**Original Research Article**

# MODIFIED CLASSES OF REGRESSION-TYPE ESTIMATORS OF POPULATION MEAN IN THE PRESENCES OF AUXILIARY ATTRIBUTE


## ABSTARCT

*The use of relevant information from auxiliary variable at the estimation stage and design stage to obtain reliable and efficient estimate is a common practice is a sample survey. But situations arise when the available auxiliary information are attribute in nature. Some existing estimators based on auxiliary attribute include; Naik and Gupta (1996), Singh et al. (2007), Singh et al. (2013), Zaman and Kadilar (2019), Zaman (2020). However, the proposed estimators by Zaman and Kadilar (2019) are less efficient when the bi-serial correlation between the study variable and auxiliary attribute is negative. While Zaman (2020) depends on an unknown parameter of the study variable* $\left(C_y\right)$*, hence the applicability of the estimator in real life situation is not possible unless if the value is estimated using a large sample which requires additional resources. In this work, the concept of regression base estimator was used to modified the estimator of Zaman and Kadilar (2019) to obtain estimators that is applicable for both negative and positive correlation. Also, the estimator of Zaman (2020) was modified to obtain estimator that is independent of unknown population parameter. The properties (Biases and MSEs) of the modified estimators were derived up to the first order of approximation using Taylor series approach. The efficiency conditions of the proposed estimation over the existing estimator considered in the study were established. The empirical studies were conducted using both existing population parameters and stimulation to investigate the efficiency of the proposed estimators over the efficiency of the existing estimators. The results revealed that the proposed estimators have minimum MSEs and higher PREs among all the competing estimators. These imply that the proposed estimators are more efficient and can produce better estimate of the population mean compared to other existing estimators considered in the study.*



**Keywords:** Auxiliary Attribute, Bias, Mean Square Error (MSE), Population Mean


## 1. Introduction

It is well known that when auxiliary information is used appropriately in probability sampling, the variances of the estimators of population parameters such as population mean, median, variance, regression coefficient, and population correlation coefficient are significantly reduced. Auxiliary information recorded from the population elements can be successfully used to design an efficient sampling design and after sample selection, to further improve the efficiency of the estimators. If auxiliary variable is available and strongly correlated with the study variable, it is possible to improve the efficiency by using ratio, product, dual-to-ratio or regression methods of estimation. The use of these techniques can considerably improve the accuracy of estimates, i.e. produce estimates that are close to the corresponding population values. There are several authors who have suggested estimators using some known population parameters of auxiliary variable(s). The first attempt was made by Cochran (1940) to investigate the problem of estimation of the population mean when auxiliary variables are present and he proposed the usual ratio estimator of population mean. Other authors like, Hartley-Ross (1954), Quenouille's (1956), Singh (1965, 1967), Abu-Dayeh et al. (2003), Kadilar and Cingi (2005), Khoshnevisan et al. (2007), Perri (2007), Singh et al. (2007a), Singh et al. (2020), Singh and Kumar (2011), Singh et al. (2015), Tailor et al. (2012), Lu (2013), Sharma and Singh (2014), Lu and Yan (2014), Verma et al. (2015), Audu and Singh (2015), Audu et al. (2020a,c), Audu et al. (2021a,b,c,e,f,g), Audu and Adewara (2017), Audu et al. (2016a,b,c), Yunusa et al (2021), Zaman et al (2021), have also considered the problem of estimating the mean of a survey variable when auxiliary variables are made available. There are many practical situations when auxiliary information is qualitative in nature, i.e, auxiliary information is available in the form of an attribute, such as the height of a person may depend on the fact that whether the person is male or female, the efficiency of a dog may depend on the particular breed of that dog, or the yield of wheat crop produced may depend on a particular variety of wheat, etc. Many authors have proposed estimators based on auxiliary attribute. Naik and Gupta (1996), Singh et al., (2007b), Singh et al., (2013), Zaman and Kadilar (2019), Zaman (2020) Audu et al. (2020b), Audu et al. (2021d) suggested estimators by using auxiliary attributes. All the above-mentioned authors assumed known population attribute ($P$). However, the estimators proposed by Zaman and Kadilar (2019) is less efficient when the bi-serial correlation between the study variable and the auxiliary attribute is negative and the unknown α in the Zaman (2020) estimator depends on an unknown parameter of the study variable $\left(C_y\right)$ which makes it impracticable in real life situations unless if estimated using a large sample which requires additional resources. To address the above weaknesses, new classes of modified estimators were proposed using regression function to transform the aforementioned estimators.

**1.1 Notations**

Y: Study Variable

$\bar{Y}$ : Population mean of variable.

$\bar{y}$ : Sample mean

N: Population Size

n: Sample size

$n'$ : Sample Size at first phase

$A=\sum_{i=1}^{N}\phi_i$ : Total number of unit in the population possessing attribute $\phi$

$a=\sum_{i=1}^{n}\phi_i$ : Total number of unit in the sample possessing attribute $\phi$

$P=\frac{A}{N}$: Population proportion attributes

$p=\frac{a}{n}$: Sample proportion of attributes

$p'=\frac{a}{n'}$: Sample proportion at first phase

$C_y=\frac{S_y}{\bar{Y}}$: Coefficient of population variation of the study variable

$C_\phi=\frac{S_\phi}{\bar{X}}$: Coefficient of variation of auxiliary attribute $\phi$

$f=\frac{n}{N}$: Sampling fraction

$B_{2(\phi)}=\frac{\mu_4}{\sigma^4}$: Coefficient of population kurtosis of the form of the auxiliary attribute $\phi$

$\rho_{y\phi}=\frac{S_{y\phi}}{S_y S_\phi}$: The population point of bi-serial correlation between the study variable y and auxiliary attribute $\phi$

$\lambda=\left(\frac{1}{n}-\frac{1}{N}\right)$: Correction Factor

$p^*=\frac{NP-np}{N-n}$: Dual to ratio in single phase.

**2. Literature Review**

Let $y_i$ be i-th characteristic of the population and $\phi_i$ is the case of possessing certain attributes. If $i$th unit has the desired characteristic, it takes the value 1; if not then the value 0. That is:

$$\phi_i=\begin{cases}1\ , & if\ ith\ unit\ of\ the\ population\ possesses\ attribute\\ 0\ , & otherwise\end{cases} \tag{2.1}$$

Let A= $\sum_{i=1}^{N}\phi_i$ and $a$ = $\sum_{i=1}^{n}=\phi_i$ be the total count of the units that possess certain attribute in the population and the sample, respectively. $P=\frac{A}{N}$ and $p=\frac{a}{n}$ denote the proportion of unit in the population and sample possessing attribute $\phi$ respectively.

Naik and Gupta (1996) defined ratio estimator denoted by $t_{NG}$ of population mean when the prior information of population proportion of units, possessing the same attribute is available which modifies ratio estimator by Cochran (1940) as in (2.2). The Bias and MSE of $t_{NG}$ are given as in (2.1) and (2.2)

$$t_{NG} = \frac{\bar{y}}{p} P,$$

$$Bias(t_{NG}) = \bar{Y}\lambda\left(C_{\phi}^{2} - \rho_{y\phi} C_{y} C_{\phi}\right) \tag{2.3}$$

$$MSE(t_{NG}) = \lambda \bar{Y}^{2}\left(C_{y}^{2} - 2\rho_{y\phi} C_{y} C_{\phi} + C_{\phi}^{2}\right) \tag{2.4}$$

However, the estimator $t_{NG}$ is less efficient when the point bi-serial correlation $\left(\rho_{y\phi}\right)$ is negative and when the difference between sample and population proportion is large due to sample information.

Singh *et al*. (2007b) modified the estimator of Naik and Gupta (1996) using exponential transformation to reduce the effect of distance between sample and population proportions. The estimator denoted $t_{STS}$ as well as its Bias and MSE as given in (2.5), (2.6) and (2.7) respectively.

$$t_{STS} = \bar{y} \exp\left(\frac{P - p}{P + p}\right) \tag{2.5}$$

The bias and MSE of estimator $t_{STS}$ are respectively given by (2.6) and (2.7)

$$Bias\left(t_{STS}\right) = \lambda \bar{Y}\left(\frac{1}{4} C_{p}^{2} - \frac{1}{2}\rho_{pb} C_{y} C_{p}\right) \tag{2.6}$$

$$MSE(t_{STS}) = \lambda \bar{Y}^{2}\left(\frac{1}{4} C_{p}^{2} - \rho_{pb} C_{y} C_{p} + C_{y}^{2}\right) \tag{2.7}$$

However; $t_{STS}$ is less efficient when the point bi-serial correlation $\left(\rho_{y\phi}\right)$ is negative.

Singh et al., (2013) proposed a ratio-product exponential type estimator which is applicable when bi-serial correlation is either negative or positive. The estimator denoted $t_{ST}$ as well as its Bias and MSE as given in (2.8), (2.9) and (2.10) respectively.

$$t_{ST} = \bar{y}\left(\alpha \exp\left(\frac{P - p}{P + p}\right) + (1 - \alpha)\exp\left(\frac{p - P}{p + p}\right)\right) \tag{2.8}$$

$$Bias(t_{ST}) = \lambda \bar{Y}\left(\frac{C_{\phi}^{2}}{8} + \rho_{y\phi} C_{y} C_{\phi}\left(\frac{1}{2} - \alpha\right)\right) \tag{2.9}$$

$$MSE(t_{ST}) = \lambda \bar{Y}^{2}\left(C_{y}^{2} + C_{\phi}^{2}\left(\frac{1}{4} + \alpha^{2} - \alpha\right) + 2\rho_{y\phi} C_{y} C_{\phi}\left(\frac{1}{2} - \alpha\right)\right) \tag{2.10}$$

$$K_{P} = \rho_{y\phi}\frac{C_{y}}{C_{\phi}}$$

$$\alpha = \frac{2K_{p} + 1}{2} = \alpha_{0}(say)$$

$$MSE(t_{ST})_{\min} = \lambda \bar{Y}^{2} C_{y}^{2}\left(1 - \rho_{y\phi}^{2}\right) \tag{2.11}$$

However, $t_{ST}$ is a function of $\alpha$ which depends on unknown parameter of the study population $C_y$ (coefficient of variation)

Zaman and Kadilar (2019) proposed family of ratio exponential estimators using known information of auxiliary attribute. The estimator denoted by $t_{ZK}$ as well as it Bias and MSE are given in (2.12), (2.13) and (2.14) respectively

$$t_{ZK} = \bar{y}\exp\left(\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right) \tag{2.12}$$

$$Bias\left(t_{ZKi}\right) = \lambda\bar{Y}\left(\theta_i^2 C_\phi^2 - \theta_i \rho_{y\phi} C_y C_\phi\right) i = 1,2,.........9 \tag{2.13}$$

$$MSE(t_{ZKi}) = \lambda\bar{Y}^2\left(\theta_1^2 C_\phi^2 - 2\theta_i \rho_{y\phi} C_y C_\phi + C_y^2\right) i = 1,2.........9 \tag{2.14}$$

Where $\theta_1 = \frac{P}{2\left(P+\beta_2(\phi)\right)}$, $\theta_2 = \frac{P}{2\left(P+C_\phi\right)}$, $\theta_3 = \frac{P}{2\left(P+\rho_{\phi y}\right)}$; $\theta_4 = \frac{\beta_2(\phi)P}{2\left(\beta_2(\phi)P+C_\phi\right)}$; $\theta_5 = \frac{C_\phi P}{2\left(C_\phi P+\beta_2(\phi)\right)}$; $\theta_6 = \frac{C_\phi P}{2\left(C_\phi P+\rho_{y\phi}\right)}$; $\theta_7 = \frac{\rho_{y\phi} P}{2\left(\rho_{y\phi} P+C_\phi\right)}$ ; $\theta_8 = \frac{\beta_2(\phi)P}{2\left(\beta_2(\phi)P+\rho_{y\phi}\right)}$ ; $\theta_9 = \frac{\rho_{y\phi} P}{2\left(\rho_{y\phi} P+\beta_2(\phi)\right)}$

However, the estimator of $t_{ZK}$ is less efficient when the point bi-serial correlation $\left(\rho_{y\phi}\right)$ is negative.

Zaman (2020) suggested an improved class of estimator for the population mean by modifying Zaman and Kadilar (2019) as in (2.15). The Bias and MSE of $t_{NG}$ are given as in (2.15) and (2.16).

$$t_{Zi} = \bar{y}\left(\frac{p}{P}\right)^\alpha \exp\left(\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right) \tag{2.15}$$

$$MSE(t_{Zi}) = \bar{Y}^2\left(\alpha^2 C_\phi^2 + \theta_i^2 C_\phi^2 - 2\alpha\theta_i C_\phi^2 + C_y^2 + 2\alpha\rho_{y\phi} C_y C_\phi - 2\theta_i C_y C_\phi\right); i = 0,1,2,...,9 \tag{2.16}$$

$$MSE_{\min}(\bar{y}_{Zi}) \cong \frac{1-f}{n}\bar{Y}^2 C_y^2\left(1-\rho_{y\phi}^2\right) \tag{2.17}$$

$$\alpha_{opt} = \frac{\theta_i C_\phi - \rho_{y\phi} C_y}{C_\phi}; i = 0,2,....,9$$

However, the estimator $t_{Zi}$ is a function $\alpha$ which depends on unknown parameters $\rho_{y\phi}$ and $C_y$

## 3. Suggested estimators

Having studied the estimator of Zaman and Kadilar (2019) and Zaman (2020), the improved class of estimators denoted by $t_{pi}$ and $t_{qi}$ for the population mean are proposed as follows.

$$t_{pi}=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{\left(kP+l\right)-\left(kp+l\right)}{\left(kP+l\right)+\left(kp+l\right)}\right)\quad i=1,2,............9 \tag{3.1}$$

$$t_{qi}=\frac{\left(\bar{y}+b_{\phi}\left(P-p\right)\left(kP+l\right)\right)}{kp^{*}+l}\exp\left(\frac{\left(kp^{*}+l\right)-\left(kP+l\right)}{\left(kp^{*}+l\right)+\left(kP+l\right)}\right)\quad i=0,1,2,............9 \tag{3.2}$$

Where $p^{*}=\dfrac{NP-np}{N-n}$

**Table 1: Members of the proposed estimator $t_{pi}$**

| Estimators | Values of $k\ and\ l$ | |
|---|---|---|
| | $k$ | $l$ |
| $t_1=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{P-p}{P+p+2\rho_{y\phi}}\right)$ | 1 | $\beta_2(\phi)$ |
| $t_2=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{P-p}{P+p+2C_{\phi}}\right)$ | 1 | $C_{\phi}$ |
| $t_3=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{P-p}{P+p+2\rho_{y\phi}}\right)$ | 1 | $\rho_{y\phi}$ |
| $t_4=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{\beta_2(\phi)(P-p)}{\beta_2(\phi)(P+p)+2C_{\phi}}\right)$ | $\beta_2(\phi)$ | $C_{\phi}$ |
| $t_5=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{C_{\phi}(P-p)}{C_{\phi}(P+p)+2\beta_2(\phi)}\right)$ | $C_{\phi}$ | $\beta_2(\phi)$ |
| $t_6=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{C_{\phi}(P-p)}{C_{\phi}(P+p)+2\rho_{y\phi}}\right)$ | $C_{\phi}$ | $\rho_{y\phi}$ |
| $t_7=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{\rho_{y\phi}(P-p)}{\rho_{y\phi}(P+p)+2C_{\phi}}\right)$ | $\rho_{y\phi}$ | $C_{\phi}$ |
| $t_8=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{\beta_2(\phi)(P-p)}{\beta_2(\phi)(P+p)+2\rho_{y\phi}}\right)$ | $\beta_2(\phi)$ | $\rho_{y\phi}$ |
| $t_9=\left(\bar{y}+b_{\phi}\left(P-p\right)\right)\exp\left(\frac{\rho_{y\phi}(P-p)}{\rho_{y\phi}(P+p)+2\beta_2(\phi)}\right)$ | $\rho_{y\phi}$ | $\beta_2(\phi)$ |

**Table 2: Members of the proposed estimator $t_{qi}$**

| Estimators | Values of $k$ and $l$ | |
|---|---|---|
| | $k$ | $l$ |
| $t_0 = \dfrac{\bar{y}+b_\phi(P-p)(kP)}{p^*}\exp\left(\dfrac{p^*-P}{P+p^*}\right)$ | 1 | 0 |
| $t_1 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(P+\rho_{y\phi}\right)\right)}{p^*+\rho_{y\phi}}\exp\left(\dfrac{p^*-P}{P+p^*+2\rho_{y\phi}}\right)$ | 1 | $\beta_2(\phi)$ |
| $t_2 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(P+C_\phi\right)\right)}{p^*+C_\phi}\exp\left(\dfrac{p^*-P}{P+p^*+2C_\phi}\right)$ | 1 | $C_\phi$ |
| $t_3 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(P+\rho_{y\phi}\right)\right)}{p^*+\rho_{y\phi}}\exp\left(\dfrac{p^*-P}{P+p^*+2\rho_{y\phi}}\right)$ | 1 | $\rho_{y\phi}$ |
| $t_4 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(\beta_2(\phi)P+C_\phi\right)\right)}{\beta_2(\phi)p^*+C_\phi}\exp\left(\dfrac{\beta_2(\phi)(p^*-P)}{\beta_2(\phi)(P+p^*)+2C_\phi}\right)$ | $\beta_2(\phi)$ | $C_\phi$ |
| $t_5 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(C_\phi P+\beta_2(\phi)\right)\right)}{C_\phi p^*+\beta_2(\phi)}\exp\left(\dfrac{C_\phi(p^*-P)}{C_\phi(P+p^*)+2\beta_2(\phi)}\right)$ | $C_\phi$ | $\beta_2(\phi)$ |
| $t_6 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(C_\phi P+\rho_{y\phi}\right)\right)}{C_\phi p^*+\rho_{y\phi}}\exp\left(\dfrac{C_\phi(p^*-P)}{C_\phi(P+p^*)+2\rho_{y\phi}}\right)$ | $C_\phi$ | $\rho_{y\phi}$ |
| $t_7 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(\rho_{y\phi}P+C_\phi\right)\right)}{\rho_{y\phi}p^*+C_\phi}\exp\left(\dfrac{\rho_{y\phi}(p^*-P)}{\rho_{y\phi}(P+p^*)+2C_\phi}\right)$ | $\rho_{y\phi}$ | $C_\phi$ |
| $t_8 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(\beta_2(\phi)P+\rho_{y\phi}\right)\right)}{\beta_2(\phi)p^*+\rho_{y\phi}}\exp\left(\dfrac{\beta_2(\phi)(p^*-P)}{\beta_2(\phi)(P+p^*)+2\rho_{y\phi}}\right)$ | $\beta_2(\phi)$ | $\rho_{y\phi}$ |
| $t_9 = \dfrac{\left(\bar{y}+b_\phi(P-p)\left(\rho_{y\phi}P+\beta_2(\phi)\right)\right)}{\rho_{y\phi}p^*+\beta_2(\phi)}\exp\left(\dfrac{\rho_{y\phi}(p^*-P)}{\rho_{y\phi}(P+p^*)+2\beta_2(\phi)}\right)$ | $\rho_{y\phi}$ | $\beta_2(\phi)$ |

## 3.1 Properties (Bias and MSE) of $t_{pi}$ and $t_{qi}$

Let $S$ be the set of n-paired observations $\left(y_i,\phi_i\right)$ drawn with SRSWOR from a set of population $\Omega$ of size $N$. To obtain the properties of $t_{pi}$ and $t_{qi}$ the following error terms are defined.

$e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}, \quad e_1 = \frac{p - P}{P}$ Such that, $\bar{y} = \left(1 + e_0\right)\bar{Y}, \quad p = (1 + e_1)P,$

The expectation (E) of the error terms were obtained as in (3.3)

$$\left.\begin{aligned} &\mathrm{E}\left(e_0\right) = \mathrm{E}\left(e_1\right) = 0, \qquad \mathrm{E}\left(e_0^2\right) = \lambda C_y^2, \\ &\mathrm{E}\left(e_1^2\right) = \lambda C_\phi^2, \quad \mathrm{E}\left(e_0 e_1\right) = \lambda \rho_{y\phi} C_y C_\phi, \end{aligned}\right\} \tag{3.3}$$

Express $t_1$ and $t_2$ in term of error terms $e_1$ and $e_2$

$$t_1 = \left(\bar{Y} + \bar{Y}e_0 - b_\phi e_1 P\right)\exp\left(\frac{-kP}{2\left(kP + l\right)} e_1 \left(1 + \frac{kP}{2\left(kP + l\right)} e_1\right)^{-1}\right) \tag{3.4}$$

$$t_2 = \left(\bar{Y} + \bar{Y}e_0 - b_\phi e_1 P\right)\left(1 - \frac{kPJ}{\left(kP + l\right)} e_1\right)^{-1} \exp\left(\frac{-kPJ}{\left(kP + l\right)} e_1 \left(1 - \frac{kPJ}{2\left(kP + l\right)} e_1\right)^{-1}\right) \tag{3.5}$$

Simplify (3.4) and (3.5) up to the first order of approximation, to obtain

$$t_1 = \left(\bar{Y} + \bar{Y}e_0 - b_\phi e_1 P\right)\exp\left(-\varpi_1 e_1 + {\varpi_1}^2 e_1^2\right) \tag{3.6}$$

$$t_2 = \left(\bar{Y} + \bar{Y}e_0 - b_\phi e_1 P\right)\left(1 - J\varpi_2 e_1\right)^{-1} \exp\left(-J\varpi_2 e_1 - J^2 \varpi_1 \varpi_2 e_1^2\right) \tag{3.7}$$

Where $\varpi_1 = \left(\frac{kP}{2\left(kP + l\right)}\right), \ \varpi_2 = \left(\frac{kP}{kP + l}\right)$ and $J = \frac{n}{N - n}$.

Simplify (3.6) and (3.7), to obtain (3.8) and (3.9)

$$t_1 = \bar{Y} + \bar{Y}e_0 - \bar{Y}\varpi_1 e_1 + \frac{3}{2}\varpi_1^2 \bar{Y} e_1^2 - \varpi_1 \bar{Y} e_0 e_1 - b_\phi e_1 P + b_\phi P e_1^2 \tag{3.8}$$

$$t_2 = \bar{Y} + \bar{Y}e_0 - \bar{Y}\left(J^2 \varpi_1 \varpi_2 - \frac{J^2 \varpi_2^2}{2}\right) e_1^2 - b_\phi P e_1 \tag{3.9}$$

Subtract $\bar{Y}$ from both sides of equation (3.8) and (3.9) to obtain (3.10) and (3.11)

$$t_1 - \bar{Y} = \bar{Y}e_0 - \left(\bar{Y} + b_\phi P\right) e_1 + \left(\frac{3}{2}\varpi_2^2 \bar{Y} + b_\phi \varpi_2 P\right) e_1^2 - \bar{Y}\varpi_2 e_0 e_1 \tag{3.10}$$

$$t_2 - \bar{Y} = \bar{Y}e_0 - \bar{Y}\left(J^2 \varpi_1 \varpi_2 - \frac{J^2 \varpi_2^2}{2}\right) e_1^2 - b_\phi P e_1 \tag{3.11}$$

Take expectation of (3.10) and (3.11) and apply result of (3.3), the Bias of $t_p$ and $t_q$ are obtained as

$$Bias\left(t_{pi}\right)=\lambda\left(\left(\frac{3}{2}\varpi_2^2\bar{Y}+b_\phi\varpi_2 P\right)C_\phi^2-\bar{Y}\varpi_2\rho_{y\phi}C_yC_\phi\right)\quad i=1,2,3.......9 \qquad (3.12)$$

$$Bias\left(t_{qi}\right)=-\bar{Y}\lambda\left(J^2\varpi_1\varpi_2-\frac{J^2\varpi_2^2}{2}\right)C_\phi^2\quad i=0,1,2,......9 \qquad (3.13)$$

Square (3.12) and (3.13) up to the first order of approximation, take expectations and apply the result of (3.3), the MSE of $t_1$ and $t_2$ are obtained as

$$MSE\left(t_1\right)=\bar{Y}^2\lambda\left(C_y^2+\left(\varpi_1+\frac{b_\phi P}{\bar{Y}}\right)^2C_\phi^2-2\left(\varpi_1+\frac{b_\phi P}{\bar{Y}}\right)\rho_{y\phi}C_yC_\phi\right) \qquad (3.14)$$

$$MSE\left(t_2\right)=\lambda\left(\bar{Y}^2C_y^2+b_\phi^2P^2C_\phi^2-2\bar{Y}b_\phi P\rho_{y\phi}C_yC_\phi\right) \qquad (3.15)$$

### 3.2 Efficiency comparison of $t_{pi}$ and $t_{qi}$

The efficiency conditions of the proposed estimators $t_{pi}$ and $t_{qi}$ over some existing estimators of Naik and Gupta (1996), Singh et al. (2007), Singh et al. (2013), Zaman and Kadilar (2019) and Zaman (2020) were established in this section.

i. $$MSE\left(t_{pi}\right)-MSE(t_{NG})<0\ \Rightarrow\ \left|\rho_{y\phi}\right|>C_\phi\left(\varpi_1+\frac{b_\phi P}{\bar{Y}}+1\right)/\,2C_y \qquad (3.16)$$

$$MSE\left(t_{qi}\right)-MSE(t_{NG})<0\ \Rightarrow\ \left|\rho_{y\phi}\right|>C_\phi\left(b_\phi P+\bar{Y}\right)/\,2\bar{Y}C_y \qquad (3.17)$$

ii. $$MSE\left(t_{pi}\right)-MSE(t_{STS})<0\ \Rightarrow\ \left|\rho_{y\phi}\right|>C_\phi\left(2\left(\varpi_1+\frac{b_\phi P}{\bar{Y}}\right)+1\right)/\,4C_y \qquad (3.18)$$

$$MSE\left(t_{qi}\right)-MSE(t_{STS})<0\ \Rightarrow\ \left|\rho_{y\phi}\right|>C_\phi\left(2b_\phi P+\bar{Y}\right)/\,4\bar{Y}C_y \qquad (3.19)$$

iii. $$MSE\left(t_{pi}\right)-MSE(t_{ST})_{\min}<0\ \Rightarrow\ \left|\rho_{y\phi}\right|>C_\phi\left(\varpi_1+\frac{b_\phi P}{\bar{Y}}\right)/\,C_y \qquad (3.20)$$

$$MSE\left(t_{qi}\right)-MSE(t_{ST})_{\min}<0\ \Rightarrow\ \left|\rho_{y\phi}\right|>C_\phi b_\phi P\bar{Y}\,/\,C_y$$
(3.21)

iv. $$MSE\left(t_{pi}\right)-MSE(t_{ZKi})<0\ \Rightarrow\ \left|\rho_{y\phi}\right|>C_\phi\left(\varpi_1+\frac{b_\phi P}{\bar{Y}}+\theta_i\right)/\,2C_y \qquad (2.22)$$

$$MSE\left(t_{qi}\right)-MSE(t_{ZKi})<0 \quad \Rightarrow \quad \left|\rho_{y\phi}\right|>C_{\phi}\left(b_{\phi}P+\bar{Y}\theta_i\right)/\,2\bar{Y}C_y \tag{3.23}$$

v. $$MSE\left(t_{pi}\right)-MSE(t_{Zi})_{\min}<0 \;\Rightarrow\; \left|\rho_{y\phi}\right|>C_{\phi}\left(\varpi_1+\frac{b_{\phi}P}{\bar{Y}}\right)/\,C_y \tag{3.24}$$

$$MSE\left(t_{qi}\right)-MSE(t_{Zi})_{\min}<0 \quad \Rightarrow \quad \left|\rho_{y\phi}\right|>C_{\phi}b_{\phi}P\,/\,\bar{Y}C_y \tag{3.25}$$

If conditions (3.16), (3.18), (3.20), (3.22), (3.24) and (3.17), (3.19), (3.21), (3.23), (3.25) are satisfied respectively, then the proposed estimators $t_{pi}$ and $t_{qi}$ are more efficient than the of Naik and Gupta (1996), Singh et al. (2007), Singh et al. (2013), Zaman and Kadilar (2019) and Zaman (2020) respectively.

## 4. Empirical Study

In this section, empirical studies were conducted to examine the superiority of the proposed estimators over some estimators considered in the study.

### 4.1 Efficiency Comparison

Empirical Study using existing population parameters (Sukhatme and Sukhatme 1970).
Population 1: The data is defined as follows:

$$y = \textit{the number of villages in the circles}$$

$$\phi_i=\begin{cases}1, & \textit{if a circle consist of more than five villages}\\ 0 & \textit{otherwise}\end{cases}$$

$N$ =89, $n$=20, $\bar{Y}$ =3.3596, $P$ =0.1236, $b_{\phi}$ =3.492, $C_y$ =0.6008, $C_{\phi}$ =2.6779, $\rho_{y\phi}$ = 0.766 (Sukhatme and Sukhatme 1970).
Population 2: The data is defined as follows:

$$y = \textit{the number of teachers}$$

$$\phi_i=\begin{cases}1, & \textit{if the number of teachers is more than } 60\\ 0 & \textit{otherwise}\end{cases}$$

$N$ =111, $n$=30, $\bar{Y}$ =29.279, $P$ =0.117, $b_{\phi}$ =3.898, $C_y$ =0.872, $C_{\phi}$ =2.758, $\rho_{y\phi}$ = 0.797

**Table 3 MSEs and PREs of Proposed and Existing Estimators Using Data from Pop. 1**

| **Estimators** | **MSEs** | **PREs** |
|---|---|---|
| Sample Mean $\bar{y}$ | 0.1579298 | 100 |
| Naik and Gupta (1996) $t_{NG}$ | 2.217078 | 7.123331 |
| Singh et al. (2007) $t_{STS}$ | 0.4031122 | 39.17762 |
| Singh et al. (2013) $t_{ST}$ | 0.06526354 | 241.9878 |
| Zaman and Kadilar (2019) $t_{ZKi}$ | | |

| | | |
|---|---|---|
| $t_{ZK1}$ | 0.1404135 | 112.4748 |
| $t_{ZK2}$ | 0.1356671 | 116.4098 |
| $t_{ZK3}$ | 0.09815459 | 160.899 |
| $t_{ZK4}$ | 0.09819807 | 160.8278 |
| $t_{ZK5}$ | 0.1171256 | 134.838 |
| $t_{ZK6}$ | 0.06664636 | 236.9669 |
| $t_{ZK7}$ | 0.1404316 | 112.4603 |
| $t_{ZK8}$ | 0.06548182 | 241.1811 |
| $t_{ZK9}$ | 0.1442428 | 109.4888 |
| Zaman (2020) $t_Z$ | 0.06526354 | 241.9878 |
| Propose Estimators $t_{pi}$ | | |
| $t_{11}$ | 0.0661802 | 238.636 |
| $t_{12}$ | 0.06679036 | 236.456 |
| $t_{13}$ | 0.08040544 | 196.4168 |
| $t_{14}$ | 0.08037597 | 196.4888 |
| $t_{15}$ | 0.07114321 | 221.9886 |
| $t_{16}$ | 0.1366728 | 115.5532 |
| $t_{17}$ | 0.0661782 | 238.6432 |
| $t_{18}$ | 0.167143 | 94.48783 |
| $t_{19}$ | 0.06581011 | 239.978 |
| Proposed Estimator $t_q$ | 0.06526354 | 241.9878 |

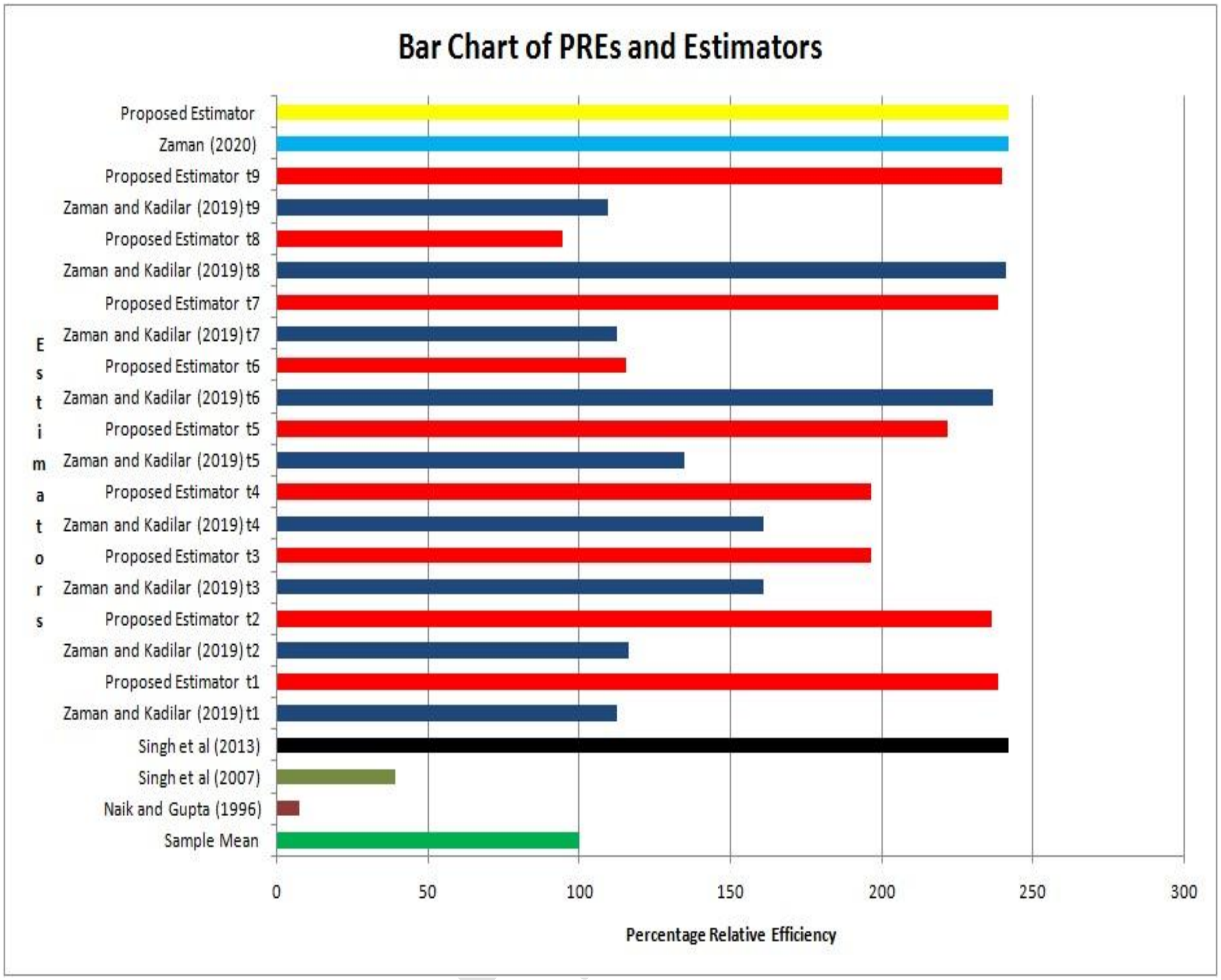


**Figure 1:** Represent the Bar Chart of PREs and Estimator using population 1

**Table 4 MSEs and PREs of Proposed and Existing Estimators Using Data from Pop. 2**

| Estimators | MSEs | PREs |
|---|---|---|
| Sample Mean $\bar{y}$ | 15.85573 | 100 |
| Naik and Gupta (1996) $t_{NG}$ | 94.532 | 16.77287 |
| Singh et al. (2007) $t_{STS}$ | 15.54034 | 102.0295 |
| Singh et al. (2013) $t_{ST}$ | 5.784028 | 274.1296 |
| Zaman and Kadilar (2019) $t_{ZKi}$ | | |
| $t_{ZK1}$ | 14.72468 | 107.6813 |
| $t_{ZK2}$ | 14.29484 | 110.9192 |
| $t_{ZK3}$ | 11.38913 | 139.2181 |
| $t_{ZK4}$ | 10.98268 | 144.3703 |
| $t_{ZK5}$ | 13.03175 | 121.67 |
| $t_{ZK6}$ | 7.630395 | 207.797 |
| $t_{ZK7}$ | 14.59097 | 108.6681 |
| $t_{ZK8}$ | 6.561422 | 241.6508 |
| $t_{ZK9}$ | 14.94357 | 106.1041 |
| Zaman (2020) $t_{Z}$ | 5.784028 | 274.1296 |
| Propose Estimators $t_{pi}$ | | |
| $t_{11}$ | 5.817701 | 272.5429 |
| $t_{12}$ | 5.849699 | 271.0521 |
| $t_{13}$ | 6.4338 | 246.4443 |
| $t_{14}$ | 6.582441 | 240.8792 |
| $t_{15}$ | 6.015808 | 263.5678 |
| $t_{16}$ | 9.077466 | 174.6713 |
| $t_{17}$ | 5.826441 | 272.1341 |
| $t_{18}$ | 11.03681 | 143.6623 |
| $t_{19}$ | 5.805672 | 273.1076 |
| Proposed Estimator $t_{q}$ | 5.784028 | 274.1296 |

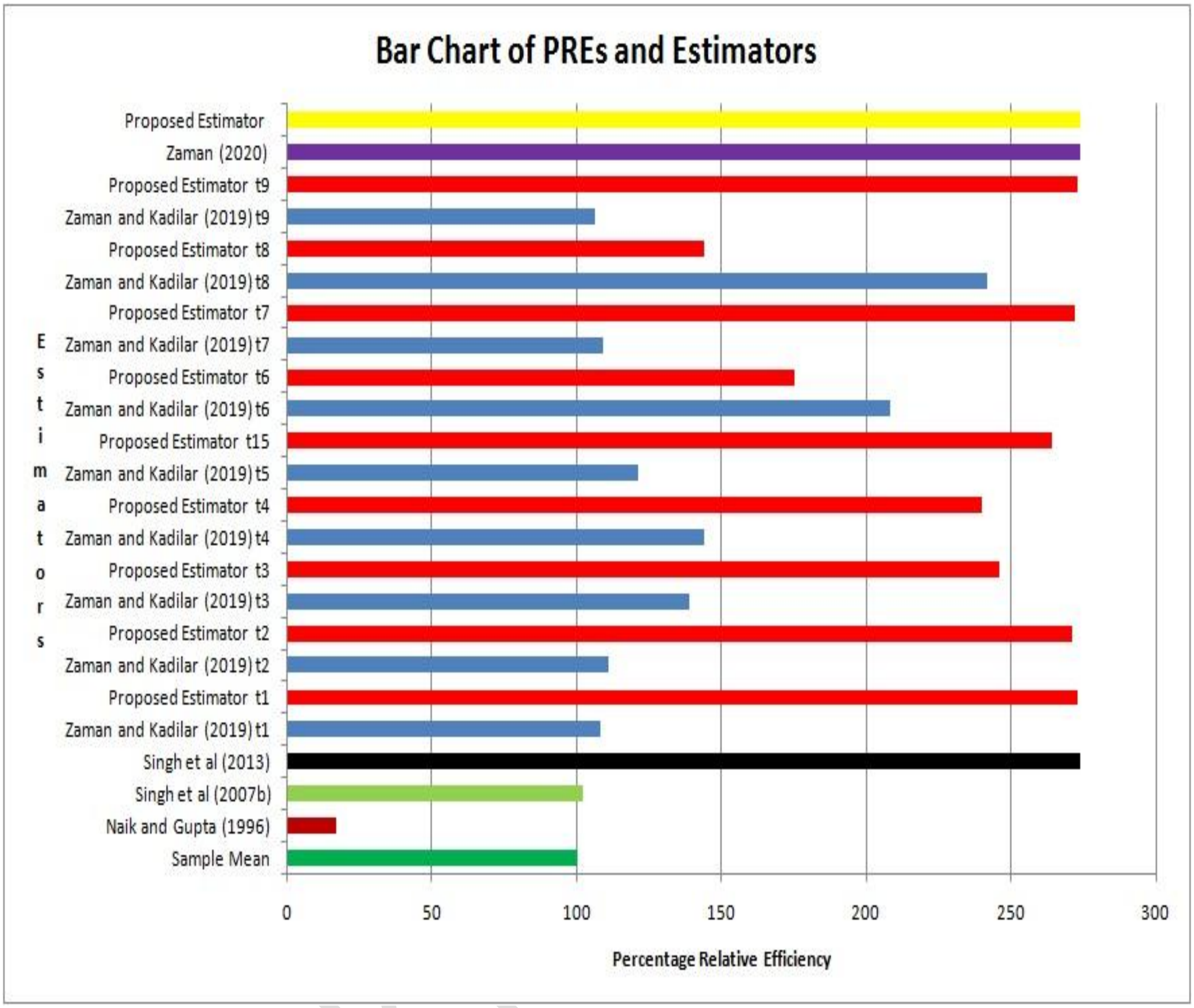


**Figure 2:** Represent the Bar Chart of PREs and Estimator using population 1

Tables 3 and 4 show the results of MSEs and PREs of the proposed estimators and that of some existing estimators using populations 1 and 2 respectively. The results revealed that the proposed estimators have minimum MSEs and higher PREs compared to conventional and other related estimators considered in this study. This implies that the proposed estimators are more efficient and can produce better estimate of the population parameters than the existing estimators. The results were also presented using Figures 1 and 2.

### 4.3 Empirical Study using Stimulated data

In this sub-section, stimulation studies were conducted to assess the performance of the proposed estimators over other estimators considered in the study. Data of size 500 units were generated for study population using functions defined in Table 5 Sample of size 50 were selected by method of Simple Random Sampling without replacement (SRSWOR) 1000 times. The Biases, MSEs and PREs of the considered estimators were computed using (4.1), (4.2) and (4.3).

$$Bias(T) = \frac{1}{1000}\sum_{j=1}^{1000}\left(T - \bar{Y}\right) \tag{4.1}$$

$$MSE(T) = \frac{1}{1000}\sum_{j=1}^{1000}\left(T - \bar{Y}\right)^2 \tag{4.2}$$

$$PREs(T) = \left(\frac{MSE(t_O)}{MSE(T)}\right)\times 100 \tag{4.3}$$

Where $T$ are any of the exiting or proposed estimators.

**Table 5: Population used for Simulation Study**

| Models | Auxiliary variable (x) | Study Variable (y) |
|---|---|---|
| 1 | $X \sim rben(500, 0.7)$ | $\mathrm{Y} = 10 + 40X + \mathrm{e}$, |
| 2 | | $\mathrm{Y} = 10 - 40X + \mathrm{e}$, where, $\mathrm{e} \sim (0, 4)$ |

**Table 6: Biases, MSEs and PREs of the Proposed and some Existing Estimators using model 1 with the Correlation ( $\rho_{y\phi}$ = 0.9775665)**

| Estimators | Biases | MSEs | PREs | Estimators | Biases | MSEs | PREs |
|---|---|---|---|---|---|---|---|
| Sample Mean $\bar{y}$ | 0.05549 | ] 64.07467 | 100 | $t_{Z8}$ | 0.1808972 | 2.925655 | 2190.097 |
| Naik and Gupta (1996) $t_{NG}$ | 1.005351 | 14.26301 | 449.2368 | $t_{Z9}$ | 0.2185038 | 2.93286 | 2184.716 |
| Singh et al. (2007b) $t_{STS}$ | 0.110366 | 8.312005 | 770.869 | **Proposed Estimators** $t_{Pi}$ | | | |
| Singh et al. (2013) $t_{ST}$ | 268.518 | 7221.504 | 0.8872759 | $t_{31}$ | 0.384611 | 0.931629 | 2185.634 |
| **Zaman and Kadilar(2019)** $t_{ZKi}$ | | | | $t_{32}$ | 0.6210778 | 3.015386 | 2124.925 |
| $t_{ZK1}$ | -0.21617 | 41.50172 | 154.3904 | $t_{33}$ | 0.5174373 | 2.973801 | 2154.639 |
| $t_{ZK2}$ | -0.17660 | 28.8395 | 222.1768 | $t_{34}$ | 0.7598552 | 3.083468 | 2078.006 |
| $t_{ZK3}$ | -0.178769 | 34.07176 | 188.058 | $t_{35}$ | 0.300506 | 2.911192 | 2200.977 |
| $t_{ZK4}$ | -0.218887 | 22.60134 | 283.4994 | $t_{36}$ | 0.4149634 | 2.940186 | 2179.272 |
| $t_{ZK5}$ | -0.213759 | 46.63139 | 137.4067 | $t_{37}$ | 0.6145036 | 3.012516 | 2126.948 |
| $t_{ZK6}$ | -0.195723 | 39.73171 | 161.2683 | $t_{38}$ | 0.6570131 | 3.031632 | 2113.538 |
| $t_{ZK7}$ | -0.139944 | 29.15678 | 219.7591 | $t_{39}$ | 0.3792218 | 2.930175 | 2186.718 |
| $t_{ZK8}$ | -0.189597 | 27.14009 | 236.0886 | **Proposed Estimators** $t_{qi}$ | | | |
| $t_{ZK9}$ | -0.219134 | 41.82048 | 153.2136 | $t_{40}$ | 0.04764066 | 2.877944 | 2226.404 |
| **Zaman (2020)** $t_{Zi}$ | | | | $t_{41}$ | 0.04764457 | 2.877942 | 2226.406 |
| $t_{Z0}$ | 0.3810892 | 2.970742 | 2156.857 | $t_{42}$ | 0.04764409 | 2.877942 | 2226.406 |
| $t_{Z1}$ | 0.2168675 | 2.932537 | 2184.957 | $t_{43}$ | 0.04764436 | 2.877942 | 2226.406 |
| $t_{Z2}$ | 0.180437 | 2.925648 | 2190.102 | $t_{44}$ | 0.04764359 | 2.877943 | 2226.405 |
| $t_{Z3}$ | 0.1879284 | 2.927082 | 2189.029 | $t_{45}$ | 0.04764465 | 2.877942 | 2226.406 |
| $t_{Z4}$ | 0.1908273 | 2.927097 | 2189.018 | $t_{46}$ | 0.04764454 | 2.877942 | 2226.406 |
| $t_{Z5}$ | 0.2465373 | 2.938672 | 2180.396 | $t_{47}$ | 0.04764411 | 2.877942 | 2226.406 |
| $t_{Z6}$ | 0.2083266 | 2.930878 | 2186.193 | $t_{48}$ | 0.04764398 | 2.877943 | 2226.405 |
| $t_{Z7}$ | 0.1805225 | 2.925674 | 2190.083 | $t_{49}$ | 0.04764458 | 2.877942 | 2226.406 |

**Table 7: Biases, MSEs and PREs of the Proposed and some Existing Estimators using Model 2 with Correlation ( $\rho_{y\phi}$ =-0.9791406)**

| Estimators | Biases | MSEs | PREs | Estimators | Biases | MSEs | PREs |
|---|---|---|---|---|---|---|---|
| Sample Mean $\bar{y}$ | -0.585156 | 68.99155 | 100 | $t_{Z8}$ | -683.4372 | 1024063 | 0.006737044 |
| Naik and Gupta (199 ) $t_{NG}$ | 0.1426181 | 3.077269 | 2241.973 | $t_{Z9}$ | -0.6003564 | 2.982463 | 2313.241 |
| Singh et al. (2007b) $t_{STS}$ | 0.06668405 | 21.67639 | 318.2797 | **Proposed Estimators** $t_{pi}$ | | | |
| Singh et al. (2013) $t_{ST}$ | -672.4029 | 6016708 | 0.001146666 | $t_{31}$ | -0.3273392 | 2.809121 | 2455.984 |
| **Zaman and Kadilar(2019)** $t_{ZKi}$ | | | | $t_{32}$ | -0.6951248 | 2.81572 | 2391.174 |
| $t_{ZK1}$ | -0.2132775 | 51.79144 | 133.2104 | $t_{33}$ | 2.895604 | 5.412566 | 1274.655 |
| $t_{ZK2}$ | -0.052035 | 42.01574 | 164.2041 | $t_{34}$ | -0.700835 | 2.946906 | 2341.152 |
| $t_{ZK3}$ | -8.672273 | 334.4563 | 20.62797 | $t_{35}$ | -0.241003 | 2.790299 | 2472.551 |
| $t_{ZK4}$ | 0.01186464 | 36.86736 | 187.1345 | $t_{36}$ | 1.132527 | 3.089483 | 2233.11 |
| $t_{ZK5}$ | -0.2885746 | 55.6611 | 123.9493 | $t_{37}$ | 66.30169 | 4400.433 | 1.567835 |
| $t_{ZK6}$ | -2.669514 | 140.9721 | 48.93988 | $t_{38}$ | -252.9175 | 149222.7 | 0.04623397 |
| $t_{ZK7}$ | -1039.578 | 1150628 | 0.00599599 | $t_{39}$ | 0.8997042 | 2.959664 | 2331.061 |
| $t_{ZK8}$ | -1395.647 | 4542102 | 0.001518934 | **Proposed Estimators** $t_{qi}$ | | | |
| $t_{ZK9}$ | -2.11172 | 123.0889 | 56.05017 | $t_{40}$ | 0.03228859 | 2.761002 | 2498.787 |
| **Zaman (2020)** $t_{Zi}$ | | | | $t_{41}$ | -0.1555318 | 2.67584 | 2498.791 |
| $t_{Z0}$ | 0.1190468 | 2.844141 | 2425.743 | $t_{42}$ | 0.03228801 | 2.760998 | 2498.79 |
| $t_{Z1}$ | 0.2357007 | 2.851608 | 2419.391 | $t_{43}$ | 0.0322362 | 2.760946 | 2498.837 |
| $t_{Z2}$ | 0.2598529 | 2.855944 | 2415.718 | $t_{44}$ | 0.03228811 | 2.760999 | 2498.79 |
| $t_{Z3}$ | -4.228685 | 8.862594 | 778.4578 | $t_{45}$ | 0.03228787 | 2.760998 | 2498.791 |
| $t_{Z4}$ | 0.2539563 | 2.855833 | 2415.812 | $t_{46}$ | 0.03228559 | 2.760994 | 2498.794 |
| $t_{Z5}$ | 0.2159155 | 2.848987 | 2421.617 | $t_{47}$ | -3.955063 | 939.421 | 7.34405 |
| $t_{Z6}$ | -0.890612 | 3.13349 | 2201.748 | $t_{48}$ | 0.01424415 | 2.771623 | 2489.211 |
| $t_{Z7}$ | -631.3627 | 334987.7 | 0.02059525 | $t_{49}$ | 0.03228681 | 2.760996 | 2498.792 |

Table 6 and 7 show the results of Biases, MSEs and PREs of the proposed estimators and that of the existing estimators using stimulated data. The result revealed that the proposed estimators has minimum MSEs and higher PREs compared to conventional and other related estimators considered in this study. This implies that the proposed estimators are more efficient and can produce better estimate of the population parameters than the existing estimators.

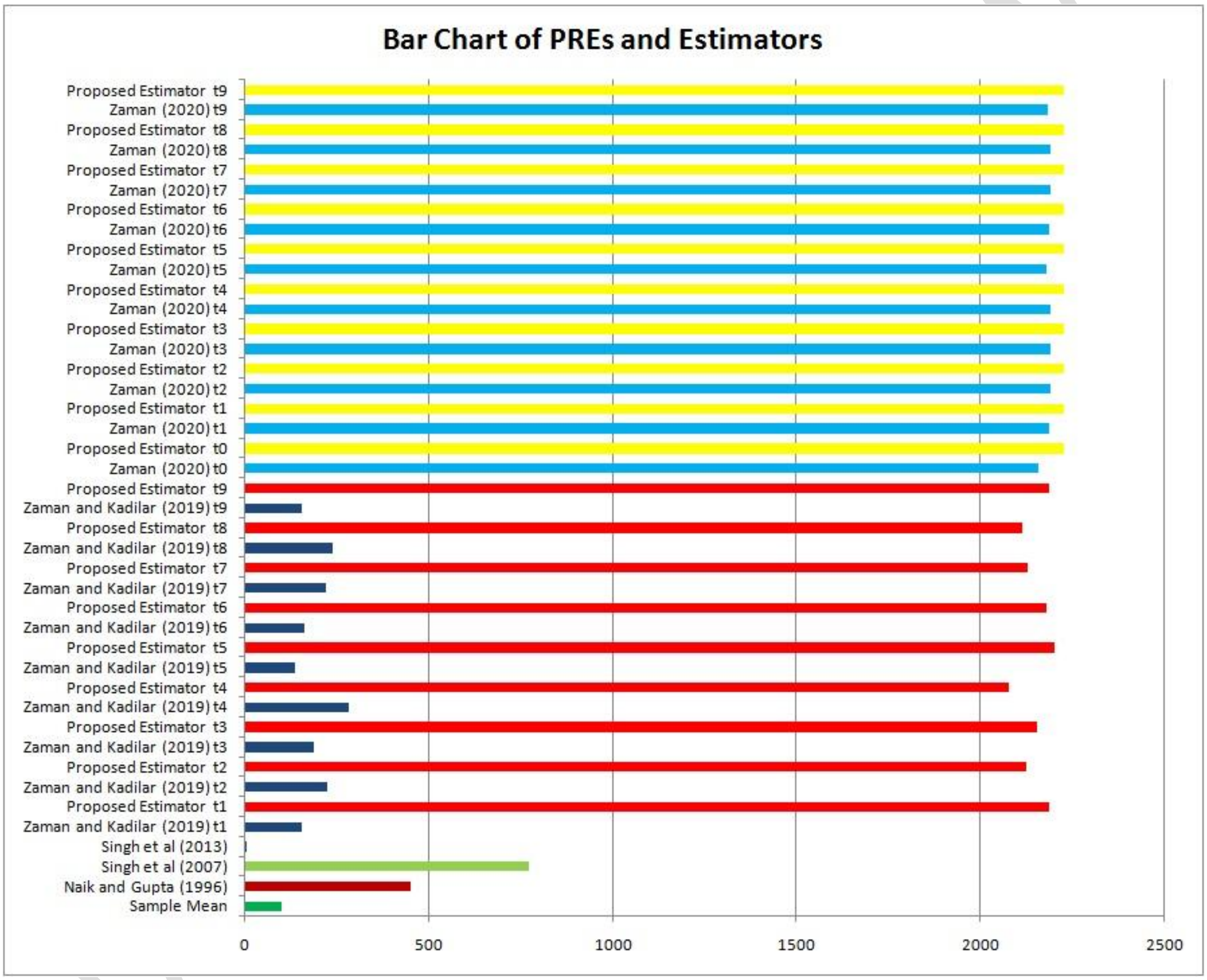


**Figure 3:** Represent the Bar Chart of PREs and Estimator with the correlation ( $\rho_{y\phi}$ =0.9775665)

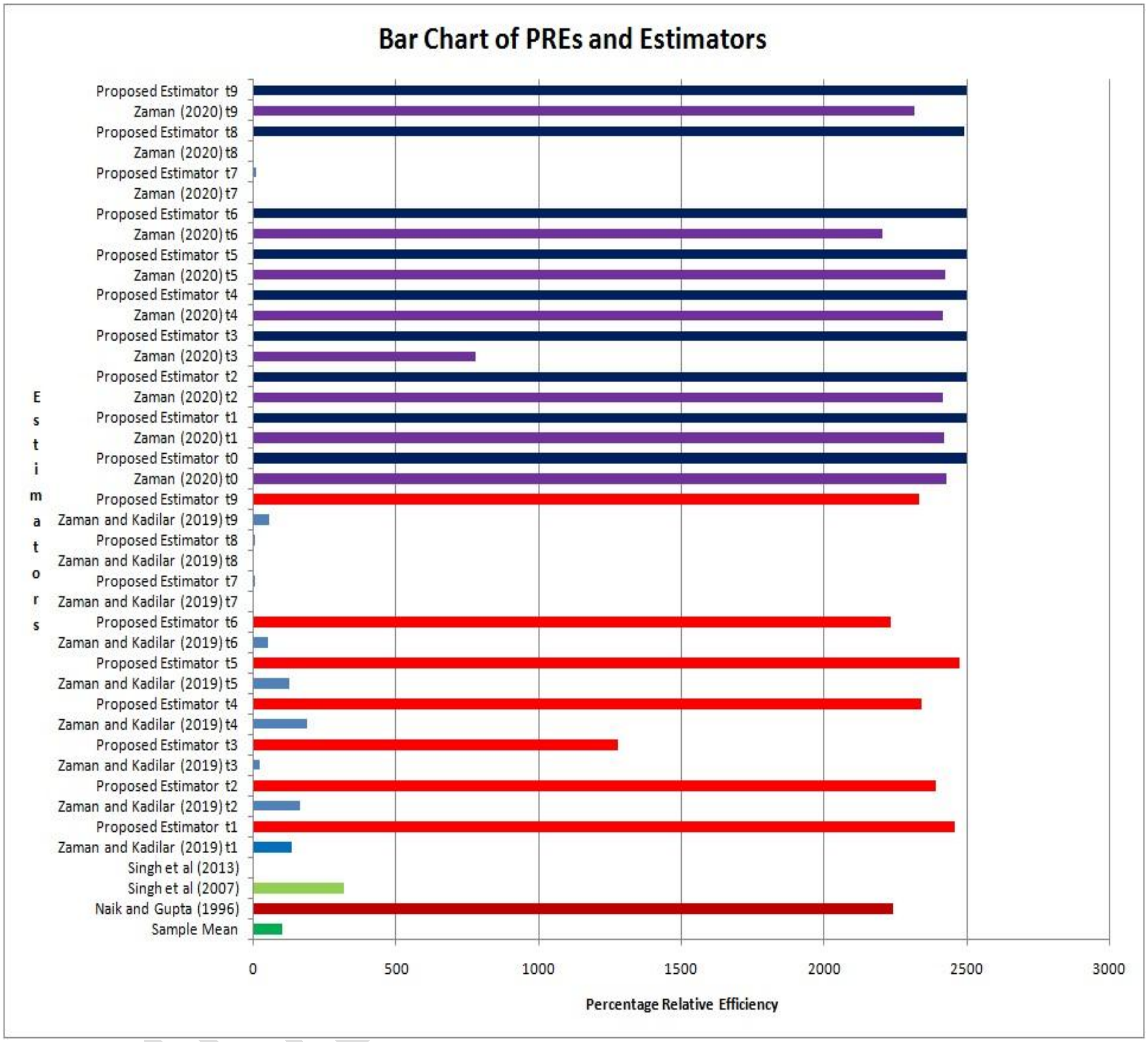


**Figure 4:** Represent the Bar Chart of PREs and Estimator with the correlation ( $\rho_{y\phi}$ =-0.9791406)

## 5. Conclusion

From the results of the empirical studies, it revealed that the suggested estimators have minimum MSEs compared to other competing estimators considered in literature. Hence, the suggested estimators demonstrated high level of efficiency over the other estimators considered in the study. In the other words, the suggested estimators have higher chance of producing estimate that is closer to the true value of the population mean than other estimators considered in the literature of this study and therefore recommended for use in practice.

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