Generalized Estimators for Finite Population Variance Using Measurable and Affordable Auxiliary Character

Abstract

In this paper, a generalized exponential-type estimator for estimating the population variance using measurable and affordable auxiliary character in single phase sampling has been proposed. Some special cases of the proposed generalized estimator with $\alpha = 1$ and $\alpha = 2$ have also been discussed. The expressions for the mean square error and bias of the proposed generalized estimator have been derived. The proposed generalized estimator has been compared theoretically with the existing estimators and the conditions under which the proposed estimators are better than some existing estimators have also been given. Numerical examples with five real data sets shows that both of the proposed estimators are more efficient than the estimators considered in literature. Thus, the estimator with $\alpha = 2$ perform better than the estimator with $\alpha = 1$.

Keywords: Variance estimator, Population variance, Auxiliary character, MSE, Efficiency.

1. Introduction

In survey sampling, the use of auxiliary characters is frequently recognized to increase the accuracy or precision of the population characteristics estimation procedures. Many researchers have considered the use of auxiliary characters in different forms to construct efficient estimators for population parameters (See Sisodia and Dwivedi [1], Murthy [2], Yadav and Kadilar [3], Singh and Tailor [4], Bahl and Tuteja [5], Singh et al. [6], Singh et al [7], Kadilar and Cingi [8], Singh and Solanki [9], Sahai and Ray [10], Srivastava and Jhajj [11], Ahmed et al. [12], Audu and Adewara [13], Audu et al. [14], Muili et al. [15], Khoshnevisan et al. [16], Singh and Audu [17], Ahmed et al.[18] and Audu and Singh [19], Das and Tripathi [20], Das and Tripathi [21], Patel and Rina [22], Rajyaguru and Gupta [23], Rajyaguru and Gupta [24], Archana and Rao [25], Singh et al. [26], Audu et al. ([27]-[32]), Singh et al. [33], Muili et al. [34], Ishaq et al. [35], Audu et al. [36], Zakari et al. [69]). The problem of estimating finite population variance is an important issue where it is difficult to control variability in application. Research in the manufacturing industries, pharmaceutical laboratories, agriculture and biological experiments is deeply confronted with this problem and therefore the objective results appear to be uncontrollable. As a result, most of the time researchers are interested in the variation of their products. In order to measure variations within study variable y, the problem of estimating the population variance $\hat{S}_y^2$ also received considerable attention from the survey sample statisticians. The issue of constructing effective population variance estimators was discussed by various authors such as Liu [47], Das & Tripathi [42], Grover [44], Isaki [45], Upadhaya & Singh [60], Singh & Singh [55], Ahmed et al. [37], Al-Jararha & Ahmed [40], Ahmed et al. [38], Kadilar &

Cingi [46], Bahl & Tuteja [41], Upadhyaya et al. [61], Ahmed and Singh [39], Swain [59], Subramani & Kumarapandiyan [58], Shabbir & Gupta [52], Singh & Malik [53], Yadav et al. [65], Yadav & Kadilar [66], Ahmed et al. ([12], [18]), Muili et al. ([48], [62]-[64]). These authors proposed different types of efficient estimators for estimating finite population variance using one or more auxiliary information under different sampling schemes.

In this paper, we considered the problem of variance estimation and consequently, we proposed an improved ratio-product type-exponential estimator of finite population variance that includes additional information in the form of an auxiliary variable.

2. Materials and Methods

Let $\Omega = (\Omega_1, ..., \Omega_N)$ be a finite population of size N and let $(y_{i,} x_i)$ be the value of the study variable Y and the auxiliary variable X on i^{th} unit $\Omega_i, i = 1, ..., N$. Let $\bar{Y}$ and $\bar{X}$ be population means of Y and X respectively (Zakari et al., [68]). We assume that the population mean $\bar{X}$ and the population variance S_x^2 of X are known. Let $s_y^2 = (n-1)^{-1} \sum_{i=1}^{n} (y_i - \bar{y})$ and $s_x^2 = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})$ be the unbiased estimators of $S_y^2 = (N-1)^{-1} \sum_{i=1}^{N} (y_i - \bar{y})$ and $S_x^2 = (N-1)^{-1} \sum_{i=1}^{N} (y_i - \bar{y})$ respectively and the correlation coefficient between Y and X be ρ_{xy}. Also, let $C_y = S_y / \bar{Y}$ and $C_x = S_x / \bar{X}$ be the coefficients of variation of the study variable Y and the auxiliary variable X.

In order to derive the bias and *MSE* of the estimator, we consider the following relative error terms:

Let $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$ with first and second degrees expectations as $E(e_0) = E(e_1) = 0$, $E(e_0^2) = \theta(\beta_{2(y)} - 1) = V_{40}$ $E(e_1^2) = \theta(\beta_{2(x)} - 1) = V_{04}$, $E(e_0 e_1) = \theta(\lambda_{22} - 1) = V_{22}$ and $\theta = \left(\frac{1}{n} - \frac{1}{N}\right)$. Here $\beta_{2(y)}$ and $\beta_{2(x)}$ are the population coefficient of kurtosis of y and x respectively.

The conventional unbiased estimator of variance is given by

$$S_y^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2 \qquad (1)$$

The variance of $\hat{S}_y^2$ is given by

$$Var(\hat{S}_y^2) \cong S_y^4 V_{40} \qquad (2)$$

Ratio estimator of the population variance $\hat{S}_R^2$ due to Isaki [45] is given by

$$\hat{S}_R^2 = s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \qquad (3)$$

The *MSE* of $\hat{S}_R^2$, to first degree of approximation, is given by

$$MSE(\hat{S}_R^2) \cong S_y^4\left[V_{40} + V_{04} - 2V_{22}\right] \tag{4}$$

The usual regression estimator proposed is given by:

$$\hat{S}_{\mathrm{Re}g}^2 = s_y^2 + b_{(s_y^2, s_x^2)}\left(S_x^2 - s_x^2\right) \tag{5}$$

where $b_{(s_y^2, s_x^2)}$ is the sample regression coefficient whose population regression coefficient.

The *MSE* of $\hat{S}_{Reg}^2$, to first order of approximation, is given by

$$MSE(\hat{S}_{\mathrm{Re}g}^2) \cong S_y^4 V_{40}(1 - \rho_{(s_y^2, s_x^2)}^2) \tag{6}$$

Where $\rho_{(s_y^2, s_x^2)} = V_{22} / \sqrt{V_{40}V_{04}}$, is the population correlation coefficient between *y* and *x*.

Singh et al.,[57] difference type estimator is given by

$$\hat{S}_d^2 = k_1 s_y^2 + k_2(S_x^2 - s_x^2) \tag{7}$$

Where k_1 and k_2 are unknown constants, whose values are to be determined.

The minimum *MSE* of $\hat{S}_d^2$, to first order of approximation, at optimum values $k_{1(opt)} = V_{04} / \left(V_{04} + V_{40}V_{04} - V_{22}^2\right)$ and $k_{2(opt)} = S_x^2 V_{22} / \left(S_y^2\left(V_{04} + V_{40}V_{04} - V_{22}^2\right)\right)$ is given by

$$MSE(\hat{S}_d^2) \cong \frac{S_y^4 V_{40}(1 - \rho_{(s_y^2, s_x^2)}^2)}{1 + V_{40}(1 - \rho_{(s_y^2, s_x^2)}^2)} \tag{8}$$

The ratio type exponential estimator for the population variance defined by (Bahl & Tuteja [41]) is given as:

$$\hat{S}_{BT}^2 = s_y^2 \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right) \tag{9}$$

The *MSE* of $\hat{S}_{BT}^2$, to first order of approximation, is given by

$$MSE(\hat{S}_{BT}^2) \cong S_y^4\left[V_{40} + 4^{-1}V_{04} - V_{22}\right] \tag{10}$$

Shabbir & Gupta [52] proposed the following estimator for $\hat{S}_y^2$is given by

$$\hat{S}_{SG}^2 = \left[k_3 s_y^2 + k_4\left(S_x^2 - s_x^2\right)\right]\exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right) \tag{11}$$

Where k_3 and k_4 are unknown constants, whose values are to be determined. The minimum *MSE* of $\hat{S}_{SG}^2$, to first order of approximation, at optimum values

$$k_{3(opt)} = \frac{V_{04}}{8}\left(\frac{8 - V_{04}}{V_{04} + V_{40}V_{04} - V_{22}^2}\right)$$

and

$$k_{2(opt)} = \frac{S_y^2}{8S_x^2}\left(\frac{-4V_{40} + V_{04} + 8V_{22} - V_{22}V_{04} + = 4V_{40}V_{04} - 4V_{22}^2}{(V_{04} + V_{40}V_{04} - V_{22}^2)}\right)$$

is given by

$$MSE(\hat{S}^2_{SG})_{\min} \cong \frac{S_y^4}{64}\left(\frac{-4V_{04}^2 - 16V_{40}(1-\rho^2_{(s_y^2,s_x^2)})\left(V_{04}-4\right)}{1+V_{40}(1-\rho^2_{(s_y^2,s_x^2)})}\right) \tag{12}$$

Swain [59] proposed a generalized estimator for population variance is given by

$$\hat{S}^2_{SW} = s_y^2\left[k\left(\frac{S_x^2}{s_x^2}\right)^q + (1-k)\left(\frac{s_x^2}{S_x^2}\right)^h\right]^{\delta} \tag{13}$$

Where *k*, *q*, *h* and δ = (1, −1) are real and free parameters to be chosen suitably. The minimum *MSE* of $\hat{S}^2_{SW}$, to first order of approximation, at optimum value $k = (\delta h + (V_{22}/V_{04}))/(\delta(g+h))$, is given by

$$MSE(\hat{S}^2_{SW})_{\min} \cong S_y^4 V_{40}(1-\rho^2_{(s_y^2,s_x^2)}) \tag{14}$$

Yadav et al. [65] proposed the general class of estimators for $\hat{S}^2_y$, given by

$$\hat{S}^2_{YG} = \left[k_5 s_y^2 + k_6\left(S_x^2 - s_x^2\right)\right]\left\{\lambda\left(\frac{aS_x^2+b}{as_x^2+b}\right) + (1-\lambda)\exp\left(\frac{a(S_x^2-s_x^2)}{a(S_x^2+s_x^2)+2b}\right)\right\} \tag{15}$$

Where k_5 and k_6 are suitably chosen constants, λ can takes values 0 or 1 and *a*, *b* be the known population parameters of the auxiliary variable.

$$k_{5(opt)} = \left(\frac{1-8^{-1}g^2(1+3\lambda+4\lambda^2)V_{04}}{1-4^{-1}g^2\lambda(1+3\lambda)V_{04}^2 + V_{40}(1-\rho^2_{(s_y^2,s_x^2)})}\right),$$

$$k_{6(opt)} = \frac{S_y^2}{S_x^2}\left(\frac{1}{2}g(1+\lambda) + k_{5(opt)}\left(\frac{V_{22}}{V_{04}} - g(1+\lambda)\right)\right)$$

is given by

$$MSE_{\min}\left(\hat{S}^2_{YG}\right) \cong S_y^4\left\{\left(1-\frac{1}{4}g^2(1+\lambda)^2 V_{04}\right) - \frac{1-8^{-1}g^2(1+3\lambda+4\lambda^2)V_{04}}{1-4^{-1}g^2\lambda(1+3\lambda)V_{04}^2 + V_{40}(1-\rho^2_{(s_y^2,s_x^2)})}\right\} \tag{16}$$

Where $g = aS_x^2/(aS_x^2+b)$. The minimum *MSE* of $\hat{S}^2_{YG}$, to first order of approximation at *(*λ, *a*, *b)* = *(*1, 1, 0*)*, is

$$MSE\left(\hat{S}^2_{YG}\right)_{\min} \cong S_y^4\left(\frac{S_y^{-4}MSE(\hat{S}^2_{\mathrm{Re}g})(1-V_{04})}{1-V_{04}+S_y^{-4}MSE(\hat{S}^2_{\mathrm{Re}g})}\right) \tag{17}$$

Yadav & Kadilar [66] proposed ratio-product-ratio type estimator for $\hat{S}^2_{YK}$, given by

$$\hat{S}^2_{YK} = s_y^2\left\{\alpha_1\left[\frac{(1-\beta_1)s_x^2+\beta_1 S_x^2}{\beta_1 s_x^2 + (1-\beta_1)S_x^2}\right] + (1-\alpha_1)\left[\frac{\beta_1 s_x^2 + (1-\beta_1)S_x^2}{(1-\beta_1)s_x^2+\beta_1 S_x^2}\right]\right\} \tag{18}$$

Where α_1 and β_1 are appropriate chosen constants.

The minimum *MSE* of $\hat{S}^2_{YK}$, to first order of approximation at $(\alpha_{1(opt)}, \beta_{1(opt)}) = \left(1/2, 1/2\right)$, is

$$MSE(\hat{S}^2_{YK}) \cong S_y^4 V_{40}, \tag{19}$$

and at $(\alpha_1,\beta_1)=(((V_{04}-V_{22})/2V_{04}),0),$ is given by

$$MSE(\hat{S}^2_{YK})_{\min}\cong S_y^4V_{40}(1-\rho^2_{(s_y^2,s_x^2)}). \tag{20}$$

Singh & Malik [53] proposed an improved estimator for $\hat{S}_y^2$, is given by

$$\hat{S}^2_{SM}=s_y^2\left[k_7+k_8\left(S_x^2-s_x^2\right)\right]\exp\left(\psi\frac{a(S_x^2-s_x^2)}{a(S_x^2+s_x^2)+2b}\right), \tag{21}$$

Where k_7 and k_8 are suitably chosen constants, ψ takes values +1 and -1 for ratio and product type estimators and *a*, *b* be the known population parameters of the auxiliary variables.

$$k_{7(opt)}=\frac{1}{4}\left(\frac{-12V_{04}V_{22}+V_{04}^2+16V_{22}^2-8V_{04}-2V_{40}^2}{V_{04}^2-4V_{04}V_{22}+8V_{22}^2-2V_{40}V_{04}-2V_{04}-V_{40}^2}\right)$$

and

$$k_{8(opt)}=\frac{-1}{4S_x^2}\left(\frac{-6V_{04}V_{22}+V_{04}^2+8V_{22}^2-4V_{04}+8V_{22}-8V_{04}V_{22}+4V_{04}V_{40}}{V_{04}^2-4V_{04}V_{22}+8V_{22}^2-2V_{40}V_{04}-2V_{04}-V_{40}^2}\right)$$

is given by

$$MSE(\hat{S}^2_{SM})_{\min}\cong\frac{S_y^4}{64}\left[\frac{V_{04}\left\{V_{04}(V_{04}+8V_{22})+16(V_{04}-4)MSE(\hat{S}^2_{\mathrm{Re}g})+16V_{22}(V_{22}-V_{04})\right\}}{-V_{04}(1+V_{40}+2V_{22})+4V_{22}^2}\right] \tag{22}$$

Yaqub & Shabbir [67] proposed an improved class of estimators for $\hat{S}_y^2$, is given by

$$\hat{S}^2_{YS}=s_y^2\left[k_9+k_{10}\left(S_x^2-s_x^2\right)\right]\left(\frac{aS_x^2+b}{as_x^2+b}\right)\left\{\frac{1}{2}\exp\left(\frac{a(S_x^2-s_x^2)}{a(S_x^2+s_x^2)+2b}\right)+\frac{1}{2}\exp\left(\frac{a(s_x^2-S_x^2)}{a(s_x^2+S_x^2)+2b}\right)\right\} \tag{23}$$

Where k_9 and k_{10} are suitably chosen constants and *a* and *b* be the known population parameters of the auxiliary variable.

$$k_{9(opt)}=\frac{V_{04}}{2}\left(\frac{1+7(1-V_{04})}{V_{04}^2+4V_{04}(1-V_{04})+4V_{40}V_{04}-4V_{22}^2}\right)$$

and

$$k_{10(opt)}=\frac{S_y^2}{2S_x^2}\left(\frac{V_{22}+7V_{22}(1-V_{04})-8V_{04}(1-V_{04})+8V_{40}V_{04}-8V_{22}^2}{V_{04}^2+4V_{04}(1-V_{04})+4V_{40}V_{04}-4V_{22}^2}\right),$$

is given by

$$MSE(\hat{S}^2_{YS})_{\min}\cong\frac{S_y^4}{16}\left(\frac{64(1-V_{04})S_y^{-4}MSE(\hat{S}^2_{\mathrm{Re}g})-V_{04}^2}{V_{04}+4(1-V_{04})+4S_y^{-4}MSE(\hat{S}^2_{\mathrm{Re}g})}\right) \tag{24}$$

Muneer et al. [50] proposed an improved class of estimators for $\hat{S}_y^2$, is given by

$$\hat{S}^2_M=s_y^2\left[k_{11}\left(\frac{S_x^2}{s_x^2}\right)+k_{12}\left(\frac{s_x^2}{S_x^2}\right)\right]\exp\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right) \tag{25}$$

Where k_{11} and k_{12} are suitably chosen constants. The minimum *MSE* of $\hat{S}_M^2$ to first order of approximation, at optimum values

$$k_{11(opt)} = \frac{1}{8}\left(\frac{16V_{22}^{2} + 6V_{22}V_{40} - 24V_{22}V_{04} - 16V_{40}V_{04} - V_{04}^{2} - 16V_{22} - 8V_{04}}{16_{22}^{2} - 16V_{22}V_{04} - 4V_{40}V_{04} + V_{04}^{2} - 4V_{04}} \right)$$

and

$$k_{12(opt)} = \frac{1}{8}\left(\frac{48V_{22}^{2} - 16V_{22}V_{40} - 72V_{22}V_{04} + 16V_{40}V_{04} + 21V_{04}^{2} + 16V_{22} - 24V_{04}}{16_{22}^{2} - 16V_{22}V_{04} - 4V_{40}V_{04} + V_{04}^{2} - 4V_{04}} \right)$$

is given by

$$MSE(\hat{S}_{M}^{2})_{\min} \cong \frac{S_{y}^{4}}{16}\left(\frac{\begin{array}{l} 64V_{22}^{2}V_{40} - 48V_{22}^{2}V_{04} - 128V_{22}V_{40}V_{04} + 48V_{22}V_{04}^{2} + 64V_{40}V_{04}^{2} - 48V_{40}V_{04}^{2} \\ -9V_{22}^{3} - 64V_{22}^{2} - 48V_{40}V_{04}^{2} \end{array}}{16_{22}^{2} - 16V_{22}V_{04} - 4V_{40}V_{04} + V_{04}^{2} - 4V_{04}} \right) \tag{26}$$

3.0 Proposed Estimator

In this section, a new generalized mixture estimator is suggested as:

$$\hat{S}_{\Pr op}^{2} = \left\{ d_{1}s_{y}^{2}\left[\frac{1}{2}\left(\frac{S_{x}^{2}}{s_{x}^{2}} + \frac{s_{x}^{2}}{S_{x}^{2}} \right) \right]^{\alpha} + d_{2}\left(S_{x}^{2} - s_{x}^{2} \right) \right\} \exp\left\{ \frac{S_{x}^{2} - s_{x}^{2}}{S_{x}^{2} + s_{x}^{2}} \right\} \tag{27}$$

where d_i $(i = 1,2)$ are unknown constants whose values to be determined later, and α is suitably chosen constant.

Expressing the estimator $\hat{S}_{RPR}^{2}$ in terms of e_i $(i = 0,\ 1)$ we can write (27) as

$$\hat{S}_{\Pr op}^{2} = \left\{ d_{1}S_{y}^{2}\left(1+e_{0}\right)\frac{1}{2^{\alpha}}\left[\left(1+e_{1}\right)^{-1} + 1 + e_{1} \right]^{\alpha} - d_{2}S_{x}^{2}e_{1} \right\} \exp\left\{ \frac{-S_{x}^{2}e_{1}}{2S_{x}^{2} + S_{x}^{2}e_{1}} \right\} \tag{28}$$

Expanding the RHS of (30) to the first order of approximation, multiplying out and neglecting the terms of e’s greater than two, we get

$$\hat{S}_{\Pr op}^{2} - S_{y}^{2} = \left(d_{1}-1\right)S_{y}^{2} + d_{1}S_{y}^{2}\left(e_{0} - 2^{-1}e_{1} - 2^{-1}e_{0}e_{1} + Ae_{1}^{2} \right) - d_{2}S_{x}^{2}\left(e_{1} - 2^{-1}e_{1}^{2} \right) \tag{29}$$

Where $A = \left[2^{-1}\alpha + 3 \times 2^{-3} \right]$

By taking expectation of (32), the bias of the proposed generalized estimator, up to the first order of approximation, is given by:

$$Bias\left(\hat{S}_{\Pr op}^{2} \right) = \left(d_{1}-1\right)S_{y}^{2} + d_{1}S_{y}^{2}\left(AV_{04} - \frac{1}{2}V_{22} \right) + \frac{d_{2}}{2}S_{x}^{2}V_{04} \tag{30}$$

By squaring equation (32) and keeping terms only up to the first order of approximation, we have:

$$MSE\left(\hat{S}_{\Pr op}^{2} \right) = \left\{ \begin{array}{l} \left(d_{1}-1\right)^{2}S_{y}^{4} + d_{1}^{2}S_{y}^{4}\left[V_{40} - 2V_{22} + \left(2A + \frac{1}{4} \right)V_{04} \right] + d_{2}^{2}S_{x}^{4}V_{04} \\ -d_{1}S_{y}^{4}\left(2AV_{04} - V_{22} \right) - 2d_{1}d_{2}S_{y}^{2}S_{x}^{2}\left(V_{22} - V_{04} \right) - d_{2}S_{y}^{2}S_{x}^{2}V_{04} \end{array} \right\} \tag{31}$$

Minimizing Equation (37) with respect to d_1 and d_2, we get the optimum values of d_1 and d_2 i.e.

$$d_{1(opt)}=\frac{1+\left(A-\frac{1}{2}\right)V_{04}}{1+\left(2A-\frac{3}{4}\right)V_{04}+\left(V_{40}-\frac{V_{22}^2}{V_{04}}\right)}\text{ and } d_{2(opt)}=\frac{S_y^2}{S_x^2}\left\{\frac{1}{2}-d_{1(opt)}\left(1-\frac{V_{22}}{V_{04}}\right)\right\}$$

Substituting the optimum values of d_1 and d_2 in Equation (37), we get the minimum *MSE* of $\hat{S}^2_{\mathrm{Pr}op}$ as:

$$MSE\left(\hat{S}^2_{\mathrm{Pr}op}\right)_{\min}=S_y^4\left\{\left(1-\frac{V_{04}}{4}\right)-\frac{\left[1+\left(A-\frac{1}{2}\right)V_{04}\right]^2}{\left[1+\left(2A-\frac{3}{4}\right)V_{04}+\left(V_{40}-\frac{V_{22}^2}{V_{04}}\right)\right]}\right\} \tag{32}$$

Special Cases:

For $\alpha=1$ this proposed estimator becomes

$$\hat{S}^2_{\mathrm{Pr}op(1)}=\left\{d_1 s_y^2\left[\frac{1}{2}\left(\frac{S_x^2}{s_x^2}+\frac{s_x^2}{S_x^2}\right)\right]+d_2\left(S_x^2-s_x^2\right)\right\}\exp\left\{\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right\} \tag{33}$$

The optimum values of d_1 and d_2, for this estimator, are given by:

$$d_{1(opt)}=\frac{1+\frac{3}{8}V_{04}}{1+V_{04}+\left(V_{40}-\frac{V_{22}^2}{V_{04}}\right)}\text{ and } d_{2(opt)}=\frac{S_y^2}{S_x^2}\left\{\frac{1}{2}-d_{1(opt)}\left(1-\frac{V_{22}}{V_{04}}\right)\right\}$$

The minimum mean squared error, up to the first order of approximation, is given by:

$$MSE\left(\hat{S}^2_{\mathrm{Pr}op(1)}\right)_{\min}=S_y^4\left\{\left(1-\frac{V_{04}}{4}\right)-\frac{\left[1+\frac{3}{8}V_{04}\right]^2}{\left[1+V_{04}+\left(V_{40}-\frac{V_{22}^2}{V_{04}}\right)\right]}\right\} \tag{34}$$

When $\alpha=2$, the proposed estimator becomes

$$\hat{S}^2_{\mathrm{Pr}op(2)}=\left\{d_1 s_y^2\left[\frac{1}{2}\left(\frac{S_x^2}{s_x^2}+\frac{s_x^2}{S_x^2}\right)\right]^2+d_2\left(S_x^2-s_x^2\right)\right\}\exp\left\{\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right\} \tag{35}$$

The optimum values of d_1 and d_2, for this estimator, are given by:

$$d_{1(opt)}=\frac{1+\frac{7}{8}V_{04}}{1+2V_{04}+\left(V_{40}-\frac{V_{22}^2}{V_{04}}\right)}\text{ and } d_{2(opt)}=\frac{S_y^2}{S_x^2}\left\{\frac{1}{2}-d_{1(opt)}\left(1-\frac{V_{22}}{V_{04}}\right)\right\}$$

The minimum mean squared error, up to the first order of approximation, is given by:

$$MSE\left(\hat{S}^2_{\mathrm{Pr}op(2)}\right)_{\min} = S_y^4\left\{\left(1-\frac{V_{04}}{4}\right)-\frac{\left[1+\ \frac{7}{8}V_{04}\right]^2}{\left[1+2V_{04}+\left(V_{40}-\frac{V_{22}^2}{V_{04}}\right)\right]}\right\} \tag{36}$$

3.1 Theoretical Efficiency comparisons

In this section, theoretical efficiency conditions of proposed estimator over some existing estimators were established

(i) $Var(\hat{S}_y^2)-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-V_{40}-4^{-1}V_{04})-C^2}{B}\right)<0 \tag{37}$$

Where $B=1+\left(2A-\frac{3}{4}\right)V_{04}+\left(V_{40}-\frac{V_{22}^2}{V_{04}}\right)$ and $C=1+\left(A-\frac{1}{2}\right)V_{04}$

(ii) $MSE(\hat{S}_R^2)-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B\left(1-V_{40}-1.25V_{04}+2V_{22}\right)-C^2}{B}\right)<0 \tag{38}$$

(iii) $[MSE(\hat{S}^2_{\mathrm{Re}g}), MSE(\hat{S}^2_{SW})_{\min}\, or MSE(\hat{S}^2_{YK})_{\min}]-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-4^{-1}V_{04}-V_{40}(1-\rho^2_{(s_y^2,s_x^2)}))-C^2}{B}\right)<0 \tag{39}$$

(iv) $MSE(\hat{S}_d^2)-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-4^{-1}V_{04})-C^2}{B}-\frac{V_{40}(1-\rho^2_{(s_y^2,s_x^2)})}{S_y^{-4}[1+V_{40}(1-\rho^2_{(s_y^2,s_x^2)})]}\right)<0 \tag{40}$$

(v) $MSE(\hat{S}^2_{BT})-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-V_{40}-2^{-1}V_{04}+V_{22})-C^2}{B}\right)<0 \tag{41}$$

(vi) $MSE(\hat{S}^2_{SG})_{\min}-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-4^{-1}V_{04})-C^2}{B}-\frac{4V_{04}^2+16V_{40}(1-\rho^2_{(s_y^2,s_x^2)})(V_{04}-4)}{(V_{40}(1-\rho^2_{(s_y^2,s_x^2)})-1)}\right)<0 \tag{42}$$

(vii) $MSE(\hat{S}^2_{YG})_{\min}-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-4^{-1}V_{04})-C^2}{B}-\frac{S_y^{-4}MSE(\hat{S}^2_{\mathrm{Re}g})(1-V_{04})}{1-V_{04}+S_y^{-4}MSE(\hat{S}^2_{\mathrm{Re}g})}\right)<0 \tag{43}$$

(viii) $MSE(\hat{S}^2_{SM})_{\min}-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-4^{-1}V_{04})-C^2}{B}-\frac{D}{64E}\right)<0 \tag{44}$$

Where $D=V_{04}(V_{04}(V_{04}+8V_{22})+16(V_{04}-4)MSE(\hat{S}^2_{\mathrm{Re}g})+16V_{22}(V_{22}-V_{04}))$ and

$E=-V_{04}(1+V_{40}+2V_{22})+4V_{22}^2$

(ix) $MSE(\hat{S}^2_{YS})_{\min}-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-4^{-1}V_{04})-C^2}{B}-\frac{64(1-V_{04})S_y^{-4}MSE(\hat{S}^2_{\mathrm{Re}g})-V_{04}^2}{16(V_{04}+4(1-V_{04})+4S_y^{-4}MSE(\hat{S}^2_{\mathrm{Re}g}))}\right)<0 \tag{45}$$

(x) $MSE(\hat{S}^2_{M})_{\min}-MSE(\hat{S}^2_{\mathrm{Pr}op})_{\min}>0$, if

$$\left(\frac{B(1-4^{-1}V_{04})-C^2}{B}-\frac{F}{16G}\right)<0 \tag{46}$$

Where

$F=64V_{22}^2V_{40}-48V_{22}^2V_{04}-128V_{22}V_{40}V_{04}+48V_{22}V_{04}^2+64V_{40}V_{04}^2-48V_{40}V_{04}^2-9V_{22}^3-64V_{22}^2-48V_{40}V_{04}^2$

and $G=16_{22}^2-16V_{22}V_{04}-4V_{40}V_{04}+V_{04}^2-4V_{04}$

4.0 Results and Discussion

To observe the performance of our proposed estimator with respect to other considered estimators, the following data sets for numerical comparisons, which earlier used by many authors in literature were used.

Table 1. Source and Description of Data Sets

Data set	Source	Y	X
1	Gujarati [43]	Per capita consumption of chickens in pounds in United Stats from 1960–1982.	Real disposable income per capita in dollars in United Stats from 1960–1982.
2	Mukherjee et al. [49]	Number of literate persons in the village.	Number of workers in the village.
3	Singh and Mangat [54]	Leaf area for the newly developed strain of wheat.	The weight of leaves.
4	http://www.osservatorionazionalerifiuti.it (2004)	Total amount (tons) of recyclable-waste collection in Italy in 2003.	Number of inhabitants living in Italy in 2003.
5	Murthy [51]	Output for 80 factories in a region.	Fixed capital.

Table 2. Parameters of the Data Sets

Parameters	Data 1	Data 2	Data 3	Data 4	Data 5
N	23	64	39	103	80
n	5	19	14	40	20
$\bar{Y}$	39.699	5.549	26.8433	62.6212	51.8264

$\bar{X}$	1035.065	141.500	106.2	556.5541	11.2646
S_y^2	54.360	2.277	38.9900	8345.7177	336.9757
S_x^2	381735.000	5772.670	124.1286	372300.473	70.6634
$\rho_{(s_y^2, s_x^2)}$	0.8277	0.2971	0.8899	0.6570	0.7941
$\beta_{2(y)}$	2.030	2.773	2.4032	37.1279	2.2667
$\beta_{2(x)}$	2.696	2.341	2.9930	17.8738	2.8664
λ_{22}	2.094	1.458	2.4882	17.2220	2.2209
θ	0.1565	0.0370	0.0458	0.0153	0.0375
V_{40}	0.1612	0.0656	0.0643	0.5525	0.0475
V_{04}	0.2655	0.0496	0.0913	0.2581	0.0699
V_{22}	0.1713	0.0169	0.0682	0.2481	0.0457

The following expression is used for the computations of percentage relative efficiency *(PREs)* of the proposed and considered estimators:

$$PRE = \frac{Var(\hat{S}_y^2)}{MSE(i) \quad or \quad MSE_{min}(i)} * 100, \quad i = \hat{S}_y^2, \hat{S}_R^2, \hat{S}_{Reg}^2, \hat{S}_d^2, \hat{S}_{BT}^2 \dots \hat{S}_{Prop}^2$$

Table 3. MSE and PRE values of different estimators with respect to $\hat{S}_y^2$

Estimators	Data 1		Data 2		Data 3		Data 4		Data 5	
	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
$\hat{S}_y^2$	476.347	100.00	1.763	100.00	97.751	100.00	38517005.7	100.00	5393.752	100.00
$\hat{S}_R^2$	248.516	191.676	2.186	80.664	29.188	334.895	21940066.5	175.556	2952.369	182.693
$\hat{S}_{Reg}^2$	149.753	318.089	1.608	109.676	20.303	481.453	21913070.0	175.772	2000.996	269.554
$\hat{S}_d^2$	142.529	334.209	1.517	116.237	20.036	487.883	16668844.7	231.072	1966.346	274.304
$\hat{S}_{BT}^2$	166.293	286.451	1.641	107.438	28.771	339.763	25736046.3	149.662	2188.728	246.433
$\hat{S}_{SG}^2$	131.232	362.981	1.561	112.962	19.329	505.722	17930714.1	214.811	1934.228	278.858
$\hat{S}_{SW}^2$	149.753	318.089	1.608	109.676	20.303	481.453	21914066.5	175.556	2000.996	269.554
$\hat{S}_{YG}^2$	140.087	340.036	1.513	116.580	20.009	488.528	15388327.7	250.301	1963.791	274.661
$\hat{S}_{YK}^2$	149.753	318.089	1.608	109.676	20.303	481.453	21914066.5	175.556	2000.996	269.554
$\hat{S}_{SM}^2$	112.932	421.800	1.497	117.768	18.612	525.198	24877706.5	154.823	1865.461	289.137
$\hat{S}_{YS}^2$	125.346	380.026	1.494	118.092	19.316	506.053	14438388.6	266.768	1919.254	281.034
$\hat{S}_M^2$	51.405	926.643	1.412	124.947	14.456	671.993	13829282.0	278.518	1631.372	330.627
$\hat{S}_{Prop(1)}^2$	44.406	1072.71	0.272	648.162	13.479	725.210	11861495.0	324.723	1608.521	335.324
$\hat{S}_{Prop(2)}^2$	33.285	1431.12	0.250	705.200	3.1788	3075.09	7970518.00	483.243	1091.940	493.960

Note: The minimum *MSE* value of $\hat{S}^2_{YG}$ is given in Table 3 for $(\lambda, a, b) = (1, 1, 0)$.

Table 3 shows the results of mean square error (MSE) values and PREs for the proposed and some existing estimators with respect to sample variance. The efficiency comparison of the proposed estimator has been made with the other estimators considered in the study and the results revealed that the performance of the proposed estimators $\hat{S}^2_{Prop(1)}$ and $\hat{S}^2_{Prop(2)}$ is better than all other considered estimators.

5. Conclusion

In this study, a new generalized exponential-type estimator for estimating the finite population variance $\hat{S}^2_y$ using the auxiliary information in simple random sampling was suggested. Expressions for bias and *MSE* of the estimators were derived up to first degree order approximation. The theoretical efficiency conditions under which the proposed class of estimators are better than the existing estimators were presented. In Table 3, the classes of the proposed estimator $\hat{S}^2_{Prop}$ are compared with usual variance estimator and other considered estimators numerically. Evidence from the study revealed that the proposed classes of estimators are more efficient than the existing estimators.

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