Original Research Article

Holographic Dark Energy and Dark Matter Interaction in Anisotropic Bianchi Type-V Universe

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Abstract

In this study, we considered a cosmological model of Holographic Dark Energy (HDE) interacting with Dark Matter (DM) in the case of anisotropic Bianchi type-V Universe model. As the Universe's expansion factor as called scale factor is used a special exponential form that is proposed recently by Silva [1] [Progress in Physics, 10(2), (2014), 93-97]. The corresponding cosmological parameters are calculated and the effective equation of states (EoS) are calculated with anisotropic additions. The results are analyzed to explain the evolution of the universe at the matter dominated era and DE dominated era and then the results are compared with the observational and theoretical studies in the literature.

Keywords: dark energy, dark matter, interaction, anisotropy

1. Introduction

The idea of our universe being in the process of cosmic acceleration is an important issue of our day that is supported with observations and theoretical approaches. This expansion has been explained and measured by Supernovae Type-Ia (SNIa) results [3]-[4], temperature fluctuations in Cosmic Microwave Background (CMB) [5], baryon acoustic oscillations, measurements of the Large Scale Structure (LSS) of the universe, the Hubble constant and the Sloan Digital Sky Survey (SDSS). Following the major results of the mentioned studies, the universe is dominated by DE at present and this situation is causing cosmic acceleration with positive energy density but negative pressure that violates the strong energy conditions and the violation brings a reverse gravitational effect. However, the exotic nature of DE is still unknown. Therefore the unknown nature of alternative DE models are studied. Some of them are a cosmological constant model [6], quintessence model[7], k-essence model [8], tachyon [9], quintom [10], braneworld [11], dilatonic ghost condensate [12], quartessence [13], Chaplygin gas [14], holographic models [15], Chaotic Universe Theory [16] and so fort. The major aim of these studies is to find an answer to the question of 'Why is the universe expanding faster than it should be?' and to understand the nature of the cosmic acceleration and the evolution of the universe. In these works generally universe is accepted as flat, homogenous and isotropic described by Friedmann-Robertson-Walker (FRW) [17] metric but it is possible that the early universe not be exactly uniform. Recent observational data reveals that FRW models are leading to spatially homogeneous and isotropic universe and there are some theoretical works which support the model best suited for the declaring of large scale structure of present day universe [18]. However the recent cosmological observations promote that in the early universe era FRW models are not describing a correct matter description and the results support the existence of an anisotropic background approaching to isotropic one [5]. The anisotropy of the DE in the Bianchi Type space times is the simplest model with anisotropic background and is useful in forming ellipsoidality of the Universe, and this model is also consistent with the observed CMBR anisotropies. Since 1960s many authors had investigated Bianchi type spacetimes in different theories [19]. Bianchi Type-I cosmological model considering a mixture of perfect fluid and DE is studied as an simplest generalization of FRW flat space time and it plays important role in explaining the phenomenal formation of galaxies in the early universe phase. Bianchi Type-V generalizes the open ($k = -1$) FRW model which contains both viscosity and heat flow with spatially homogeneity.

In cosmology, the interaction between the dark components is thought to give information about the evolution and expansion history of the universe. There are lots of works in the literature [20]-[21] such as different DE models interacts with DM and with unknown component of DE. These approaches aims to describe an energy flow between mentioned components. In these works the analysis of DE is generally made by the equation of the state parameter (EoS) where $w_{DE} = P_{DE}/\rho_{DE}$ where P_{DE} and ρ_{DE} are the pressure and energy density of the DE, respectively. The EoS parameter plays crucial importance of solving two famous fine tuning cosmological problems: cosmological constant [25]-[24] and coincidence problem [23]-[26]. There are also some works about DE-DM interaction at anisotropic background, such as holographic Bianchi Type-V DE model in Saez-Ballester scalar tensor theory of gravitation is discussed [27], HDE model with linearly varying deceleration parameter are discussed spatially homogeneous and anisotropic Bianchi Type-V Universe that is filled with interacting DE and HDE [28], non-interacting HDE with linearly varying deceleration parameter in Bianchi type-I and V Universe and interacting HDE in Bianchi Type-II are analyzed [29], LRS Bianchi Type-V DE model in a scalar tensor theory of gravitation are discussed [30], Bianchi Type-II MHRDE model in a self-creation cosmology [27], Bianchi Type-V I0 anisotropic MHRDE cosmological model [31], so fort on. Hence in recent years, observational data supports the anomalies in the maps of CMB therefore anisotropic models rise in importance. HDE models are studied by many authors in anisotropic space-times. Understanding of the early-time inflation and late time acceleration of the universe are important topics in cosmology. The works also have been trying to find a better explaining of the early stages of evolution of the universe. In this study we have considered a cosmological model of HDE interacting with DM anisotropic Bianchi Type-V Universe for a stretched exponential form of the scale factor, and the results compared with the literature. In this Letter we have considered Holographic Dark Energy (HDE) model which is applying of Holgraphic principle to DE. Holographic principle was supported by black hole thermodynamics. This principle says that the universe can be coded as two dimensional information on the surface area of the horizon. The model takes the characteristic length scale L as the size of the universe. And the energy density is given in this model $\rho_{DE} = 3c^2 m_p^2 L^{-2}$, where c is speed of light and m_p^2 is planck reduced mass. In this study we have taken the cosmological model of HDE interacting with DM in the background of an anisotropic Bianchi Type-V Universe for a stretched exponential form of the scale factor to model the evolution of some physical parameters in the an isotropic universe, and the results compared with the literature. It is known that in the framework of FRW cosmology, the phantom like DE is strong enough to cause universe ends in the big rip and to prevent the formation of the black hole and in the phantom energy dominated universe case the expansion scale factor grows rapidly. The stretched exponent form of the scale factor is chosen in this work can be important to describe the evolution of the universe because of the evolution of the scale factor versus cosmic time. It is including the current values of the Hubble parameter and current universe age. For this model the interaction between HDE and DM is taken into account with anisotropic additions and the results show that HDE generate phantom like equation of state. This results is in contrast with some other works of HDE discussed in the literature [20, 21, 22]. The model studied in this Letter may provide a unification of the inflation generated by phantom model so tends the late time phantom acceleration of the universe which may be a key element for early time as well as in the late time acceleration of the universe. A transition from HDE to phantom DE phase with phantom-like EoS parameter with the decreasing rate of change over cosmic time may be more accurate approach to explaining the evolution of the universe consistent with the current observational results.

Outline of this letter is following: In Section 2, HDE and DM interactions are given for anisotropic Bianchi type-V Universe. First of all the important cosmological parameters and the corresponding EoS parameter calculated with anisotropic additions. In Section 3, The results are compared with the theoretical and recent observational results. Finally, some concluding remarks are given in Section 4.

2. Holographic Dark Energy Dark Matter Interaction

The holographic principle was first suggested by Hooft [32] and Susskind [33] for the black hole physics that contains the effective local quantum field theories with degrees of freedom. In this principle the entropy scales extensively for an effective quantum field theory in a box of size L with UV cut-off represented by λ and the principle assumes that the entropy of a system scales with surface area not volume not with its volume. When applying of the principle to DE problem a new DE model is showed up and it is called HDE model. According this HDE model the volume space can be seemed as 2D line encoded on the lower boundary to the region, so IR cut-off related to DE can be said the size of the corresponding horizon. The energy density of HDE model is defined by

$$\rho_{DE} = 3c^2 M_p^2 H^2 \tag{1}$$

where M_p is the Planck mass, c is the speed velocity and H is the Hubble parameter with function of time and then depends on L with $H = L^{-1}$ and can be find $H = \dot{a}/a$. In this study we have chosen the scale of $M_p c = 1$ and also we have chosen apparent horizon with the radius of L.

$$L = ar(t) \tag{2}$$

where a is the average scale factor that indicates the relative size of the universe with function of time. In this study we consider spatially homogeneous and anisotropic Bianchi Type-V Universe that can be described by the line element

$$ds^2 = -dt^2 + R_1^2 dx^2 + e^{2\alpha x}(R_2^2 dy^2 + R_3^2 dz^2) \tag{3}$$

R_1, R_2, and R_3 are the corresponding metric functions of cosmic time t where α is a constant. The average scale factor of the space time can be defined as a=$(R_1R_2R_3)^3$ in case of the corresponding metric functions, therefore the average Hubble's parameter can be interpreted as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_2}{R_2}) \tag{4}$$

over dot indicates the derivative with respect to the cosmic time t and $H_i = \dot{R}_i/R_i$. The Einstein's field equations;

$$R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij}^{(DM)} - T_{ij}^{(DE)}) \tag{5}$$

$T_j^{(DM)i}$ and $T_j^{(DE)i}$ are the energy momentum tensors of DM and DE with $\Lambda(t)$ cosmological constant, g_{ij} the metric tensor, R_{ij} is the Ricci tensor and R is the Ricci scalar. From the energy conservation

$$T_{;j}^{(DM)ij} - T_{;j}^{(DE)ij} = 0 \tag{6}$$

These are given by

$$T_j^{(DM)i} = diag[-\rho_{DM}, P^{DM}, P^{DM}, P^{DM}] \tag{7}$$

and

$$T_j^{(DE)i} = diag[-\rho_{DE}, P_x^{DE}, P_y^{DE}, P_z^{DE}] \tag{8}$$

with the relation of $P_{DE} = w_{DE}\rho_{DE}$ the energy momentum tensor of DE is given by

$$T_j^{(DE)i} = diag[-1, w_{DE_x}, w_{DE_y}, w_{DE_z}]\rho_{DE} \tag{9}$$

where w is the EoS parameter of DE with the components of w_{DE_x}, w_{DE_y} and w_{DE_z} are function of the time and they are the directional of w along

x, y and z coordinate axes, respectively. By the modifying EoS parameter we assume that the skewness parameters of γ_x, γ_y and γ_z and retyping (9)

$$T_j^{(DE)i} = diag[-1, (w_{DE} + \gamma_x), (w_{DE} + \gamma_y), (w_{DE} + \gamma_z)] \tag{10}$$

γ is representing the deviation from w_{DE} on x,y and z axes. Using of (3) , (5), (6) and (10) the corresponding field equations are derived as

$$\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \frac{\dot{R}_3\dot{R}_1}{R_3R_1} - \frac{3\alpha^2}{R_1 2} = (\rho_{DE} + \rho_{DM}) \tag{11}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} - \frac{\alpha^2}{R_1^2} = -(w_{DE} + \gamma_z)\rho_{DE} \tag{12}$$

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} - \frac{\alpha^2}{R_1^2} = -(w_{DE} + \gamma_x)\rho_{DE} \tag{13}$$

$$\frac{\ddot{R}_3}{R_3} + \frac{\ddot{R}_1}{R_1} + \frac{\dot{R}_3\dot{R}_1}{R_3R_1} - \frac{\alpha^2}{R_1^2} = -(w_{DE} + \gamma_y)\rho_{DE} \tag{14}$$

$$\frac{2\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = 0 \tag{15}$$

By using the equation of (15) we can get;

$$R_1^2 = R_2R_3 \tag{16}$$

Assuming $R_2 = R_3^{\varsigma}$ and the relation between the scale factor, and ς is a positive constant and preserves the anisotropic character of the space time. The metric functions are assumed a power law relation with cosmic time in an attempt to find exact solutions of the set of field equations. The metric functions are calculated by using (16)

$$R_1 = a, \quad R_2 = a^{\frac{2\varsigma}{\varsigma+1}}, \quad R_3 = a^{\frac{2}{\varsigma+1}} \tag{17}$$

Recently introduced a special exponential form of the scale factor by Silva [1];

$$a = \exp\left[\frac{H_0T_0}{\beta}\left(\frac{t}{T_0}\right)^{\beta} - 1\right] \tag{18}$$

where β is a constant. H_0 is the current value of Hubble parameter and T_0 the current age of the universe. Equation of (18) is assumed to be describing the expansion of our universe from the beginning. With subtracting the equation of (18) to (17)

$$R_1 = a, \quad R_2 = a^{\frac{2\varsigma}{\varsigma+1}}, \quad R_3 = a^{\frac{2}{\varsigma+1}} \tag{19}$$

the directional Hubble parameters can be calculated by $H_i = \dot{R}_i/R_i$;

$$\begin{aligned} H_1 &= H_0 t^{-1+\beta} {T_0}^{1-\beta} \\ H_2 &= \frac{H_0 \varsigma t^{-1+\beta} {T_0}^{1-\beta}}{1+\varsigma} \\ H_3 &= \frac{H_0 t^{-1+\beta} {T_0}^{1-\beta}}{1+\varsigma} \end{aligned} \tag{20}$$

the time dependent Hubble parameter is calculated by putting (20) to (4)

$$H = H_0 t^{-1+\beta} T_0^{1-\beta} \tag{21}$$

the time dependent deceleration parameter $q = -a.\ddot{a}/\dot{a}$ can be found by using (18)

$$q = -\left(\frac{\beta - 1}{H_0 T_0} \left(\frac{t}{T_0} \right)^{-\beta} + 1 \right). \tag{22}$$

The time dependent skewness parameters can be calculated by putting (19) to (12), (13) and (14)

$$\begin{aligned} \gamma_x = {} & \exp\left[\frac{-H_0(-1+t^\beta)}{{T_0}^{1-\beta}} \right] \left[\frac{\alpha^2 {T_0}^2}{3{H_0}^2} \right] - 2\frac{\exp\left[(\frac{H_0(1-t^2)}{2T_0})(1 + \frac{2\varsigma}{1+\varsigma}) \right] \varsigma t^{-2+2\beta} {T_0}^{4-2\beta}}{9(1+k)} \\ & - 2\frac{\exp\left[(\frac{H_0(-1+t^2)}{2T_0})(\frac{4\varsigma}{1+\varsigma}) \right] \varsigma t^{-2+\beta}(-1+\beta){T_0}^{3-\beta}}{9H_0(1+\varsigma)} \\ & - 2\frac{\exp\left[(\frac{H_0(-1+t^\beta){T_0}^{1-\beta}}{2T_0})(\frac{-4}{1+\varsigma}) \right] t^{-2+\beta}(-1+\beta){T_0}^{3-\beta}}{9H_0(1+\varsigma)} - w_{DE} \end{aligned} \tag{23}$$

$$\gamma_y = \exp\left[\frac{-H_0(-1+t^{\beta})}{{T_0}^{1-\beta}}\right]\left[\frac{\alpha^2{T_0}^2}{3{H_0}^2} - \frac{(-1+\beta)}{H_0{T_0}^{-3+\beta}}\right] - 2\frac{\exp\left[(\frac{H_0(1-t^2)}{2T_0})(1+\frac{2\varsigma}{1+\varsigma})\right]t^{-2+2\beta}{T_0}^{4-2\beta}}{9(1+\varsigma)}$$
$$- 2\frac{\exp\left[(\frac{H_0(-1+t^2)}{2T_0})(\frac{4}{1+\varsigma})\right]t^{-2+\beta}(-1+\beta){T_0}^{3-\beta}}{9H_0(1+\varsigma)} - w_{DE} \tag{24}$$

$$\gamma_z = \exp\left[\frac{-H_0(-1+t^{\beta})}{{T_0}^{1-\beta}}\right]\left[\frac{\alpha^2{T_0}^2}{3{H_0}^2} - \frac{(-1+\beta)}{H_0{T_0}^{-3+\beta}}\right] - 2\frac{\exp\left[(\frac{H_0(1-t^2)}{2T_0})(1+\frac{2\varsigma}{1+\varsigma})\right]\varsigma t^{-2+2\beta}}{9(1+\varsigma){T_0}^{-4+2\beta}}$$
$$- 2\frac{\exp\left[(\frac{H_0(-1+t^2)}{2T_0})(\frac{4\varsigma}{1+\varsigma})\right]\varsigma t^{-2+\beta}(-1+\beta)}{9H_0(1+\varsigma){T_0}^{-3+\beta}} - w_{DE} \tag{25}$$

The physical quantities named as cosmological parameters that are important in cosmology. θ is the expansion scalar and it formulates as;

$$\theta = 3H = 3H_0t^{-1+\beta-1}T_0^{1-\beta} \tag{26}$$

The scalar expansion $\theta > 0$ indicates that the model yields expanding in nature. The shear scalar which is a measure of how the expansion rate differs in different directions is represented as σ^2

$$\sigma^2 = \frac{1}{2}(\sum_{i=1}^{n} H_i^2 - \frac{1}{3}\theta^2) \tag{27}$$

can be calculated by using (21) and (26)

$$\sigma^2 = \frac{H_0^2(-1+k)^2t^{-2+2\beta}T_0^{2-2\beta}}{(1+k)^2} \tag{28}$$

As can be seen the rate of σ^2/θ^2 does not of depend on the cosmic time that it implies that the model does not pass isotropy. The corresponding average anisotropy parameter is the measure of the magnitude of anisotropy in the expansion rate is given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}(\frac{H_i - H}{H})^2 \tag{29}$$

using (20) and (21)

$$\Delta = \frac{2(-1+\varsigma)^2}{3(1+\varsigma)^2} \tag{30}$$

We assume the universe in which the DE and DM are interacting to each other and the total energy density satisfies the continuity equation, therefore we can rewrite (6) to find the continuity equation

$$\begin{aligned} \dot{\rho}_{DE} + \dot{\rho}_{DM} + \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right)(\rho_{DE}(1+w_{DE}) + \rho_{DM}) \\ + \left(\gamma_x \frac{\dot{R}_1}{R_1} + \gamma_y \frac{\dot{R}_2}{R_2} + \gamma_z \frac{\dot{R}_3}{R_3}\right)\rho_{DE} = 0 \end{aligned} \tag{31}$$

we can rewrite by separating the equation of (31) for non-interaction case

$$\dot{\rho}_{DE} + 3H(1+w_{DE})\rho_{DE} + (\gamma_x H_x + \gamma_y H_y + \gamma_z H_z)\rho_{DE} = 0 \tag{32}$$

$$\dot{\rho}_{DM} + 3H\rho_{DM} = 0 \tag{33}$$

for the interaction case

$$\dot{\rho}_{DE} + 3H(1+w_{DE})\rho_{DE} + (\gamma_x H_x + \gamma_y H_y + \gamma_z H_z)\rho_{DE} = -Q \tag{34}$$

$$\dot{\rho}_{DM} + 3H\rho_{DM} = Q \tag{35}$$

where Q represent the overall conservation of the energy momentum tensor which indicates a transfer from DE component to DM component and vice versa. If we choose to take the coupling term depend on the density of DE;

$$Q = 3b^2 H\rho_{DE} \tag{36}$$

where b is the coupling constant.Then we have used (1), (34) and (36) to get the corresponding EoS parameter of DE;

$$w_{DE} = -1 - b^2 - \frac{2\dot{H}}{3H^2} - \frac{(\gamma_x H_x + \gamma_y H_y + \gamma_z H_z)}{3H} \tag{37}$$

the directional EoS Parameters can be calculated by using (19), (20), (23), (24) and (25)

$$w_{DE} = -1 - b^2 + \frac{-2t^{\beta}(1+\beta){T_0}^{-1+\beta}}{3H_0} - \frac{1}{3t^2}\exp\left[\frac{-4H_0(-1+t^{\beta}){T_0}^{-1+\beta}}{\beta}\right]{T_0}^{-2\beta}$$
$$-3\exp\left[\frac{-4H_0(-1+t^{\beta}){T_0}^{-1+\beta}}{\beta}\right]t^2 w {T_0}^{2\beta}$$
$$+\exp\left[\frac{2H_0(-1+t^{\beta}){T_0}^{-1+\beta}}{\beta}\right]{T_0}^{\beta}(-2H_0t^{\beta}(-1+\beta)T_0 + 9t^2\alpha^2{T_0}^{\beta})$$
$$-\frac{4\exp\left[\frac{3H_0(-1+t^{\beta}){T_0}^{-1+\beta}}{\beta}(3-\frac{2\varsigma}{1+\varsigma})\right]\varsigma t^{2\beta}}{(1+\varsigma)^2{H_0}^{-2}{T_0}^{-2}} - \frac{2\exp\left[\frac{H_0(-1+t^{\beta}){T_0}^{-1+\beta}}{\beta}(\frac{1+3\varsigma}{1+\varsigma})\right](1+3\varsigma)t^{2\beta}}{(1+\varsigma)^2{H_0}^{-2}{T_0}^{-2}}$$
$$-\frac{2\exp\left[\frac{H_0(-1+t^{\beta}){T_0}^{-1+\beta}}{\beta}(\frac{4\varsigma}{1+\varsigma})\right](-1+\beta)(1+3\varsigma)^2t^{\beta}}{(1+\varsigma)^2{H_0}^{-1}{T_0}^{-\beta-1}}$$
$$-\frac{2\exp\left[\frac{H_0(-1+t^{\beta}){T_0}^{-1+\beta}}{\beta}(\frac{4}{1+\varsigma})\right](-1+\beta)(1+\varsigma^2)t^{\beta}}{(1+\varsigma)^2{H_0}^{-1}{T_0}^{-\beta-1}} \tag{38}$$

It can be seen from the (38) the directional EoS parameter depends on the cosmological time and the coupling constant.

3. Numerical Results and Discussion

Bianchi Type-V Universes are generalization of the open FRW model that is isotropic. In this study Bianchi Type-V metric is considered with the thought that may be important in the unknown nature of the DE. The scale factor is taken a special form that is proposed by Silva. Again this study was considered under two important assumptions: one of them is interaction of DE and DM which are two fluids in the anisotropic universe, the second is that the universe was a holographic wrapped by L. Under these two assumptions the Einstein field equations are calculated and some important cosmological parameters are calculated. Then, the equation of state of DE are calculated by using the modified continuity equation which includes its interaction and anisotropic additives. Obtained numerical solutions are compared observational results and isotropic models in the literature.

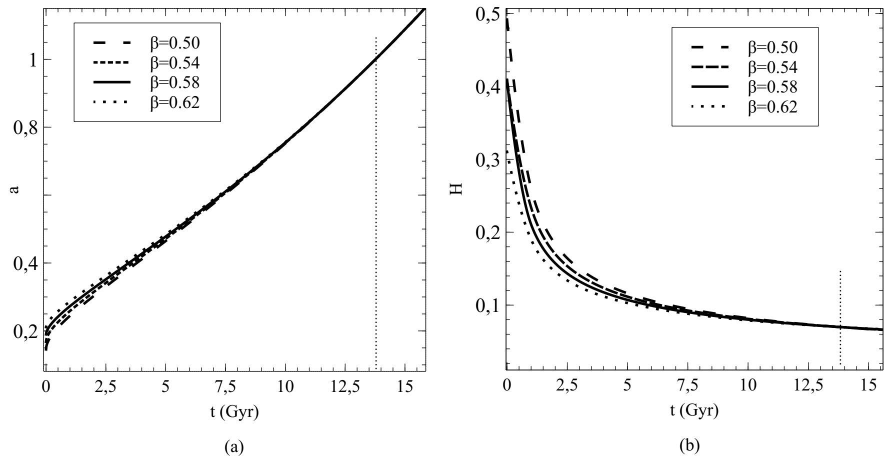

Figure 1: (a) The scale factor with function of cosmic time at different β values where $H_0 = 0.07\ Gyr^{-1}$ and $T_0 = 13.77\ Gyr$ (b) The Hubble parameter with function of cosmic time at different β values where $H_0 = 0.07\ Gyr^{-1}$ and $T_0 = 13.77\ Gyr$

Firstly, the chosen scale factor is examined versus the cosmic time, then plotted in the Fig. 1 (a). As we know that the scale factor behaves increasing in an expanding universe and explains how the universe expanded over time [34]. In the standard solutions the scale factor is changed by $a(t) \propto t^{2/3}$, $a(t) \propto t^{1/3}$ ve $a(t) \propto e^{Ht}$. In addition changing of the density function is given by $\rho \propto a^{-4}$ for radiation, $\rho \propto a^{-3}$ for pressureless matter and ρ_0^{Λ} for Λ. The second variation of the scale factor may give the information about whether the expanding universe is decreasing or increasing. It can be said that for matter and radiation the expansion of the universe is decreasing and for Λ the expansion of the universe is increasing. By looking at the relationship between the scale factor and density, it is possible to say that in the early stage of the universe the matter and radiation is dominated but in the late stage of the universe there are an energy source that force the universe expanding, for example Λ. With the mentioned informations we back to our results, as seen from the Fig. 1 (a), the change of the scale factor versus cosmic time shows that the universe is decreasing phase of the expansion in the early stage of the universe and increasing phase of the expansion in the late stage of the universe, therefore the expansion of the universe is accelerating. Silva calculated the value of β is 0.58 in the conditions of $H_0 = 0.07$ Gyr^{-1} and

$T_0 = 13.77$ Gyr [2]. Overlapping of these curves is occurring approximately at the 6.650 Gyr. The special form of the scale factor is actually say that the increased phase of the expansion of the universe began before the present time. At the same time studies are showed that the value of the present scale factor is 1 [35]. The results are in accordance with the theoretical results. By thinking the present age of the universe is changeable the Hubble parameter is plotted at the different β values. As can be seen from the Fig. 1 (b) obtained curves in the different β values is getting same values at the present time. As we know that the Hubble parameter give information about expansion speed of the universe, in other words how quickly a point in a certain distance moves away from us. In the Fig. 1 (b) it can be seen that the Hubble parameter is getting the same value but not the constant value at the present time with the increasing value of the scale factor at the different β values. If we remember the Hubble parameter is depended the scale factor by $\dot{a}/a$, the expansion of the universe causes the increasing behavior of the scale factor, therefore it causes $\dot{a} > 0$. because of the the accelerated expansion of the universe the value of $\dot{a}$ is increased. It means that the Hubble parameter is decreasing with time. As mentioned the standard model say that the Hubble parameter takes the constant value by dominating an energy source. It is possible to say that in an anisotropic universe model where the interactions of DE and DM are taken into consideration, the universe will continue to expand in the late times of the universe. Overlapping of these curves is occurring approximately at the 6.650 Gyr. It can be said that accelerated expansion occurs independent form the Hubble parameter and the present age of the universe.

Thirdly, in the Fig. 2 (a) the change of the shear scale versus cosmic time is plotted at the different β values. The shear scalar gives us the deviation from the average expansion rate of the universe [36]. In this model as the values of the shear scalar starts of the huge values at the early stage of the universe it is displaying decreasing behavior until the present time. In the near future is approaching a constant value but not to zero. The rate of the shear and expansion scalar is taken $lim_{t\to\infty}\sigma^2/\theta = 0$. This result showed that the model would approach isotropic behavior at the late stage of the universe. It is consistent with results of other theoretical studies [37, 38]. In the Fig. 2 (b) the change of the deceleration parameter versus cosmic time is plotted at the different β values. The deceleration parameter is a dimensionless measure of the cosmic expansion, so it gives the rate of expansion of the universe. In the Fig. 2 (b) the value of the deceleration parameter takes big values form zero at the early stage of the universe. However it takes less than zero values an the present time. These change of the deceleration parameter q show that in the early stages of the universe, the phase from which the expansion decreasing to the increasing phase of expansion [39]. Separating apart of these curves is occurring approximately at the 3.21 Gyr. This result means that the speed of the expansion of the universe will be affected by the change of Hubble parameter and the age of the universe. At the same time, the present values of the deceleration parameter are in consistent with the work of Gómez [40].

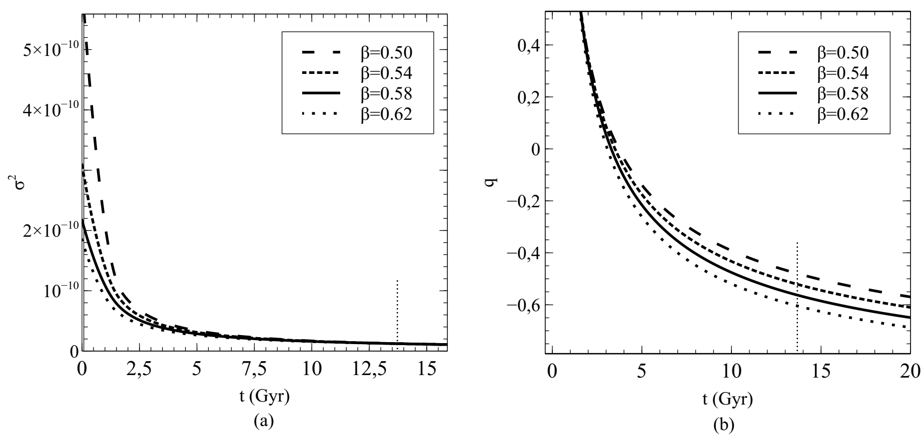

Figure 2: (a) Expansion of the shear scalar versus cosmic time with the different β values where $H_0 = 0.07\ Gyr^{-1}$ and $T_0 = 13.77\ Gyr$ (b) The deceleration parameter with function of cosmic time at different β values where $H_0 = 0.07\ Gyr^{-1}$ and $T_0 = 13.77\ Gyr$

Finally, in the Fig. 3 the change of EoS parameter of DE versus cosmic time is plotted at the different β values. As mentioned before there are different DE models according to different EoS parameters. In the Fig. 3 it is seen that the EoS parameter is in the range of -1 to -2 at the present time, that is, it exceeds the value of -1. This result is the same as the phantom DE model. Under this assumption of the interaction of the holographic universe and HDE with DM, the EoS parameter goes to the phantom model in an anisotropic universe. It shows that the universe will continue to accelerated expansion.

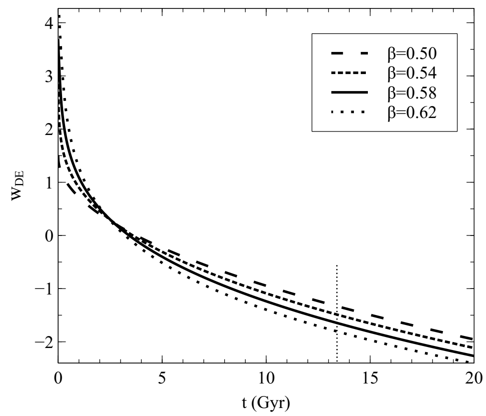

Figure 3: The evolution of the EoS parameter versus cosmic time with different β values where $w = -1.1$, $H_0 = 0.07\ Gyr^{-1}$ and $T_0 = 13.77\ Gyr$

4. Concluding Remarks

In this study, we have worked with anisotropic Bianchi type-V Universe in which Holographic model of DE interacts with DM. First of all as the expansion factor of universe is considered a special form proposed by Silva recently. Some cosmological parameters important for comparing with the observational results (H, Δ, σ^2, q) are calculated and plotted versus cosmic time. And then we have calculated the EoS parameter of corresponding DE by considering HDE-DM interaction with anisotropic additions. The results are analyzed in detail and we have concluded that this special form of the scale factor play important role to describe the evolution of the universe and to reach the results that are consistent with observational and theoretical results.

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