Review Article

A COPULA-BASED APPROACH FOR MODELLING THE DEPENDENCE BETWEEN INFLATION AND EXCHANGE RATE IN KENYA

Abstract

In this study, we used Kenyan data to model, by using the copula approach, the relationship between inflation and the exchange rate. For each univariate series of returns, serial dependence was modeled using ARMA+GARCH. Both for inflation and exchange rate, it was found that the student t distribution was the best marginal distribution. Then, before we estimate the copula, we transformed the standardized residuals from those marginal distributions (student t) into uniform over the range $[0, 1]$. For the copula estimation, a parametric test was applied. It was found that the Gumbel copula worked best to capture the dependence. Gumbel's copula successfully captures an upper tail dependence. Moreover,the time-varying dependence was also studied using change-point detection. We found that there is a change in the nature of dependence over the period under consideration.

Keywords: Copula, ARMA+GARCH, time-varying dependence, inflation, exchange rate

1 Introduction

Macroeconomic variables which comprehend inflation and exchange rates are crucial in the financial system of any country. Inflation and exchange rates capture significant signs of the overall performance of a financial system as a whole. A few researchers have tried to demonstrate the dynamics of dependence between inflation and exchange rate. As per [1], inflation is the first concern for national banks as it indicates of an increase or decrease in price in an economy. [2]and [3] argue that having high inflation prompts lower investment funds for people and furthermore makes light of an economy's global competitiveness. According to [4], every country that operates under a fixed exchange rate regime tends to see a decline in inflation. [5] and [6] both contended that having a stable exchange rate improves the effectiveness of the monetary policy and in addition decreases inflation.

The literature offers several outcomes when modeling the relationship between inflation and exchange rate. In terms of both country and data periods, these various outcomes vary. According to this theory, the importation of products and materials required for production is how the exchange rate and inflation are related. It has been shown in a few past studies that the dependence on macroeconomic indicators has an existing relationship. Different methods have been employed to establish this relationship, such as the autoregressive distributed lag (ARDL) approach used by [7], the Markov switching regression, and vector autoregression (VAR) approaches used by [8] and the cointegration approach used by [6]. It is important to remember, meanwhile, that these authors used a type of multivariate time series analysis to identify the dynamical dependency between inflation and exchange rate..
The approaches suggested by these authors were not appropriate for the general case when the multivariate distributions might not have the same distribution as the marginal densities. The copula approach makes it possible to overcome this challenge. With the use of the copula, any two marginals can be linked to their bivariate distribution. The copula can potentially create the joint distribution of two random variables given their marginal distributions. Because of this characteristic, the copula is well-known and highly desirable in statistics.

In finance, the use of copula has been well-known over the last few years. A copula is, by definition, a multivariate cumulative distribution function for which each variable's marginal probability distribution is uniform on the interval $[0, 1]$ [9]. Copulas are used to demonstrate how random variables depend on one another. There are two common families of copula

1. To capture symmetric dependence, elliptical copulas are appropriate. The Gaussian and t-copulas are two examples of elliptical copulas, respectively.
2. Tail dependence can be captured using Archimedean copulas. The Clayton, Gumbel, and Frank copulas are common examples of archimedean copulas.

The copula permits the mixing of all univariate marginal distributions, even though they are not necessarily coming from the same distribution family. As the number of dimensions rises, a particular class of copula models known as "elliptical copula" exhibits the trait of rising in complexity far more slowly than existing multivariate probability models. According to [10], copulas are exceedingly generic, covering a variety of multivariate models that already exist and provide a framework for creating many more. In comparison to the probabilistic models currently utilized in macroeconomics, copula models have more advantages that make them more suitable for use in empirical analysis.

Two researchers [11] and [12] used copulas to model the relationship between inflation and exchange rate. When modeling the relationship between inflation and exchange rates, [12] used data from European banks between 2000 and 2016; [11] used Ghanaian data between 2000 and 2018.
Our research departs from their research in three ways.:

1. For estimating the marginal distribution, we employed the ARMA + GARCH model as opposed to [11] who used GARCH. [11] used GARCH because it is well known to capture the volatility and [12] used SARIMA because their data exhibited the presence of seasonality. The conditional mean is known to be captured by the ARMA model before the GARCH model is used.
2. As a result of using data from different country and period, our methodology was different from theirs.
3. Our study investigated the time- varying dependence. Time variation in the dependence parameters of financial variables is the correlation between two variables which may be varying with time.

Among all researchers who have modeled the dependence between inflation and exchange rate, none of them has taken into account time-varying dependence in their research. This research seeks to bridge this gap. Analyzing the change in dependence between two variables for specified time periods can be useful for understanding how the exchange rate affects inflation during certain markets, cycles, crises, or target events.

The remaining sections are arranged as follows: Section 2 presents the methodology used for this research. Results are found in Section 3. Finally, Section 4 presents the work's conclusion.

2 Methodology

2.1 Introduction

This section explains our modeling approach for the dependence between inflation and exchange rate. Most importantly, we discuss the models of copula that were utilized to capture the dependencies and the ARMA + GARCH model that was used for the selection of the marginal distributions. We describe the change point detection that was used to capture time-varying dependence. In this study, we used R software for data analysis.

2.2 Data

Data were collected from the Central Bank of Kenya's website, centralbank.go.ke, where we collected monthly data on inflation and exchange rate (Kenya Shillings on the US dollar). The data covered the years 2005 to 2020.

2.3 Copula Theory and Dependence Measure

A bivariate copula is a function $C : [0,1]^2 \to [0,1]$ with the following properties:

1. $domC = [0,1]^2$
2. C is both 2- increasing and grounded
3. For every $(u,v) \in [0,1]^2$, c(u,1)=u and c(1,v)=v

Theorem 1 (Sklar'theorem) *Assume that F and G have a joint distribution H and are marginals. Then there is a copula C with:*

$$H(x,y) = C(G(x), F(y)) \tag{2.1}$$

The theorem demonstrates that each joint distribution can be decomposed into its marginal distribution and a copula, which reflects the dependence between the marginals. [13].

Corollary 1.1 *The corollary states that*

$$C(u,v) = H(F^{-1}(u), G^{-1}(v)) \tag{2.2}$$

i.e. a copula function, is a multivariate cdf.

Bivariate copulas that are frequently used include Gaussian, student t, Clayton, Frank, and Gumbel [14]. The Gumbel copula exhibits a strong right tail dependence but fails to capture the lower tail dependence. Each tail of the Frank copula exhibits symmetric dependence. Dependence in the lower tail is captured by the Clayton copula. While the Gaussian copula is unable to capture tail dependence, the student t copula can. [15].

The relationship between two variables is shown by a dependence measure. There are three widely used methods for evaluating dependence: linear correlation, spearman Rho, and

Kendall's tau. Kendall's tau is particularly popular in copula analyses [11]. With Copula C, Kendall's tau for any two random variables X and Y can be written as:

$$T_C = 4\int\int_{[0,1]^2} C(u,v)dC(u,v) - 1 \tag{2.3}$$

for $u, v \in [0,1]$.

2.4 Formulation of a bivariate copula

The data must be transformed into log returns before we can examine the relationship of inflation and exchange rate. Let

$$R_t = log(\frac{X_t}{X_{t-1}}) \tag{2.4}$$

$$P_t = log(\frac{Y_t}{Y_{t-1}}) \tag{2.5}$$

where X_t is inflation and R_t is the log returns of inflation. Y_t represents exchange rate and P_t is the exchange rate log returns.

2.4.1 ARMA model

The ARMA (p,q) model is a mixture of two linear models i.e. AR and MA. In order to decide which order p,q of the ARMA model is suitable for a series, we used the AIC (or BIC) throughout a subset of values for p,q. We then looped over all pairwise values of $p \in (1,2,3,4,5,6,7,8,9)$ and $q \in (1,2,3,4,5,6,7,8,9)$ and calculated the AIC and BIC. The model with the lowest AIC and BIC is the one we selected.

2.4.2 GARCH model

The conditional expectation of a process given the historical data is modeled using ARMA models, according to [16]. The conditional variance based on historical data is constant in an ARMA model, though. Due to ARMA's inability to account for volatility, the GARCH model was developed to help detect the dependence structure. A common technique for modeling time series with conditional heteroskedastic errors is the GARCH model.

GARCH is an extension of the ARCH model that contains a moving average with the autoregressive component[16]. The GARCH model (p,q) is :

$$a_t = \sigma_t \epsilon_t$$

where

$$\sigma_t = \sqrt{\omega + \sum_{i=1}^{p} \alpha_i {a^2}_{t-1} + \sum_{i=1}^{q} \beta_i {\sigma^2}_{t-1}} \tag{2.6}$$

With a stationary mean and variance, the process a_t is uncorrelated. σ_t is the volatility where ω, α and β are parameters.

2.4.3 Distribution of Margins

Before we assume any marginal distribution, we need to verify if they are normal. We used shapiro-Wilk test and Anderson-Darling test.

The parsimonious GARCH (1, 1) model will be used to find the best marginal distribution if the two tests show that they are not normally distributed. Based on the AIC and BIC criterion, the best marginal distribution will be chosen.

Let R_t and P_t be the log returns for inflation and exchange rate modelled as

$$R_t = ARMA(r, s) + GARCH(1, 1) \tag{2.7}$$

$$P_t = ARMA(r, s) + GARCH(1, 1) \tag{2.8}$$

The marginal distributions which will be considered:

1. Student t Distribution

$$f(x) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi r(\frac{v}{2})}}(1 + \frac{x^2}{v})^{-\frac{(v+1)}{2}} \tag{2.9}$$

where v is the degree of freedom and Γ is the gamma function.

2. Skew Normal

$$f(x) = \frac{2}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}} \int_{-\infty}^{\frac{\alpha(x-u)^2}{\sigma}} \frac{1}{\sqrt{2\pi}}e^{\frac{-t^2}{2}} dt \tag{2.10}$$

Where μ is location parameter, σ is scale parameter, and α is the shape parameter.

3. Laplace Distribution

$$f(x|\mu, b) = \frac{1}{2b}e^{(\frac{-|x-\mu|}{b})} \tag{2.11}$$

Where μ is the location parameter and b is the scale parameter.

4. Standardized Normal Inverse Gaussian distribution

$$f(x) = \frac{\alpha\sigma K_1(\alpha\sqrt{(x-\mu)^2 + \sigma^2}}{\pi(x-\mu)^2 + \sigma^2}e^{\sigma_Y + B(x-\mu)} \tag{2.12}$$

Where μ is the location, α is tail heaviness, B is asymmetry parameter, and σ is scale parameter.

5. Skew Student t Distribution

$$f(x; \mu, \sigma, \lambda, p, q) = \frac{p}{2v\sigma q^{\frac{1}{p}} B(\frac{1}{p}, q)(\frac{|x-\mu+m|^p}{q(v\sigma^p)(\lambda(x-\mu)+m)^p})^{\frac{1}{p}+q}} \tag{2.13}$$

Where B is the beta function, μ is the location parameter, $\sigma > 0$ is the scale parameter, $-1 < \lambda < 1$ is the skewness parameter, and $p > 0$ and $q > 0$ are the parameters that control the kurtosis. m and v are not parameters.

6. Normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2} \tag{2.14}$$

where μ is the mean (location) and σ is the variance.

2.4.4 Copula Models

We employed the elliptical and Archimedean families of copulas to describe the relationship between inflation and exchange rate.

1. Symmetric dependence can be captured using elliptical copulas. The Gaussian copula and the t-copula are two common types of elliptical copulas. When compared to Gaussian copula, the t-copula has the benefit of being able to capture tail dependency in both the upper and lower tail.

- Gaussian copula: For a given $\rho \in [-1, 1]$, the gaussian copula with parameter matrix ρ

$$C(u, v)_\rho = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v)) \tag{2.15}$$

where Φ^{-1} is the inverse CDF of a standard normal and ϕ_ρ corresponds to the joint CDF of the bivariate normal distribution with mean zero and covariance matrix equal to the correlation matrix ρ

- t-copula: The bivariate t-copula of X is given as

$$C^t(u, v : \rho\nu) = t^2_{\rho\nu}(t_\nu^{-1}(u), t_\nu^{-1}(v)) \tag{2.16}$$

where $t_\nu(x)$ is t-distribution with ν degrees of freedom and $t^2_{\rho,\nu}(x, y)$ is a bivariate t-distribution with correlation ρ.

2. For capturing tail dependence, archimedean copulas are appropriate. BB1 copula (Clayton-Gumbel copula), BB6 copula (Joe-Gumbel copula), BB7 copula (Joe-Clayton copula), and BB8 copula (Joe-Frank copula) are all examples of Archimedean copula. In contrast to Clayton and Gumbel copulas, the Frank copula is symmetric. The lower tail dependence is captured by the Clayton copula, the upper tail dependence is captured by the Gumbel copula, and the top tail addition is captured by the Joe copula. The asymmetric tail dependence is captured by the BB6 (Joe-Gumbel)copula.

- Clayton Copula

$$C(u_1, u_2) = ({u_1}^\theta + {u_2}^\theta - 1)^{\frac{-1}{\theta}} \tag{2.17}$$

where $C(.,.)$ is the bivariate copula with dependence parameter $\theta > 0$.

- Gumbel copula

$$C_\theta(u, v) = exp[-(\sum_i(-\log u)^\theta + (-\log v)^\theta)^{\frac{1}{\theta}}] \tag{2.18}$$

where $1 \leq \theta < \infty$

- Frank copula : is a symmetric Archimedean copula given as

$$C_\theta(u, v) = -\frac{1}{\theta}\ln(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}) \tag{2.19}$$

where $\theta \in (-\infty, \infty)\backslash\{0\}$.

- Joe copula

$$C(u, v) = 1 - [(1-u)^\theta + (1-v)^\theta - (1-u)^\theta(1-v)^\theta]^{\frac{1}{\theta}} \tag{2.20}$$

where $\theta \in [1, \infty)$

2.4.5 Estimation of the Parameters

The estimate of copula parameters can indeed be divided into parametric (such as maximum likelihood), semiparametric (such as the maximum pseudo-likelihood technique, SCOMDY (Semiparametric Copula-Based Multivariate Dynamic Models), etc.), and non-parametric methods. IFM (Inference Function for Marginal) is a parametric test that requires two-step maximum likelihood. Any fitting method of a univariate probability distribution is used to first fit the marginal distributions for each random variable. The copula parameter is calculated in the second stage using the maximum likelihood approach [17]. The maximum likelihood method, a parametric approach, is used in the first stage to estimate the copula's parameters. However, using order statistics from each sample of data, a nonparametric approach, the CDF (Non-Exceedance Probabilities) from the marginal distribution are estimated [17].

The maximum pseudo-likelihood method combines parametric and nonparametric approaches.

Suppose g(.) and f(.) are our marginal densities. θ_1 and θ_2 are their respective parameters. θ_3 depends on the copula. Let's say we have a sample pair of data with $(x_i, y_i), i = 1, , n$ of size n. The joint distribution's log-likelihood is denoted by:

$$L(\theta_1, \theta_2, \theta_3) = \sum_{i=1}^{n} \log g(x_i; \theta_1) + \sum_{i=1}^{n} \log f(y_i; \theta_2) + \sum_{i=1}^{n} \log c(G(x_i; \theta_1), F(y_i; \theta_2); \theta_3) \quad (2.21)$$

Where C (.) is the copula to be estimated. c is the copula density, which is the the derivative of C with respect to each of its arguments u and v

$$c(u, v) = \frac{\partial C(u, v)}{\partial u \partial v}$$

The maximum likelihood will be used to estimate the parameters of the copula from this log-likelihood function.

2.5 Time-varying dependence

Change-point detection is a well-established and important problem in time series analysis [18]; [19]; and [20]. The goal of change point detection, as its name suggests, is to determine if and when unexpected distribution changes occur in a time series. These changes are important in a variety of sectors, including environmental science, economics, medical, finance, etc... Finding the start point and end point—also known as the change points—is the aim of change point detection.

We employed the copula method in this work to identify the change points [21]. A copula is a frequently used tool for explaining the relationship structure of data. The copula's parameters display the level of dependence. We dynamically fit the copula to the data and locate the change points where we add data one at a time. The copula's parameters will remain constant if there is no event occurring. We considered that an event will have long-lasting effects on the dependence. When a particular event occurs and influences positively the dependence between two variables, the copula parameter will exhibit a positive correlation and the fitted parameters will not remain constant. This particular event will mark the beginning of the change in dependence which will be called start point. The parameter increases as more data are added at the starting point. The data properties will determine this change in parameters. Because new data are added after the start point, the parameter maintains an upward trend. The parameter decreases when additional data is added at the endpoint. The properties of the data will also affect this decrease in parameters. More data are added after the endpoint and the parameter keeps trending downward.
On the other hand, if a particular event occurs and impacts negatively the dependence between two variable, the copula parameter will exhibit a negative correlation and the fitted parameters will not also remain constant.This particular event will mark the beginning of the change in dependence which will be called start point. For this case, the parameter keeps trending downward, when the data is added from the starting point. The data properties will determine this change in parameters. The parameter keeps trending upward when additional data is added at the endpoint.
First, it can handle unbalanced panel data, which other techniques can only rarely handle. Second, it can recognize several change points at once.

Procedure for identifying change points

- Step 1: Choose an appropriate copula for the entire data. Here, we need to know which copula capture the dependence between Inflation and Exchange rate. We discussed above how to estimate copula.

- Step 2: Dynamically fit the chosen copula to the data. Once the correct copula has been found, the data will be dynamically fitted to the chosen copula. It can be done in one of two ways: either by fitting the selected copula to the data backward or forward. We decided to fit the selected copula to the data backward in this study. Data are added backward one at a time starting at the beginning.
  - Fit the chosen copula to the data containing t_1 in Subset 1.
  - data from t_2 is added to subset 1 to form subset 2;
  - data from t_3 is added to subset 2 to form subset 3; and so on.

  A set of fitted parameters a, including a_1, a_2, a_3,..., a_n, will be obtained at the end.
- Step 3 is to determine the change points. After obtaining the fitted parameters in step 2, we plotted the parameters with time where the change points could be determined. The start point is the time when the fitted parameter is not constant and the endpoint is the time when the fitted parameter becomes again constant (stable).

3 RESULTS AND DISCUSSION

3.1 Introduction

The methodology's application to our data on inflation and exchange rates is reviewed in this section. For the data analysis, R is used. First, we get descriptive statistics and data visualization. To find the marginal distribution of inflation and exchange rate, we then apply the ARMA + GARCH model. Then, we establish the standardized residuals' marginal probability distribution. Our uniform marginals, which we utilize to estimate the copulas, are obtained by transforming the marginal distribution. With the use of change-point detection based on copula, we continue our investigation into the time-varying dependence.

3.2 Preliminary analysis

A plot of our variables is shown along with some descriptive statistics. From 2005 to 2020, monthly data on inflation and the exchange rate were obtained from the Central Bank of Kenya. Table 1 shows that the inflation standard deviation is 4.2849 and the exchange rate standard deviation is 13.1566. This outcome demonstrates a significant level of volatility. This outcome validates our decision to model the univariate margins using GARCH.

After converting the data into log returns, the two plots in figure 1 and figure 2 below demonstrate that there is a lot of variation as well, supporting the conclusions we made above. We used a seasonal subseries plot, a special method for displaying seasonality, because figure 2 doesn't show a trend pattern. The seasonal subseries plot displays the seasonal differences (between group patterns) as well as the within-group patterns quite well [22]. We may conclude that there is no seasonality because the means of each month for both inflation and exchange are relatively close in figures 3 and 4. Since neither time series displays trend or seasonality pattern, they are literally stationary.

Table 1: Descriptive Statistics

Statistics	Inflation	Exchange rate
Mean	7.662	87.58
Standard deviation	4.2849	13. 1566
Minimum	1.850	61.90
Maximum	19.720	110.59

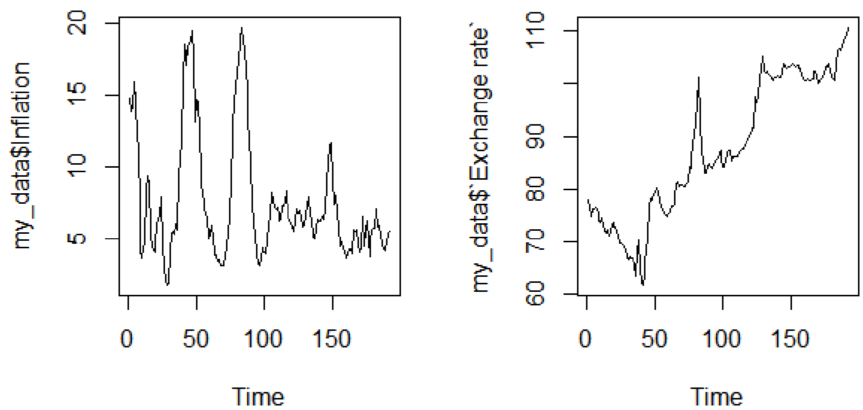


Figure 1: Plot of monthly inflation rate and monthly exchange rate

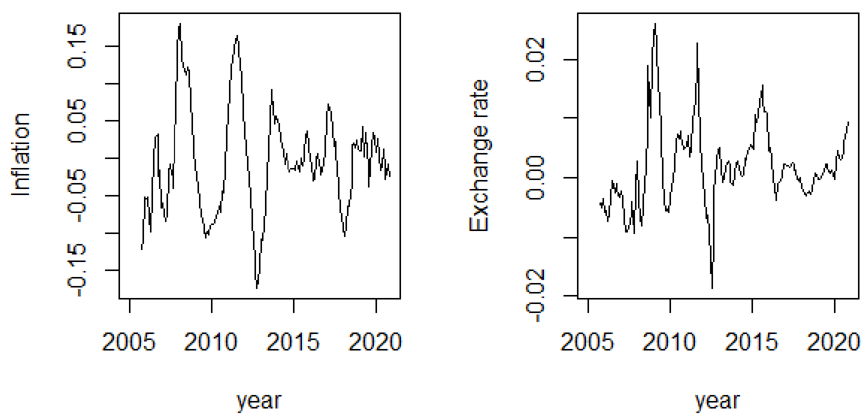


Figure 2: Plot of log returns for inflation and exchange rate

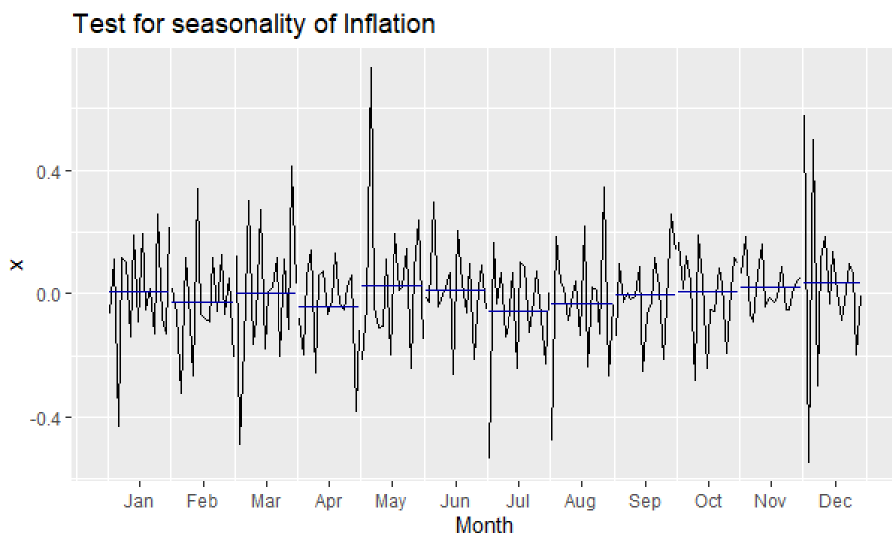


Figure 3: Seasonal Subseries Plot for Inflation

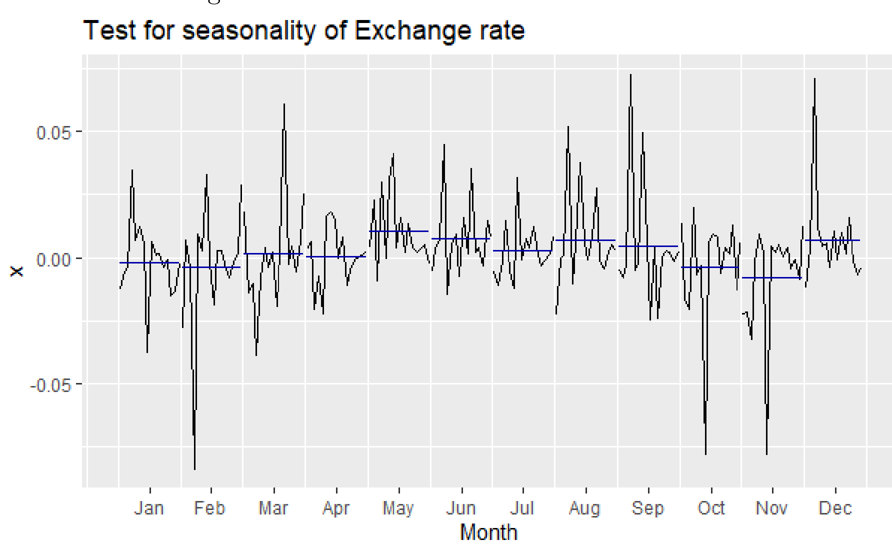


Figure 4: Seasonal Subseries Plot for Exchange rate

3.3 Test for dependence

The increase in the exchange results in cheaper domestic goods for foreign consumers, leading to the rise in exports and total demand and costs (prices). The rate of inflation rises as the exchange rate rises. We can draw the conclusion that inflation and the exchange rate are related. For this reason, before introducing copula, we begin by determining whether there is dependence between them. In this study, the Kendall and Spearman tests were employed to determine whether inflation and exchange are truly significantly dependent at the 5% level of significance.

These studies demonstrated that there is a dependence between them, but they were unable to reveal the nature of this dependence, including whether it is symmetric, asymmetric, or tail dependent. That is why we introduce the copula.

Table 2: Test for dependence

Pair	Kendaul's tau	Spearman'rho	P-value
μ_1, μ_2	0.0924	0.1378	0.05762

3.4 Formulation of bivariate copula

3.4.1 ARMA (p, q) model

Two linear models, AR and MA, are combined to form the ARMA (p, q) model. In time series, we observe two things when we try to fit a time series model. First, the passed values are used in AR models. We can figure out what our next point might be by observing a series of past points. Second, we analyze the past prediction errors, called the MA model. ARMA allows us to fit a nice model that analyze both past values and past forecast errors.
By using the log returns data, figure 5 and figure 6 show that there is a presence of serial correlation since some lags are not falling within the confidence limit which support the decision of using ARMA model.
The ARMA (4,6) model was determined to be the best model for inflation from Table 3 since it has the lowest AIC and BIC. Figure 7's ACF and PACF indicate that there is no serial correlation, supporting the ARMA (4,6) model as the best for inflation. Since it has the lowest AIC and BIC, the ARMA (1,1) model was found to be the best model for exchange rate from table 4. Figure 8's ACF and PACF indicate that there is no serial correlation, supporting the ARMA (1,1) model as the best for the exchange rate.

Figure 5: ACF and PACF of inflation

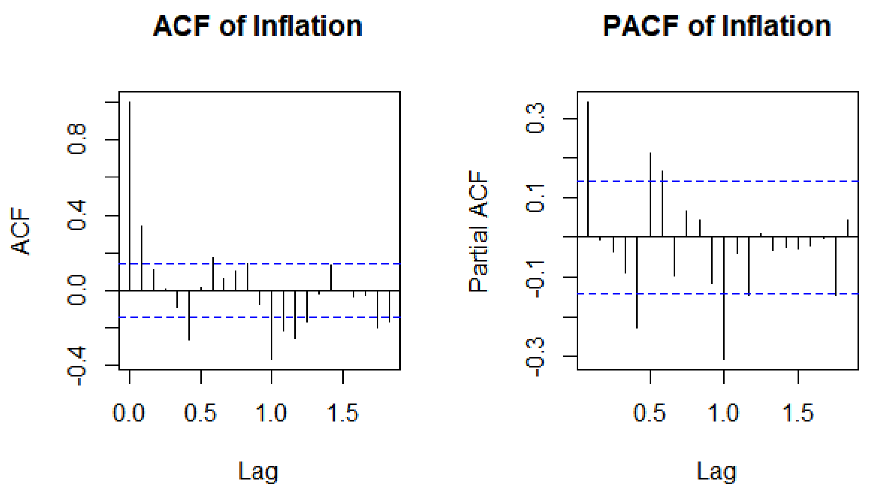


Figure 6: ACF and PACF of exchange rate

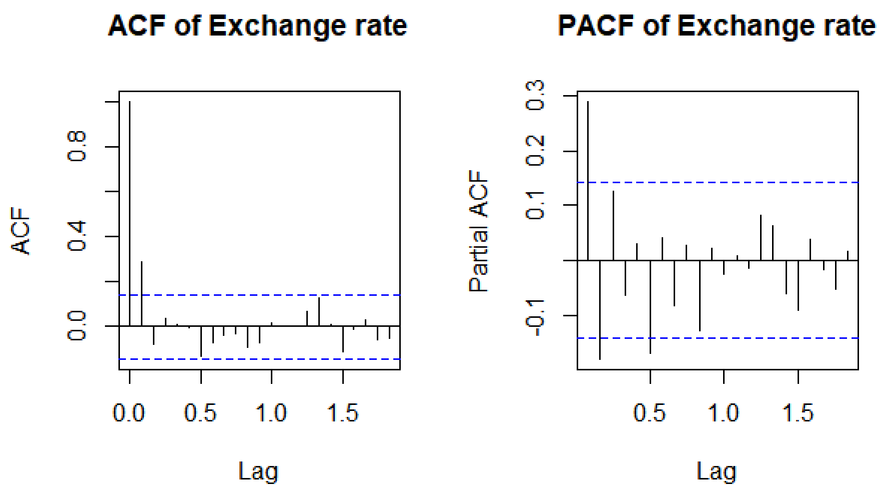


Figure 7: ACF and PACF of inflation residuals

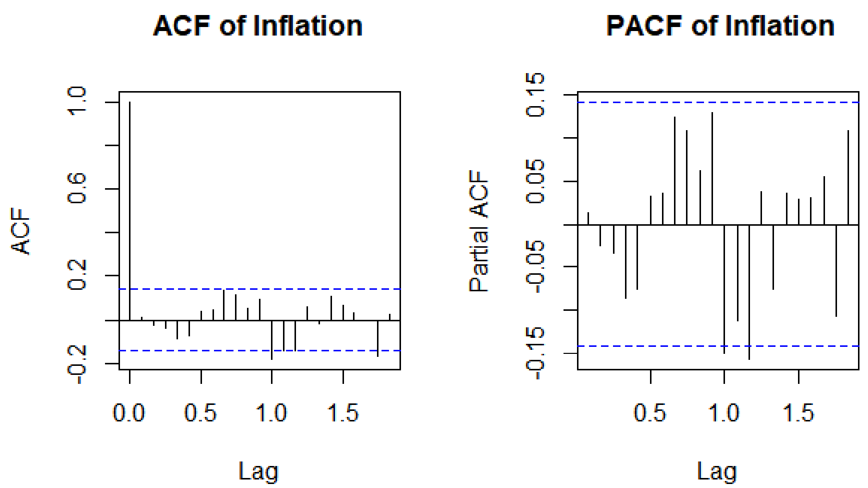


Figure 8: ACF and PACF of Exchange rate residuals

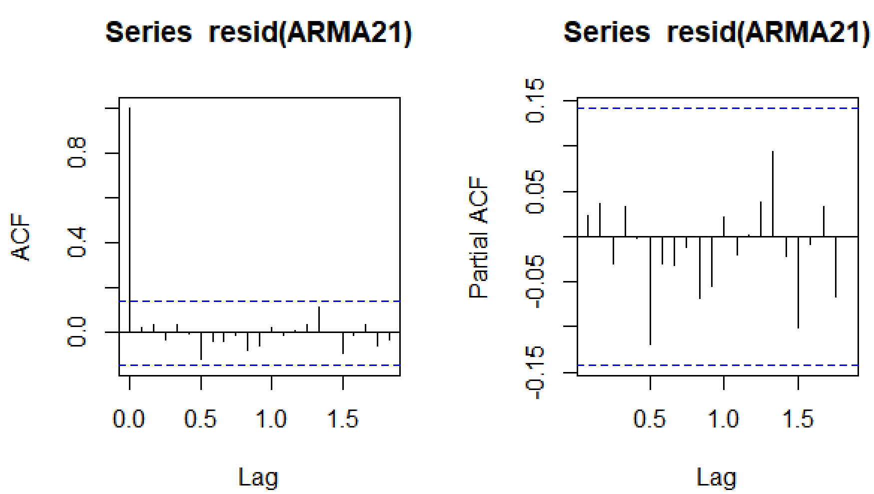


Table 3: ARMA (4,6) model for inflation

Coefficients	Estimates	Standard Error
α_0	-0.0023	0.005
α_1	1.2654	0.0843
α_2	-0.6930	0.1086
α_3	1.1721	0.1075
α_4	-0.7956	0.0893
β_1	-1.0481	
β_2	0.4840	0.1494
β_3	-1.2614	0.1370
β_4	0.6444	0.0944
β_5	-0.1961	0.1360
β_6	0.3778	0.0864
ACF	-166.54	
BIC	-127.5137	

Table 4: ARMA (1,1) model for exchange rate

Coefficients	Estimates	Standard Error
α_0	-0.0019	$0,0016$
α_1	0.8344	0.0843
β_1	-0.6930	0.0679
ACF	-992.63	
BIC	-979.6187	

3.4.2 Test for heteroskedasticity

Before we move to GARCH, we'd like to check if there's a presence of heteroskedasticity. Heteroskedasticity takes place while the variance isn't constant over time. After getting the ARMA model for both inflation and rate of exchange, we extracted the residuals from ARMA(4,6) and ARMA(1,1) and so square them in R. The square residuals were used to run ACF and PACF for the heteroscedasticity test. Figure 9 and Figure 10 show signs of nonlinear serial dependence or the presence of "ARCH effects". Since there's a presence of "ARCH effects", it supports our decision to use GARCH during this study.

Figure 9: Test for heteroskedasticity of exchange rate

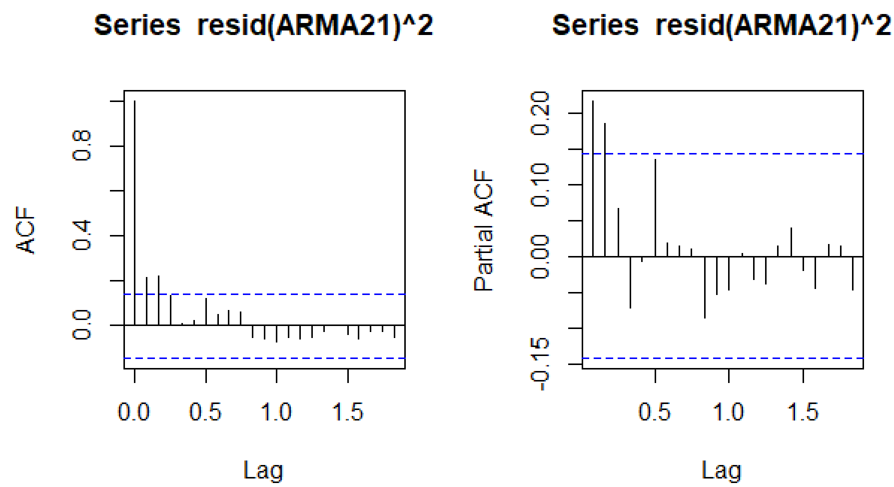
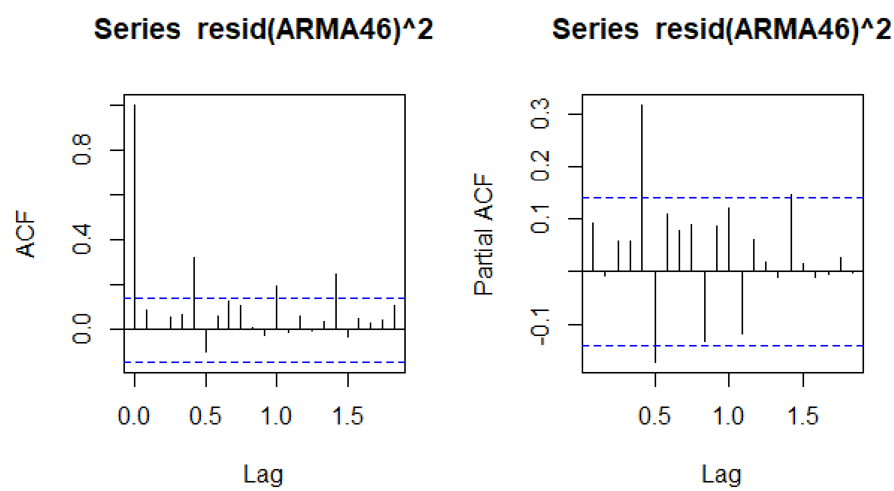


Figure 10: Test for heteroskedasticity inflation residuals



3.4.3 Normality test

After fitting the ARMA model to the log return data, we extracted the residuals. We extracted the residuals after fitting the ARMA model to the log return data. We must first determine whether the extracted residuals follow normal distribution before assuming any of the distributions mentioned in the methodology section. In this study, we employed the Shapiro-Wilk and Anderson-Darling tests, and we discovered that the variables were not normally distributed since we have enough evidence to reject the null hypothesis. The numbers in the table are the p-values of Shapiro-Wilk and Anderson-Darling.

H0: They are normally distributed
H1: They are not normally distributed

Table 5: Normality test of our standardized residuals

Data	Shapiro-Wilk	Anderson-Darling
Inflation	0.0007199	0.0216
Exchange rate	$9.99e^{-11}$	$2.263e^{-12}$

3.4.4 MARGINAL DISTRIBUTION

The copula model configuration requires accurate marginal distribution specification. If the model of the marginal distributions has not been correctly specified, the copula model will be incorrectly specified, which prevents the probability integral transforms from being i.i.d. For the development of the copula model, testing for marginal distribution models is crucial. The GARCH model, by definition, is a popular method for modeling time series with conditional heteroscedastic errors. Additionally, to obtain the best fitted marginal distribution, [11], amongst many others, used the parsimonious GARCH (1, 1) model.

Let X_t and Y_t be the log returns for inflation and exchange rate modelled as

$$X_t = \omega_0 X_{t-1} + \omega_1 z_t \tag{3.1}$$

$$Y_t = \omega_0 Y_{t-1} + \omega_1 z_t \tag{3.2}$$

where $z_t \backsim GARCH(1,1)$

Before, we fit the GARCH (1,1), we first extracted the residuals from ARMA (4,6) model for inflation and ARMA (1,1) for exchange rate. We then fitted GARCH (1, 1) model to each marginal distribution shown from the table 6 to table 17 for each variable. With the exception of the inflation mean and scale and exchange rate mean, all of the parameters were statistically significant at the 5% level of significance. Based on AIC and BIC, it was determined that the student t distribution seemed to have the best fit for both inflation and exchange rates. So now we can transform the probability distributions into a uniform distribution on interval [0, 1].

4.2.3.1. Inflation

Table 6: Student t distribution for GARCH $(1,1)$

	Estimates	Std. error	t-value	P-value
μ	0.0123555	0.0090096	1.371	0.1703
σ	0.0009156	0.0008419	1.088	0.2768
shape	6.6941319	3.0048023	2.228	0.0259
α_1	0.0691856	0.0409580 1.689	0.0912	
β_1	0.8834817	0.0657508 13.437	$< 2e^{-16}$	
AIC	-1.129523			
BIC	-1.044385			

Table 7: Skew normal distribution for GARCH $(1,1)$

	Estimates	Std. error	t-value	P-value
μ	0.0126688	0.0093708	1.352	0.1764
σ	0.0006239	0.0004413	1.414	0.1574
skew	0.9281859	0.0806352	11.511	$< 2e^{-16}$
α_1	0.0677574	0.0297290	2.279	0.0227
β_1	0.8986787	0.0367861	24.430	$< 2e^{-16}$
AIC	-1.093000			
BIC	-1.007862			

Table 8: Laplace distribution for GARCH (1, 1)

	Estimates	Std. error	t-value	P-value
μ	0.0128143	0.0094743	1.353	0.1762
σ	0.0006912	0.0005943	1.163	0.2448
shape	1.4461332	0.1926909	7.505	$6.15e^{-14}$
α_1	0.0666996	0.0359236	1.857	0.0634
β_1	00.8961738	0.0495649	18.081	$< 2e^{-16}$
AIC	-1.121537			
BIC	-1.036399			

Table 9: Standardized Normal Inverse Gaussian distribution for GARCH

	Estimates	Std. error	t-value	P-value
μ	0.0111195	0.0094412	1.178	0.2389
σ	0.0008369	0.0007448	1.124	0.2612
shape	2.1960855	1.4051386	1.563	0.1181
skew	-0.0635886	0.1511667	-0.421	0.6740
α_1	0.0657540	0.0383662	1.714	0.0866
β_1	0.8894209	0.0600397	14.814	$< 2e^{-16}$
AIC	-1.119568			
BIC	-1.017402			

Table 10: skew student t distribution for GARCH $(1, 1)$

	Estimates	Std. error	t-value	P-value
μ	0.0115173	0.0094332	1.221	0.2221
σ	0.0009100	0.0008295	1.097	0.2726
shape	6.7699335	3.0969865	2.186	0.0288
skew	0.9681459	0.1008987	9.595	$< 2e^{-16}$
α_1	0.0675937	0.0404184	1.672	0.0945
β_1	0.8849395	0.0649446	13.626	$< 2e^{-16}$
AIC	-1.119555			
BIC	-1.017389			

Table 11: Normal distribution for GARCH $(1, 1)$

	Estimates	Std. error	t-value	P-value
μ	0.0130306	0.0093372	1.396	0.1628
σ	0.0005809	0.0004210	1.380	0.1676
α_1	0.0690300	0.0297705	2.319	0.0204
β_1	0.9002370	0.0354008	25.430	$< 2e^{-16}$
AIC	-1.099641			
BIC	-1.031530			

4.2.3.2. Exchange rate

Table 12: Student t distribution for GARCH (1, 1)

	Estimates	Std. error	t-value	P-value
μ	$-2.671e^{-04}$	$6.820e^{-04}$	-0.392	0.695331
σ	$3.020e^{-05}$	$1.608e^{-05}$	1.879	0.060268
shape	3.433	$9.353e^{-01}$	3.670	0.000242
α_1	$5.866e^{-01}$	$2.700e^{-01}$	2.172	0.029825
β_1	$4.987e^{-01}$	$1.051e^{-01}$ 4.744	$2.09e^{-06}$	
AIC	-5.743149			
BIC	-5.658011			

Table 13: Skew normal distribution for GARCH (1, 1)

	Estimates	Std. error	t-value	P-value
μ	$-1.345e^{-04}$	$9.148e^{-04}$	-0.147	0.883097
σ	$5.041e^{-05}$	$1.568e^{-05}$	3.214	0.001307
skew	1.172	$9.347e^{-02}$	12.543	$< 2e^{-16}$
α_1	$4.717e^{-01}$	$1.276e^{-01}$	3.698	0.000218
β_1	$3.982e^{-01}$	$1.039e^{-01}$	3.833	0.000127
AIC	-5.608560			
BIC	-5.523422			

Table 14: Laplace distribution for GARCH (1, 1)

	Estimates	Std. error	t-value	P-value
μ	$1.051e^{-02}$	$9.842e^{-03}$	1.067	0.2858
σ	$1.468e^{-03}$	$1.145e^{-03}$	1.282	0.1998
shape	1.447	$1.902e^{-01}$	7.609	$2.78e^{-14}$
α_1	$9.358e^{-02}$	$4.872e^{-02}$	1.921	0.0548
β_1	$1.000e^{-08}$	$7.287e^{-02}$	0.000	1.0000
β_1	$1.000e^{-08}$			
β_2	$8.242e^{-01}$			
AIC	-1.1083620			
BIC	-0.9891688			

Table 15: Standardized Normal Inverse Gaussian distribution for GARCH

	Estimates	Std. error	t-value	P-value
μ	$-1.376e^{-04}$	$7.952e^{-04}$	-0.173	0.8626
σ	$3.638e^{-05}$	$1.799e^{-05}$	2.023	0.0431
shape	1.000	$6.182e^{-01}$	1.617	0.1058
skew	$1.527e^{-01}$	$1.449e^{-01}$	1.054	0.2919
α_1	$3.852e^{-01}$	$1.760e^{-01}$	2.189	0.0286
α_2	$4.866e^{-01}$	$3.180e^{-01}$	1.530	0.1260
β_1	$2.189e^{-01}$	$1.746e^{-01}$	1.253	0.2101
β_2	$1.000e^{-08}$			
AIC	-5.726097			
BIC	-5.589876			

Table 16: skew student t distribution for GARCH

	Estimates	Std. error	t-value	P-value
μ	$-2.671e^{-04}$	$8.654e^{-04}$	0.309	0.757583
σ	$2.946e^{-05}$	$1.588e^{-05}$	1.855	0.063536
shape	3.463	$9.573e^{-01}$	3.617	0.000298
skew	1.039	$1.091e^{-01}$	9.523	$< 2e^{-16}$
α_1	$5.850e^{-01}$	$2.691e^{-01}$	2.174	0.029718
β_1	$5.005e^{-01}$	$1.047e^{-01}$	4.781	$1.74e^{-06}$
AIC	-5.733788			
BIC	-5.631622			

Table 17: Normal distribution for GARCH (1, 1)

	Estimates	Std. error	t-value	P-value
μ	$-2.671e^{-04}$	$9.029e^{-04}$	-0.296	0.767366
σ	$5.618e^{-05}$	$1.614e^{-05}$	3.481	0.000500
α_1	$4.727e^{-01}$	$1.315e^{-01}$	3.594	0.000325
β_1	$3.786e^{-01}$	$1.036e^{-01}$	3.655	0.000257
AIC	-5.597566			
BIC	-5.529456			

4.2.3.3.Uniform transformation

Every variable's marginal probability distribution is uniform in copula over the range $[0, 1]$. To obtain uniform random variables on the range $[0, 1]$, we must convert the student t distribution's marginals of inflation and exchange rate. A copula function, as mentioned above, is represented by the notation C (F(x), G(y)), where F and G are the cumulative density function (cdf) of the univariate marginal. We need to transform the pdf into cdf before the estimation of the copula. We use pt(x) in R to get the value of the cdf function at point x where we need to specify the degree of freedom parameters for each variable.
Before the transformation, we fitted the the student t distribution to the residuals extracted from the parsimonious GARCH (1,1). But we need to determine the degree of freedom since the form of student t is determined by its degree of freedom. To estimate the degree of freedom, we used the metRology package which helps us to estimate the parameters of Student t distribution.

4.2.3.3.1. Inflation

From the table 18, the results show that the estimated df for inflation was 5, and which provides also a good fit as shown in the Q-Q plot. After estimating the degree of freedom, we used pt(x) to get the value of the cdf function at point x. The table 19 shows that the marginal distribution (student t)has been transformed into a cdf and is uniform over the interval[0,1].

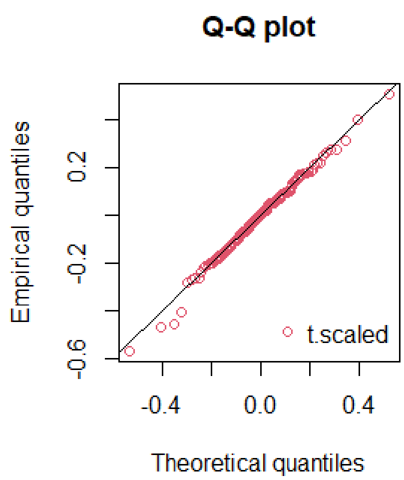
Table 18: Estimation parameters of Student t distribution

	Estimates	Std. error
df	4.994722664	1.748747949
mean	-0.004262268	0.009355979
s.d	0.111688828	0.009670249

Table 19: Uniform transformation for inflation

Min	Mean	Max
0.2977	0.4972	0.6828

Figure 11: Q-Q plot of the student t distribution for the inflation



4.2.3.3.2. Exchange Rate

The results from the table 20 show that the estimated df for exchange rate was 2, and which provides also a good fit as shown in the Q-Q plot. After estimating the degree of freedom, we used pt(x) to get the value of the cdf function at point x. The table 21 shows that the student t distribution has been transformed into a cdf and is uniform over the interval [0,1].

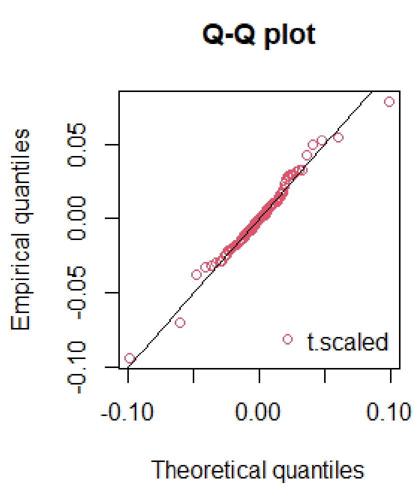
Table 20: Estimation parameters of Student t distribution

	Estimates	Std. error
df	2.2986272327	0.4769908281
mean	-0.0001801039	-0.0008479315
s.d	0.0092743497	0.0009217361

Table 21: Uniform transformation for Exchange rate

Min	Mean	Max
0.4670	0.5001	0.5277

Figure 12: Q-Q plot of the student t distribution for the exchange rate



3.4.5 Goodness of fit for marginal distributions

Checking that the marginal distributions are correct is necessary before proceeding with the estimation of the copula. If they are mispecified, therefore the construction of the copula model will be incorrect.

H0: ARCH effects are not present.
H1: ARCH effects are present.

First, we checked if there is presence of ARCH effects or volatility after fitting the GARCH(1,1). Table 22 demonstrates that there are no ARCH effects or volatility in the residuals derived from the marginal distributions since there is insufficient evidence to reject the null hypothesis. These results support our decision of using GARCH (1, 1) for capturing the volatility.

H0: The two marginal distributions' probability transforms are not uniform.
H1: The two marginal distributions' probability transforms are uniform.

Secondly, we checked if the transformation of each marginal distribution is uniform. According to the previous sentence, each variable's marginal probability distribution is uniform in copula over the range $[0, 1]$. We used Kolmogorov–Smirnov tests in this study to test if the marginal distributions are uniform. Since there is sufficient evidence to reject H0, Table 23's p-values from the KS.test reveal that each transformation of the two marginal distributions (student t) is uniform over the range [0, 1].

As a result, the finding provides strong evidence that our marginal distributions (student t) are accurate. Also their transformations were uniform over the interval [0,1]. Therefore, we can estimate and capture the dependence structure of inflation and exchange using copula.

Table 22: Goodness of fit for marginal distribution

Residuals	Statistics	LM Arch test
Inflation	13.97977	0.3020018
Residuals	2.705796	0.9972887

Table 23: Uniformity test on the interval [0,1]

Probability transforms	D	p-value
μ_1, μ_2	0.39791	$1.471e^{-13}$

3.5 Estimation of copula

In this section, the results of the estimation of our selected copulas parameters will be presented. We estimated the parameters using the maximum likelihood estimation technique in R software .The following copulas were estimated: Gaussian, Student t, Clayton, Gumbel, Joe, Clayton-Gumbel, Frank, Joe-Gumbel, Joe-Clayton, and Joe-Frank. We employed the vinecopula package, which offers an easy way to choose the best copula using BIC and AIC. Table 24's Gumbel copula, which has a low AIC and BIC, was the best at capturing the relationship between our variables. Inflation and exchange rate have an upper tail dependence that the Gumbel copula captures. Gumbel's copula indicates that, in extreme cases, the exchange rate can affect inflation in an economy by showing higher dependency in the upper than in the lower tail. According to Kendall'Tau, there was an 8% dependency between inflation and the exchange rate.

Table 24: Estimation of copula

Copula	θ_1	θ_2	AIC	BIC	Kendall'Tau
Gaussian	0.99		-576.16	-572.91	0.08
Student t	0.99	3.9	-589.13	-582.63	0.08
Clayton	17.22		-589.9	-586.65	0.08
Gumbel	12.15		-590.77	-587.52	0.08
Frank	35		-590.16	-586.91	0.08
Joe	17.92		-590.04	-586.79	0.08
Clayton-Gumbel	2.72	5.35	-588.64	-582.13	0.08
Joe-Gumbel	2.71	6	-588.71	-582.2	0.08
Joe-Clayton	5	6	-373.06	-366.56	0.08
Joe-Frank	6	1	-365.37	-358.87	0.08

3.6 Time-varying dependence

By definition, time-varying or time volatility refers to fluctuations in volatility over different time periods. Analyzing the change in dependence between inflation and exchange rate by specified time periods can be useful for understanding how the exchange rate can affect inflation during certain markets, cycles, crises, or target events. The change-point detection will help us to know when the change starts and when it ends. It will help us to know the cause of that change.
The first step was to select the best copula and Gumbel copula was found to be the best fitting model. The second step was to fit the selected copula (Gumbel) dynamically to the data backward. It means we fit Gumbel copula from 2005 to 2020 one at a time. First, we fit Gumbel copula to data of 2005. Secondly, we add data of 2006. Then, we add data of 2007 and so on. A parameter will be shown when the Gumbel copula isfitted to the data. This parameter displays the degree to which inflation and exchange rates are correlated. Table 25 displays the parameters that were fitted. When the Gumbel copula is fitted to data from 2005 to 2007, for instance, the fitted parameter is 7.890587 in the fourth row and second column. When the Gumbel copula is fitted to data from 2005 to 2016, the fitted parameter is 12.86238 in the 13th row and second column.
The last step was to identify the change points. To identify the start point and end point, we need to analyze the trend of the fitted parameters. We found in section 3.5, the dependence between inflation and exchange rate was 8%. From 2005 to 2007, the fitted parameters are relatively stable because the dependence between inflation and exchange was approximately 8%. When we added data of 2008, the fitted parameters increase. So, 2008 can be considered as the start point. The fitted parameters keep the upward trend as more data are added until 2020 where the dependence between inflation and exchange rate moves from 9% to 12%. When we add data of 2020, the fitted parameters decrease where the dependence between inflation and exchange becomes again approximately 8%. Therefore, 2020 might be seen as the year when the relationship between inflation and the exchange rate stabilizes.

The Gumbel copula's parameters, which can be seen in figure 13, show how closely inflation and the exchange rate are related. Since we discovered above that their dependency was at 8%, the relationship between inflation and exchange rate was constant from 2005 to 2007. From 2008 to 2016, the trend increases where the dependence between inflation and exchange increased from 8% to 12%. According to [23], there was a huge increase in inflation. From 2008 to 2011, there was a depreciation of Kenya shilling due to the post-election violence that the country faced. The depreciation was due to high international oil prices and along with the decrease in capital inflows in Kenya. The post-election violence was affected by that increase in inflation in 2008. From 2009 to 2010, inflation decreased due to recovery from the post-election. But in 2011, the inflation increases due to oil and food prices, bad weather, and depreciation of the Kenya shillings [24]. A change in dependency occurred as a result of the depreciation of the Kenyan Shilling, which indicates that inflation is now more or less affected by changes in the exchange rate.
But in 2018, there is a sudden decline in dependence. According to [25] and [26], there was a decline in both inflation and exchange rate from 2017 to 2018 as a result of a decrease in consumer prices as well as a restriction on the central bank's domestic supply. Due to lockdown measures, imports decreased and exports increased in 2020. [27] said that the imports decreased due to disruption of sea cargo trade with countries; while the exports increased due to the rise in food exports. Increased exports cause Kenya's currency to appreciate, which tends to lower inflation. We can draw the conclusion that changes in exchange rates significantly affect the economy [28]. These lockdown measures in 2020 can explain this sudden decrease in dependence between inflation and exchange.

Table 25: The dynamically fitted parameters of Gumbel copula

Time points	Fitted parameters
2005	8.361464
2006	8.35358
2007	7.890587
2008	9.125635
2009	9.764374
2010	10.3558
2011	10.75607
2012	11.30622
2013	11.58336
2014	12.04071
2015	12.50386
2016	12.86238
2017	12.74033
2018	12.6429
2019	12.12512
2020	7.73489

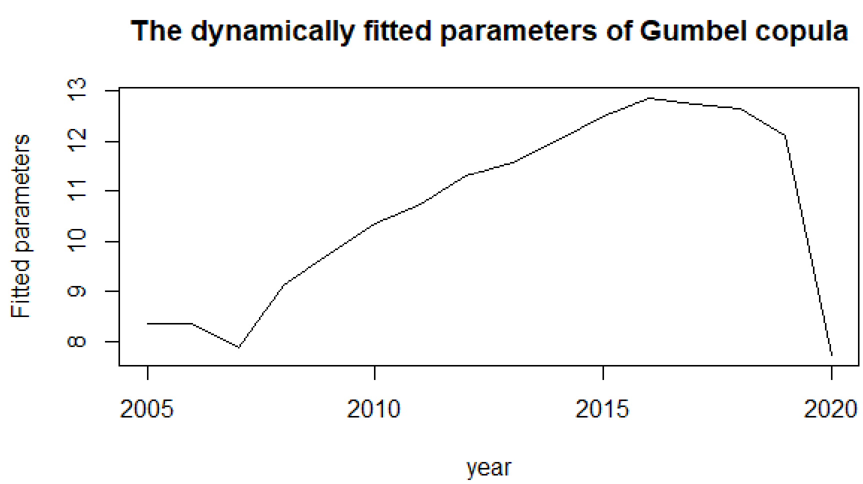


Figure 13: Trend of the fitted parameters

4 Conclusion and Recommendations

The purpose of this study was to use the copula to model how inflation and exchange rates are connected. Between 2005 and 2020, we used monthly statistics from the Central Bank of Kenya. We discovered that the student t marginal distribution was the best one for both inflation and exchange rate. The Gumbel copula was likewise discovered to be the most effective at capturing their dependence. Their dependence was approximately 8% according to Kendall'Tau. This finding suggests that, although there are numerous other factors that can influence inflation, the exchange rate can help to stabilize prices to some extent.

In this study, we also considered time varying dependence using change point detection techniques. The change point detection was done using the three steps procedure. The first step was to choose an appropriate copula for the entire data (Gumbel copula). The second step was to fit Gumbel copula to data progressively. The third step was to find the change points. We found that there are two change points which start from 2008 and end to 2020. This change was due to depreciation of Kenya shillings. We can conclude that there is indeed a change in dependence between the two variables with time.
We recommend to the future researchers to consider studying time varying dependence between those two variables and investigate also the change in copula parameters in values with time. Additionally, we suggest that additional macroeconomic variables be included in the modeling of the relationship between inflation and exchange rate.

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