Original Research Article

EXISTENCE AND UNIQUENESS OF SOLUTION OF MAGNETOHYDRODYNAMIC BUOYANCY DRIVEN FLOW PAST A STRETCHING SHEET UNDER THE INFLUENCE OF VARIABLE VISCOSITY

Abstract

The focus of the present study is to examine existence and uniqueness of solutions to governing flow equations arising from magnetohydrodynamic free convective flow of incompressible viscous fluid past a stretching surface under the influence of variable viscosity. Thermal radiation effect is considered in the heat equation. Also, the governing dimensional partial differential equations of the flow and energy are non-dimensionalized and converted into system of ordinary differential equations. Theorems on existence and uniqueness of solutions to system of equations governing the flow model under consideration are stated and validated.

Keywords
Magnetohydrodynamic, Free Convection, Temperature Dependent Viscosity, Existence and Uniqueness, Radiation.

1 Introduction

Magnetohydrodynamics (MHD) is a branch of fluid dynamics which studies the behavior of an electrically-conducting fluid in motion. Various areas of application of MHD fluid flow include electrostatic filters, purification of crude oil, MHD pumps, cooling reactors, MHD power generation etc. Ryoichi et al. [1] studied the application of MHD to flow of an electrolyte in an electrode-cell with short rectangular channel. Pakmor et al. [2] discussed numerical solutions of moving-mesh code in the presence of applied magnetic field. The combination of Lagrangian and Eularian methods was used in a single computation technique. Opanuga et al. [3] used Adomian decomposition method to solve the problem of Joule heating effects on MHD flow of reactive viscous fluid in a porous medium. Gedik et al. [4] numerically investigated MHD pressure induced flow through a pipe in the absence of slip conditions. A numerical simulation based on the finite volume method was employed by Azimi-Boulali et al. [5] to investigate a 3D model of Newtonian magnetohydrodynamic fluid flow. Zaman et al. [6] presented the exact solution of unsteady MHD fluid flow in a channel under the influence of heat transfer and slip conditions.

The study of natural convection flow and heat transfer past a stretching surface is of great importance in engineering and industry. A vast number of researchers have investigated convective heat transfer in the presence of several fluid properties. Rao et al. [7] studied the feature of MHD flow and convective heat transfer of nanofluid with chemical reaction effect. Mendal et al. [8] discussed the effects of mass diffusion and thermal radiation on laminar unsteady free convective fluid flow in a vertical channel. Miroshnichenko and Sheremet [9] investigated the unsteady free convection flow of viscous fluid in the presence of heat transfer. Samuel [10] presented analytical solutions of chemical reaction and melting heat transfer effects on steady incompressible MHD Newtonian fluid flow in the presence of buoyancy force. Saravanan and Chinnasamy [11] studied the impacts of heat transfer on buoyancy driven flow of a viscous incompressible fluid under the influence of thermal radiation. Satya et al. [12] considered MHD two dimensional unsteady flow of a Newtonian fluid in a porous medium in the presence of heat generation and chemical reaction. Khalil-Ur-Rehman and Malik [13] examined mixed convection flow of Eyring-Powell fluid past a stretching cylindrical surface with heat generation/absorption effects. Adegbie and Fagbade [14] presented the problem of convective heat transfer flow of viscous fluid in the presence of thermal rediation and explored the qualitative properties of solution of ordinary differential equations arising from such flow model. Samuel and Ajayi [15] performed a numerical study of the radiation effect on buoyancy driven flow of non-Newtonian power law fluid past a catalytic surface. Ali et al. [16] utilized Galerkin weighted residual finite-element technique to analyze problems of heat transfer flow of nanofluid in the presence of natural convection. Zaidi and Ahmad [17] studied the effects of heat generation on buoyancy driven flow of viscous fluid through an inclined microchannel with slip conditions. Adegbie et al. [18] analytically examined the impacts of pertinent fluid parameters on free convective flow of Newtonian fluid past a continuous moving surface. Recently, Khan et al. [19] investigated heat generation and natural convection effects on incompressible nanofluids along a sphere in the presence of Brownian motion and thermophoresis. Anwar et al. [20] investigated MHD free convective flow of an optically thick Casson fluid in the presence of heat generation.

Having reviewed the literature above, it is obvious that none of the authors studied the existence and uniqueness of solutions to flow model arising from buoyancy driven MHD flow of Newtonian fluid with variable viscosity past a continuous moving plate, which is the objective of the present investigation. The classical model for thermal radiation is based on Cogley et al. [21]. Qualitative properties such as existence and uniqueness of solution are investigated [14, 22].

2 Mathematical Formulation

A two-dimensional laminar free convection and heat transfer flow of an electrically conducting incompressible viscous fluid past a stretching surface is examined. The fluid viscosity is assumed to vary exponentially with temperature. Also, Thermal radiation effect is incorporated into heat equation. The hall and induced magnetic field impacts are not considered. Furthermore, viscous dissipation and chemical reactions effects are neglected. The wall temperature T_w is assumed to be $T = T_\infty + Ax$ where A is a constant and T_∞ is the free stream temperature. The stretching velocity is expressed as $u_w(x) = bx$ where b is a positive constant. With the aforementioned assumptions coupled with Boussinesq approximations, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) + g\beta(T - T_\infty) - \frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

subject to the following boundary conditions

$$y = 0: \quad u = u_w(x) = bx, \quad v = 0, \quad T = T_\infty + Ax, \tag{4}$$

$$y \to \infty: \quad u \to 0, \quad T \to T_\infty, \tag{5}$$

where B_o denotes magnetic field strength, u is the velocity component along x direction, v is the velocity component along y direction, g is the gravitational acceleration, ρ is the density of the fluid, μ is the fluid dynamic viscosity, κ is the fluid thermal conductivity, ν is the kinematic viscosity, β is the thermal expansion coefficient, σ is the fluid electrical conductivity, c_p is the fluid specific heat at constant pressure, u_w is velocity with which the plate is moving, T_w is the surface temperature of the moving surface. q_r is the radiative heat flux which based on Cogley et al. [21] is expressed as

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)\Gamma \tag{6}$$

where

$$\Gamma = \int_0^\infty k_\Gamma(\partial e_{b\lambda}/\partial T)d\lambda, \tag{7}$$

λ is the wave length, k_Γ is the absorption coefficient and $e_{b\lambda}$ is the Plank function.

Also, the fluid viscosity is assumed to be a function of temperature and it is expressed as

$$\mu(T) = \mu_r e^{-\alpha(T - T_\infty)}, \tag{8}$$

where α is a constant and μ_r is the reference fluid velocity.

By introducing the following similarity transformations in equations (1)-(3),

$$\eta = \left(\frac{b}{\nu}\right)^{\frac{1}{2}}, \quad u = bxf', \quad v = -(\nu b)^{\frac{1}{2}} f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{9}$$

we obtain the following dimensionless equations,

$$f''' - \epsilon f''\theta' + e^{\epsilon\theta}\left[G_r\theta - Mf' - (f')^2 + ff''\right] = 0, \tag{10}$$

$$\theta'' - P_r[\theta f' - \theta' f] - P_r R_a \theta = 0, \tag{11}$$

the transformed boundary conditions are

$$\eta = 0: \quad f(0) = 0 \quad f^{'}(\eta) = 1 \quad \theta(0) = 1, \tag{12}$$

$$\eta \to \infty: \quad f^{'}(\infty) \to 0, \quad \theta(\infty) \to 0, \tag{13}$$

where $R_a = \frac{4\Gamma}{bc_p\rho}$ is the Radiation parameter, $P_r = \frac{\mu_\infty C_p}{k_\infty}$ is the Prandtl number, $M = \frac{\sigma B_0{}^2}{\rho_b}$ is the magnetic parameter, $\epsilon = \frac{T - T_\infty}{T_w - T_\infty}$ is the variable viscosity parameter, $G_r = \frac{g\beta(T_w - T_\infty)}{b^2 c}$ is the Grashof number and $\epsilon = \alpha(T_w - T_\infty)$ is the variable viscosity parameter.

3 Reduction of Dimensionless Governing Equations to System of First Order Ordinary Differential Equations

The dimensionless governing equations (10) - (11) together with boundary conditions (12) - (13) are reduced to system of first order nonlinear autonomous ordinary differential equations as shown below.
Suppose

$$f = x_1, \tag{14}$$

$$\frac{df}{d\eta} = \frac{dx_1}{d\eta} = x_2, \tag{15}$$

$$\frac{d^2 f}{d\eta^2} = \frac{d}{d\eta}\left(\frac{df}{d\eta}\right) = \frac{dx_2}{d\eta} = x_3, \tag{16}$$

$$\frac{d^3 f}{d\eta^3} = \frac{d}{d\eta}\left(\frac{d^2 f}{d\eta^2}\right) = \frac{dx_3}{d\eta} = \epsilon x_3 x_5 - e^{\epsilon x_4}[G_r x_4 - M x_2 - x_2^2 + x_1 x_3], \tag{17}$$

$$\theta = x_4, \tag{18}$$

$$\frac{d\theta}{d\eta} = \frac{x_4}{d\eta} = x_5, \tag{19}$$

$$\frac{d^2\theta}{d\eta} = \frac{d}{d\eta}\left(\frac{d\theta}{d\eta}\right) = \frac{dx_5}{d\eta} = P_r(x_4 x_2 - x_5 x_1) + P_r R_a x_4. \tag{20}$$

The next step is to carry out mathematical manipulation and simplification in order to combine our equations.

$$\frac{dx_1}{d\eta} = x_2,\ x_1(0) = f_w, \tag{21}$$

$$\frac{dx_2}{d\eta} = x_3,\ x_2(0) = 1, \tag{22}$$

$$\frac{dx_3}{d\eta} = \epsilon x_3 x_5 - e^{\epsilon x_4}[G_r x_4 - M x_2 - x_2^2 + x_1 x_3]\ x_3(0) = \alpha, \tag{23}$$

$$\frac{dx_4}{d\eta} = x_5,\ x_4(0) = 1, \tag{24}$$

$$\frac{dx_5}{d\eta} = P_r(x_4 x_2 - x_5 x_1) + P_r R_a x_4,\ x_5(0) = \beta \tag{25}$$

4 Properties of Solutions

The purpose of this section is to formulate theorems on existence and uniqueness of solution of ordinary differential equations of the flow model (10) and (11) subject to boundary conditions (12) and (13). Furthermore, we want to check if the solution to the flow problem exist(s) and to also know if it is unique.

4.1 Existence and uniqueness theorems

Theorem 1 *Let D denotes the region in (n+1)-dimensional space, one dimension for t and n dimensions for the vector X, such that*
$|t - t_0| \leq,\ \ ||x - x_0|| \leq b,$
and suppose that $f(t, x)$ satisfy Lipschitz condition

$$||f(t, x_1) - f(t, x_2)|| \leq K||x_1 - x_2||, \tag{26}$$

whenever the pair (t, x_1) *and* (t, x_2) belong to D, where K is a positive constant, then there is a constant $\delta > 0$ such that there exists a unique continuous vector solution $X(t)$ of the system (14) - (25) in the interval $|t - t_0| \leq \delta$.

Before leaving this theorem, we should briefly discuss the condition (26). It is easy to see that (26) is implied by the inequality

$$||f(t, x_{11}, X_{12}, \ldots, x_{1n}) - f(t, x_{21}, x_{22}, \ldots, x_{2n})|| \leq K_1 \sum_{j=1}^{n} ||x_{1j} - x_{2j}||. \tag{27}$$

For some number K_1. This fact follows from the double inequality

$$\frac{1}{n} \sum_{j=1}^{n} |x_j| \leq ||x_j|| \leq \sum_{j=1}^{n} |x_j|, \tag{28}$$

which is an immediate consequence of the definition of $||X||$.
Another condition that implies (26) is

$$||f(t, x_{11}, X_{12}, \ldots, x_{1n}) - f(t, x_{21}, x_{22}, \ldots, x_{2n})|| \leq K_2 max_j |x_{1j} - x_{2j}|; i = 1, 2, 3, \ldots, n \tag{29}$$

The two inequalities (27) and (29) are useful since it is often very difficult to verify inequality (26) directly.

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If the partial derivative $\frac{\partial f_i}{\partial x_j}$ $i, j = 1, 2, 3, \ldots, n$, are continuous in D and also bounded on D and condition (27) and (29) both follow from the mean theorem of differential calculus.

We now turn to the main purpose of this section which is to show the existence and uniqueness of solutions of the theorem formulated below.

Theorem 2 *: Let* $\alpha_i (i = 1, 2) \geq 0,\ 0 \leq x_1 < a_1,\ 0 \leq x_2 < a_2,\ b_1 \leq x_3 < a_3,\ 0 \leq x_4 < a_4,\ b_2 \leq x_5 < a_5$ *where* $a_i (i = 1, 2, \ldots, 5)$ *and* $b_i (i = 1, 2)$ *are positive constants. Then, there exists a unique solution of problem (10) and (11) subject to the boundry conditons (12) and (13).*

Proof: Considering the system of first order ordinary differential equations (21) - (25) which represent (10) and (11) subject to the boundary conditions (12) and (13). In compact form we have

$$\begin{pmatrix} \frac{dx_1}{d\eta} \\ \frac{dx_2}{d\eta} \\ \frac{dx_3}{d\eta} \\ \frac{dx_4}{d\eta} \\ \frac{dx_5}{d\eta} \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \\ T_1 \\ x_5 \\ T_2 \end{pmatrix}, \tag{30}$$

satisfying the boundary conditions:

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \\ x_5(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \alpha_1 \\ 1 \\ \beta \end{pmatrix}, \tag{31}$$

where

$$\left.\begin{array}{rc} T_1 = & \epsilon f''(\eta)\theta' - e^{\epsilon\theta}[G_r\theta - Mf'(\eta) - (f'(\eta))^2 + f(\eta)f''(\eta)], \\ T_2 = & P_r(x_4x_2 - x_5x_1) + P_rR_ax_4, \end{array}\right\} \tag{32}$$

and $\alpha_i (i = 1, 2)$ are guess values that must satisfy the boundary conditions.
Let

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2, x_3, x_4, x_5) \\ f_2(x_1, x_2, x_3, x_4, x_5) \\ f_3(x_1, x_2, x_3, x_4, x_5) \\ f_4(x_1, x_2, x_3, x_4, x_5) \\ f_5(x_1, x_2, x_3, x_4, x_5) \end{pmatrix} \tag{33}$$

Now,

$$f_1 = x_2$$

$$\left|\frac{\partial f_1}{\partial x_1}\right| = \left|\frac{\partial f_1}{\partial x_2}\right| = \left|\frac{\partial f_1}{\partial x_4}\right| = \left|\frac{\partial f_1}{\partial x_5}\right| = 0 \tag{34}$$

$$\left|\frac{\partial f_1}{\partial x_2}\right| = 1 \tag{35}$$

$$f_2 = x_3$$

$$\left|\frac{\partial f_2}{\partial x_1}\right| = \left|\frac{\partial f_2}{\partial x_2}\right| = \left|\frac{\partial f_2}{\partial x_4}\right| = \left|\frac{\partial f_2}{\partial x_5}\right| = 0$$

$$\left|\frac{\partial f_2}{\partial x_3}\right| = 1$$

$$f_3 = \epsilon x_3x_5 - e^{\epsilon x_4}[G_rx_4 - Mx_2 - x_2^2 + x_1x_3]$$

$$\left|\frac{\partial f_3}{\partial x_1}\right| = |-e^{\epsilon x_4}x_3| = |-1|\,|e^{\epsilon x_4}x_3| \leq P_1 \tag{36}$$

$$\left|\frac{\partial f_3}{\partial x_2}\right| = |-e^{\epsilon x_4}(-M - 2x_2)| = |e^{\epsilon x_4}||-(M + 2x_2)| \leq P_2 \tag{37}$$

$$\left|\frac{\partial f_3}{\partial x_3}\right| = |\epsilon x_5 - e^{\epsilon x_4}x_1| \leq |\epsilon x_5| + |-1||e^{\epsilon x_4}x_1| \leq P_3 \tag{38}$$

$$\left|\frac{\partial f_3}{\partial x_4}\right| = |-e^{\epsilon x_4}G_r - \epsilon G_rx_4e^{\epsilon x_4}| \leq |-1||e^{\epsilon x_4}G_r + \epsilon G_rx_4e^{\epsilon x_4}| \leq P_4 \tag{39}$$

$$\left|\frac{\partial f_3}{\partial x_5}\right| = |\epsilon x_3| \leq P_5 \tag{40}$$

$$f_4 = x_5 \tag{41}$$

$$\left|\frac{\partial f_4}{\partial x_1}\right| = \left|\frac{\partial f_4}{\partial x_2}\right| = \left|\frac{\partial f_4}{\partial x_3}\right| = \left|\frac{\partial f_4}{\partial x_4}\right| = 0$$

$$\left|\frac{\partial f_4}{\partial x_5}\right| = 1 \tag{42}$$

$$f_5 = P_r(x_4x_2 - x_5x_1) + P_rR_ax_4 \tag{43}$$

$$\left|\frac{\partial f_5}{\partial x_1}\right| = |P_r(-x_5)| \leq |-1||P_rx_5| \leq P_6 \tag{44}$$

$$\left|\frac{\partial f_5}{\partial x_2}\right| = |P_rx_2| \leq P_7 \tag{45}$$

$$\left|\frac{\partial f_5}{\partial x_3}\right| = 0 \tag{46}$$

$$\left|\frac{\partial f_5}{\partial x_4}\right| = |P_rx_2 + P_rR_a| = |P_r(x_2 + R_a)| \leq P_8 \tag{47}$$

$$\left|\frac{\partial f_5}{\partial x_5}\right| = |P_r(-x_1)| \leq |-1||P_rx_1| \leq P_9 \tag{48}$$

where $P_1 - P_9$ are real constants.

Clearly from equations (34) - (48), it is obvious that $\left|\frac{\partial f_i}{\partial x_j}\right| \leq K \; \forall i, j = 1, \ldots, 5$ is bounded and there exists K such that $K = max(0, 1, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9)$ and $0 < K < \infty$. Therefore $f_i(x_1, x_2, x_3, x_4, x_5)$ is Lipchitz continuous. Hence, there exist a unique solution of the system of non-linear coupled ordinary differential equation (10) and (11) subject to boundary conditions (12) and (13).

5 Conclusion

In the present study, existence and uniqueness of solutions to problem of natural convection flow of viscous fluid over stretching surface with variable viscosity and magnetic field effects is presented. Theorems on existence and uniqueness of solutions are formulated to analyze the flow equations under consideration. Findings indicate that the solution of the flow model exists and unique.

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