

The Application of Homotopy Perturbation Method: An Overview

ABSTRACT

As an effective method for solving the linear and nonlinear equations, homotopy perturbation method has been applied to numerous problems. In this work, we analyze the researches on the application of homotopy perturbation method. And some conclusions can be obtained that the homotopy perturbation method (1) is most commonly applied to ordinary differential equations; (2) is combined with the technology of transform for more complicated equations; (3) is rapidly convergent, highly accurate, computational simple when compared to other similar methods; (4) should prescribe the number of terms, time span, time step carefully.

Keywords: Homotopy perturbation method, models, Application mode, Application effect, Shortcoming

1. INTRODUCTION

The homotopy perturbation method (HPM), proposed by Chinese mathematician He in 1998, is an analytical technique for differential equations at first [1]. The basic principle of HMP is to combine the traditional perturbation method and the homotopy technique to transform a difficult solving problem into a simple one. Therefore, in recent years, this method has been applied extensively to various models in the field of physics and engineering including not only the differential equations [1-74].

Although there are numerous publications on the application of the HPM, there are hardly ever reviews about them. To this end, this work is to collect the researches on the application of the HPM and analyze them to clarify the previous researches and give some suggestions to the further study.

In order to elaborate the current situation of the application of the HPM to the largest degree, we shall consider some significant problems of the application the method: What models did HPM apply to? Which models were solved most usually by HPM method? How was the HPM method used for solving problems? What was the effect of HPM method in application? And is there any weakness of the HPM in the case when it was utilized. By answering these questions, a detailed introduction of the HPM is presented which may offer a reference to further investigate the application of the method.

2. BASIC IDEA OF HOMOTOPY PERTURBATION METHOD

To illustrate the basic ideas of the HPM method, we consider the following nonlinear differential equation which can be integral or differential equations [2]

$$L(u) = 0 \quad (1)$$

Where L is any integral or differential operator. We construct a convex homotopy functional $H(u, p)$ with an embedding parameter (homotopy parameter) $p \in [0, 1]$ as follows

$$H(u, p) = (1 - p)F(u) + pL(u) \quad (2)$$

Where $F(u)$ is a functional operator with a solution v_0 , which can be obtained easily.

It is clear that for $H(u, p) = 0$, we have two boundary conditions

$$H(u, 0) = F(u), H(u, 1) = L(u)$$

This shows that $H(u, p)$ traces an implicitly defined curve continuously from a starting function $H(v_0, 0)$ to the final function $H(f, 1)$. The embedding parameter monotonically increases from zero to unity as the trivial problem $F(u) = 0$ continuously deforms the original problem $L(u) = 0$. Expanding u in Taylor series with respect to p , one has

$$u = \sum_{i=0}^{\infty} p^i u_i = u_0 + pu_1 + p^2 u_2 + \dots \quad (3)$$

If $p \rightarrow 1$, then (3) corresponds to (2) and becomes the approximate solution of the form

$$f = \lim_{p \rightarrow 1} u = \sum_{i=0}^{\infty} u_i = u_0 + u_1 + u_2 + \dots \quad (4)$$

In general, series (4) is convergent for most of the cases and also the rate of convergence is dependent on operator L . For more details on the convergence of HPM we refer to [3,4].

3. THE APPLICATION OF HPM

We take “homotopy perturbation method”, “homotopy perturbation technique” and “homotopy perturbation approach” as keywords to search for literatures on the websites of China National Knowledge Infrastructure, Baidu Academic Search, Google Academic Search and the websites of web of SCI. Then 107 papers were found. And after excluding the literatures

irrelevant to the application of HPM, there were 74 papers finally. We analyzed these researches on the solved models, patterns, effect and the shortcomings of the application of HPM.

3.1 The models solved by HPM

Once the HPM method was generated, it was applied to solve a Lighthill equation and a Duffing equation, which belong to nonlinear ordinary differential equations [1]. Then, HPM was also applied to solve various types of models governed by equations.

For example, Ji-Huan He et al. applied HPM to the generalized N/MEMS oscillators, Duffing oscillator, Fangzhu oscillator, nonlinear oscillators with coordinate-dependent mass, which were all elucidated by ordinary differential equations(ODEs) [5-8]. And there were many researches on its application to partial differential equations(PDEs) [2,9-33]. Gurmeet Kaur et al. applied HPM to the fragmentation as well as aggregation population balance equation[12]. Sumit Gupta et al. applied it for solving convection-diffusion equations [13]. Hassan Kamil Jassim used it to obtain the solution of Newell-Whitehead Segel equation [18]. Additionally to the models governed by ODEs and PDEs, the fractional equations can be also solved by HPM [21,34-49]. Yasir KHAN et al. extended the application of HPM to obtain the analytical solutions to Klein-Gordon fractional partial differential equation [2]. Chun-Fu WEI applied it to solve the non-linear and singular fractional Lane-Emden type equations [20]. Asma Ali Elbeleze et al. tried to obtain the solution of fractional Black-Scholes equation [36]. Other models such as integral equations and delay differential equations were considered to be solved by HPM in recent years[50-68]. Samad Noeiaghdam et al. utilized it to studied the second kind linear Volterra integral equations with adiscontinuous kernel [51]. The solution of a delay differential equation was presented by means of a HPM and some numerical illustrations were given by Fatemeh Shakeri, Mehdi Dehghan [54].

Based on the analysis above, we conclude that the models of application of HPM comprise ODEs, PDEs, fractional equations, integral equations and other models. Thus we divided the 74 researches into 5 types according to the models they solve. The numbers of the researches with different types of models solved by HPM are shown in Table 1.

Table 1. The numbers of the researches with different models

Model type	ODEs	PDEs	Fractional equations	Integral equations	Other models
Number of the reseaches	27	26	18	4	5

**In some studies, HPM is applied to solve multiple model problems in one paper.*

As can be seen from table 1, HPM method is applied to solve ODEs the most times, followed by PDEs and fractional equations, and it is rarely applied to integral equations or other models.

3.2 The way the HPM method is applied

Purely HPM was applied to various problems without any change or modification in the early

time. We had found that 39 researches among the selected literatures applied HPM only [2,6-8,12,14,15,20-24,26,28,30-32,35,36,42,43,48,51-55,57,59-69].

However, there is a prevailing trend to improvement of the current HPM in many ways. A HPM coupled with the variational iteration method by Yong-Ju Yang and Shun-Qin Wang was utilized to study local fractional nonlinear oscillators [37]. In the research of Ali Demir and his team, the homotopy was constructed based on the decomposition of a source function [11]. And a new method called multistage HPM which treated the HPM as an algorithm in a sequence of intervals for finding accurate approximate solutions was proposed and applied to some problems [56,70-72].

The hybrid methods combining the HPM with other approaches have been attractive for solving more complicated problems, for examples the combination of it with the technology of certain transform. Since the Laplace transform enables a differential equations to be a algebraic equation that is easy to solve, it has been incorporated into HPM in numerous applications, such as to find the solutions of the time fractional wave equations, the convection-diffusion equations, the Klein-Gordon equations, the Newell-Whitehead-Segel equations and so on [5,16,18,25,27,34,38,50,73]. Additionally, there were other transform patterns combined with HPM. For instance, a method derived by combining Elzaki transform and HPM was presented and applied to the system of nonlinear PDEs [19,49,33]. The Sumudu transform was also considered to be an additional technique to HPM for some nonlinear fractional equations [40,41,47].

3.3 The effect of the application of HPM

In order to investigate the effect of HPM in solving problems, we analyzed the comments on the its application in the found researches. Yasir KHAN et al. mentioned that the method was capable of reducing the volume of the computational work as comparing to the classical methods [34]. Erdem Cuce and Pinar Mert Cuce's results have revealed that the HPM was a very practical and reliable approach that presents an accurate approximate solution [59]. M. Fathizadeh et al. concluded that it gave series solutions that converged very rapidly in finding the analytical solutions for boundary value problems [10]. The research of Yong-Ju Yang and Shun-Qin Wang showed that it worked well on the local fractional differential equations, which avoided cumbersome computational works [37]. Moreover, HPM was also demonstrated to be simple and accurate extremely for the exact numerical solution in a wide range of values of oscillation amplitude [26]. When applied to Volterra's Integro-differential equation, HPM did not require small parameters in the equations, which may eliminate the limitations of the standard perturbation methods [60]. It was also found that HPM was applied in a direct way without using linearization, transformation, discretization or restrictive assumptions [2,10,13,14,59,70,74].

At the same time, there are researchers who made comparisons of HPM with other methods to obtain the particular properties of the application of HPM to better use it in certain cases. A detailed comparative study between perturbation method and HPM showed that the errors of HPM increased with a lower rate and the it became more accurate as the rate of nonlinearity was higher [17]. Compared with Adomian decomposition method, HPM did not use Adomian polynomials to find the analytical solutions for integral equations and needed less computation

[22,52,66]. And few approaches for a generalized oscillatory system for N/MEMS comprising energy balance method and spreading residual harmonic balance method are considered for comparison with HPM by Naveed Anjum and Ji-Huan He, the results revealed that HPM gave better accuracy than other methods [5].

According to the analysis above, we can conclude some common characters of the application of HPM listed here: HPM (1) has a rapid speed to calculate; (2) has less computational volume compared to other methods; (3) has similar high accuracy with other methods or even better; (4) can solve the equations without additional assumptions.

3.4 The shortcomings of the application of HPM.

Although HPM has many advantages in application, the shortcomings can be inevitable, which are supposed to be learned so that one may avoid these problems or further improve the algorithm when applying HPM.

Throughout the 74 researches selected, only a small number of them have pointed out the drawbacks of HPM. It found that more components of a expression obtained by HPM must be taken into account to achieve more accurate solutions, which may make the solving process complicated [14,61]. M.S.H. Chowdhury and I. Hashimstill noted that care had to be taken on the choice of time span, time step and the number of terms used when HPM was applied to highly chaotic systems such as the Chen system [15]. When HPM was employed to a sixth-order boundary value problem involving a parameter c , the numerical results showed that the solutions obtained were in poor agreement with the exact solutions for large values of c [52]. L. Cveticanin found that the theoretical reasons why HPM was successfully used for solving pure nonlinear differential equation was hard to explain [26].

4. CONCLUSIONS

In this work, we have selected and analyzed 74 researches in the four essential aspects of HPM involving the models it tried to solve, its application mode, its application effect and its shortcomings. Consequently, some conclusions can be summarized about the application of HPM as follows:

- (1) The models of application of HPM include ODEs, PDEs, fractional equations, integral equations and other models, in which the number of ODEs is the largest followed closely by that of PDEs.
- (2) Half of the researches on the application of the HPM offer an improvement of the method. And the hybrid methods combining HPM with the technology of transform have become increasingly popular as they can intensify its capability of solving complicated equations.
- (3) Regarding to the performance of the application of HPM, many researchers described it as a rapidly convergent, highly accurate, computational simple method when compared to other similar methods.
- (4) The disadvantage of the application of HPM is that the number of terms of the expansion of intermediate process, time span, time step should be chosen carefully, otherwise HPM is not feasible.

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