Original Research Article

ON THE EFFICIENCY OF MODIFIED REGRESSION-TYPE ESTIMATORS USING ROBUST REGRESSION SLOPES AND NON-CONVENTIONAL MEASURES OF DISPERSION

ABSTARCT

Supplementary variables associated with the study variables have been identified to be helpful in improving the efficiency of ratio, product and regression estimators both at planning and estimation stages. The existing regression-based estimators are functions of regression slopes and known auxiliary variables which are sensitive to outliers. Zaman & Bulut (2018) and Zaman (2019) addressed the issue of regression slopes in the aforementioned estimators using robust regression slopes like Huber-M, Hampel-M, Least Trimmed Squares (LTS) and Least Absolute Deviation (LAD). However, their estimators still utilized known auxiliary functions which are also sensitive to outliers or extreme values. Similarly, Yadav and Zaman (2021) suggested non-conventional robust parameters of auxiliary variable which are robust against outliers. However, the problems of effects of outliers on regression slopes were not considered. To modify proposed estimators of Zaman & Bulut (2018) and Zaman (2019) estimators using robust non-conventional measures and Yadav and Zaman (2021) estimators by using robust regression slopes. The properties (Biases and MSEs) of the modified estimators were derived up to the first order of approximation using Taylor series approach. The efficiency conditions of the proposed estimation over the existing estimator considered in the study were established. The empirical studies were conducted using both existing population parameters and stimulation to investigate the efficiency of the proposed estimators over the efficiency of the existing estimators. The results revealed that the proposed estimators have minimum MSEs and higher PREs among all the competing estimators. These imply that the proposed estimators are more efficient and can produce better estimate of the population mean compared to other existing estimators considered in the study.

Keywords: Outliers, Estimators, Robust Measures, Auxiliary Variable, Population Mean

1. Introduction

The use of supplementary (auxiliary) information has been widely discussed in sampling theory. Auxiliary variables are in use in survey sampling to obtain improved sampling designs and to achieve higher precision in the estimates of some population parameters such as the mean or the variance of the variable under study. This information may be used at both the stage of designing (leading for instance, to stratification, systematic or probability proportional to size sampling designs) and estimation stage. It is well established that when the auxiliary information is to be used at the estimation stage, the ratio, product and regression methods of estimation are widely used in many situations. In survey sampling literature, a great variety of techniques for using auxiliary information by means of ratio, product and regression methods has been used. A wide variety of estimators have been proposed, following different ideas, and linking together ratio, product or regression estimators, each one exploiting the variables one at a time Okafor (2002). However, often the characteristic Y under study is closely related to an auxiliary variable X, and summary data on X, such as the population mean $\bar{X}$ and the population variance S_x^2, which are readily available. In such a situation it is convenient to consider estimators of the population mean $\bar{Y}$ and population variance S_y^2 that use information about X. Those estimators are generally more efficient than those based on a sample of Y alone if the correlation between X and Y is strong. Many modifications of the ratio and product estimators have been done to improve their efficiency by using a number of known parameters of the auxiliary variable such as the coefficient of variation C_x, the coefficient of kurtosis $\beta_2(x)$, standard deviation δ_x, the coefficient of skewness $\beta_1(x)$, the correlation coefficient between the study variable and an auxiliary variable ρ_{yx}. Sisodia and Dwivedi (1981) have suggested a modified ratio estimator using the coefficient of variation C_x of an auxiliary variable X for estimating the population mean $\bar{Y}$. Upadhyaya and Singh (1999) suggested another modified ratio estimator using a linear combination of the coefficient of variation C_x and coefficient of the kurtosis $\beta_2(x)$. Singh and Tailor (2003) proposed another estimator using the correlation coefficient ρ_{yx} between X and Y. By using the population variance S_x^2 of an auxiliary variable X, Singh (2003) proposed another modified ratio estimator. Also, Singh used a linear combination of the coefficient of kurtosis $\beta_2(x)$ and standard deviation δ_x, and the coefficient of skewness $\beta_1(x)$ for estimating the population mean of the study variable $\bar{Y}$. Motivated by Singh (2003), Yan and Tian (2010) used a linear combination of the coefficient of kurtosis $\beta_2(x)$, coefficient of skewness $\beta_1(x)$, coefficient of variation C_x of the auxiliary variable X. More recently, Subramani and Kumarapandiyan (2013) suggested a new modified ratio estimator using known population median M_d of an auxiliary variable. Subramani and Kumarapandiyan (2012, 2013) have also suggested modified ratio estimators using the known median and the coefficient of kurtosis, median and coefficient of skewness, median and the coefficient of variation, and median and the coefficient of correlation. Other authors that had work in this direction include Hartley-Ross (1954), Quenouille's (1956), Singh (1965, 1967), Naik and Gupta (1991), Kadilar and Cingi (2003), Singh and Espejo (2003), Shabbir and Yaab (2003), Abu-Dayeh et al. (2003), Kadilar and Cingi (2005), Jhajj et al. (2006), Khoshnevisan et al. (2007), Perri (2007), Singh et al. (2007), Gupta and Shabbir (2008), Sharma and Taylor (2010), Subramani and Kumarapandiyan (2012), Subramani and Kumarapandiyan (2013), Subramani and Kumarapandiyan (2014), Singh and Kumar (2011),Singh et al. (2015), Tailor et al. (2012), Lu (2013), Sharma and Singh (2014), Lu and Yan (2014), Verma et al. (2015), Audu and Adewara (2017a,b), Audu and Muili (2019), Muili et al. (2020), Singh et al. (2020), Audu et al. (2020a,b,c,d), Audu et al. (2021a,b,c,e,f,), Audu et al. (2016a,b,c), Yunusa et al (2021), Zaman et al (2021), Audu and Singh (2021).

Regression estimator is used to estimate the population characteristics such as population mean, total and variance when the regression line of y on **x** does not pass through the origin but makes an intercept along the y-axis. Many modifications of the regression type estimators have been done to improve their efficiency by using a number of known parameters of the auxiliary variable such as the coefficient of variation C_x, the coefficient of kurtosis $\beta_2(x)$, standard deviation δ_x, the coefficient of skewness $\beta_1(x)$, and the correlation coefficient between the study variable and an auxiliary variable ρ_{yx}. Shabbir & Gupta (2010) proposed a regression ratio type exponential estimator by combining Rao's (1991) and Bedi's (1996) estimators. Following these works, Grover & Kaur (2011) introduced a regression exponential type estimator. Ozgul & Cingi (2014) proposed a new class of exponential regression cum ratio estimator using functions of any known population parameters of the auxiliary variable, such as standard deviation, coefficient of variation, coefficient of skewness, coefficient of kurtosis and coefficient of correlation of the auxiliary variable for the estimation of finite population mean. Several authors like Kadilar and Cingi (2004), Kadilar and Cingi (2006), Subramani and Kumarapandiyan (2006), Abid et al. (2016), Subzar et al. (2017), Subzar et al. (2018a), Subzar et al. (2018b), Subzar et al. (2018c) have proposed some regression-based estimators which utilized known functions of auxiliary variables. However, these auxiliary parameters are sensitive to outliers or extreme values that do present in the population distributions. Outliers are observations that are distant from other observations in the population. They tend to inflate average deviation of the entire observations from central values. When there are outliers in data, the auxiliary functions like Kurtosis, Skewness, Coefficient of variation, standard deviation etc, will be affected and consequently the efficiency of the estimators which utilize these functions will drastically reduce. Some of the regression estimators in the above paragraph are function of these auxiliary functions. Similarly, regression slope in the regression estimators is also sensitive to outliers and its effect will decreases the efficiency of the estimators. Zaman & Bulut (2018), Zaman (2019) suggested robust regression slopes like Hampel M, Huber M, LTS and LAD methods which are robust against outliers as an alternative to regression slope in the regression estimators of the previous authors. However, the problem of effects of outliers on auxiliary functions in the previous studies was not addressed. Similarly, Yadav and Zaman (2021) suggested non-conventional robust parameters of auxiliary variable which are robust against outliers. However, the problems of effects of outliers on regression slopes were not considered. This current study focused on the modification of robust regression estimators using robust non-conventional measures (Gini's mean, Downton's method, and probability weighted moment) and robust regression slopes simultaneously to address the effect of outliers on auxiliary functions and regression slopes respectively.

1.1 Symbols and Notations

Under this section various notations have been shown which have been used throughout the study.

N- Population Size

n- Sample Size

f = n/N- Sampling fraction

Y- Study Variable (Primary variable)

X- Auxiliary Variable (Secondary variable)

$$R = \frac{\bar{Y}}{\bar{X}}$$

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$$ - Population Mean of Study Variable Y

$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ - Population Mean of Auxiliary Variable X

$S_y = \sqrt{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2 / \left(N-1\right)}$ - Population Standard Deviation of Study Variable Y

$S_x = \sqrt{\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2 / \left(N-1\right)}$ - Population Standard Deviation of Auxiliary Variable X

$\rho_{yx} = S_{yx} / \left(S_y S_x\right)$ - Population Correlation Coefficient between Study and Auxiliary Variables

$S_{yx} = \sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right) / \left(N-1\right)$ - Population Covariance between Study and Auxiliary Variables

$C_y = S_y / \bar{Y}$ - Population Coefficient of Variation of Study Variable Y

$C_x = S_x / \bar{X}$ - Population Coefficient of Variation of Auxiliary Variable X

$u_r = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^r$

$\beta_{1(x)} = \frac{u_3}{u_2^{3/2}}$ - Population Coefficient of Skewness of Auxiliary Variable X

$\beta_{2(x)} = \frac{u_4}{u_2^2}$ - Population Coefficient of Kurtosis of Auxiliary Variable X

$HL = Median\left[(X_j + X_k)/2, 1 \le j \le k \le N\right]$ - Hodges-Lehmann Estimator

$MR = \frac{X_{(1)} - X_{(N)}}{2}$ - Population mid-range of Auxiliary Variable

$G = \frac{4}{N-1}\sum_{i=1}^{N}\left(\frac{2i-N-1}{2N}\right)X_{(i)}$ -Gini's Mean Difference for Auxiliary Variable

$D = \frac{2\sqrt{\pi}}{N-1}\sum_{i=1}^{N}\left(i - \frac{N+1}{2N}\right)X_{(i)}$ - Downton's Method for Auxiliary Variable

$QD = \frac{Q_3 - Q_1}{2}$ - Population Quartile Deviation of Auxiliary Variable

$DM = \frac{D_1 + D_2 + ... + D_9}{9}$ - Decile Mean for Auxiliary Variable

$TM =$ - Trim Mean for Auxiliary Variable

$S_{pw} = \frac{\sqrt{\pi}}{N^2}\sum_{i=1}^{N}\left(2i - N - 1\right)X_{(i)}$ - = Probability Weighted Moments for Auxiliary Variable

$R_h = \frac{\bar{Y}}{\bar{X}_h}, \; h = 1, 2, ...r$

$S_{x_h}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(X_h - \bar{X}_h\right)^2, \qquad h = 1, 2, ...r$

$$S_{yx_h}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(X_{hi}-\bar{X}_h\right)\left(Y_{hi}-\bar{Y}_h\right), \quad h=1,2,...r$$

$$\varphi_h=\frac{A_h\bar{X}_h}{A_h\bar{X}_h+B_h}, \quad h=1,2,...r$$

$$\bar{X}_h=\frac{1}{N_h}\sum_{i=1}^{N_h}X_{hi}, \quad h=1,2,...r$$

A_j & B_j are any of coefficients of variation, skewness, kurtosis and standard deviation of auxiliary variable X_j.

2. Literature Review

Several authors have proposed different regression-type estimators using different auxiliary information. The notable ones include Kadilar and Cingi (2004), Kadilar and Cingi (2006), Subramani and Kumarapandiyan (2006), Abid et al. (2016), Subzar et al. (2017), Subzar et al. (2018a), Subzar et al. (2018b) and Subzar et al. (2018c).

Recently, regression-type estimators have been studied using robust regression slopes and non-conventional robust measures of dispersion.

2.1 *Some Existing Regression Estimators with robust regression slopes*

Zaman & Bulut (2018) proposed ratio-type estimators using Hampel M, Huber MM, LTS, and LAD methods instead of coefficients of slope in ratio estimators. The suggested estimators are modified as below,

$$\bar{y}_{ZB1}=\frac{\bar{y}+\alpha_{rbst(zb)}(\bar{X}-\bar{x})}{\bar{x}}\bar{X} \tag{1}$$

$$\bar{y}_{ZB2}=\frac{\bar{y}+\alpha_{rbst(zb)}(\bar{X}-\bar{x})}{\bar{x}+C_x}(\bar{X}+C_x) \tag{2}$$

$$\bar{y}_{ZB3}=\frac{\bar{y}+\alpha_{rbst(zb)}(\bar{X}-\bar{x})}{\bar{x}+\beta_2(x)}[\bar{X}+\beta_2(x)] \tag{3}$$

$$\bar{y}_{ZB4}=\frac{\bar{y}+\alpha_{rbst(zb)}(\bar{X}-\bar{x})}{\bar{x}\beta_2(x)+C_x}[\bar{X}\beta_2(x)+C_x] \tag{4}$$

$$\bar{y}_{ZB5}=\frac{\bar{y}+\alpha_{rbst(zb)}(\bar{X}-\bar{x})}{\bar{x}C_x+\beta_2(x)}[\bar{X}C_x+\beta_2(x)] \tag{5}$$

where $\alpha_{rob(zb)}$ are coefficients of slope obtained from Hampel M, Huber MM, LTS, and LAD methods.

$$MSE(\bar{y}_{ZBi})=\frac{1-f}{n}\left(R_{KCi}^2S_x^2+2B_{rob(zb)}R_{KCi}S_x^2+B_{rob(zb)}^2S_x^2-2R_{KCi}S_{xy}-2B_{rob(zb)}S_{xy}+S_y^2\right) \tag{6}$$

where $R_{KC1}=\frac{\bar{Y}}{\bar{X}}$, $R_{KC2}=\frac{\bar{Y}}{(\bar{X}+C_x)}$, $R_{KC3}=\frac{\bar{Y}}{(\bar{X}+\beta_2)}$, , $R_{KC4}=\frac{\bar{Y}\beta_2}{(\bar{X}\beta_2+C_x)}$, $R_{KC5}=\frac{\bar{Y}C_x}{(\bar{X}C_x+\beta_2)}$

Zaman (2019) adopted transformation techniques to the work of Zaman and Bulut (2018) and then proposed a general form of estimators as:

$$t_Z = \mu \frac{y + \alpha_{rbst(zb)}(\bar{X} - x)}{x}\bar{X} + (1-\mu)\frac{y + \alpha_{rbst(zb)}(\bar{X} - x)}{(x w_1 + w_2)}(\bar{X} w_1 + w_2) \qquad (7)$$

where μ is a real constant to be determined such that the MSE of t_Z is minimum. $w_1 \neq 0$ and w_2 are either real number or the function of known parameters like C_x and $\beta_2(x)$.

$$MSE(t_Z) \cong \frac{1-f}{n}\left(S_y^2 + \psi_m^2 S_x^2 - 2\psi_m S_{xy}\right) m = 1,2,3,4 \qquad (8)$$

where $\psi_m = \mu(\phi_{rbst(zb)} + R) + (1-\mu)(\phi_{rbst(zb)} + \lambda_{KC(m+1)}), \mu = \frac{B_{reg} + \phi_{rbst(zb)} + \lambda_{KC(m+1)}}{\lambda_{KC(m+1)} - R}, B_{reg} = \frac{\rho_{xy} S_y}{S_x}$

2.2 *Some Existing Regression Estimators with non-conventional robust measures of dispersion.*

Yadav and Zaman (2021) suggested the following class of estimators of population mean using some conventional and non-conventional parameters of auxiliary variable along with the information on the size of the sample as,

$$t_{p1} = \frac{y + b(\bar{X} - x)}{(x + \varpi_j)}(\bar{X} + \varpi_j)\; i = 1,2,...,8 \; and \; j = 1,2,...,8 \qquad (9)$$

$$t_{p2} = \frac{y + b(\bar{X} - x)}{(x\rho + \varpi_j)}(\bar{X}\rho + \varpi_j),\; i = 9,10,...,16 \; and \; j = 1,2,...,8 \qquad (10)$$

$$t_{p3} = \frac{y + b(\bar{X} - x)}{(x C_x + \varpi_j)}(\bar{X} C_x + \varpi_j),\; i = 17,18,...,24 \; and \; j = 1,2,...,8 \qquad (11)$$

$$MSE(t_{pi}) = \frac{1-f}{n}\left(R_{pi}^2 S_x^2 - (1-\rho^2) S_y^2\right), \qquad i = 1,2,...,24 \qquad (12)$$

where,

$$R_{pi} = \frac{\bar{Y}}{\bar{X} + \varpi_j},\; i = 1,2,...,8 \; and \; j = 1,2,...,8$$

$$R_{pi} = \frac{\bar{Y}\rho}{\bar{X}\rho + \varpi_j},\; i = 9,10,...,16 \; and \; j = 1,2,...,8$$

$$R_{pi} = \frac{\bar{Y} C_x}{\bar{X} C_x + \varpi_j},\; i = 17,18,...,24 \; and \; j = 1,2,...,8$$

$$\varpi_1 = (QD + n),\; \varpi_2 = (DM + n),\; \varpi_3 = (TM + n),\; \varpi_4 = (MR + n),\; \varpi_5 = (HL + n),\; \varpi_6 = (G + n),$$
$$\varpi_7 = (D + n),\; \varpi_8 = (S_{pw} + n)$$

3. Proposed Estimators

The suggested estimators are as (13) and (14)

$$t_{1k} = \frac{[y + \alpha_{rbst(zb)}(\bar{X} - x)]}{(\eta_i x + \eta_j)}(\eta_i \bar{X} + \eta_j) \qquad (13)$$

$$t_{2k} = \lambda \frac{[y + \alpha_{rbst(zb)}(\bar{X} - x)]}{x}\bar{X} + (1-\lambda)\frac{[y + \alpha_{rbst(zb)}(\bar{X} - x)]}{(\eta_i x + \eta_j)}(\eta_i \bar{X} + \eta_j) \qquad (14)$$

where

$\eta_i, \eta_j, i,j=1,2,3, \eta_i \neq \eta_j \in \{G\times n, D\times n, S_{pw}\times n\}$ $k=1,2,...,6$

3.1 Biases and MSEs of the Classes of Proposed Estimator $t_{1k}, k=1,2,3,4,5,6$

Theorem 1: The bias & MSE of the suggested classes of estimators $t_{1k}\ (t_{1k}, k=1,2,3,4,5,6)$ to first order of approximation are

$$Bias(t_{1k})=\frac{1-f}{n}\left(\frac{\eta_i}{\eta_i\bar{X}+\eta_j}\left(\left(\alpha_{rbst(zb)}+\frac{\bar{Y}\eta_i}{(\eta_i\bar{X}+\eta_j)}\right)S_X^2-S_{XY}\right)\right) \quad (15)$$

$$MSE(t_{1k})=\frac{1-f}{n}\left(S_Y^2-2A_1S_{XY}+A_1^2S_X^2\right) \quad (16)$$

where, $A_1=\alpha_{rbst(zb)}+\frac{\eta_i\bar{Y}}{(\eta_i\bar{X}+\eta_j)}$

Proof: Bias and MSE (t_{1k}) can be defined up to first order of approximation

$$Bias(t_{ik})=2^{-1}\frac{1-f}{n}\left(\sum_{i=1}^{q}\sum_{j=1}^{q}\Delta_{ij}\left(\hat{\theta}_i-\theta_i\right)\left(\hat{\theta}_j-\theta_j\right)\right) \quad (17)$$

Where q is the number of the sample means in the estimator, $q=2, \hat{\theta}_1=\bar{y}, \hat{\theta}_2=\bar{x}, \theta_1=\bar{Y},$ $\theta_2=\bar{X}$ and $\Delta_{ij}=\frac{\partial^2 t_{1k}}{\partial\hat{\theta}_i\partial\hat{\theta}_j}\Big|_{\bar{y}=\bar{Y},\bar{x}=\bar{X}}$

Simplify (17),

$$Bias(t_{1k})=2^{-1}\psi_{n,N}\left\lfloor\Delta_{11}S_Y^2+\Delta_{22}S_X^2+2\Delta_{12}S_{XY}\right\rfloor \quad (18)$$

where

$$\Delta_{11}=\frac{\partial^2 t_{1k}}{\partial\bar{y}^2}\Big|_{\bar{y}=\bar{Y},\bar{x}=\bar{X}}=\partial\left(\eta_i\bar{X}+\eta_j\Big/\left(\eta_i\bar{x}+\eta_j\right)\right)\Big/\partial\bar{y}\Big|_{\bar{y}=\bar{Y},\bar{x}=\bar{X}}=0 \quad (19)$$

$$\Delta_{12}=\Delta_{21}=\frac{\partial^2 t_{1k}}{\partial\bar{y}\partial\bar{x}}\Big|_{\bar{y}=\bar{Y},\bar{x}=\bar{X}}=\partial\left(-\eta_i\bar{X}+\eta_j\Big/\left(\eta_i\bar{x}+\eta_j\right)\right)\Big/\partial\bar{x}\Big|_{\bar{y}=\bar{Y},\bar{x}=\bar{X}}=\frac{\eta_i}{(\eta_i\bar{X}+\eta_j)} \quad (20)$$

$$\Delta_{22}=\frac{\partial^2 t_{1k}}{\partial\bar{x}^2}\Big|_{\bar{y}=\bar{Y},\bar{x}=\bar{X}}=\partial\left(-\frac{\left(\eta_i\bar{X}+\eta_j\right)\left(\hat{\alpha}_{rbst(zb)}\left(\eta_i\bar{x}+\eta_j\right)+\left(\bar{y}+\hat{\alpha}_{rbst(zb)}\left(\bar{X}-\bar{x}\right)\right)\right)}{\left(\eta_i\bar{x}+\eta_j\right)^2}\right)\Big/\partial\bar{x}\Big|_{\bar{y}=\bar{Y},\bar{x}=\bar{X}}$$

$$=2\eta_i\left(\frac{\alpha_{rbst(zb)}}{\eta_i\bar{X}+\eta_j}+\frac{\bar{Y}\eta_i}{(\eta_i\bar{X}+\eta_j)^2}\right) \quad (21)$$

Substitute (19), (20) and (21) in (18), then (15) is obtained.

$$MSE(t_{1k})=\Delta\Sigma\Delta^T \text{ for } (k=1,2,3,4,5,6) \quad (22)$$

where Δ is a 1×2 matrix and Σ is a 2×2, variance-covariance matrix defined as:

$$\Delta=\left(\frac{\partial t_{1k}}{\partial y}\Big|_{x=\bar{X},y=\bar{Y}} \quad \frac{\partial t_{1k}}{\partial x}\Big|_{x=\bar{X},y=\bar{Y}}\right),\ \Sigma=\begin{pmatrix}\psi_{n,N}S_Y^2 & \psi_{n,N}\rho S_Y S_X\\ \psi_{n,N}\rho S_Y S_X & \psi_{n,N}S_X^2\end{pmatrix},\ \psi_{n,N}=\frac{1-f}{n} \text{ and } f=\frac{n}{N}$$

From definition, we have,

$$\frac{\partial t_{1k}}{\partial y}\Big|_{y=\bar{Y},x=\bar{X}}=\frac{\eta\bar{X}+\eta_j}{\eta x+\eta_j}\Big|_{y=\bar{Y},x=\bar{X}}=1 \tag{23}$$

$$\frac{\partial t_{1k}}{\partial x}\Big|_{y=\bar{Y},x=\bar{X}}=-\frac{\left(\eta\bar{X}+\eta_j\right)\left(\hat{\alpha}_{rbst(zb)}\left(\eta x+\eta_j\right)+\left(y+\hat{\alpha}_{rbst(zb)}\left(\bar{X}-x\right)\right)\right)}{\left(\eta x+\eta_j\right)^2}\Bigg|_{y=\bar{Y},x=\bar{X}}$$

$$\frac{\partial t_{1k}}{\partial x}\Big|_{y=\bar{Y},x=\bar{X}}=-\left(\hat{\alpha}_{rbst(zb)}+\frac{\bar{Y}\eta}{\eta\bar{X}+\eta_j}\right)=-A_1(say) \tag{24}$$

$$\Delta=\left(1 \quad -A_1\right) \tag{25}$$

Substitute (25) in (22), then (16) is obtained, hence, the proof.

3.2 Biases and MSEs of Classes of the Proposed Estimator $t_{2k},\ k=1,2,3,4,5,6$

Theorem 2: The bias and MSE of the suggested estimator $t_{2k}\ (t_{2k},\ k=1,2,3,4,5,6)$ to first order of approximation are

$$Bias(t_{2k})=\frac{1-f}{n}\left(\begin{array}{l}\left(\frac{\lambda\hat{\alpha}_{rbst(zb)}}{\bar{X}}+\frac{\lambda\bar{Y}}{\bar{X}^2}+\frac{(1-\lambda)\left(\hat{\alpha}_{rbst(zb)}\left(\eta\bar{X}+\eta_j\right)+\bar{Y}\right)\eta}{\left(\eta\bar{X}+\eta_j\right)^2}\right)S_X^2\\ -\frac{1}{2}\left(\frac{\lambda}{\bar{X}}+\frac{\eta(1-\lambda)}{\eta\bar{X}+\eta_j}\right)S_{XY}\end{array}\right) \tag{26}$$

$$MSE(t_{2k})=\frac{1-f}{n}\left(S_Y^2-2A_2S_{XY}+A_2^2S_X^2\right) \tag{27}$$

where $A_2=\lambda\left(\hat{\alpha}_{rbst(zb)}+R\right)+(1-\lambda)\left(\hat{\alpha}_{rbst(zb)}+\frac{\bar{Y}\eta}{\left(\eta\bar{X}+\eta_j\right)}\right),\ \lambda=\dfrac{\frac{\rho_{yx}S_y}{S_x}-\left(\hat{\alpha}_{rbst(zb)}+\frac{\bar{Y}\eta}{\left(\eta\bar{X}+\eta_j\right)}\right)}{R-\frac{\bar{Y}\eta}{\left(\eta\bar{X}+\eta_j\right)}}$

Proof: $Bias(t_{2k})$ can be defined up to first order of approximation as

$$Bias(t_{2k})=2^{-1}\frac{1-f}{n}\left(\sum_{i=1}^{q}\sum_{j=1}^{q}\Delta_{ij}\left(\hat{\theta}_i-\theta_i\right)\left(\hat{\theta}_j-\theta_j\right)\right) \tag{28}$$

Where q is the number of the sample means in the estimator, $q=2,\ \hat{\theta}_1=y,\ \hat{\theta}_2=x,\ \theta_1=\bar{Y},$ $\theta_2=\bar{X}$ and $\Delta_{ij}=\frac{\partial^2 t_{2k}^2}{\partial\hat{\theta}_i\partial\hat{\theta}_j}\Big|_{y=\bar{Y},x=\bar{X}}$

Using the above (28), we have,

$$Bias(t_{2k}) = 2^{-1}\frac{1-f}{n}\left[\Omega_{11}S_Y^2 + \Omega_{22}S_X^2 + 2\Omega_{12}S_{XY}\right] \quad (29)$$

where,

$$\Omega_{11} = \left.\frac{\partial^2 t_{2k}}{\partial y^2}\right|_{y=\bar{Y},x=\bar{X}} = \partial\left(\frac{\lambda\bar{X}}{x} + \frac{(1-\lambda)(\eta_i\bar{X}+\eta_j)}{(\eta_i x+\eta_j)}\right)\Big/\partial y\Bigg|_{y=\bar{Y},x=\bar{X}} = 0 \quad (30)$$

$$\Omega_{12} = \Omega_{21} = \left.\frac{\partial^2 t_{2k}}{\partial y\partial x}\right|_{y=\bar{Y},x=\bar{X}} = \partial\left(\frac{\lambda\bar{X}}{x} + \frac{(1-\lambda)(\eta_i\bar{X}+\eta_j)}{(\eta_i x+\eta_j)}\right)\Big/\partial x\Bigg|_{y=\bar{Y},x=\bar{X}} = -\left(\frac{\lambda}{\bar{X}} + \frac{\eta_i(1-\lambda)}{\eta_i\bar{X}+\eta_j}\right) \quad (31)$$

$$\Omega_{22} = \left.\frac{\partial^2 t_{2k}}{\partial x^2}\right|_{y=\bar{Y},x=\bar{X}} = \frac{\partial}{\partial x}\left(\begin{array}{l} -\lambda\bar{X}\left(\dfrac{\hat{\alpha}_{rbst(zb)}x + \left(y+\hat{\alpha}_{rbst(zb)}(\bar{X}-x)\right)}{x^2}\right) \\ -(1-\lambda)(\eta_i\bar{X}+\eta_j)\left(\dfrac{\hat{\alpha}_{rbst(zb)}(\eta_i x+\eta_j) + \left(y+\hat{\alpha}_{rbst(zb)}(\bar{X}-x)\right)\eta_i}{(\eta_i x+\eta_j)^2}\right)\end{array}\right)\Bigg|_{y=\bar{Y},x=\bar{X}}$$

$$\Omega_{22} = \frac{2\lambda\hat{\alpha}_{rbst(zb)}}{\bar{X}} + \frac{2\lambda\bar{Y}}{\bar{X}^2} + \frac{2(1-\lambda)\left(\hat{\alpha}_{rbst(zb)}(\eta_i\bar{X}+\eta_j)+\bar{Y}\right)\eta_i}{(\eta_i\bar{X}+\eta_j)^2}$$

(32)

Substitute (30), (31) and (32) in (29), (26) is obtained.

$$MSE(t_{2k}) = \Omega\Sigma\Omega^T \quad (33)$$

where Ω is a 1×2 matrix and Σ is a 2×2 variance-covariance matrix defined as:

$$\Omega = \left(\left.\frac{\partial t_{2k}}{\partial y}\right|_{x=\bar{X},y=\bar{Y}} \quad \left.\frac{\partial t_{2k}}{\partial x}\right|_{x=\bar{X},y=\bar{Y}}\right),\ \Sigma = \begin{pmatrix}\psi_{n,N}S_Y^2 & \psi_{n,N}\rho S_Y S_X \\ \psi_{n,N}\rho S_Y S_X & \psi_{n,N}S_X^2\end{pmatrix},\ \psi_{n,N} = \frac{1-f}{n} \text{ and } f = \frac{n}{N}$$

Using the above definition, we obtained

$$\left.\frac{\partial t_{2k}}{\partial y}\right|_{y=\bar{Y},x=\bar{X}} = \left.\frac{\eta_i\bar{X}+\eta_j}{\eta_i x+\eta_j}\right|_{y=\bar{Y},x=\bar{X}} = 1 \quad (34)$$

$$\left.\frac{\partial t_{2k}}{\partial x}\right|_{y=\bar{Y},x=\bar{X}} = \left.\left(\begin{array}{l} -\lambda\bar{X}\left(\dfrac{\hat{\alpha}_{rbst(zb)}x+\left(y+\hat{\alpha}_{rbst(zb)}\left(\bar{X}-x\right)\right)}{x^{2}}\right) \\ -(1-\lambda)\left(\eta_{i}\bar{X}+\eta_{j}\right)\left(\dfrac{\hat{\alpha}_{rbst(zb)}\left(\eta_{i}x+\eta_{j}\right)+\left(y+\hat{\alpha}_{rbst(zb)}\left(\bar{X}-x\right)\right)\eta_{i}}{\left(\eta_{i}x+\eta_{j}\right)^{2}}\right) \end{array} \right)\right|_{y=\bar{Y},x=\bar{X}}$$

$$\left.\frac{\partial t_{2k}}{\partial x}\right|_{y=\bar{Y},x=\bar{X}} = -\left(\lambda\left(\hat{\alpha}_{rbst(zb)}+R\right)+(1-\lambda)\left(\hat{\alpha}_{rbst(zb)}+\frac{\bar{Y}\eta_{i}}{\left(\eta_{i}\bar{X}+\eta_{j}\right)}\right)\right)=-A_{2}\,(say)$$

(35)

$$\Omega=\left(1\ -A_{2}\right) \quad (36)$$

Substitute (36) in (33), then (27) is obtained.

Differentiate (27) with respect to λ, equate the result to zero and solve for λ, the optimum value of λ is obtained as

$$\lambda=\frac{\rho_{yx}S_{y}}{S_{x}}-\left(\hat{\alpha}_{rbst(zb)}+\frac{\bar{Y}\eta_{i}}{\left(\eta_{i}\bar{X}+\eta_{j}\right)}\right)\Big/\left(R-\frac{\bar{Y}\eta_{i}}{\left(\eta_{i}\bar{X}+\eta_{j}\right)}\right) \quad (37)$$

4. Empirical Study

In this section, simulation studies were conducted to assess the performance of the estimators of the proposed with respect to Zaman & Bulut (2018) and Zaman (2019) estimators. Data of size 1000 units were generated for study populations using function defined in Table 1. Samples of size 100 units were selected randomly 10,000 times by method of simple random sampling without replacements (SRSWOR). The MSEs and PREs of the considered estimators were computed using (42) and (43).

$$MSE(T)=\frac{1}{10000}\sum_{d=1}^{10000}\left(T-\bar{Y}\right)^{2} \quad (38)$$

$$PRE(T)=\left(\frac{MSE(\bar{y})}{MSE(T)}\right)\times 100 \quad (39)$$

where T is any of the proposed and existing estimators, $MSE(\bar{y})=\frac{1-f}{n}S_{y}^{2}$

Table 1: Populations Used for Empirical Study on Zaman & Bulut (2018), Zaman Audu (2019) and Proposed Estimators t_{1k} and t_{2k}

Populations	**Auxiliary variable (x)**	**Methods for Robust regression slope $\left(\alpha_{rbst(zb)}\right)$**	**Study variable (y)**
I	$X \sim pois(0.1)$	Huber MM	$Y = 20X^2 + e,$ $where, e \sim (0,4)$
		Hampel M	
		Least Trimmed Squares (LTS)	
		Least Absolute Deviation (LAD)	
II	$X \sim exp(0.7)$	Huber MM	
		Hampel M	
		Least Trimmed Squares (LTS)	
		Least Absolute Deviation (LAD)	

Table 2: MSEs and PREs of the Proposed and Some Existing Estimators using Population I under Huber MM Method

Estimators	MSEs	PREs	Estimators	MSEs	PREs	Estimators	MSEs	PREs
Sample Mean $\bar{y}$	4.156443	100	t_{p5}	0.3708674	1120.736	t_{p22}	0.3694504	1125.034
Zaman & Bulut (2018)			t_{p6}	0.3702848	1122.499	t_{p23}	0.3693983	1125.193
y_{ZB1}	6.432489	64.6164	t_{p7}	0.3702075	1122.733	t_{p24}	0.369398	1125.194
y_{ZB2}	0.3794472	1095.394	t_{p8}	0.3702069	1122.735	**Proposed Estimator** t_{1k}		
y_{ZB3}	0.3905998	1064.118	t_{p9}	7.476133	55.59616	t_{11}	0.3642255	1141.173
y_{ZB4}	0.4454944	932.9955	t_{p10}	7.476133	55.59616	t_{12}	0.364227	1141.168
y_{ZB5}	0.3804599	1092.479	t_{p11}	7.476133	55.59616	t_{13}	0.3652286	1138.039
Zaman (2019)			t_{p12}	0.3708734	1120.718	t_{14}	0.3644196	1140.565
t_{ZM1}	3084.196	0.1347658	t_{p13}	0.3708734	1120.718	t_{15}	0.3652373	1138.012
t_{ZM2}	3169.226	0.1311501	t_{p14}	0.3703114	1122.418	t_{16}	0.3644284	1140.538
t_{ZM3}	2239.978	0.1855573	t_{p15}	0.3702362	1122.646	**Proposed Estimator** t_{2k}		
t_{ZM4}	3092.96	0.134384	t_{p16}	0.3702356	1122.648	t_{21}	0.3644099	1140.596
Yadav and Zaman (2021)			t_{p17}	7.476133	55.59616	t_{22}	0.3644092	1140.598
t_{p1}	7.476133	55.59616	t_{p18}	7.476133	55.59616	t_{23}	0.365297	1137.826
t_{p2}	7.476133	55.59616	t_{p19}	7.476133	55.59616	t_{24}	0.3646913	1139.715
t_{p3}	7.476133	55.59616	t_{p20}	0.3705853	1121.589	t_{25}	0.365303	1137.807
t_{p4}	0.3708674	1120.736	t_{p21}	0.3705853	1121.589	t_{26}	0.364699	1139.691

Table 3: MSEs and PREs of the Proposed and Some Existing Estimators using Population I under Hampel M Method

Estimators	MSEs	PREs	Estimators	MSEs	PREs	Estimators	MSEs	PREs
Sample Mean $\bar{y}$	4.156443	100	t_{p5}	0.3708674	1120.736	t_{p22}	0.3694504	1125.034
Zaman & Bulut (2018)			t_{p6}	0.3702848	1122.499	t_{p23}	0.3693983	1125.193
y_{ZB1}	6.372284	65.2269	t_{p7}	0.3702075	1122.733	t_{p24}	0.369398	1125.194
y_{ZB2}	0.3813039	1090.061	t_{p8}	0.3702069	1122.735	**Proposed Estimator** t_{1k}		
y_{ZB3}	0.3931105	1057.322	t_{p9}	7.476133	55.59616	t_{11}	0.3639457	1142.05
y_{ZB4}	0.4404676	943.6433	t_{p10}	7.476133	55.59616	t_{12}	0.3639444	1142.054
y_{ZB5}	0.3823841	1086.981	t_{p11}	7.476133	55.59616	t_{13}	0.3655552	1137.022
Zaman (2019)			t_{p12}	0.3708734	1120.718	t_{14}	0.3644557	1140.452
t_{ZM1}	3073.428	0.135238	t_{p13}	0.3708734	1120.718	t_{15}	0.3655662	1136.988
t_{ZM2}	3158.318	0.1316031	t_{p14}	0.3703114	1122.418	t_{16}	0.3644695	1140.409
t_{ZM3}	2231.113	0.1862946	t_{p15}	0.3702362	1122.646	**Proposed Estimator** t_{2k}		
t_{ZM4}	3082.177	0.1348541	t_{p16}	0.3702356	1122.648	t_{21}	0.3641949	1141.269
Yadav and Zaman (2021)			t_{p17}	7.476133	55.59616	t_{22}	0.3641926	1141.276
t_{p1}	7.476133	55.59616	t_{p18}	7.476133	55.59616	t_{23}	0.3654195	1137.444
t_{p2}	7.476133	55.59616	t_{p19}	7.476133	55.59616	t_{24}	0.3646557	1139.827
t_{p3}	7.476133	55.59616	t_{p20}	0.3705853	1121.589	t_{25}	0.3654268	1137.422
t_{p4}	0.3708674	1120.736	t_{p21}	0.3705853	1121.589	t_{26}	0.3646662	1139.794

Table 4: MSEs and PREs of the Proposed and Some Existing Estimators using Population I under LTS Method

Estimators	MSEs	PREs	Estimators	MSEs	PREs	Estimators	MSEs	PREs
Sample Mean $\bar{y}$	4.222585	100	t_{p5}	0.3566033	1184.113	t_{p22}	0.3549082	1189.768
Zaman & Bulut (2018)			t_{p6}	0.3559334	1186.341	t_{p23}	0.3548256	1190.045
y_{ZB1}	6.824481	61.87408	t_{p7}	0.3558433	1186.642	t_{p24}	0.354825	1190.047
y_{ZB2}	0.3655567	1155.111	t_{p8}	0.3558425	1186.644	**Proposed Estimator** t_{1k}		
y_{ZB3}	0.3771683	1119.549	t_{p9}	7.907988	53.39646	t_{11}	0.3498859	1206.846
y_{ZB4}	0.4342889	972.2987	t_{p10}	7.907988	53.39646	t_{12}	0.3498877	1206.84
y_{ZB5}	0.3666092	1151.795	t_{p11}	7.907988	53.39646	t_{13}	0.3508672	1203.471
Zaman (2019)			t_{p12}	0.35661	1184.09	t_{14}	0.350055	1206.263
t_{ZM1}	3258.69	0.1295792	t_{p13}	0.35661	1184.09	t_{15}	0.350876	1203.441
t_{ZM2}	3345.678	0.1262101	t_{p14}	0.3559643	1186.238	t_{16}	0.3500637	1206.233
t_{ZM3}	2396.451	0.1762016	t_{p15}	0.3558768	1186.53	**Proposed Estimator** t_{2k}		
t_{ZM4}	3267.653	0.1292238	t_{p16}	0.355876	1186.533	t_{21}	0.3493191	1208.805
t_{p2}			t_{p17}	7.907988	53.39646	t_{22}	0.349316	1208.815
t_{p3}	7.907988	53.39646	t_{p18}	7.907988	53.39646	t_{23}	0.3508725	1203.453
	7.907988	53.39646	t_{p19}	7.907988	53.39646	t_{24}	0.3499135	1206.751
	7.907988	53.39646	t_{p20}	0.3562807	1185.185	t_{25}	0.3508815	1203.422
t_{p4}	0.3566033	1184.113	t_{p21}	0.3562807	1185.185	t_{26}	0.3499267	1206.706

Table 5: MSEs and PREs of the Proposed and Some Existing Estimators using Population I under LAD Method

Estimators	MSEs	PREs	Estimators	MSEs	PREs	Estimators	MSEs	PREs
Sample Mean $\bar{y}$	4.156443	100	t_{p5}	0.3708674	1120.736	t_{p22}	0.3694504	1125.034
Zaman & Bulut (2018)			t_{p6}	0.3702848	1122.499	t_{p23}	0.3693983	1125.193
y_{ZB1}	6.283581	66.14767	t_{p7}	0.3702075	1122.733	t_{p24}	0.369398	1125.194
y_{ZB2}	0.3843207	1081.504	t_{p8}	0.3702069	1122.735	**Proposed Estimator** t_{1k}		
y_{ZB3}	0.3970974	1046.706	t_{p9}	7.476133	55.59616	t_{11}	0.3637952	1142.523
y_{ZB4}	0.4332848	959.2866	t_{p10}	7.476133	55.59616	t_{12}	0.3637898	1142.54
y_{ZB5}	0.3855012	1078.192	t_{p11}	7.476133	55.59616	t_{13}	0.3663035	1134.699
Zaman (2019)			t_{p12}	0.3708734	1120.718	t_{14}	0.3647732	1139.459
t_{ZM1}	3057.532	0.1359411	t_{p13}	0.3708734	1120.718	t_{15}	0.3663178	1134.655
t_{ZM2}	3142.217	0.1322774	t_{p14}	0.3703114	1122.418	t_{16}	0.3647945	1139.393
t_{ZM3}	2218.023	0.1873941	t_{p15}	0.3702362	1122.646	**Proposed Estimator** t_{2k}		
t_{ZM4}	3066.259	0.1355542	t_{p16}	0.3702356	1122.648	t_{21}	0.3640346	1141.771
Yadav and Zaman (2021)			t_{p17}	7.476133	55.59616	t_{22}	0.36403	1141.786
t_{p1}	7.476133	55.59616	t_{p18}	7.476133	55.59616	t_{23}	0.365744	1136.435
t_{p2}	7.476133	55.59616	t_{p19}	7.476133	55.59616	t_{24}	0.3647531	1139.522
t_{p3}	7.476133	55.59616	t_{p20}	0.3705853	1121.589	t_{25}	0.3657529	1136.407
t_{p4}	0.3708674	1120.736	t_{p21}	0.3705853	1121.589	t_{26}	0.3647675	1139.477

Table 6: MSEs and PREs of the Proposed and Some Existing Estimators using Population II Huber MM Method

Estimators	MSEs	PREs	Estimators	MSEs	PREs	Estimators	MSEs	PREs
Sample Mean $\bar{y}$	1836.21	100	t_{p5}	317.1903	578.8987	t_{p22}	317.1807	578.9162
Zaman & Bulut (2018)			t_{p6}	317.1828	578.9124	t_{p23}	317.1868	578.9051
y_{ZB1}	448.7248	409.2064	t_{p7}	317.1897	578.8998	t_{p24}	317.1869	578.905
y_{ZB2}	320.2487	573.3701	t_{p8}	317.1898	578.8996	**Proposed Estimator** t_{1k}		
y_{ZB3}	371.7055	493.9961	t_{p9}	317.2267	578.8323	t_{11}	322.3614	569.6124
y_{ZB4}	418.3125	438.9566	t_{p10}	317.1826	578.9127	t_{12}	322.3915	569.5592
y_{ZB5}	373.1852	492.0374	t_{p11}	317.1911	578.8973	t_{13}	316.5761	580.0218
Zaman (2019)			t_{p12}	317.1713	578.9334	t_{14}	319.1135	575.4099
t_{ZM1}	780669.8	0.2352096	t_{p13}	317.1848	578.9087	t_{15}	316.5586	580.0539
t_{ZM2}	3518880	0.05218168	t_{p14}	317.179	578.9193	t_{16}	319.0654	575.4966
t_{ZM3}	16132.41	11.38212	t_{p15}	317.1844	578.9095	**Proposed Estimator** t_{2k}		
t_{ZM4}	3554406	0.05166012	t_{p16}	317.1844	578.9094	t_{21}	313.1458	586.3756
Yadav and Zaman (2021)			t_{p17}	317.2339	578.819	t_{22}	313.1444	586.3782
t_{p1}	317.2425	578.8035	t_{p18}	317.1848	578.9087	t_{23}	313.4932	585.7258
t_{p2}	317.1874	578.9039	t_{p19}	317.1943	578.8914	t_{24}	313.3141	586.0607
t_{p3}	317.1982	578.8843	t_{p20}	317.1711	578.9337	t_{25}	313.4947	585.723
t_{p4}	317.1709	578.9341	t_{p21}	317.1873	578.9042	t_{26}	313.3169	586.0553

Table 7: MSEs and PREs of the Proposed and Some Existing Estimators using Population II Hampel M Method

Estimators	MSEs	PREs	Estimators	MSEs	PREs	Estimators	MSEs	PREs
Sample Mean $\bar{y}$	3158.129		t_{p5}	693.949	455.0952	t_{p22}	693.9596	455.0883
Zaman & Bulut (2018)			t_{p6}	693.9608	455.0875	t_{p23}	693.9522	455.0931
y_{ZB1}	746.0382	423.32	t_{p7}	693.9532	455.0925	t_{p24}	693.9522	455.0932
y_{ZB2}	690.1397	457.6072	t_{p8}	693.9532	455.0925	**Proposed Estimator** t_{1k}		
y_{ZB3}	912.0371	346.272	t_{p9}	693.9435	455.0988	t_{11}	685.5707	460.6569
y_{ZB4}	721.3057	437.835	t_{p10}	693.9592	455.0885	t_{12}	685.5436	460.6751
y_{ZB5}	910.5493	346.8378	t_{p11}	693.9498	455.0947	t_{13}	694.6124	454.6606
Zaman (2019)			t_{p12}	694.0527	455.0272	t_{14}	689.4219	458.0837
t_{ZM1}	1608305	0.1963637	t_{p13}	693.9554	455.091	t_{15}	694.6604	454.6291
t_{ZM2}	7058832	0.04474011	t_{p14}	693.9689	455.0822	t_{16}	689.4966	458.034
t_{ZM3}	36196.35	8.724991	t_{p15}	693.9605	455.0877	**Proposed Estimator** t_{2k}		
t_{ZM4}	7029147	0.04492905	t_{p16}	693.9604	455.0878	t_{21}	682.65	462.6278
Yadav and Zaman (2021)			t_{p17}	693.9494	455.095	t_{22}	682.6474	462.6296
t_{p1}	693.9481	455.0958	t_{p18}	693.9512	455.0938	t_{23}	683.2897	462.1947
t_{p2}	693.9522	455.0932	t_{p19}	693.9445	455.0982	t_{24}	682.9635	462.4154
t_{p3}	693.9451	455.0978	t_{p20}	694.0474	455.0307	t_{25}	683.2924	462.1929
t_{p4}	694.0482	455.0302	t_{p21}	693.9482	455.0958	t_{26}	682.9688	462.4119

Table 8: MSEs and PREs of the Proposed and Some Existing Estimators using Population II LTS Method

Estimators	MSEs	PREs	Estimators	MSEs	PREs	Estimators	MSEs	PREs
Sample Mean $\bar{y}$	3125.882	100	t_{p5}	653.4654	478.3546	t_{p22}	653.3262	478.4565
Zaman & Bulut (2018)			t_{p6}	653.3142	478.4653	t_{p23}	653.415	478.3915
y_{ZB1}	861.7203	362.749	t_{p7}	653.4011	478.4017	t_{p24}	653.4158	478.3909
y_{ZB2}	1309.538	238.701	t_{p8}	653.4019	478.4011	**Proposed Estimator** t_{1k}		
y_{ZB3}	2073.677	150.741	t_{p9}	653.7517	478.1451	t_{11}	1265.203	247.0656
y_{ZB4}	921.7388	339.1288	t_{p10}	653.3299	478.4538	t_{12}	1264.893	247.1261
y_{ZB5}	2069.443	151.0495	t_{p11}	653.4523	478.3642	t_{13}	1344.188	232.548
Zaman (2019)			t_{p12}	652.8283	478.8214	t_{14}	1303.352	239.834
t_{ZM1}	1621003	0.1928362	t_{p13}	653.3734	478.422	t_{15}	1344.531	232.4886
t_{ZM2}	7141032	0.04377353	t_{p14}	653.2415	478.5186	t_{16}	1304.007	239.7135
t_{ZM3}	53299.14	5.864788	t_{p15}	653.3174	478.463	**Proposed Estimator** t_{2k}		
t_{ZM4}	7105956	0.0439896	t_{p16}	653.318	478.4625	t_{21}	620.1533	504.0498
Yadav and Zaman (2021)			t_{p17}	653.9266	478.0172	t_{22}	620.1533	504.0499
t_{p1}	653.9015	478.0356	t_{p18}	653.4297	478.3807	t_{23}	620.1959	504.0152
t_{p2}	653.4155	478.3911	t_{p19}	653.5734	478.2755	t_{24}	620.1658	504.0397
t_{p3}	653.5561	478.2882	t_{p20}	652.8466	478.808	t_{25}	620.1962	504.015
t_{p4}	652.844	478.8099	t_{p21}	653.4807	478.3434	t_{26}	620.1661	504.0394

Table 9: MSEs and PREs of the Proposed and Some Existing Estimators using Population II LAD Method

Estimators	**MSEs**	**PREs**	**Estimators**	**MSEs**	**PREs**	**Estimators**	**MSEs**	**PREs**
Sample Mean $\bar{y}$	3158.129	100	t_{p5}	693.949	455.0952	t_{p22}	693.9596	455.0883
Zaman & Bulut (2018)			t_{p6}	693.9608	455.0875	t_{p23}	693.9522	455.0931
y_{ZB1}	712.7123	443.1141	t_{p7}	693.9532	455.0925	t_{p24}	693.9522	455.0932
y_{ZB2}	709.4701	445.1391	t_{p8}	693.9532	455.0925	**Proposed Estimator** t_{1k}		
y_{ZB3}	987.546	319.7956	t_{p9}	693.9435	455.0988	t_{11}	700.8432	450.6185
y_{ZB4}	696.2509	453.5906	t_{p10}	693.9592	455.0885	t_{12}	700.7872	450.6545
y_{ZB5}	985.8308	320.352	t_{p11}	693.9498	455.0947	t_{13}	717.0097	440.4583
Zaman (2019)			t_{p12}	694.0527	455.0272	t_{14}	708.1954	445.9403
t_{ZM1}	1598412	0.1975792	t_{p13}	693.9554	455.091	t_{15}	717.0877	440.4104
t_{ZM2}	7032856	0.04490535	t_{p14}	693.9689	455.0822	t_{16}	708.3292	445.8561
t_{ZM3}	36626.47	8.622531	t_{p15}	693.9605	455.0877	**Proposed Estimator** t_{2k}		
t_{ZM4}	7003182	0.04509563	t_{p16}	693.9604	455.0878	t_{21}	682.7649	462.5499
Yadav and Zaman (2021)			t_{p17}	693.9494	455.095	t_{22}	682.7632	462.5511
t_{p1}	693.9481	455.0958	t_{p18}	693.9512	455.0938	t_{23}	683.1824	462.2673
t_{p2}	693.9522	455.0932	t_{p19}	693.9445	455.0982	t_{24}	682.9699	462.4112
t_{p3}	693.9451	455.0978	t_{p20}	694.0474	455.0307	t_{25}	683.1842	462.2661
t_{p4}	694.0482	455.0302	t_{p21}	693.9482	455.0958	t_{26}	682.9733	462.4088

Tables 2, 3, 4 and 5 shows the results of MSEs and PREs of the proposed estimators and that of the existing estimators using data from population I under Huber MM, Hampel M, LTS and LAD methods respectively. The result revealed that the proposed estimators has minimum MSEs and higher PREs compared to conventional and other related estimators considered in this study. This implies that the proposed estimators are more efficient and can produce better estimate of the population parameters than the existing estimators.

Table 6 shows the results of MSEs and PREs of the proposed estimators and that of the existing estimators using data from population II under Huber MM method. The results revealed that the proposed estimators have minimum MSEs and higher PREs compared to other existing estimators with the exception of some members of proposed estimators $t_{11}, t_{12}, t_{14},$ and t_{16} which performed below Zaman (2019) and Yadav & Zaman (2021). This implies that the proposed estimators $t_{13}, t_{15}, t_{21}, t_{22}, t_{23}, t_{24}, t_{25}$ and t_{26} are more efficient and can produce better estimate of the population parameters than the existing estimators.

Table 7 shows the results of MSEs and PREs of the proposed estimators and that of the existing estimators using data from population II under Hampel M method. The result revealed that the proposed estimators have minimum MSEs and higher PREs compared to other existing estimators with the exception of some members of proposed estimators $t_{13}$and t_{15}which performed below Zaman & Bulut (2018), Zaman (2019) and Yadav & Zaman (2021). This implies that the proposed estimators $t_{11}, t_{12}, t_{14}, t_{16}\, t_{21}, t_{22}, t_{23}, t_{24}, t_{25}$ and t_{26} are more efficient and can produce better estimate of the population parameters than the existing estimators.

Table 8 shows the results of MSEs and PREs of the proposed and other estimators under population II using Least Trimmed squares (LTS) method. The proposed classes of estimator t_{2k}, $(k=1,2,3,4,5,6)$ outperformed all the existing estimators considered in the study. The results also revealed that the proposed classes of estimator t_{1k}, $(k=1,2,3,4,5,6)$ are more efficient than the existing sample mean, Zaman & Bulut (2018) estimators y, y_{ZB2}, y_{ZB3} and y_{ZB5} Zaman (2019) estimators with the exception of Zaman & Bulut (2018) y_{ZB1}, y_{ZB4} and y_{ZB6} Yadav & Zaman (2021) estimators.

Table 9 shows the results of MSEs and PREs of the proposed and other estimators under population II using Least Absolute Deviation (LAD) method. The proposed classes of estimator t_{2k}, $(k=1,2,3,4,5,6)$ outperformed all the existing estimators considered in the study. The results also revealed that the proposed classes of estimator t_{1k}, $(k=1,2,3,4,5,6)$ are more efficient than the existing sample mean, Zaman & Bulut (2018) estimators y, and y_{ZB4} Zaman (2019) estimators with the exception of Zaman & Bulut (2018) $y_{ZB1}, y_{ZB2}, y_{ZB3}, y_{ZB5}$ and y_{ZB6} Yadav & Zaman (2021) estimators.

5. Conclusion

From the empirical results, it was revealed that the suggested estimators have minimum MSE compared to other competing estimators considered in the study. Hence, the suggested estimators demonstrated high level of efficiency over the other estimators considered in the study. In the other words, the suggested estimators have higher chance of producing estimate that is closer to the true value of the population mean than other estimators considered in the literature of this study. The suggested estimators are recommended for use in the estimation of population means of any variable of interest especially when the study and auxiliary variables are highly associated or correlated.

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