramatically alter the stability of convection, which in turn affects heat and mass transfer rates. Using the mathematical model of Darcy ***KALLA*** [7] studied thermosolutal convection within a porous cavity saturated with a binary fluid in a rectangular cavity tilted at an angle and subjected to heat flows heated from below. As a result, the convective flow induced by thermal forces is highly dependent on the thermal Rayleigh number $R_T$. ***BENISSAAD*** and ***OUAZAA*** [8] used Darcy's model to make the analytical and numerical study of natural bi-diffusive convection in a porous medium saturated with a binary fluid and confined in a rectangular cavity. The horizontal walls of the cavity are subjected to a uniform flow of heat while the vertical walls are considered adiabatic and impermeable. These authors have shown that the control parameters significantly influence flow, heat and mass transfer. Through Brinkmann's mathematical model, ***AKOWANOU*** and ***DEGAN*** [9] studied convective transfer in a rectangular cavity, filled with a porous medium saturated with an incompressible electrically conductive fluid and subjected to a transverse magnetic field. The side walls of the cavity are subjected to differential heating. It appears that the convective flow is greatly influenced by the anisotropy parameters in permeability of the porous layer and by the effect of the applied transverse magnetic field.

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Likewise, the rate of heat transfer in the porous medium increases when the permeability in the horizontal direction is higher than that prevailing in the vertical direction. ***ATTIA*** et ***al*** [10] studied the suppression of thermosolutal instabilities by the action of a magnetic field. The geometry of the considered problem is a rectangular enclosure filled with an aqueous solution. The problem situation is therefore governed by a system of Maxwell's equations and hydrodynamic equations. The horizontal walls are adiabatic, impermeable and non-conductive of electricity while the two vertical walls are maintained at two uniform and constant, but different temperatures. One of these walls is connected to an anode and the other wall to a cathode. As a result, the application of a magnetic field ($Ha \neq 0.0$) made it possible to suppress thermosolutal instabilities and to control heat and mass transfers. Likewise, it is observed that this increase in the magnetic field represented by the Hartmann number has the effect of reducing the amplitude of the oscillations, especially for Ha = 100 where these oscillations disappear completely and the flow becomes stable. ***OUAZAA*** [11] through Darcy's mathematical model studied thermosolutal convection in porous media confined in a rectangular cavity and saturated with a binary fluid. The horizontal walls of the cavity are subjected to uniform heat fluxes $q'$ and species $j'$ while the vertical walls are considered adiabatic and impermeable. It appears that the flow intensity and the mass and heat transfer rates are significantly influenced by the thermal Rayleigh number. ***BENISSAAD*** et ***al*** [12] studied analytically the thermosolutal natural convection in anisotropic porous medium confined in a rectangular enclosure with horizontal walls supposed to be impermeable and adiabatic using the mathematical model Darcy-Brinkman-Forcheimer. Constant and uniform temperature and concentration gradients are imposed on the vertical walls. From their study, it appears that the flow increases with an increase in the Darcy number which increases with the Rayleigh number. The tilt angle of the permeability axes greatly influences heat and mass transfer rates. Using the Darcy-Brinkmann model, ***HADIDI*** [13] studied two-dimensional thermosolutal convection in a porous cavity arranged vertically. The side walls are subjected to uniform temperature and concentration conditions while the horizontal walls are adiabatic and impermeable. The results of this investigation show that the variation in the permeability of the two layers has a very appreciable effect on the flow structure and the transfers. Through the mathematical formulation of Darcy-Brinkman-Forchheimer, ***SAFI*** [14] proceeded to the study of bi-diffusive convection in an anisotropic porous medium with for geometry used, an anisotropic porous medium saturated by a binary fluid, supposed incompressible, confined in a rectangular enclosure and arranged horizontally. The vertical walls of the cavity are subjected to constant temperatures and concentrations (Dirichlet-type boundary conditions), while the

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horizontal walls are kept impermeable and adiabatic. It was concluded that thermal anisotropy significantly affects heat and mass transfers as well as heat and mass transfers increase with Rayleigh number. ***BEN KHRIDLA*** and ***BENZID*** [15] made the analytical study of natural thermosolutal convection in a Fluid medium in a rectangular cavity of languor L ', height H' and depth W 'filled by a binary fluid. The horizontal walls of the cavity are subjected to constant flows of heat (q ') and mass (J'), on the other hand the vertical walls are considered adiabatic and impermeable, the phenomenon of thermosolutal natural convection is governed by mathematical equations deduced from the Navier-Stokes model. As a result, the Nusselt number is directly influenced by the intensity of the convection ($\Psi 0$) however the Sherwood number always takes higher values in the case of convection induced by the Soret effect only.

In this study, we will look at the heat and mass transfer induced by thermosolutal convection in an anisotropic horizontal porous cavity in permeability and under the effect of the transverse external magnetic field. The aim is to study the influence of the magnetic field and anisotropy on convective flow and the transfer of heat and mass.

## 2. Governing Equations

The physical model considered in Figure 1 is that of a rectangular enclosure with flat walls. The soil, interface between the different media, contained in the enclosure constitutes a porous medium saturated with a polluting water mixture (cadmium) similar to a binary fluid and seat of the heat transfer of heat and mass (particles of pollutants) to these hydro-systems by the phenomenon of convection. The porous medium is anisotropic in permeability, the directions of which are oriented obliquely with respect to the vertical axis at an angle $\boldsymbol{\theta}$. The porous medium-enclosure system is subjected to a uniform and transverse magnetic field. The porous matrix is also subjected to a constant heat flow such that the upper horizontal wall subjected to heat from the sun and the lower part to geothermal heat. From the onset of heating, the anisotropic porous medium becomes the site of the thermosolutal transfer phenomenon that we will study.

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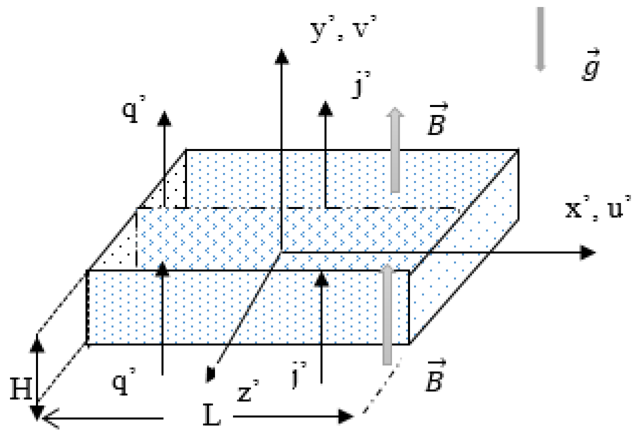


**Figure 1**: Physical model and coordinates system.

The regime considered here is the steady state with flow developed in the porous channels. The equations of continuity, motion, energy and concentration are written:

$$\nabla \vec{V}' = 0 \quad (1)$$

$$V' = \frac{\bar{\bar{K}}}{\mu}\left[\nabla \vec{P'} + \rho' \vec{g} + \vec{J} \wedge \vec{B}\right] \quad (2)$$

$$\nabla \vec{J'} = 0 \; ; \vec{J} = \gamma\left(-\nabla \emptyset + V'\vec{B}\right) \quad (3)$$

$$\left(\rho C_p\right)_m \frac{\partial T'}{\partial t} + \left(\rho C_p\right)_f \nabla.\left(\vec{V'}T'\right) = k\nabla^2 T' \quad (4)$$

$$\varepsilon \frac{\partial T'}{\partial t} + \nabla.\left(\vec{V'}C'\right) = k\nabla^2 (DC') \quad (5)$$

where $\bar{\bar{K}}$ represents the permeability tensor of the porous medium in the axis system shown in Figure (1). It is a tensor of order 2 which is written according to the coordinate axis system :

$$\bar{\bar{K}} = \begin{bmatrix} K_1 \cos\theta^2 + K_2 \sin\theta^2 & (K_2 - K_1)\sin\theta\cos\theta \\ (K_2 + K_1)\sin\theta\cos\theta & K_2\cos\theta^2 + K_1\sin\theta^2 \end{bmatrix} \quad (6)$$

The porous medium being electrically isolated, then the electric field is zero everywhere. On this basis we have :

$$\nabla \emptyset = 0 \quad (7)$$

Likewise

$$\vec{J'} = \gamma\left(\vec{V'} \wedge \vec{B}\right) \quad (8)$$

Using the coordinates $(u', v')$ of the filtration rate $V'$ defined in the plane $(Ox', Oy')$ illustrated in Figure 1, the equations (1), (2), (3), (4) and (5) describing the phenomenon of convection in an anisotropic porous medium in permeability, formulated as primitive variables are written:

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$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (9)$$

$$\left.\begin{aligned} au' - cv' &= \frac{K_1}{\mu}\left(-\frac{\partial P'}{\partial x'} - \gamma u' B^2\right) \\ -cv' + bv' &= \frac{K_1}{\mu}\left(-\frac{\partial P'}{\partial y'} - \rho \vec{g}\right) \end{aligned}\right\} \quad (10)$$

$$\sigma \frac{\partial T'}{\partial t'} + u' \frac{\partial T'}{\partial x'} + v' \frac{\partial T'}{\partial y'} = \alpha \left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) \quad (11)$$

$$\varepsilon \frac{\partial T'}{\partial t'} + u' \frac{\partial C'}{\partial x'} + v' \frac{\partial C'}{\partial y'} = D \left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2}\right) \quad (12)$$

With :

$$\left.\begin{aligned} a &= cos^2\theta + K^* sin^2\theta \\ b &= sin^2\theta + K^* cos^2\theta \\ c &= (1 - K^*) sin\theta cos\theta \\ K^* &= K_1/K_2 \,, k^* = k_{x'}/k_{y'} \end{aligned}\right\} \quad (13)$$

$$\sigma = \frac{(\rho C_p)_m}{(\rho C_p)_f} \ ; \ \rho = \rho_0[1 - \beta_T(T' - T_0) - \beta_C(C' - C_0)$$

The dimensionless variables not only have a simplifying advantage of the equations but also they allow a better physical interpretation of the phenomenon studied.

The normalization scale factors used for the quantities of interest are :

$$\left.\begin{aligned} (x, y) &= \frac{(x', y')}{H'} & C &= \frac{C' - C_0}{\Delta C'} \\ (u, v) &= \frac{(u', v')H'}{\alpha_f} & \Delta C' &= \frac{J' H'}{D} \\ \Psi &= \frac{\Psi'}{\alpha_f} & \Delta T' &= q' H'/k \\ & & T &= \frac{T' - T_0}{\Delta T'} \end{aligned}\right\} \quad (15)$$

By introducing these dimensionless quantities into the conservation equations for mass (9), motion (10), energy (11) and concentration (12), we obtain respectively:

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (16)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \quad (18)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{1}{Le}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \quad (19)$$

The feeding of the equation system reveals the control parameters of the problem : $\mathbf{Ha}=\mathbf{B}(K_1\gamma/\mu)^{1/2}$is the Hartmann number, $Ra_H=K_1 g\beta H\Delta T/(\alpha\mu)$is the Rayleigh number of the cavity based on the height , $N=\beta_C\Delta C'/\beta_T\Delta T'$ is the ratio of the thermal and solutal volume forces , $Le=\alpha_f/D$is the Lewis number which represents the ratio of the thermal diffusivity to the mass diffusivity of the saturated porous medium and the constants $\alpha$ ,$\beta$ , $\gamma$

Based on the parallel flow approximation and assuming that $u=\frac{\partial\Psi}{\partial y}$and$v=-\frac{\partial\Psi}{\partial y}$

$$(a+Ha^2)\frac{\partial^2\Psi}{\partial y^2}+b\frac{\partial^2\Psi}{\partial x^2}+2c\frac{\partial^2\Psi}{\partial x\partial y}=-Ra_H\left(\frac{\partial T}{\partial x}+N\frac{\partial C}{\partial y}\right) \quad (20)$$

$$\nabla^2 T=\frac{\partial\Psi}{\partial y}*\frac{\partial T}{\partial x}-\frac{\partial\Psi}{\partial x}*\frac{\partial T}{\partial y} \quad (21)$$

$$\nabla^2 C=Le\left(\frac{\partial\Psi}{\partial y}*\frac{\partial C}{\partial x}-\frac{\partial\Psi}{\partial x}*\frac{\partial C}{\partial y}\right) \quad (22)$$

**a°) Hydrodynamic boundary conditions**

$$\left.\begin{matrix} *\ on\ vertical\ walls \\ x=\mp\frac{L}{2},\quad u=0,\qquad \Psi=0 \\ *\ on\ horizontal\ walls \\ x=\mp\frac{H}{2},\quad v=0,\qquad \Psi=0 \end{matrix}\right\} \quad (23)$$

**b°) Thermal conditions**

$$\left.\begin{matrix} *\ on\ vertical\ walls \\ x=\mp\frac{L}{2},\quad u=0,\qquad \Psi=0 \\ *\ on\ horizontal\ walls \\ x=\mp\frac{H}{2},\quad v=0,\qquad \Psi=0 \end{matrix}\right\} \quad (24)$$

**c°) The thermal and mass boundary conditions**

When fluxes are applied to vertical and horizontal walls,

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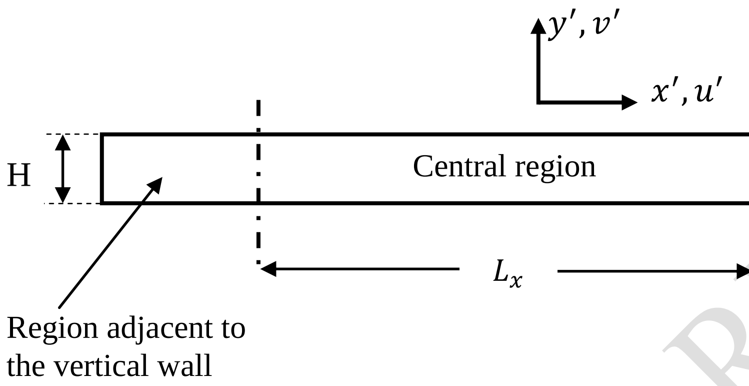
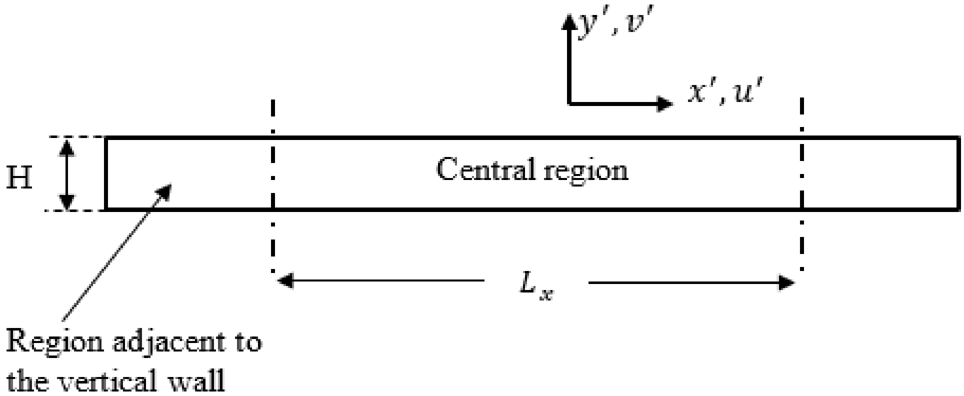
$$\left.\begin{array}{c} \boldsymbol{for\ x = \mp \frac{A}{2}\ , \qquad \frac{\partial T}{\partial x} = \frac{\partial C}{\partial x} = 0} \\ \boldsymbol{for\ y = \mp \frac{A}{2}\ , \qquad \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = -1} \\ \boldsymbol{With\ A = \frac{H}{L}\ represents\ the\ aspect\ ratio\ of\ the\ cavity} \end{array}\right\} \quad (25)$$

**3. Solution by perturbation method**

In this part, let's take a look at the central region of the cavity :




$L_x$ et $H$ are the characteristic scales of the variables x and y in the central region of the cavity :

$$y \sim H\ ;\ x \sim L_x$$

According to the analysis on the basis of equation (21) we can write

$$\frac{\boldsymbol{u}}{\boldsymbol{v}} \sim \frac{L_x}{\boldsymbol{H}} \text{ so } \frac{\boldsymbol{u}}{L_x} \sim \frac{\boldsymbol{v}}{H} \text{ and since A} = \frac{\boldsymbol{H}}{\boldsymbol{L}} \ll 1 \text{ and } \mathrm{L} > L_x \ \ so\ \frac{\boldsymbol{L}}{H} \gg \frac{L_x}{H} \text{ where } \frac{\boldsymbol{u}}{\boldsymbol{v}} \gg 1 \quad (26)$$

From the above, we infer that the flow in the central region of the cavity is developed in the horizontal Ox region as has been discussed in detail in the past by Cormack et al. [12], **Vasseur et al.** [3]. So we have : u = u(y) ; v = 0 ; T = T(y), C= C(y) et $\Psi = \Psi$ (y)

So we have

**3.1. Expression of current function, flow velocity, and temperature and concentration profiles**

$$(a + Ha^2)\frac{d^2\Psi}{dy^2} = -Ra_H\left(\frac{dT}{dx} + N\frac{dC}{dx}\right) \quad (27)$$

$$\frac{d\Psi}{dy} * \frac{dT}{dx} = \frac{d^2T}{dy^2} \quad (28)$$

$$\frac{d\Psi}{dy} * \frac{dC}{dx} = \frac{1}{Le}\frac{d^2C}{dy^2} \quad (29)$$

The temperature and concentration profiles are defined as follows

$$T(x,y) = C_T x + \theta_T(y) \quad (30)$$

and

$$C(x,y) = C_S x + \theta_S(y) \quad (31)$$

where $C_T$ and $C_s$ are coefficients expressing the temperature and concentration gradients; $\theta_T$ and $\theta_s$ are dimensionless temperatures and concentrations. So we have

$$(a + Ha^2)\frac{d^2\Psi}{dy^2} = -Ra_H(C_T + NC_S) \quad (32)$$

$$\frac{d^2\theta_S}{dy^2} = LeC_S\frac{d\Psi}{dy} \quad (34)$$

By referring to the hydrodynamic boundary conditions we obtain the expression of the current function : :

$$\Psi = \Psi_0(1 - 4y^2) \quad (35)$$

with

$$\Psi_0 = \frac{Ra_H}{8}\frac{C_T + NC_S}{(a + Ha^2)} \quad (36)$$

In addition, the flow velocity of the following fluid is written therein :

$$u = -8\Psi_0 y \quad (37)$$

As for the temperature and concentration profiles, we have:

$$T(x,y) = C_T x + \frac{C_{T\Psi_0}}{3}(3y - 4y^3) - y \quad (38)$$

And

$$C(x,y) = C_S x + \frac{LeC_{S\Psi_0}}{3}(3y - 4y^3) - y \quad (39)$$

Temperature and concentration gradients $C_T$ et $C_S$ write

$$C_T = \frac{4\beta\Psi_0}{3(2\beta + \Psi_0^2)} \; and \; C_S = \frac{4\beta Le\Psi_0}{3(2\beta + Le^2\Psi_0^2)} \text{ with } \beta = \frac{15}{16} \quad (40)$$

By combining the equation of the current function at the center of gravity with the terms of the temperature and concentration gradients we have

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$$\boldsymbol{\Psi_0} = \frac{Ra_H}{8(a + Ha^2)}\left[\frac{4\beta\Psi_0}{3\left(2\beta + {\Psi_0}^2\right)} + N\frac{4\beta Le\Psi_0}{3\left(2\beta + Le^2{\Psi_0}^2\right)}\right] \tag{41}$$

From (40) we get a fifth order equation in terms of the current function $\Psi_0$,

$$\Psi_0(Le^4{\Psi_0}^4 - 2\beta Le^2{\Psi_0}^2 P_1 - \beta^2 P_2 = 0 \tag{42}$$

$$\left.\begin{array}{l} P_1 = \dfrac{1}{(a + Ha^{2)}}\left[\dfrac{Ra_H}{12}Le(N + Le) - (a + Ha^2)(1 + Le^2)\right] \\ P_2 = \dfrac{1}{(a + Ha^{2)}}\left[\dfrac{Ra_H}{3}Le^2(1 + NLe) - \ 4Le^2(a + Ha^2)\right] \\ \dfrac{6(a + Ha^2)}{Le^2} \neq 0 \end{array}\right\} \tag{43}$$

Equation (42) indicates a possibility of five solutions, one of which is zero, the zero solution corresponds to the state of the fluid at rest

$$\Psi_0 = 0 \tag{44}$$

Les quatre autres solutions sont les solutions convectives données par :

$$\Psi_0 = \ \mp\frac{\sqrt{\beta}}{Le}\left[P_1 \ \mp \ \sqrt{P_1^2 + P_2}\right]^{\frac{1}{2}} \tag{45}$$

The signs ($\pm$) on the outside of the hook indicate the counterclockwise or clockwise direction of the convective flow while the signs ($\pm$) inside the hook indicate the two possible convective solutions of our flow. As shown by Mamou [14] the sign (+) in our solution indicates that the flow is stable and the sign (-) corresponds to an unstable solution

### 3.2.LES TAUX DE TRANSFERT DE CHALEUR ET DE MASSE

The thermal and mass transfers are expressed respectively by the number of Nusselt and Sherwood

**- Nusselt number**

$$N_u = \frac{1}{\Delta T}\text{with} \quad \Delta T = T\left(0, -\frac{1}{2}\right) - T\left(0 - \frac{1}{2}\right)T_0 \quad x = 0 \tag{46}$$

From where

$$N_u = \frac{6(\Psi_0^2 + 2\beta\ )}{\Psi_0^2 + 12\beta} \tag{47}$$

- **Sherwood number**

$$Sh = \frac{1}{\Delta C}\text{with} \quad \Delta C = C\left(0, -\frac{1}{2}\right) - C\left(0 - \frac{1}{2}\right)T_0 \quad x = 0 \tag{48}$$

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From where

$$\mathbf{Sh} = \frac{\mathbf{6(\ \Psi_0^2 + 2\beta\ )}}{\mathbf{\Psi_0^2\ Le^2 + 12\beta}} \tag{49}$$

### *4. ANALYSIS AND DISCUSSION OF TH RESULTS*

To describe the phenomenon of the transfer and diffusion of contaminants through the soil, we have considered the soil as a porous and anisotropic medium in permeability. Since liquid is a mixture of liquid and

pollutants, the water-soluble cadmium around the flow is why it takes the name of binary fluid. The whole binary fluid and porous medium is not only subjected to a constant heat flow between solar radiation and geothermal heat but also under the effect of the earth's magnetic field.

The steady-state behavior of the heat and mass transfer generated by the phenomenon of the diffusion of contaminants in the presence of the anisotropy parameters and of the magnetic field is analyzed by solving analytically and then numerically the system of nonlinear ordinary differential equations thanks to the "MATHALAB version 15.0" calculation software.

Figure 2.1 shows the behavior of the binary fluid in flow theme under different values of Ha.

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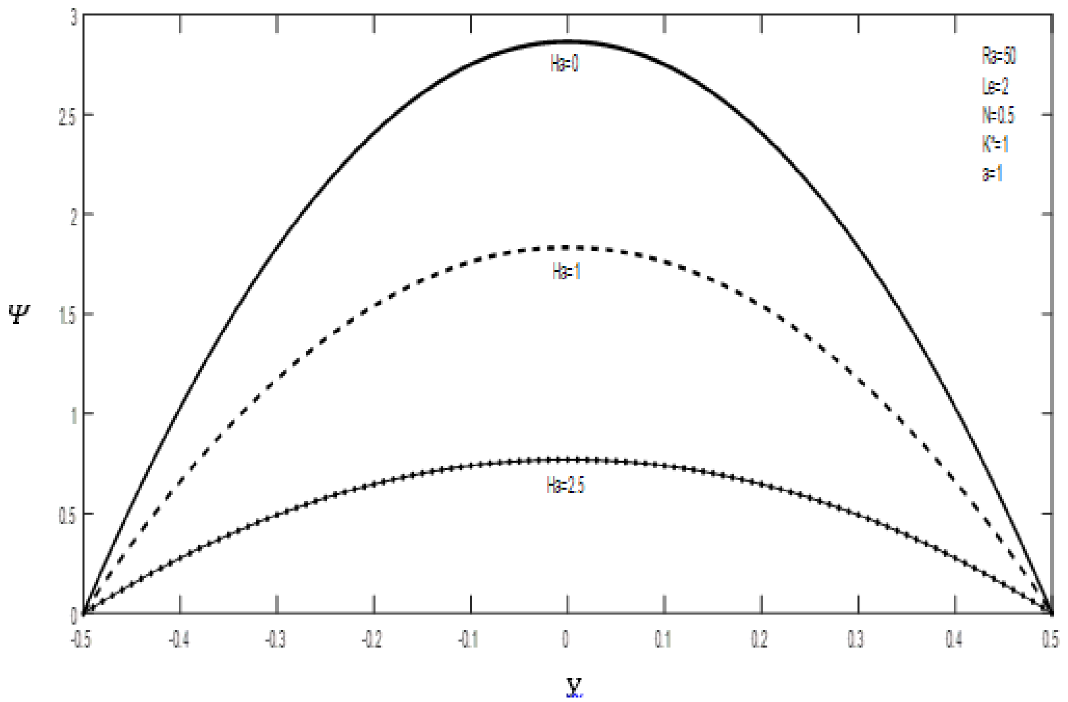


**Figure.2.1**. Variation of the current function (Ψ) as a function of y for different value of Ha

The various current function profiles observed show a gradual attenuation of the convective flow as the application of the magnetic field becomes significant. This proves that the magnetic field considerably reduces the intensity of the flow and gradually until it disappears. Moreover considering the same values (Ra = 50; Le = 2; N = 0.5; $\theta$ = 45° and Ha = 0) Mamou [14] found Ψ0 = 2.837; Kalla [6] found Ψ0 = 2.847; Ouazaa [5] obtained Ψ0 = 2.865 and for our present study we obtained Ψ0 = 2.8648. Thus we notice a satisfactory agreement with just small differences.

Figure 2.2 shows the influence of the anisotropy ratio on the flow intensity. We find that the more the anisotropy ratio increases, the less the flow intensity decreases. We can then conclude that the permeability anisotropy significantly affects the flow of the fluid.

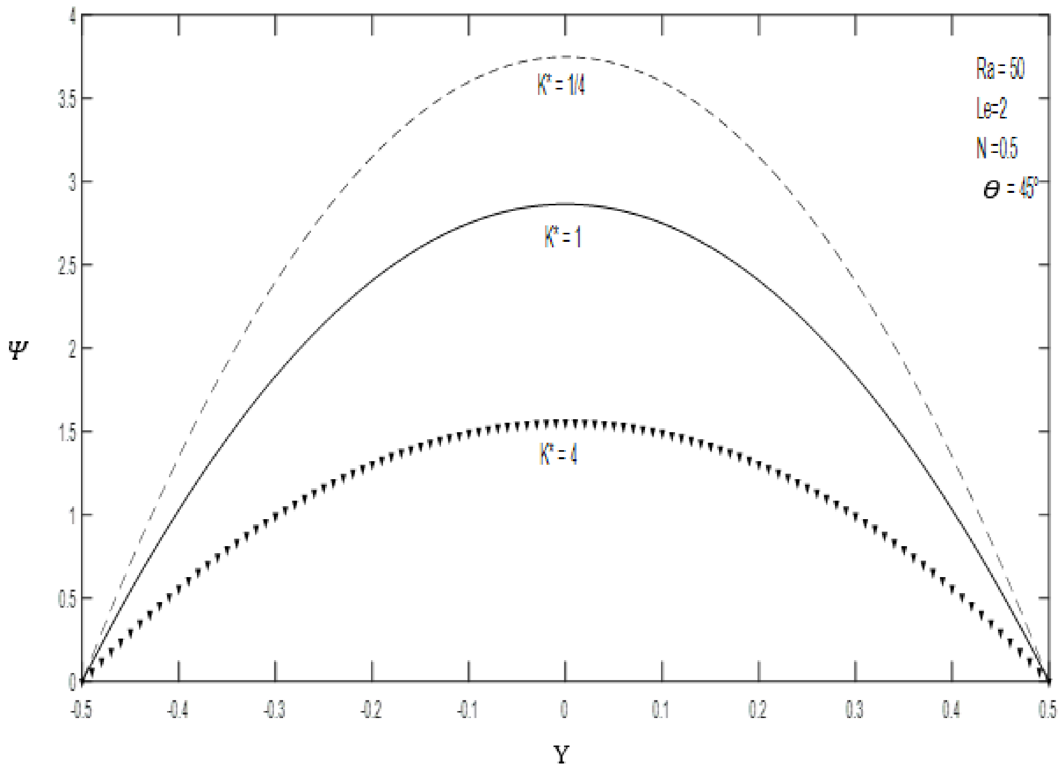


**Figure 2.2 :** Influence of permeability anisotropy on binary fluid flow

Figure 2.3.a shows us the temperature and concentration profiles in the median plane (x = 0) of the cavity for Ra = 50; N = 2; Le = 2; Ha = 0; 1; 2.5. It can be seen that the heat and mass transfer is zero at the center of the cavity and it is

also symmetrical. The first curve (T (Ha = 0), C (Ha = 0)) is the reference (Ouazaa) [5]. As the Hartmann number is varied, heat and mass transfers also change. We deduce from this fact that the magnetic field favors the transfer of heat and mass.

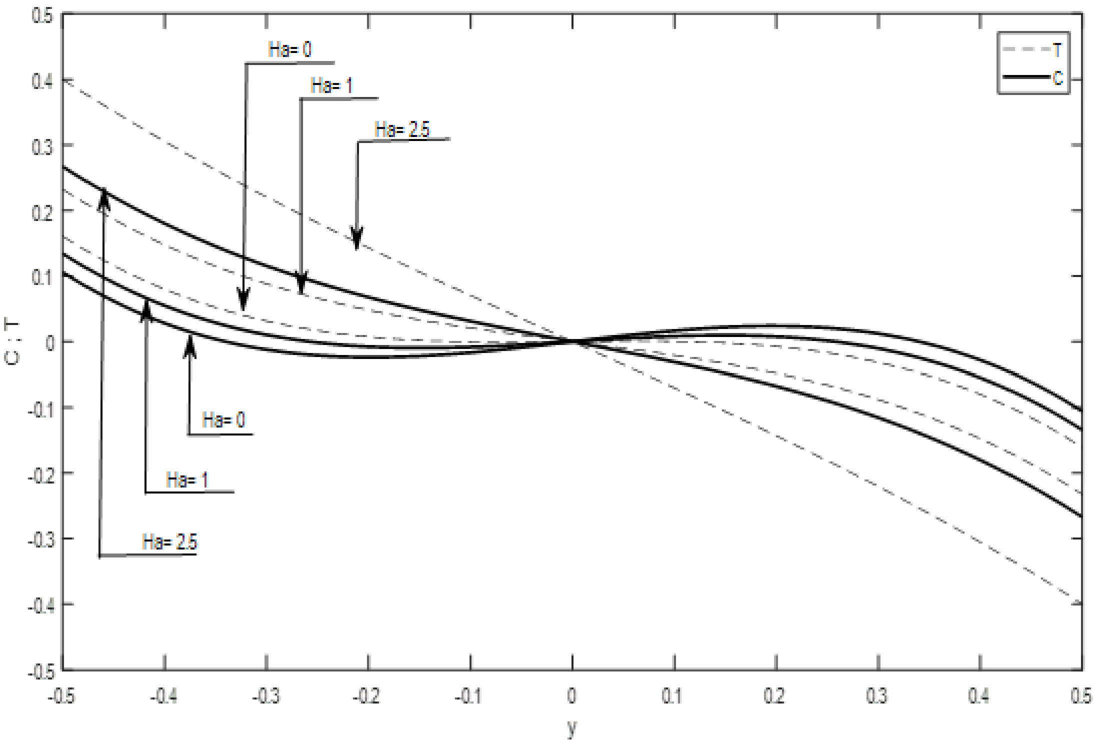


**Figure2.3.a :**Temperature and concentration profiles at x = 0 for different values of Ha

Figure 2.3.b shows us the influence of the Rayleigh number on the flow for different values of volume forces. The cavity being subjected to a constant and differential heat flow on the horizontal face we therefore have:

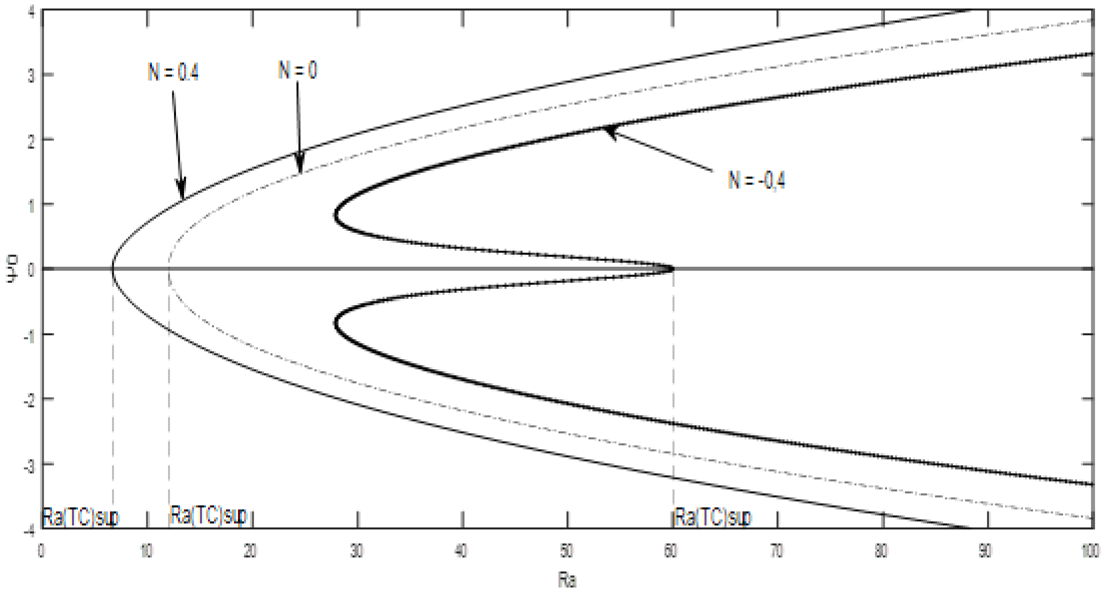


**Figure2.3. b) :**Bifurcation diagram Ψ0 (Ra) for Le = 2 ; N = 0 ; N = 0.4 and N = -0.4

* when N = 0, the solutal volume force is zero, which means that the flow within this cavity is controlled by the flow of heat. Thus we notice a fork-type bifurcation which occurred for a critical Rayleigh number 12, a gold value already predicted by **Nield** [1] based on the theory of linear stability. This is the value from which the convection is started. For Ra <12 we are in the case of pure conduction and for Ra> 12 the intensity of the flow increases with possibilities of two opposite convective motions.

* When N = -0.4, the thermal and solutal volume forces act contrary, which means that the convection is opposite.

Figure 2.3.c shows the influence of the Rayleigh number on the heat transfer rate for different values of the volume ratio

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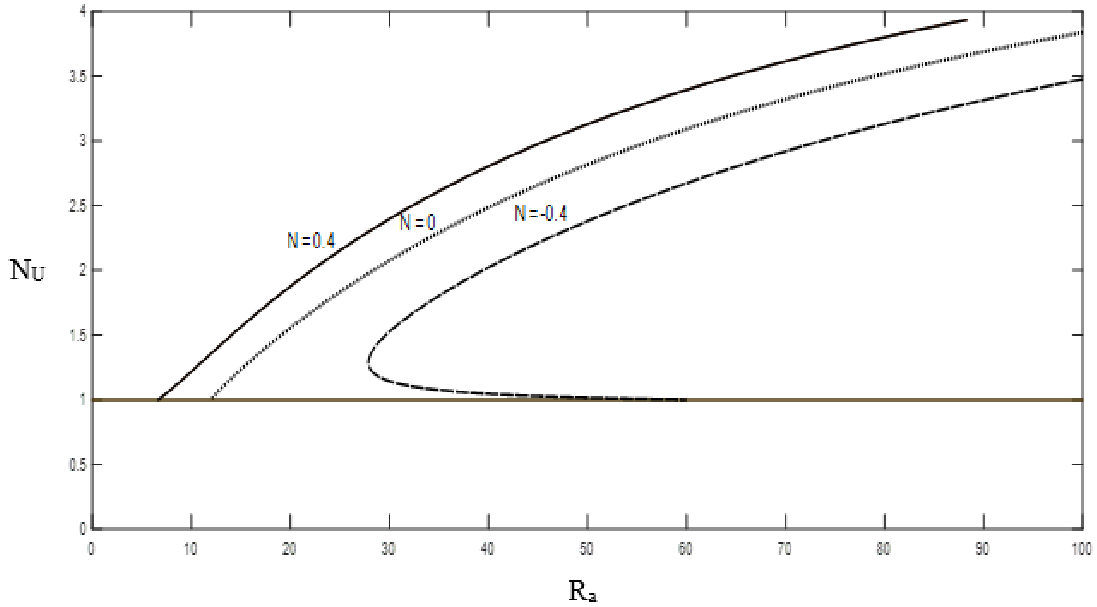


**Figure 2.3.c) :**Influence of the Rayleigh number on Nu for different aspect ratio value with A= 4, Le = 2

Here we examine the influence of Ra on the flow according to whether the volume forces are purely thermal (N = 0); cooperating (N> 0) or opposing (N <0). As well $Ra^{sup}_{TC}$ (N=0.4) = 6.7 ; $Ra^{sup}_{TC}$ (N=0) = 12 et $Ra^{sup}_{TC}$ (N=-0.4) = 29.5 represents the supercritical Rayleigh number, value of the Rayleigh number for constant Nu. We find that for all volume forces whose Rayleigh number is less than the supercritical Rayleigh ($Ra < Ra^{sup}_{TC}$ ) the heat transfer rate is equal to 1 (Nu = 1), which means that we are in the presence of pure conduction. On the other hand, for Ra greater than supercritical Ra, the transfer rate increases until it reaches an asymptote value that can be deduced from equation (45)

Figure 2.4 shows the variation of the Nusselt number as a function of the Rayleigh number for different Hartmann number.

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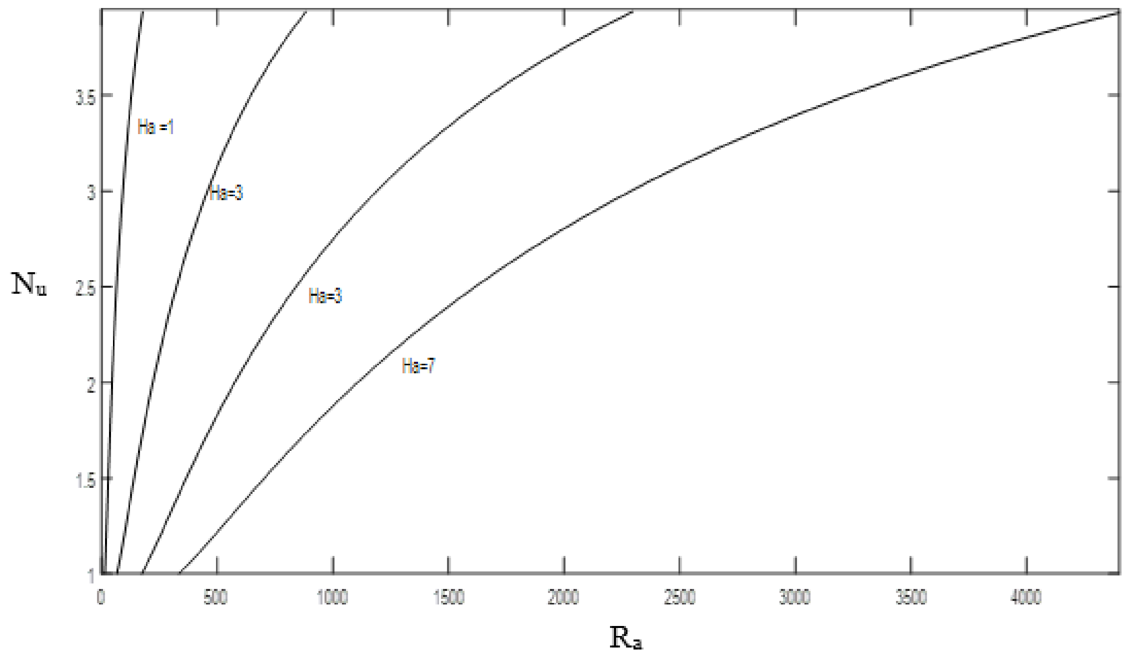


**Figure 2.4 :**Influence of the Rayleigh number on Nu for different value of Ha

For a fixed number of Ra, we notice that when Ha increases, the heat transfer rate decreases.

It is concluded that the application of a relatively large magnetic field is necessary to decrease the rate of heat.

Figure 2.5. tells us about the behavior of the mass transfer rate as a function of the Rayleigh number under the effect of anisotropy.

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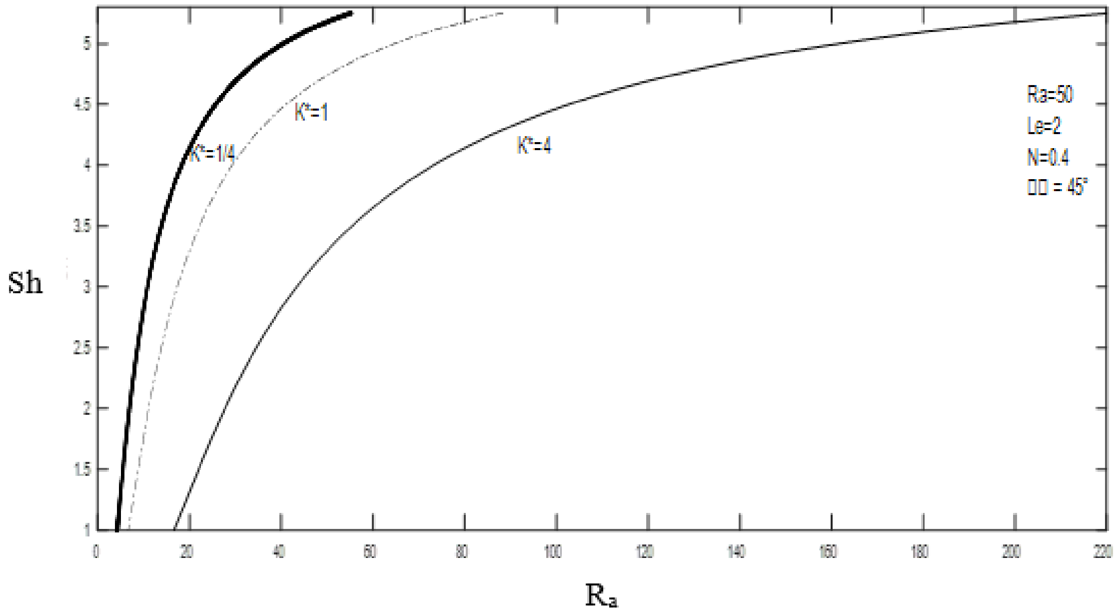


**Figure 2.5 :**Influence of the Rayleigh number on Sh for different value of K *

Figure 2.5 shows the variation of Sherwood number versus Rayleigh number for different anisotropy ratio. For a fixed number of Ra, we notice that when K * (anisotropy ratio) increases, the mass transfer rate decreases. Thus we deduce that anisotropy greatly influences the mass transfer rate.

Figures 2.6.a) and 2.6.b) show us the behavior of the heat transfer rate and the current function at the center in the center of the cavity in the presence of tilt angle under different various Rayleigh numbers and ratio of anisotropy

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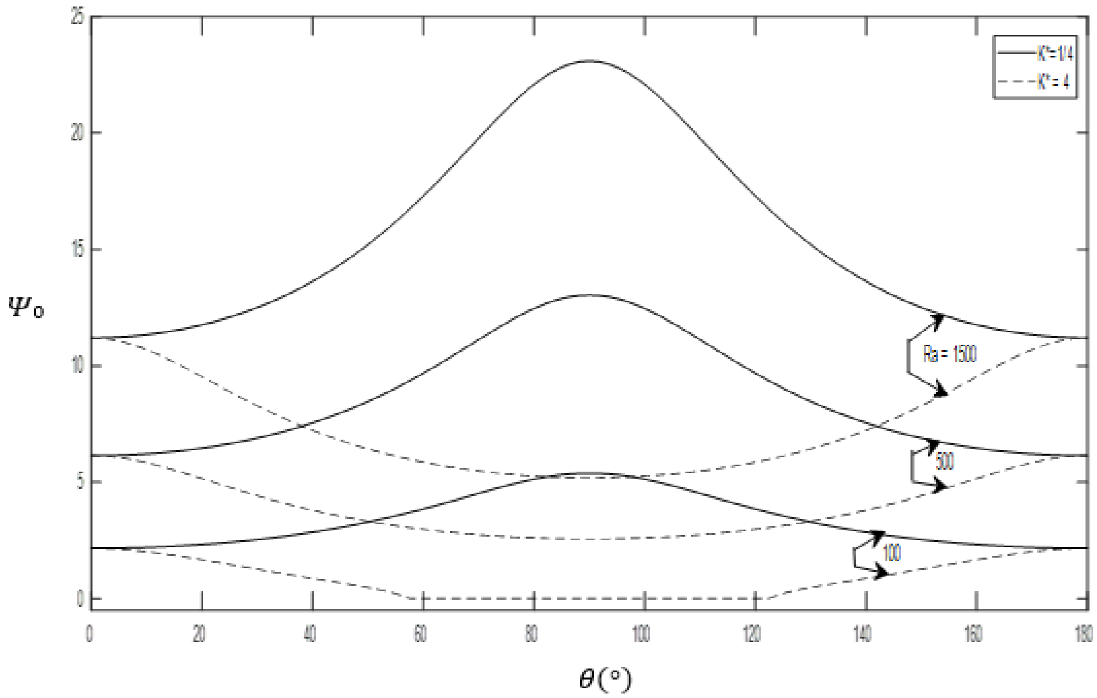


**Figure 2.6.a : )** Effects of the angle of inclination $\boldsymbol{\theta}$ and of the Rayleigh number for K * = 4 and K * = 1/4 on the current function at the center of the cavity

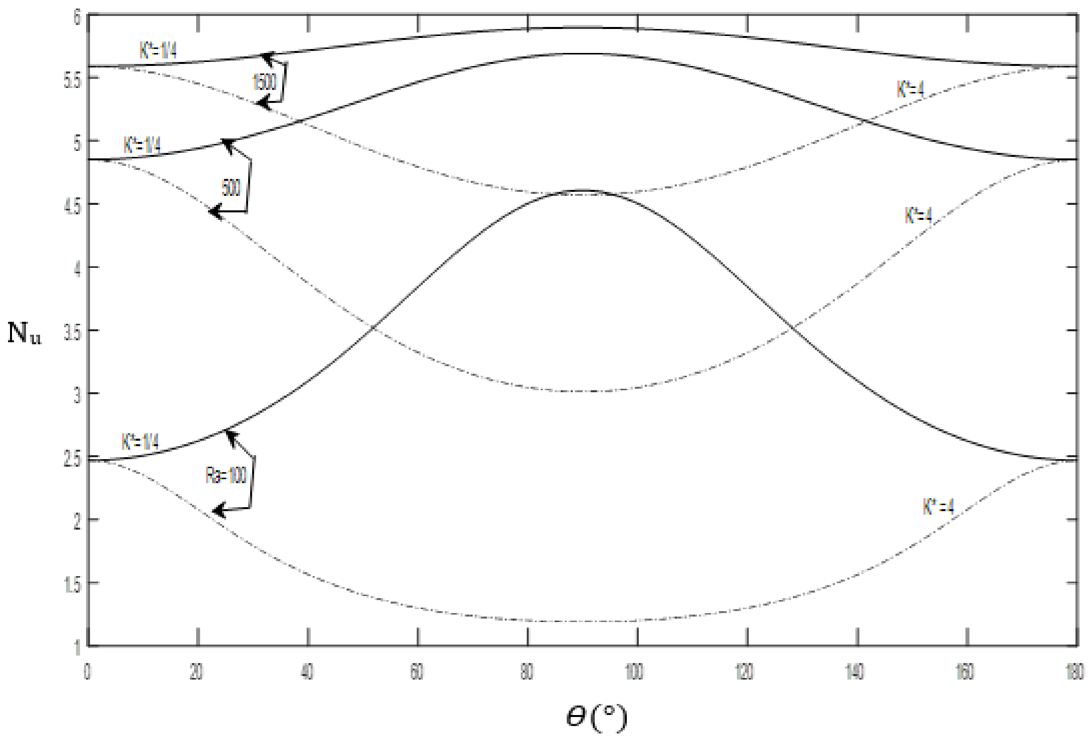


**Figure 2.6.b ) :**Effects of the angle of inclination $\boldsymbol{\theta}$ and of the Rayleigh number for K * = 4 and K * = 1/4 on the Nusselt number

From the analysis of figure 2.6.a) and 2.6.b), we notice that the heat transfer rate is maximum for $\boldsymbol{\theta}$ = 90 ° and is minimum for $\boldsymbol{\theta}$ = 180 ° for K * = 1/4 ce which means that the permeability is maximum in the vertical direction and minimum in the horizontal direction. The opposite effect is observed for K * = 4, where the intensity of the unicellular convective motion and the resulting heat transfer are minimum at $\boldsymbol{\theta}$ = 90° and maximum at $\boldsymbol{\theta}$ = 0° and 180 °. The fact that for K *> 1 (K '* <1) Nu is maximal (minimal) at $\boldsymbol{\theta}$ = 0 ° and 180 ° and minimal (maximal) at $\boldsymbol{\theta}$ = 90 °. Similar results have been reported in the past by Zhang (1993) when this author studied

natural convection in a laterally heated rectangular cavity. In all these studies, it is observed that a maximum (minimum) of heat transfer by natural convection is obtained when the orientation of the main axis of the anisotropic porous medium with the highest permeability is parallel (perpendicular) to the gravitational field. We then conclude that the rate of heat and the intensity of the flow all depend on the angle of orientation of the porous medium.

Moreover, considering the same conditions (Ra = 50; Le = 2; N = 0.5; $\theta$ = 45 ° and Ha = 0) Mamou [14] found Nu = 4.008 and Sh = 4.519; Kalla [6] found Nu = 4.062 and Sh = 4.658; Ouazaa [5] obtained Nu = 4.109 and Sh = 4.723 and for our present study we obtained Nu = 3.1090 and Sh = 4.7239. Thus we notice a satisfactory agreement with just small differences.

## 5. Conclusion

Our study focused on natural thermosolutal convection in a rectangular cavity confined by an anisotropic porous medium, saturated by a binary fluid and subjected to the effect of the transverse magnetic field. It emerges from this study that:

1) the intensity of the convective flow reflected by the current function is greatly influenced on the one hand by the effect of the transverse magnetic field and on the other hand by the anisotropy parameters in permeability of the porous layer.

2) the increase in the anisotropy ratio reduces and regulates the rates of heat and mass transfer

**COMPETING INTERESTS DISCLAIMER:**

**Authors have declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.**

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