Statistical Analysis of Flow Parameters For The Graphical Simulation Outputs

Abstract

In general, System dynamics simulation software illustrates graphical interpretations. However, the values of Parameters are needed to predict. The motivation of this study is to construct a new multivariate model to predict the flow parameters while simulating the flow under different conditions. The study is looking for the variation of streamline and develop the new multivariate model for velocity magnitude of each elliptic cylinder system. Moreover, the flow regions were divided into three regions according to behavior of streamlines. Consequently, linear regression and semiparametric regression are used to develop new models for flow regions using simulation outputs. Mean square error(MSE), Root of mean square error(RMSE) and Mean absolute percentage error(MAPE) was used to figure out the best fitted model for each flow region. It could be examined a summary statistic and variability based on the fitted smoothing curve of the velocity magnitude. Using these models, the proposed models are useful in predicting magnitude in any point of fluid flow.

Keywords: Multivariate model, Semiparametric regression, Streamlines, Linear Regression

Introduction

The field of the computational fluid dynamics(CFD) makes a significant contribution to understand the physical events that take place in the flow of fluids around and within designated objects. In addition, these events are connected to the action and interaction of phenomena convection, diffusion, boundary layers and turbulence. In the field of aerodynamics, all of these phenomena are governed by compressible Naiver-Stokes equation. Applying the fundamental laws of mechanics to a fluid gives the governing equations for a fluid. The conservation of mass equation and the conservation of momentum equation along with the conservation of momentum equation along with the conservation energy equation from a set of coupled, non-linear partial differential equations. I t is not possible to solve these equations analytically for most engineering problems. However, it is possible to obtain approximate computer-based solutions to the governing equations for a variety of engineering problems. This is the subject matter of computational fluid dynamics.

Previous studies provide, sound knowledge of the flow behaviors of flow past elliptic cylinders and semi parametric regression could be gained. The flow past cylinders have applications in engineering problems such as cooling towers, heat exchangers, chimney stacks, nuclear reactors, and offshore structures. The most of the researchers have now forced to study flow past an elliptic cylinder. Elliptic cylinders are more general geometrical configuration than the canonical circular cylinder and present a richer flow behavior characteristic of typical engineering flow configuration. D' Alessio and Dennis proposed a mathematical model for the steady, 2- dimensional flow of a viscous incompressible fluid past a cylinder in 1994[4].

Moreover, Mittal and Balachandra have presented the results of two- and three- dimensional simulations for a range of flow and geometric parameters [2]. Furthermore, the results were compared with available experimental data and it was found that important quantities like Strouhal numbers and drag coefficients match well with established values

De Silva et al., compared the numerical results with theory when one cylinder is present and the parameter relation with Reynolds number and moreover, flow behavior and the parameter relation is given when two cylinders are present [5]. Most of the scholars mainly use the numerical and computational methods to investigate and study the flow behaviors and parameter relations when elliptic cylinders are present.

Fiosina and Maksims have presented distributed parallel versions of some nonparametric and semiparametric regression models [7]. They used MapReduce paradigm and describe the algorithms in terms of SPARK data structures to parallelize the calculations. The forecasting accuracy of the proposed algorithms was compared with the linear regression model, which was the only forecasting model currently having parallel distributed realization within the SPARK framework to address big data problems. The advantages of the parallelization of the algorithm were also provided.

Ludwig et al. aimed to investigate such effects in spline-based semiparametric regression for spatial data [9]. They have discussed estimators' behavior under the traditional spatial linear regression, how the estimates changed in spatial confounding-like situations, and how selecting a proper tuning parameter for the spline could help reduce bias.

Li et al. have proposed a semiparametric model-based screening algorithm [6]. The model quantified organism-specific infection risks in individual subjects and accounts for the within-subject interdependence of the infection outcomes of different organisms and the serial correlations among the repeated assessments of the same organism. Model parameters were estimated by using a penalized likelihood method. For inference, they have developed a likelihood-based resampling procedure to compared the bivariate effect surfaces across outcomes. Simulation studies were conducted to evaluate the model fitting performance. A screening algorithm were developed using data collected from an epidemiological study of young women at increased risk of STIs.

In this study a software was used to simulate the elliptic system. The flow simulation performs calculations lay on Navier-Stokes equations to simulate the interaction of fluids with surfaces. This software gives graphical interpretations, however, the values of parameters are needed to predict. The main purpose of this study is to simulate the flow under different conditions and proposes a new multivariate model to predict the flow parameters. Consequently, linear regression and semiparametric regression are used to develop new models for flow regions using simulation outputs. Using these models, the approximate value of velocity magnitude in any point of fluid flow can be calculated. Only the graphical interpretations are presented in many simulation researches but here the new models are developed for each regions using statistical methods.

Materials and Methods

Numerical Analysis

The development of the 2-D models is described for the fluid velocity distribution for two cases. First system has developed when an elliptic cylinder was presented while two elliptic cylinder system was second one. Elliptic cylinders are kept at fixed distance form inlet.

Fig. 1: Geometry of an elliptic cylinder system

Fig. 2: Geometry of two elliptic cylinders' system

The fluid was Newtonian with constant density and viscosity resulting in the Navier-Stokes equations for laminar flow.

$$\rho(u.v)u = \nabla.\left[-\rho I + \mu\left(\nabla u + (\nabla u)^{T}\right)\right] + F \qquad (1)$$

Where,

ρ is the density

u is the fluid velocity

v is a vector

μ is the fluid dynamic viscosity

F is the external forces

I is the identity matrix

For laminar flow the boundary conditions were zero velocity (no-slip) on the outer sphere, constant angular velocity on the inner sphere. For the wall function condition was applied. If we want to give a function, we must define units. The laminar flow interface, found under the single-phase flow branch. We can use this interface to compute velocity and pressure field for the single-phase fluid in the laminar flow. The laminar interface can be used for stationary and time-dependent analysis.

For turbulent flow the usual extra terms involving turbulent viscosity and turbulent kinetic energy appear.

Navier-Stokes equation for turbulent flow

$$\rho\frac{\partial u}{\partial t} + \rho(u.\nabla)u = \nabla.\left[-\rho I + (\mu + \mu_{T})\left(\nabla u + (\nabla u)^{T}\right) - \frac{2}{3}\rho k I\right] + F \qquad (2)$$

Where,

ρ is the density
u is the fluid velocity
t is the time
μ is the fluid dynamic viscosity
μ_T is the turbulent viscosity
k is the turbulent kinetic energy
F is the external forces
I is the identity matrix

The turbulent flow, *k-ε* interface found under the single-phase flow> turbulent flow branch. Turbulence effects are modeled using the standard two equation *k-ε* model with reliability constants. Flow close to walls is modeled using wall function. This interface also can be used for stationary and time-dependent analyses. We cannot give two boundary condition at a wall. If we give two boundary conditions for a waIl, one condition will be overlapped. Other equations which are solved under the turbulent flow interface.

$$\rho\frac{\partial k}{\partial t}+\rho(u.\nabla)k=\nabla.\left[\left(\mu+\frac{\mu_T}{\sigma_k}\right)\nabla k\right]+p_k-\rho\varepsilon \tag{3}$$

$$\rho\frac{\partial \varepsilon}{\partial t}+\rho(u.\nabla)\varepsilon=\nabla.\left[\left(\mu+\frac{\mu_T}{\sigma_\varepsilon}\right)\nabla \varepsilon\right]+C_{\varepsilon1}\frac{\varepsilon}{k}p_k-C_{\varepsilon2}\rho\frac{\varepsilon^2}{k}=ep$$

(4)

In this equation p_k is

$$p_k=\mu_T\left[\nabla u\left(\nabla u+(\nabla u)^T\right)\right] \tag{5}$$

The turbulent viscosity was given by

$$\mu_T=\rho C_\mu\frac{k^2}{\varepsilon} \tag{6}$$

Where $C_{\varepsilon1}, C_{\varepsilon2}$ and C_μ are turbulent model constants, ε is turbulent dissipation and $\sigma_\varepsilon, \sigma_k$ are turbulent Prandtl number for *ε* and *k* respectively.

The systems describe by the Navier Stokes equation which is a partial difference equation (PDE) and cannot be solved with analytical methods. Because the vast majority of geometries and problems, these PDEs cannot be solved with analytical methods. Instead, an approximation of the equations is based on different types of discretization. These discretization methods approximate the PDEs with numerical model equations and also model equation can be solved using numerical methods. Furthermore, the finite element method (FEM) is used to compute this approximation.

Statistical Analysis

It is difficult to find general model for whole region because elliptic cylinders make serious effect on the velocity of the flow. Hence the flow region should be divided into three parts and models are fitted for each region. semiparametric regression [10] is a technique that provide the user with some flexibility in modeling complex data without maintaining stringent assumptions. With this regression, the aim is to construct a properly specified model that integrates the simplicity of parametric estimation with the flexibility provided by nonparametric splines. They are often used in situations where the fully nonparametric model may not perform well or when the researcher needs to use a parametric model but the functional form according to a subset of the regressors or the density of the errors is not known. In Semiparametric regression, a smooth function,

$$f(x)=\sum_{j=0}^{m-1}\beta_j x^j+\sum_{k=1}^{K}b_k\left|x-K_k\right|^{2m-1},m=1,2,3,... \tag{7}$$

Where,

β_j is the j^{th} regression coefficent

K_k is the location of the k^{th} knot

With

$$b=[b_1,...,b_k]^T \sim N\left(0,\sigma_b^2\Omega^{-1/2}\left(\Omega^{-1/2}\right)^T\right),\quad \Omega\equiv\left[\left|K_k-K_{k'}\right|^{2m-1}\right],1\le k.k'\le K \tag{8}$$

is estimated using penalized spline smoothing. Penalized spline smoothers come in a number of forms. In a linear spline, weights are put onto each of the splines to penalize over fitting of the data while still allowing the splines to fit the data well. The weights are determined based upon some penalization criteria. There are several options for the penalization criteria, but the easiest to implement is to choose a *C* such that:

$$\sum_{i=1}^{k}b_i^2<C \tag{9}$$

Since these criteria reduces the overall effect of individual piecewise functions and avoids over fitting the data, it is an excellent minimization criterion. This criteria is more formally stated as minimizing the equation $\left|y-X\beta\right|^2$ subject to $\beta^T D\beta\le C$, where

$$D=\begin{bmatrix}0_{2\times2} & 0_{2\times K}\\ 0_{K\times2} & I_{K\times K}\end{bmatrix} \tag{10}$$

Using Lagrange multipliers, this is equivalent to minimizing $\left|y-X\beta\right|^2+\lambda^2\beta^T D\beta$ for some $\lambda\ge0$ with respect to β.

Results and Discussion

Results

The flow pattern in the meridian plane is shown in following figure. In this case, an elliptic cylinder is fixed and the pressure difference is kept between inlet and outlet.

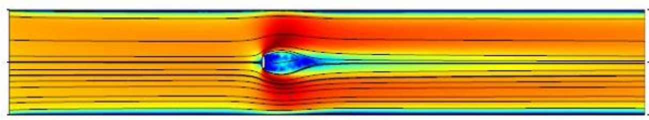


Fig. 3: The flow behavior of the system that an elliptic cylinder is presented

As we can see above figure, there is a serious effect on the velocity of the flow around elliptic cylinder. Thus we cannot find a general model for the velocity magnitude. Then the flow region should be divided into three parts as follows

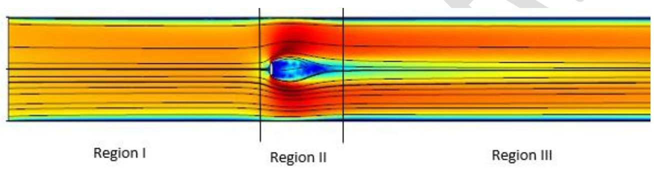
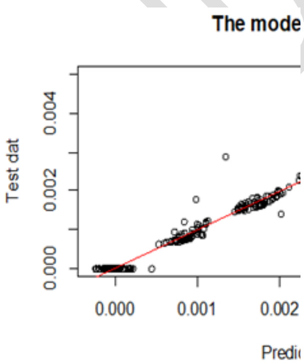
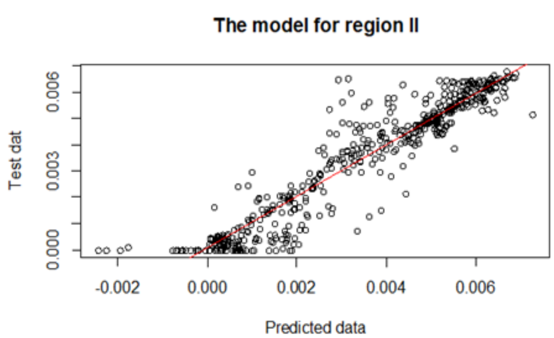


Fig.4: Three flow regions of the system that an elliptic cylinder is presented

According to the change of velocity, the flow regions are categorized. In second region, there are two stagnation points in front of elliptic cylinder, these eddies are created by centrifugal effects generated due to the pressure difference. The velocity magnitude of the flow slightly decreases with increases the distance from the inlet in third region.

According to the statistical analysis, the best model has been chosen and the cross validation has been done for each region





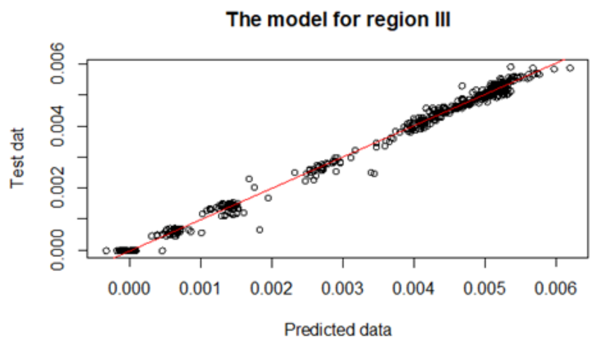


Fig. 5: The cross validations for the semiparametric regression models of first system

Further our problem can be extended to the system that two elliptic cylinders are presented. In here also there is serious changes of the velocity magnitude. It can be seen behaviors of the following streamline. So three regions had to be considered.

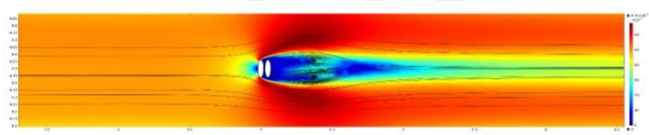


Fig. 6: The flow behavior of the system that two elliptic cylinders are presented

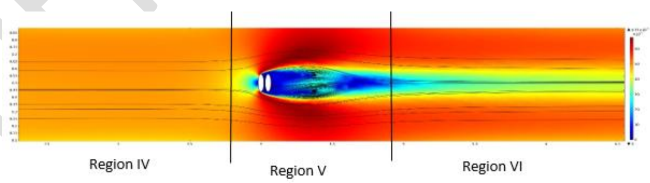


Fig. 7: Three flow regions of the system that two elliptic cylinders are presented

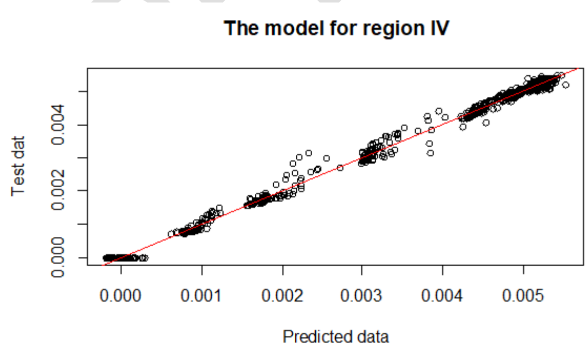
Using statistical analysis, the linear regression and the semiparametric regression are applied for the data of all regions. Then The following table could be used to find best fitted models for each region.

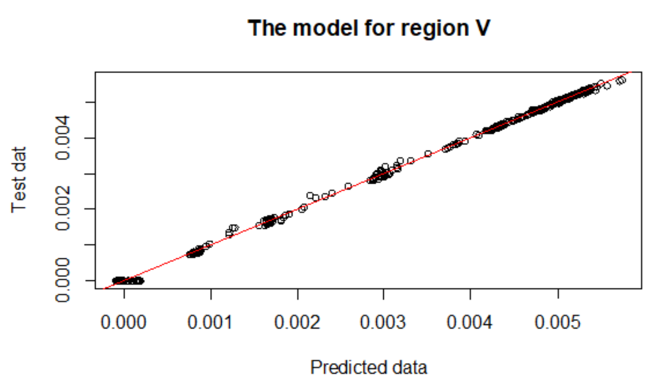
Table I: Summary of the model selection

Region	Linear Regression	Semiparametric Regression

	MSE	RMSE	MAPE	MSE	RMSE	MAPE
I	3.091299e-07	0.0005559945	0.309768	1.209647e-08	0.000109984	0.1179557
II	4.541549e-06	0.002131091	0.5910871	5.472022e-07	0.0007397312	0.3137074
III	4.414634e-07	0.0006644271	0.2602194	2.348009e-08	0.0001532321	0.1325621
IV	4.147061e-07	0.0006439767	0.3161197	1.208502e-08	0.0001099319	0.1184157
V	1.794775e-07	0.0004236478	0.2184671	1.149482e-08	0.0001072139	0.0979895
VI	4.563871e-06	0.002136322	0.7086085	5.028746e-07	0.0007091365	0.5126755

After selecting best fitted model, the cross validation has done for each region in the second system.





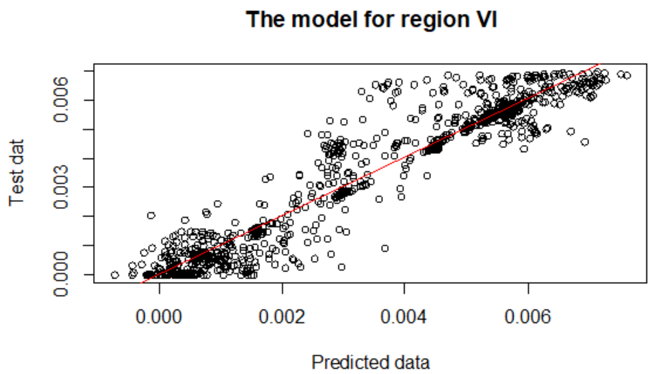


Fig. 8: The cross validations for the semiparametric regression models of the second system

Conclusion

The system of flow past elliptic cylinders can be solved numerically using software. Although this software produces graphical interpretation, the mathematical model is developed by statistical analysis. Normally the linear regression is used for the modeling, in this study Semiparametric regression provided the best results than the linear regression. The first system that an elliptic cylinder is presented was divided into three regions according to the behavior of streamlines in the flow. The most important component of the flow is velocity magnitude. Therefore, the model was illustrated for each region using R.

The value of mean square error was helped to select good model for each region, as well as RMSE and MAPE. Furthermore, the model validation proved that selected model is best model. According to the results, the semiparametric regression gives best model for each region for first system. The similar process was followed for the second system that two elliptic cylinders are presented. The semiparametric model was developed for each region in this.

References

[1] Rahni M. T., Esfashanian V., Salary M. Investigation of Flow Around a Confined Elliptical Cylinder Using the Lattice Boltzmann Method . MiddleEast Journal of Scientific Research .

[2] Mittal R., Balachandar S. Direct Numerical Simulation of Flow Past Elliptic Cylinders. , Journal of Computational Physics,1996.

[3] Perumal D. A., Kumar G. V. S., Dass A. K. Lattice Boltzmann Simulation of Viscous Flow past Elliptical Cylinder. , www.cfdl.issres.net,2012.

[4] D' Alessio S. J. D., Kocabiyik S. Numerical Simulations of the Flow Induced by a Transversely Oscillating Inclined Elliptic Cylinder. , Journal of fluid and structures ,2001.

[5] De Silva J. W. E. W., Dhananjaya K. D. E., Weliwita J. A. A Numerical Study on Viscous Flow Past Two Adjacent Elliptic cylinders. , Proceeding of the Peradeniya University International Research sessions,2017.

[6] Zhuokai Li, Hai Liu, Wanzhu Tu. Model selection in multivariate semiparametric regression., SAGE Journals .2017.

[7] Jelena Fiosina , Maksims Fiosins. Distributed Nonparametric and Semiparametric Regression on SPARK for Big Data Forecasting. Applied Computational Intelligence and Soft Computing. Volume 2017, Article ID 5134962, 13 pages.

[8] Arnab Maity ,Tatiyana V. Apanasovich. Estimation via corrected scores in general semiparametric regression models with error-prone covariates.Electron jornal of statistics.volume 5(2011).

[9] Guilherme Ludwig, Jun Zhu, Chun-Shu Chen. On fixed effects estimation in spline-based semiparametric regression for spatial data.2015-27th annual conference on applied statistics in agriculture.Department of Kansas State University.

[10] Zhuokai Li, Hai Liu, Wanzhu Tu. A sexually transmitted infection screening algorithm based on semiparametric regression models. Statistics in Medicine. Volume 34(2015).

[11] Thomas S. Shively and Thomas W. Semiparametric Regression Approach to Adjusting for Meteorological Variables in Air Pollution Trends.