

Probabilities and Probable Solutions of a Modified KdV Type Nonlinear Partial Differential Equation

ABSTRACT

The goal of this work is not only the search for the solutions of a nonlinear partial differential equation, but how to locate and choose a form of solution verifying the nonlinear partial differential equation. In this work, we use the probabilities of appearance of the pairs (n, m) linked to iB-functions for which certain terms of the range of coefficients equations are grouped together to locate and then determine the solutions of the partial differential equation of the KdV type. The pairs (n, m) when identified, indicate with precision the iB-function which will choose from the start as the solution function which we want to build. The probabilities here are essential data to select the analytical sequences of the solutions to be investigated.

Keywords: KdV equation, iB-functions, solitary wave, range equations, field of possibilities of Solutions, probabilities of the pairs, dominant pairs

1. INTRODUCTION

Most often the fundamental difficulty encountered in physical science is that of finding solutions to equations which model the dynamics of physical systems. These equations in the majority are nonlinear and even strongly nonlinear [1-14]. When the integral method is limited in the resolution, one proceeds by searching for the forced solutions[15-25]. But what is even more difficult to do is to make the choices of the solution function to introduce into the considered equation.

Within the framework of the resolution of certain types of nonlinear and dispersive partial differential equations, we have shown that the use of the iB-function [26-30] was very appropriate in this case.

But this function being multiple, that is to say varying according to three characteristic parameters n , m and α , choose with precision the values of n , m and α so that the iB-function is solution of the nonlinear partial differential equation to solve is also difficult. We realized that the most probable solution to verify the partial differential equation considered depends on the number of times that the pair (n, m) considered favors the grouping of the terms in the range equation that is a probability of appearance of the pairs (n, m) in the total number of pairs which are at the origin of the regroupings [31]. In this work, we go through the probabilities of appearance of the pairs to locate the solutions of modified nonlinear KdV equation type. The principle consists in injecting into the modified KdV equation, the solution of the form $aJ_{n,m}(\alpha x - \alpha_0 t)$, where a, α, α_0, n and m are arbitrary

constants, x the independent variable, t the temporal variable, and then listing all the pairs (n, m) for which certain terms of the coefficient range equation are grouped together and finally identify the most favorable pairs for obtaining solutions. Why use the concept of probability in the choice of pairs (n, m) ? Simply because, it makes it possible to avoid the hazardous choices of the forms of solutions to be constructed.

We organize this work in three main sections which are: Some notions on the iB-function, results and discussion which has for subsections : obtaining the coefficient range equation, the probabilities of the possibilities of grouping, the calculations of the coefficients of the terms of the range equation, the search for implicit solutions, the deduction of trigonometric solutions and the conclusion.

2. IB-FUNCTIONS

iB- implicit functions are generally defined by

$$J_{n,m} \left(\sum_{i=0}^p \alpha_i x_i \right) = \sinh^m \left(\sum_{i=0}^p \alpha_i x_i \right) / \cosh^n \left(\sum_{i=0}^p \alpha_i x_i \right), \quad (1)$$

where $J_{n,m} \left(\sum_{i=0}^p \alpha_i x_i \right)$ represents the implicit form of the function,

$\sinh^m \left(\sum_{i=0}^p \alpha_i x_i \right) / \cosh^n \left(\sum_{i=0}^p \alpha_i x_i \right)$ the explicit form of the function, α_i ($i = 0, 1, 2, \dots, p$)

represent the parameters associated with the independent variables x_i ($i = 0, 1, 2, \dots, p$), the pair $(n, m) \in R^2$ indicates the power of the function. More precisely, n is the power

of $\cosh \left(\sum_{i=0}^p \alpha_i x_i \right)$ and m the power of $\sinh \left(\sum_{i=0}^p \alpha_i x_i \right)$. This function, as defined in relation

(1), is also called the iB-functions of several variables and any derivative operation undertaken in this case is partial.

The iB-functions of a single variable is defined by

$$J_{n,m}(\alpha x) = \sinh^m(\alpha x) / \cosh^n(\alpha x), \quad (2)$$

where $J_{n,m}(\alpha x)$ represents the implicit form of the function, α represents the parameter associated with the independent variable x , the pair $(n, m) \in R^2$ indicates the power of the function.

some important transformations are given by

$$J_{n+1,m+1} = J_{n,m} J_{1,1}, \quad (3)$$

$$J_{n+2,m+2} = J_{n,m} J_{2,2}, \quad (4)$$

$$J_{n+p,m+p} = J_{n,m} J_{p,p}, \quad (5)$$

$$J_{n-p,m-p} = J_{n,m} J_{-p,-p}, \quad (6)$$

$$J_{n+p,m+p} = J_{2p,2p} J_{n-p,m-p}, \quad (7)$$

$$\begin{aligned}
J_{n,m} \cdot J_{n',m'} &= J_{n+n',m+m'}, \\
J_{n,m} \cdot J_{m,n} &= J_{m+n,m+n},
\end{aligned} \tag{8}$$

$$J_{m_1,n_1} \cdot J_{m_2,n_2} \cdots J_{m_p,n_p} = J_{m_1+m_2+\dots+m_p, n_1+n_2+\dots+n_p}, \tag{9}$$

$$\frac{dJ_{n,m}}{dx} = m\alpha J_{n-1,m-1} - n\alpha J_{n+1,m+1}, \tag{10}$$

and

$$J_{n,m}(i\alpha x) = (i)^m T_{n,m}(\alpha x), i^2 = -1. \tag{11}$$

Some of properties in its compact forms which facilitate the addition and multiplication of expressions are given by the following formulas

$$J_{n,m}(x+y) = \frac{\sum_{k'=0}^m C_m^{k'} J_{-k',m-k'}(x) J_{-m+k',k'}(y)}{\sum_{k=0}^n C_n^k J_{-n+k,k}(x) J_{-n+k,k}(y)}, \tag{12}$$

By matching $y \rightarrow -y$, we obtain

$$J_{n,m}(x-y) = \frac{\sum_{k'=0}^m (-1)^{k'} C_m^{k'} J_{-k',m-k'}(x) J_{-m+k',k'}(y)}{\sum_{k=0}^n (-1)^k C_n^k J_{-n+k,k}(x) J_{-n+k,k}(y)}, \tag{13}$$

The compact trigonometric formulas which result from formulas (12) and (13) are given by

$$J_{n,m}[i(x+y)] = \frac{\sum_{k'=0}^m C_m^{k'} J_{-k',m-k'}(ix) J_{-m+k',k'}(iy)}{\sum_{k=0}^n C_n^k J_{-n+k,k}(ix) J_{-n+k,k}(iy)}, i^2 = -1, \tag{14}$$

and

$$J_{n,m}[i(x-y)] = \frac{\sum_{k'=0}^m (-1)^{k'} C_m^{k'} J_{-k',m-k'}(ix) J_{-m+k',k'}(iy)}{\sum_{k=0}^n (-1)^k C_n^k J_{-n+k,k}(ix) J_{-n+k,k}(iy)}, i^2 = -1. \tag{15}$$

3. RESULTS AND DISCUSSION

3.1 Obtaining the main range equation

The modified KdV equation chosen to solve is of the form[32]

$$U_t + \alpha U^2 U_x + \beta U_x U_{xx} + \gamma U U_{xxx} + U_{xxxx} = 0, \tag{16}$$

where α , β and γ are nonlinear coefficients. We assign arbitrary coefficients to the above equation to obtain its following generalized form.

$$n_0 U_t + n_1 U^2 U_x + n_2 U_x U_{xx} + n_3 U U_{xxx} + n_4 U_{xxxx} = 0. \quad (17)$$

The goal being not only the resolution of the equation, but also the obtaining of the relations of constraints linking the coefficients n_i ($i=0, \dots, 4$) favoring the widening of the field of analysis of the solutions. In order to build solutions in the form

$$U(x, t) = a J_{n,m}(\alpha x - \alpha_0 t), \quad (18)$$

where a , α and α_0 are arbitrary constants, n and m the indicators of the iB-function, we set the change of variable $\xi = \alpha x - \alpha_0 t$ and equation (17) becomes

$$-n_0 \alpha_0 U_\xi + n_1 \alpha U^2 U_\xi + n_2 \alpha^3 U_\xi U_{\xi\xi} + n_3 \alpha^3 U U_{\xi\xi\xi} + n_4 \alpha^5 U_{\xi\xi\xi\xi\xi} = 0. \quad (19)$$

With regard to the terms of equation (18), we note that progress in the search for the desired solution requires to calculate the successive derivatives which constitute the equation. So the most imposing terms give

$$U^2 U_\xi = a^3 m J_{3n-1, 3m-1} - a^3 n J_{3n+1, 3m+1}, \quad (20)$$

$$U_\xi U_{\xi\xi} = A_1 J_{2n-3, 2m-3} - A_2 J_{2n-1, 2m-1} + A_3 J_{2n+1, 2m+1} - A_4 J_{2n+3, 2m+3}, \quad (21)$$

with

$$A_1 = a^2 m^2 (m-1), \quad (22)$$

$$A_2 = a^2 n m (m-1) + a^2 m^2 (n-1) + a^2 n m (m+1), \quad (23)$$

$$A_3 = a^2 n m (n+1) + a^2 n m (n-1) + a^2 n^2 (m+1), \quad (24)$$

and

$$A_4 = a^2 n^2 (n+1), \quad (25)$$

$$U U_{\xi\xi\xi} = B_1 J_{2n-3, 2m-3} - B_2 J_{2n-1, 2m-1} + B_3 J_{2n+1, 2m+1} - B_4 J_{2n+3, 2m+3}, \quad (26)$$

with

$$B_1 = a^2 m (m-1)(m-2), \quad (27)$$

$$B_2 = a^2 m (m-1)(n-2) + a^2 m^2 (n-1) + a^2 n m (m+1), \quad (28)$$

$$B_3 = a^2 n m (n-1) + a^2 n^2 (m+1) + a^2 n (n+1)(m+2), \quad (29)$$

and

$$B_4 = a^2 n (n+1)(n+2), \quad (30)$$

$$U_{\xi\xi\xi\xi\xi} = C_1 J_{n-5, m-5} - C_2 J_{n-3, m-3} + C_3 J_{n-1, m-1} - C_4 J_{n+1, m+1} + C_5 J_{n+3, m+3} - C_6 J_{n+5, m+5}, \quad (31)$$

with

$$C_1 = a m (m-1)(m-2)(m-3)(m-4), \quad (32)$$

$$C_2 = a m (m-1)(m-2)(m-3)(n-4) + a m (m-1)(m-2)^2 (n-3) + a m (m-1)(n-2)(m-1)(m-2) + a m^2 (n-1)(m-2)^2 + a n m (m+1)(m-1)(m-2), \quad (33)$$

$$\begin{aligned}
C_3 = & am(m-1)(m-2)(n-3)(n-2) + am(m-1)^2(n-2)^2 + am^2(n-1)(m-2)(n-2) \\
& + amn(m+1)(m-1)(n-2) + am^2(m-1)(n-2)(n-1) + am^3(n-1)^2 + am^2n(m+1)(n-1) \\
& + anm^2(n-1)(m+1) + amn^2(m+1)^2 + anm(n+1)(m+2)(m+1),
\end{aligned} \tag{34}$$

$$\begin{aligned}
C_4 = & am(m-1)(n-2)(n-1)n + am^2(n-1)^2n + amn^2(m+1)(n-1) \\
& + an^2m(n-1)(m+1) + an^3(m+1)^2 + an^2(n+1)(m+2)(m+1) + anm(n-1)(n+1)(m+2) \\
& + an^2(m+1)(n+1)(m+2) + an(n+1)^2(m+2)^2 + an(n+1)(n+2)(m+3)(m+2),
\end{aligned} \tag{35}$$

$$\begin{aligned}
C_5 = & anm(n-1)(n+1)(n+2) + an^2(m+1)(n+1)(n+2) + an(n+1)^2(m+2)(n+2) \\
& + an(n+1)(n+2)^2(m+3) + an(n+1)(n+2)(n+3)(m+4),
\end{aligned} \tag{36}$$

and

$$C_6 = an(n+1)(n+2)(n+3)(n+4). \tag{37}$$

The taking into account of the equations (20) to (37) in the equation (19) leads to the following equation

$$\begin{aligned}
& n_4\alpha^5 C_1 J_{n-5,m-5} - n_4\alpha^5 C_2 J_{n-2,m-3} + (n_4\alpha^5 C_3 - n_0\alpha_0 am) J_{n-1,m-1} + (n_0\alpha_0 an - n_4\alpha^5 C_4) J_{n+1,m+1} \\
& + n_4\alpha^5 C_5 J_{n+3,m+3} - n_4\alpha^5 C_6 J_{n+5,m+5} + (n_2\alpha^3 A_1 + n_3\alpha^3 B_1) J_{2n-3,2m-3} - (n_2\alpha^3 A_2 + n_3\alpha^3 B_2) J_{2n-1,2m-1} \\
& + (n_2\alpha^3 A_3 + n_3\alpha^3 B_3) J_{2n+1,2m+1} - (n_2\alpha^3 A_4 + n_3\alpha^3 B_4) J_{2n+3,2m+3} + n_1\alpha ma^3 J_{3n-1,3m-1} - n_1\alpha na^3 J_{3n+1,3m+1} = 0,
\end{aligned} \tag{38}$$

Equation (38) is the main coefficient range equation to analyze. Thus, the different values of n and m favorable to the search for solutions will be given in the following sections.

3.2 Field of possible solutions

We are looking for the values of n and m for which certain terms of equation (38) are grouped together. Thus, to obtain the values of n and m for which certain terms of equation (38) regroup, we solve the pairs of equations in n and m such that if $\lambda_i J_{n,m}$ and $\lambda_j J_{n',m'}$ (with $i \neq j$) are two terms of the equation (30) and where λ_i and λ_j are constants, we have simultaneously $n = n'$ and $m = m'$.

In this quest, we count a total of 42 pairs of equations which lead to the determination of the pairs (n, m) . Such that we have

$$n, m \in \{-8, -6, -4, -3, -2, -1, 0, 1, 2, 3, 4, 6, 8\}. \tag{39}$$

The combination of the values of n and m for which there is a grouping of the terms makes it possible to obtain a table of the possibilities of solutions comprising 169 pairs.

The 169 pairs constitute the extended field of pairs for which the search for solutions must be made. But the probabilities of appearance of pairs make it possible to determine the most probable pairs and thereby reduce the field of possibilities in order to obtain a restricted field of possibilities in which the effective research will be carried out.

3.3 Probabilities of appearance of pairs (n, m) and dominant pairs

On the 42 pairs of equations solved in n and m , the probabilities of obtaining pairs (n, m) for which certain terms of the range equation are grouped together are given by

$$P(-2, -2) = 7/42, \quad P(-3, -3) = 2/42, \quad P(-6, -6) = 2/42, \quad P(-8, -8) = 1/42, \\ P(4, 4) = 4/42, \quad P(2, 2) = 8/42, \quad P(0, 0) = 8/42, \quad P(1, 1) = 2/42, \\ P(-4, -4) = 2/42, \quad P(6, 6) = 2/42, \quad P(-1, -1) = 2/42, \quad P(3, 3) = 1/42, \\ P(8, 8) = 1/42.$$

$P(n, m)$ represents the fraction of the number of times that the couple (n, m) appears in the grouping possibilities or the probability of obtaining the solution for the pair (n, m) . The combination of the values of n and m obtained above gives the pairs of the main field or extended field of solutions research. With regard to the probabilities of appearance of the pairs, we remark that $P(0, 0) = 8/42$, $P(-2, -2) = 7/42$ and $P(2, 2) = 8/42$. Then, the pairs $(0, 0)$, $(-2, -2)$ and $(2, 2)$ are the dominant pairs.

A combination of the values of n and m for these dominant pairs forms the restricted field of search for solutions. The following table is the narrow field of the search for solutions such that the pairs which form it will examine in detail to see if they lead to solutions.

Table 1: Restricted field of possibilities

(n, m)	-2	0	2
-2	$(-2, -2)$	$(-2, 0)$	$(-2, 2)$
0	$(0, -2)$	$(0, 0)$	$(0, 2)$
2	$(2, -2)$	$(2, 0)$	$(2, 2)$

The goal of obtaining the restricted field of pairs aims to verify whether in addition to the dominant pairs, there are other pairs which lead to non-trivial solutions. Thus, in the following lines, we will solve equation (38) for the different values of the above pairs. But before going to the effective resolution, we will determine the values of $A_i (i = 1, \dots, 4)$, $B_i (i = 1, \dots, 4)$ and $C_i (i = 1, \dots, 6)$ for each pair (n, m) of the restricted field.

3.3 Calculation of the coefficients of the terms

For each couple in the restricted field of possibilities, the values of $A_i (i = 1, \dots, 4)$,

$B_i (i = 1, \dots, 4)$ and $C_i (i = 1, \dots, 6)$ are as follows.

- For $(n, m) = (-2, -2)$, we have

$$A_1 = -12a^2,$$

$$A_2 = -28a^2, A_3 = -20a^2, A_4 = -4a^2, B_1 = -24a^2, B_2 = -42a^2, B_3 = -16a^2, B_4 = 0,$$

$$C_1 = -720a, C_2 = -1488a, C_3 = -1280a, C_4 = -272a, C_5 = 0, C_6 = 0.$$

- For $(n, m) = (2, 2)$, we have

$$A_1 = 4a^2,$$

$$A_2 = 20a^2, A_3 = 28a^2, A_4 = 12a^2, B_1 = 0, B_2 = 16a^2, B_3 = 40a^2, B_4 = 24a^2, C_1 = 0, \\ C_2 = 0, C_3 = 260a, C_4 = 1232a, C_5 = 1680a, C_6 = 720a.$$

- For $(n, m) = (2, 0)$, we have

$$A_1 = 0,$$

$$A_2 = 0, A_3 = 4a^2, A_4 = 12a^2, B_1 = 0, B_2 = 0, B_3 = 16a^2, B_4 = 24a^2, C_1 = 0, C_2 = 0, \\ C_3 = 0, C_4 = 272a, C_5 = 960a, C_6 = 240a.$$

- For $(n, m) = (-2, 0)$, we have

$$A_1 = 0,$$

$$A_2 = 0, A_3 = 4a^2, A_4 = -4a^2, B_1 = 0, B_2 = 0, B_3 = 8a^2, B_4 = 0, C_1 = 0, C_2 = 0, C_3 = 0, \\ C_4 = -32a, C_5 = 4a, C_6 = 0.$$

- For $(n, m) = (2, -2)$, we have

$$A_1 = -12a^2,$$

$$A_2 = 4a^2, A_3 = -20a^2, A_4 = 12a^2, B_1 = -24a^2, B_2 = 8a^2, B_3 = -8a^2, B_4 = 24a^2, \\ C_1 = -720a, C_2 = -224a, C_3 = -32a, C_4 = 0, C_5 = 240a, C_6 = -240a.$$

- For $(n, m) = (-2, 2)$, we have

$$A_1 = 4a^2,$$

$$A_2 = -28a^2, A_3 = 28a^2, A_4 = -4a^2, B_1 = 0, B_2 = -32a^2, B_3 = 32a^2, B_4 = 0, C_1 = 0, \\ C_2 = 0, C_3 = 464a, C_4 = -512a, C_5 = 0, C_6 = 0.$$

- For $(n, m) = (0, -2)$, we have

$$A_1 = -12a^2,$$

$$A_2 = -4a^2, A_3 = 0, A_4 = 0, B_1 = -24a^2, B_2 = -16a^2, B_3 = 0, B_4 = 0, C_1 = -720a, \\ C_2 = -400a, C_3 = -280a, C_4 = 0, C_5 = 0, C_6 = 0.$$

- For $(n, m) = (0, 2)$, we have

$$A_1 = 4a^2,$$

$$A_2 = -4a^2, A_3 = 0, A_4 = 0, B_1 = -4a^2, B_2 = 0, B_3 = 0, B_4 = 0, C_1 = 0, C_2 = 0, C_3 = 24a, \\ C_4 = 0, C_5 = 0, C_6 = 0.$$

3.4 Solving the main range equation

When we fix the values of the pairs (n, m) , the resulting range equation is the secondary coefficient range equation. Thus, we look for the solutions of equation (38) for the pairs of the restricted field of possibilities.

For the pairs $(0,0), (-2,2), (2,-2), (2,0), (0,-2), (-2,0)$ and $(0,2)$ the equation admits trivial solutions so that the search for solutions is reduced only for the pairs $(-2,-2)$ and $(2,2)$.

- Case $(n,m) = (-2,-2)$

Taking into account the pair $(n,m) = (-2,-2)$ in the equation (38) we obtain

$$\begin{aligned} & \left[-720an_4\alpha^5 - 12a^2n_2\alpha^3 - 24a^2n_3\alpha^3 - 2n_1\alpha a^3 \right] J_{-7,-7} + \left[1488an_4\alpha^5 + 28a^2n_2\alpha^3 + 42a^2n_3\alpha^3 + 2n_1\alpha a^3 \right] J_{-5,-5} \\ & - \left[1280an_4\alpha^5 + 2n_0\alpha_0a + 20a^2n_2\alpha^3 + 16a^2n_3\alpha^3 \right] J_{-3,-3} - \left[2n_0\alpha_0a - 272an_4\alpha^5 - 4a^2n_2\alpha^3 \right] J_{-1,-1} = 0, \end{aligned} \quad (40)$$

Equation (40) is valid for $a \neq 0$ if and only if we have

$$360n_4\alpha^4 + (6n_2\alpha^2 + 12n_3\alpha^2)a + n_1a^2 = 0, \quad (41)$$

$$744n_4\alpha^4 + (14n_2\alpha^2 + 21n_3\alpha^2)a + n_1a^2 = 0, \quad (42)$$

$$640n_4\alpha^5 + n_0\alpha_0 + (10n_2\alpha^3 + 8n_3\alpha^3)a = 0, \quad (43)$$

and

$$n_0\alpha_0 - 136n_4\alpha^5 - 2an_5\alpha^3. \quad (44)$$

From equation (44), we obtain

$$a = (n_0\alpha_0 - 136n_4\alpha^5) / 2n_2\alpha^3, \quad n_2 \neq 0, \quad \alpha \neq 0. \quad (45)$$

The introduction of the equation (45) in the equations (41) to (44) permits to obtain the constraint relation

$$40n_2 + 61n_3 = 0. \quad (46)$$

The solution in this case is given by

$$U(\xi) = \left[(n_0\alpha_0 - 136n_4\alpha^5) / 2n_2\alpha^3 \right] J_{-2,-2}(\xi) \Rightarrow U(x,t) = \left[(n_0\alpha_0 - 136n_4\alpha^5) / 2n_2\alpha^3 \right] J_{-2,-2}(\alpha x - \alpha_0 t). \quad (47)$$

- Case $(n,m) = (2,2)$

Taking into account the pair $(n,m) = (2,2)$ in the equation (38) we obtain

$$\begin{aligned} & \left[260an_4\alpha^5 - 2n_0\alpha_0a + 4a^2n_2\alpha^3 \right] J_{1,1} + \left[2n_0\alpha_0a - 1232an_4\alpha^5 - 20a^2n_2\alpha^3 - 16a^2n_3\alpha^3 \right] J_{3,3} \\ & + \left[28a^2n_2\alpha^3 + 40a^2n_3\alpha^3 + 2n_1\alpha a^3 \right] J_{5,5} - \left[720n_4\alpha^5 + 12a^2n_2\alpha^3 + 24a^2n_3\alpha^3 + 2n_1\alpha a^3 \right] J_{7,7} = 0, \end{aligned} \quad (48)$$

Equation (48) is valid for $a \neq 0$ if and only if we have

$$130n_4\alpha^5 - n_0\alpha_0 + 2an_2\alpha^3 = 0, \quad (49)$$

$$14an_2\alpha^2 + 20an_3\alpha^2 + n_1a^2 = 0, \quad (50)$$

$$360n_4\alpha^4 + 6an_2\alpha^2 + 12an_3\alpha^2 + n_1a^2 = 0, \quad (51)$$

and

$$n_0\alpha_0 - 616n_4\alpha^5 - 10an_2\alpha^3 - 8an_3\alpha^3 = 0. \quad (52)$$

From equation (52), we obtain

$$a = (130n_4\alpha^5 - n_0\alpha_0) / 2n_2\alpha^3, n_2 \neq 0, \alpha \neq 0. \quad (53)$$

The introduction of the equation (53) in the equations (49) to (52) permits to obtain the constraint relation

$$213n_2^2 - 117n_2n_3 - 240n_3^2 = 0. \quad (54)$$

The solution in this case is given by

$$U(\xi) = \left[(130n_4\alpha^5 - n_0\alpha_0) / 2n_2\alpha^3 \right] J_{2,2}(\xi) \Rightarrow U(x,t) = \left[(130n_4\alpha^5 - n_0\alpha_0) / 2n_2\alpha^3 \right] J_{2,2}(\alpha x - \alpha_0 t). \quad (55)$$

3.5 Trigonometric solutions

One of the great peculiarities of the use of the iB-function is that it facilitates the passage from the hyperbolic form to the trigonometric form and vice versa. When we make the correspondences $\alpha \rightarrow i\alpha$ and $\alpha_0 \rightarrow i\alpha_0$ with $i^2 = -1$, we obtain respectively from equations (46) and (47), the following trigonometric solutions

$$U(x,t) = -i \left[(n_0\alpha_0 - 136n_4\alpha^5) / 2n_2\alpha^3 \right] \cot \alpha n^2 (\alpha x - \alpha_0 t), \quad (56)$$

and

$$U(x,t) = -i \left[(130n_4\alpha^5 - n_0\alpha_0) / 2n_2\alpha^3 \right] \tan^2 (\alpha x - \alpha_0 t). \quad (57)$$

4. CONCLUSION

The objective of this work was to show how, in the impossibility of using the integral methods to solve a nonlinear partial differential equation, one can proceed to choose or know the suitable form of solution. To this end, we decided to use this technique to first locate the forms of solutions and then build them by relying on the modified partial differential equations of the KdV type. For this purpose, we have considered building a solution of the form $aJ_{n,m}(\alpha x - \alpha_0 t)$ where $a, n, m, \alpha, \alpha_0$ are real constants to be determined. But the fixed values of n and m are those which make it possible to indicate with precision the solution functions. It is for this that the work to be done first and foremost consisted in determining the values of n and m for which we suspect the solutions. In this perspective, we have obtained 42 possible (n, m) pairs forming what we have called the extended field of the possibilities of solutions. But of all these pairs, only a few are more favorable to obtaining the solutions. These pairs, called dominant pairs, are identified through a high probability of presence among the pairs for which certain terms of the coefficient range equation are grouped together.

In the case of this study, we have the pairs $(0, 0)$, $(-2, -2)$ and $(2, 2)$ having respectively for probability $P(0, 0) = 8/42$, $P(-2, -2) = 7/42$ and $P(2, 2) = 8/42$. These three pairs indicated are those which are more favorable to obtaining the solutions. But in order to detect other particular solutions which do not appear at first glance, we have combined the values of n and m of the dominant pairs to make what we have called a restricted field of the possibilities of obtaining solutions.

This allowed to obtain a total of nine (n, m) pairs that we thoroughly examined in search of possible solutions. Of all these analyzes, only the dominant pairs $(-2, -2)$ and $(2, 2)$ made

it possible to have non-trivial solutions, the pair $(0,0)$ leading to a trivial solution like $U(\xi) = a$. We have deduced from the implicit solutions obtained, the trigonometric solutions by making use of the magnificent properties of iB-functions.

These results obtained confirm our predictions, namely, only the $(n,m) \neq (0,0)$ pairs having the greatest probabilities of appearance among the pairs of the field of possibilities are the most favorable to obtaining the solutions.

We can see that unlike the classical KdV equation which has the third-order dispersion term and admits a pulse-type solitary wave solution for $(n,m) = (2,0)$, the modified KdV equation has the term of dispersion of order six and as treated in this article, admits non-trivial solutions just for pairs $(-2,-2)$ and $(2,2)$, which are solitary wave solutions of the kink type.

REFERENCES

1. Bogning J R. Pulse soliton solutions of the modified KdV and Born-Infeld equations , International Journal of Modern Nonlinear Theory and Application. 2013; 2 :135-140.
2. Bogning J R, Porsezian K, Fautso Kuiaté G, Omanda H M. gap solitary pulses induced by the Modulational instability and discrete effects in array of inhomogeneous optical fibers , Physics Journal. 2015;1(3):216-224.
3. Bogning J R. Nth Order Pulse Solitary Wave Solution and Modulational Instability in the Boussinesq Equation. American Journal of Computational and Applied Mathematics.2015; 5(6):182-188.
4. Bogning J R, Fautso Kuiaté G, Omanda H M and Djeumen Tchaho C T. Combined Peakons and multiple-peak solutions of the Camassa-Holm and modified KdV equations and their conditions of obtention. Physics Journal. 2015;1(3): 367-374.
5. Bogning J R. Analytical soliton solutions and wave solutions of discrete nonlinear cubic-quintique Ginzburg-Landau equations in array of dissipative optical system,. American Journal of Computational and Applied Mathematics. 2013 ; 3(2) : 97-105.
6. Bogning J R and Kofané T C. Analytical solutions of the discrete nonlinear Schrödinger equation in arrays of optical fibers, Chaos, Solitons & Fractals. 2006;28(1):148-153.
7. Bogning J R. Sechⁿ Solutions of the generalized and modified Rosenau-Hyman Equations, Asian Journal of Mathematics and Computer Research. 2015; 9(1): 1-7.
8. Bogning J R, Djeumen Tchaho C T and Omanda H M. Combined solitary wave solutions in higher-order effects optical fibers, British Journal of Mathematics and Computer Science.2016; 13(3): 1-12.
9. Djeumen Tchaho C T, Bogning J R and Kofané T C. Modulated Soliton Solution of the Modified Kuramoto-Sivashinsky's Equation, American Journal of Computational and Applied Mathematics. 2012; 2(5): 218-224.
10. Djeumen Tchaho C T, Bogning J R and Kofane T C. Multi-Soliton solutions of the modified Kuramoto-Sivashinsky's equation by the BDK method, Far East J. Dyn. Sys.2011; 15(2): 83-98.
11. Djeumen Tchaho C T, Bogning J R and Kofane T C. Construction of the analytical solitary wave solutions of modified Kuramoto-Sivashinsky equation by the method of identification of coefficients of the hyperbolic functions, Far East J. Dyn. Sys, 2010; 14(1): 14-17.
12. Njikue R, Bogning J R and Kofane T C. Exact bright and dark solitary wave solutions of the generalized higher order nonlinear Schrödinger equation describing the propagation of ultra-short pulse in optical fiber , J. Phys. Commun. 2018; 2: 025030.
13. Bogning J R and Kofané T C. Solitons and dynamics of nonlinear excitations in the

- array of optical fibers, *Chaos, Solitons & Fractals* .2006; 27(2): 377-385.
14. Bogning J R. Exact solitary wave solutions of the (3+1) modified B-type Kadomtsev-Petviashvili family equations, *American Journal of computational and applied mathematics*. 2018; 8(5):85-92.
 15. Bogning J R, Djeumen Tchaho C T and Kofané T C. Construction of the soliton solutions of the Ginzburg-Landau equations by the new Bogning-Djeumen Tchaho-Kofané method, *Physica Scripta*. 2012; 85: 025013-025018.
 16. Bogning J R, Djeumen Tchaho C T and Kofané T C. Generalization of the Bogning-Djeumen Tchaho-Kofané Method for the construction of the solitary waves and the survey of the instabilities, *Far East J. Dyn. Sys.* 2012;20(2):101-119.
 17. Tiague Takongmo Guy and Bogning J R. Construction of solitary wave solutions of higher-order nonlinear partial differential equations modeled in a nonlinear hybrid electrical line, *American Journal of circuits, systems and signal processing*. 2018; 4(3):36-44.
 18. Tiague Takongmo Guy and Bogning J R. Construction of solitary wave solutions of higher-order nonlinear partial differential equations modeled in a modified nonlinear Noguchi electrical line, *American Journal of circuits, systems and signal processing*. 2018;4(1)8-14
 19. Tiague Takongmo Guy and Bogning J R. Construction of solitary wave solutions of higher-order nonlinear partial differential equations modeled in a nonlinear capacitive electrical line, *American Journal of circuits, systems and signal processing*. 2018; 4(2):15-22.
 20. Tiague Takongmo Guy and Bogning J R. Construction of solutions in the shape (pulse, pulse) and (kink, kink) of a set of two equations modeled in a nonlinear inductive electrical line with crosslink capacitor, *American Journal of circuits, systems and signal processing*. 2018; 4(2):28-35.
 21. Tiague Takongmo Guy and Bogning J R. (kink, kink) and (pulse, pulse) exact solutions of equations modeled in a nonlinear capacitive electrical line with capacitor, *American Journal of circuits, systems and signal processing* 2018;4(3):45-53.
 22. Tiague Takongmo Guy and Bogning J R. Solitary wave solutions of modified telegraphist equations modeled in an electrical line, *Physics Journal*, 2018;4(3):29-36.
 23. Tiague Takongmo Guy and Bogning J R. Coupled soliton solutions of modeled equations in a Noguchi electrical line with crosslink capacitor, *Journal of Physics communications*. 2018;2:105016.
 24. Rodrigue Njikue, Bogning J R and Kofané T C. higher order nonlinear Schrödinger equation family in optical fiber and solitary wave solutions, *American journal of optics, American Journal of optics and photonics*. 2018; 6(3): 31-41.
 25. Bogning J R, Djeumen Tchaho C T and Kofané T C. Solitary wave solutions of the modified Sasa- Satsuma nonlinear partial differential equation *American Journal of Computational and Applied Mathematics*. 2013; 3(2): 97-107.
 26. Bogning JR .*Mathématique: les fonctions implicites de Bogning&applications*. Editions universitaires Européenne. Germany; 2019.
 27. Bogning JR. *Mathematics for physics. The implicit Bogning functions&applications*. Lambert Academic Publishing. Germany; 2019.
 28. Bogning J R. *Mathematics for nonlinear physics :Solitary wave in the center of the resolution of dispersive nonlinear partial differential equations*, Dorrance Publishing Co; USA; 2019.
 29. Bogning JR. *Eléments de la Mécanique Analytique et de la Physique quantique*. Editions universitaires Européenne. Germany; 2020.
 30. Bogning JR. *Elements of Analytical Mechanics and Quantum Physics*. Lambert Academic Publishing. Germany; 2020.
 31. Ngouo Tchinda C and Bogning JR. solitary waves and property management of nonlinear dispersive and flattened optical fiber. *American Journal of optics and*

32. photonics. 2020; 8(1): 87-32.
Russel JS. Report on waves. Report of the fourteenth of the British association for the advancement of science.1884.

UNDER PEER REVIEW