MODELING ADDITION OF DISSIMILAR FRACTIONS: MISCONCEPTIONS OF PRE-SERVICE TEACHERS

ABSTRACT

Fractions and learning about them have been consistently difficult for students. This difficulty, grounded on the lack of conceptual knowledge, results in errors in performing operations with fractions. The descriptive study explored the errors in adding dissimilar fractions and misconceptions on modeling fractions among 265 pre-service teachers. The study utilized an open-ended question asking for the procedural computation and the modeling of the addition of two dissimilar fractions as a data-gathering instrument. The findings of the study revealed that the majority of the respondents got the required answer correctly in the question requiring procedural knowledge. In the question requiring the fractions to be modeled, only a few were able to model the fractions correctly. A big percentage of respondents either had no answer to the question or expressed the modeling of fractions in a rule. The study further revealed varied misconceptions on how adding dissimilar fractions are modeled. Recommendations on improving the conceptual knowledge of students as early as elementary grades are promoted.

Keywords: *modeling, procedural knowledge, conceptual knowledge, addition of dissimilar fractions, misconceptions, pre-service teachers*

I. INTRODUCTION

Fractions and learning about them have been consistently difficult for students. This difficulty results in misconceptions which further results in errors in performing operations with fractions. Previous researches found out that the difficulty in dealing with fractions are manifested in elementary (Gabriel et al., 2013), high school (Coetzee & Mammen, 2017; Almeda et al., 2013; Idris & Narayanan, 2011), and even college students (Alghazo & Alghazo, 2017; Cantoria, 2016). The study of Cantoria (2016) revealed the performance in solving fractions of pre-service teachers as unacceptable. Several researches have pointed out weak understanding of fraction content knowledge among pre-service teachers (Butterworth et al., 2011), the difficulty to conceptualize fractions (Ball, 1990), the difficulty to explain fractions to children, and why computation procedures work (Chinnapan, 2000), and the inability to operate fractions correctly (Becker & Lin, 2005). In the study of Cantoria (2016), 71.43% of the pre-service teachers committed an error on the item on adding two dissimilar fractions. The participants erroneously performed addition by adding numerators and denominators and reducing the sum to its lowest term. Torbeyns et al. (2015) found that difficulties with fractions are common, even among prospective teachers from various countries.

Difficulties in learning fractions could be attributed to the interrelation of the procedural and conceptual knowledge of fractions. Rittle-Johnson and Alibali (1999) differentiated conceptual knowledge as the explicit or implicit understanding of the principles ruling a domain and the interrelations between the different parts of knowledge in a domain as against procedural knowledge which can be defined as sequences of actions that are useful to solve problems. As learning procedures are not independent of learning why procedures work, a balance of procedural and conceptual knowledge is essential to learn fractions better.

The results of these studies pose an alarming situation for pre-service teachers since the pre-service teachers are the same persons who will teach the operation of fractions to their future students. If conceptual knowledge will not be strengthened to support procedural knowledge, then the cycle of the misconception on the addition of fractions will continue.

II. METHODOLOGY

The descriptive study was conducted among 265 pre-service teachers of the College of Education, University of Eastern Philippines enrolled in the Bachelor of Elementary Education program. The number is further classified into year levels: 102 first-year students, 68 second-year students, and 95 third-year students. The senior students are undertaking their practicum teaching outside the university when the study was conducted, hence, the non-participation of the fourth-year students. The respondents answered a two-part open-ended question: (1) Answer the question 3/4 + 1/2 showing the steps needed and (2) Why did your procedure work? Explain your answer on the left using a diagram, model, or an example as appropriate. The first part answered the procedural knowledge of the respondent while the second part delved into the conceptual knowledge. Answers for the questions were coded first into correct and incorrect. The incorrect answers were further coded and similarities in the errors or misconceptions were noted on the answers. The study used frequency counts and percentages as statistical tools.

III. RESULTS AND DISCUSSIONS

Procedural Knowledge

The question asking the respondents to add the dissimilar fractions 3/4 and 1/2 found out the procedural knowledge of the respondents. The application of a rule on how to add dissimilar fractions was tested in this question. Table 1 shows that the majority of the respondents in every year level got the correct answer to the question. This indicates that the respondents did not encounter many difficulties in adding dissimilar fractions. This could be verified as adding dissimilar fractions is taught as early as the elementary grades.

Table 1. Responses to the Procedural Knowledge Questions

	Year Level	f	%
Correct	First Year	73	71.57

	Second Year	57	83.82
	Third Year	65	68.42
Incorrect	First Year	26	25.49
	Second Year	11	16.18
	Third Year	19	20.00
No answer	First Year	3	2.94
	Second Year	0	0.00
	Third Year	11	11.58

Table 2 shows the errors committed by the respondents in the procedural question. Although there was a minimal percentage of committing an error in the procedural question, teachers must study how misconceptions are committed by students. The biggest number of errors committed in the procedural question is adding the numerators and denominators as if they are whole numbers independent of each other, as seen in Figure 1. This supports the study of Cantoria (2016) which found out that 71.43% of the pre-service teachers had an error on the question 1/2 + 3/4. The respondents answered 2/3 which could be derived by just adding the numerators and the denominators, resulting in 4/6 which in turn could be simplified into 2/3. Moreover, 73.68% of the same respondents had an error on the question 1/3 + 3/5, of which the process of adding the numerator and denominators were again done on the dissimilar fractions. Gabriel et al. (2013) averred that the most common error in adding dissimilar fractions is based on the natural number bias, i.e., adding or subtracting fractions as if they were natural numbers.

While only three respondents correctly started the task of representing 3/4 and 1/2 independently into models, other misconceptions noted in the addition of dissimilar fractions task is the wrong addition of fractions (Figure 2), applying division algorithm (Figure 3), wrong equivalent fractions (Figure 4), multiplying numerators and denominators (Figure 5). Lortie-Forgues, Tian, & Siegler (2015) argues that students' problems do not mean they did not know the correct procedure, and not that they had a systematic misconception, but rather that they were confused about which of several procedures was correct, which led to a mix of procedures.

Table 2. Errors in the Procedural Question

	Year Level	f	%
Added numerator and denominator	First Year	5	19.23
	Second Year	3	18.18
	Third Year	9	47.37
Wrong addition of fractions	First Year	2	7.69
	Second Year	2	18.18
	Third Year	2	10.53
Division process applied	First Year	1	3.85
	Third Year	1	5.26
Wrong equivalent fractions	First Year	2	7.69
	Third Year	4	21.06
Multiplied numerators and denominators	Second Year	6	54.54
Started the task only	First Year	3	11.54

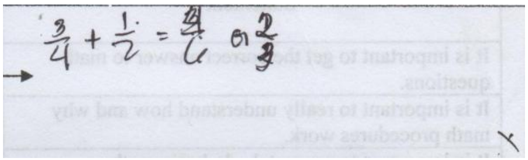

Figure 1

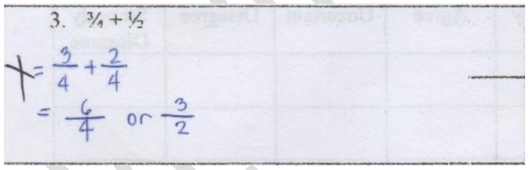

Figure 2

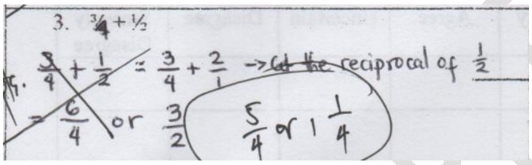

Figure 3

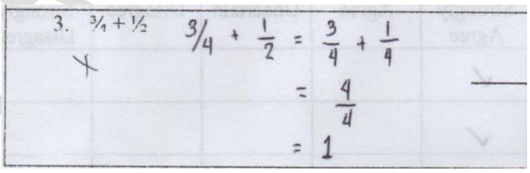

Figure 4

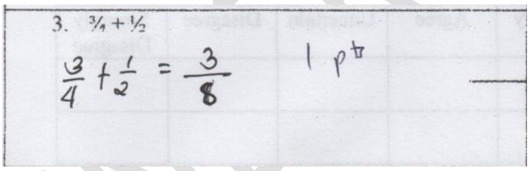

Figure 5

Conceptual Knowledge

The question asking the respondents how the procedure answered in the first part of the question worked by illustrating the addition task through a diagram, a model, or an example explored the conceptual knowledge of the respondents. Unlike the answers to the procedural question, the majority of the respondents had either no answer or an incorrect answer to the question. This indicates that the level of conceptual knowledge of the respondents is poor. The findings, in Table 3, also show that the second-year respondents registered the highest percentage of incorrect answers while the third-year respondents had the highest percentage of no answers.

Table 3. Responses to the Conceptual Knowledge Questions

	Year Level	F	%
Correct	First Year	15	14.71
	Second Year	3	4.41
	Third Year	4	4.21
Incorrect	First Year	40	39.22
	Second Year	62	91.18
	Third Year	33	28.42
No answer	First Year	47	46.08
	Second Year	3	4.41
	Third Year	58	61.05

Table 4 shows the errors committed by the respondents in terms of conceptual knowledge. A big percentage of students answered a rule when asked to represent the addition task using a diagram, a model, or another appropriate example, thus, the failure to manifest a conceptual knowledge of how the addition of dissimilar fractions could be represented more concretely. The table also shows that 18 respondents failed to present the sum of the parts as more than a whole, as seen in Figures 6 and 7, wherein the sum was not represented by a whole and a fourth. This is supported by the study of Kallai and Tzelgov (2009) which showed that adults have a mental representation of what they called a "generalized fraction" which corresponds to an "entity smaller than one" emerging from the common notation of fraction. Hence, even if the respondents correctly represented 3/4 and 1/2 as part of a whole, there was a misconception in representing the sum. Moreover, Alghazo & Alghazo (2017) found out that 83% of the pre-service teachers had the misconception that all fractions are always part of 1, never bigger than 1. Still, 12 respondents considered fractions like whole numbers wherein the numerator could be expressed concretely distinct from the numerator (Figures 8 and 9), hence, manifesting the poor conceptual knowledge in terms of what a fraction is.

Table 4. Errors in the Conceptual Knowledge Question

	Year Level	f	%
Presented the rule in adding fractions	First Year	1	2.50
	Second Year	59	95.16
	Third Year	19	57.58
Started the task only	First Year	17	42.50
	Second Year	1	1.61
	Third Year	4	12.12
Failure to present the sum as more than a whole	First Year	12	30.00
	Third Year	6	18.18
Represented fractions as whole numbers	First Year	10	25.00
	Second Year	2	3.23
Presented a wrong rule	Third Year	3	9.09
Presented a partial rule	Third Year	1	3.03

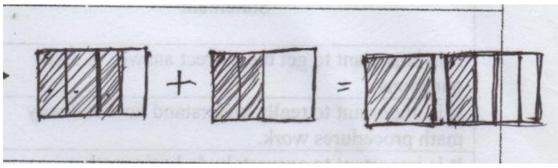

Figure 6

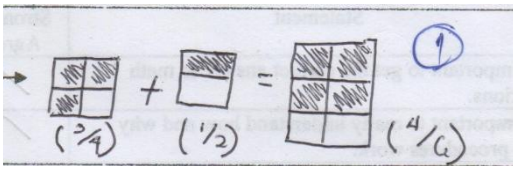

Figure 7

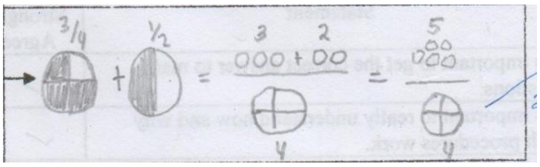

Figure 8

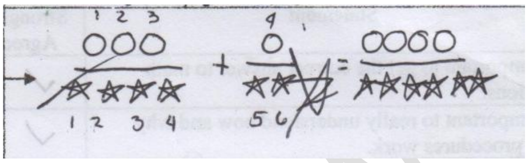

Figure 9

IV. CONCLUSIONS

The study found out the procedural and conceptual knowledge of the respondents on a given addition of dissimilar fractions question. It explored the misconceptions in adding dissimilar fractions and modeling fractions among the pre-service teachers. The pre-service teachers are knowledgeable about rules in adding dissimilar fractions, a manifestation of good procedural knowledge. However, the pre-service teachers failed to represent concretely the addition task using models, hence, a manifestation of poor conceptual knowledge. Learning about fractions ends up only in procedures and not how these rules work in a concrete situation. This becomes a reason why learners cannot even see how fractions work in real-life settings. The way fractions have been learned is common to the three-year levels as both the correct way of solving the task and misconceptions were seen as a pattern in all three-year levels. Misconceptions on procedural knowledge centered on the pre-service teachers performing addition of dissimilar fractions like adding whole numbers while the misconceptions on conceptual knowledge focused on thinking that fractions cannot be more than one and that fractions can be represented concretely just like numbers. This indicates that fractions as a multi-concept were not thoroughly learned by the respondents.

V. RECOMMENDATIONS

Based on the findings, the following recommendations are promoted. Emphasis on representing fractions in more concrete ways should be done in teaching elementary mathematics. Learning fractions is hierarchical, hence, failure to conceptualize fractions in elementary grades can be carried on through college. The teaching of procedures should go hand in hand with the teaching of conceptual knowledge so that learners could see the interrelation of procedural and conceptual knowledge. The concept of fractions as multi-dimensional should be explored by teachers in teaching fractions. Teachers should be aware of misconceptions of learners so that these misconceptions could be remedied immediately.

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