

ARIMA-GARCH MODELING OF MONTHLY CRUDE OIL PRICES VOLATILITY FROM NIGERIA

Abstract

This research work, studied the hybrid of autoregressive integrated moving average (ARIMA) and generalized autoregressive conditional heteroscedasticity models that best fitted monthly crude oil price volatility of Nigeria between January, 2010 to March, 2021. The study collected secondary data from quarterly Central Bank of Nigeria (CBN) Statistical Bulletin, June, 2021 on monthly crude oil price of Nigeria to compute the monthly crude oil price returns. The ARIMA-GARCH modeling was adopted for this work. The series was tested for stationarity using Augmented Dickey Fuller test. Several ARIMA -GARCH models were applied to the monthly crude oil price returns to ascertain the best fit model for the series. The ARIMA (2, 0, 5)-GARCH(1,4) model was selected as the best fit for the data since it has minimum values of Akaike Information Criteria and Mean Squared Errors. The forecasted period showed a crude oil price with an unstable monthly crude oil price returns. Therefore, the government of Nigeria was advised to be conservative when planning with revenue from crude oil sale in future.

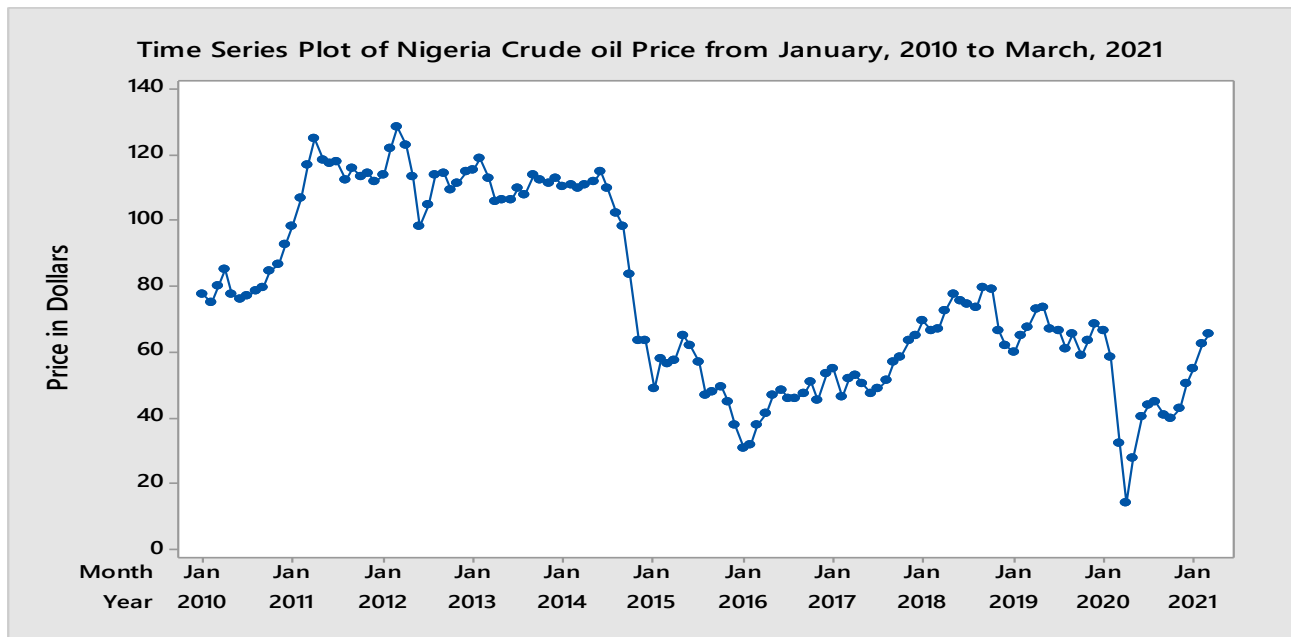
Key words: Crude Oil, ARIMA, GARCH, Model Adequacy, Forecast

1.0 Introduction

The discovery of crude oil since last century, it has continued to play a vital role in global economy because of its many uses. The continued use of crude oil as a source energy has created a strong demand for it globally. As a result, this has initiated a demand-supply factor that determines the price of this essential product. Therefore, changes or fluctuations in the price of crude oil have remained volatile over a long period of time as a result market players speculations, face-off among oil giants, covid-19 issues and sentiments of speculators.

However, in Nigeria, volatility of crude oil prices calls for a great attention, since it plays a key role in revenue generation for the Nigerian government. An increase in oil prices improves government revenue, while a decrease helps to reduce subsidy paid to maintain petroleum prices in the country.

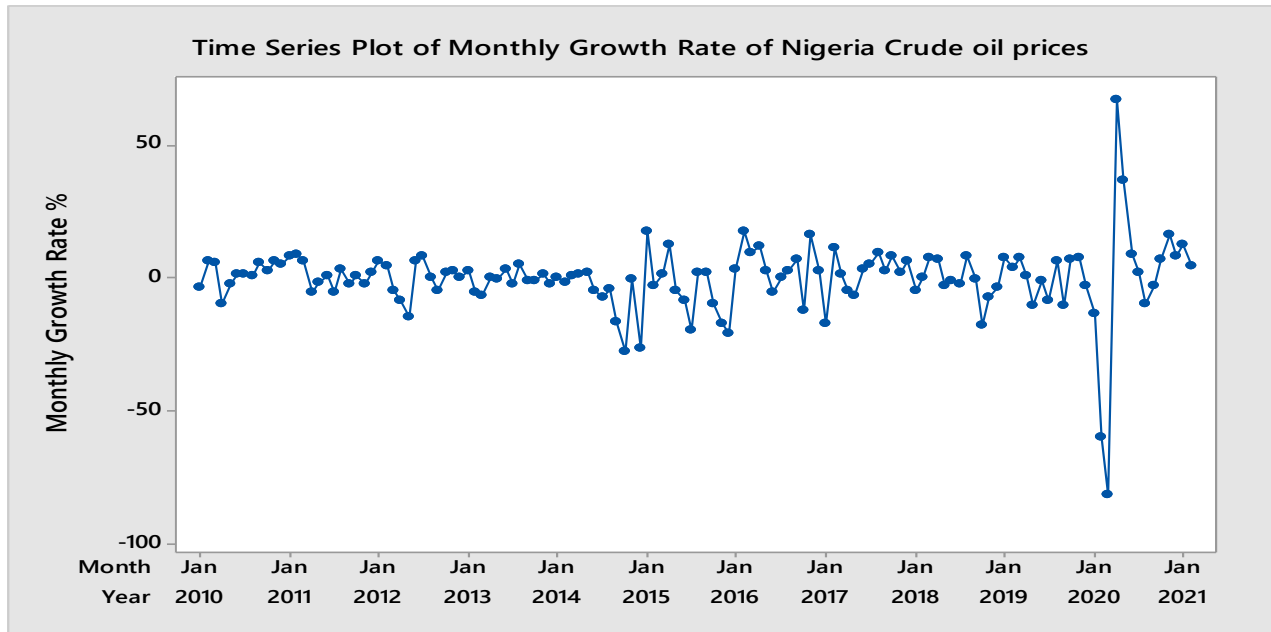
In light of the above, understanding the volatility of crude oil price and its implication become very important.



Source: Author's computation and Minitab output

Fig. 1.0: The Time Series Plot of Monthly Nigeria Crude Oil Prices (Brent) from January, 2010 to March, 2021

Fig. 1.0 shows the monthly prices of Nigeria crude oil price from January, 2010 to March, 2021 that indicates a downward trend of the crude oil price and also its instability within the period of interest.



Source: Author's computation and Minitab output

Fig. 2.0: The Time Series Plot of Monthly Price Returns of Nigeria Crude Oil from January, 2010 to March, 2021

Fig. 2.0 shows volatility of monthly price returns of Nigeria crude oil from January, 2010 to March, 2021. It depicts that there was relative stability in prices from January, 2010 up to December, 2014. However, between January, 2015 to March, 2021 the crude oil price showed volatility thereby making future predictions difficult.

However, several studies have been done on modeling the volatility of Nigeria crude oil price. Hence, the study carried out by [1], understanding crude oil price variation using time series analysis to comprehend the behavior of Nigeria crude oil variation showed that ARIMA (1,0,0) with a constant to be the model that can explain volatile monthly crude oil price in Nigeria from January, 2006 to December, 2011.

Also, [2], in their work, symmetric volatility forecast models for crude oil prices in Nigeria between January, 2003 and October, 2016 by applying ARIMA and generalized autoregressive conditional heteroscedasticity (GARCH) indicated ARIMA (1,1,0)-GARCH (1,1) modeled the stylistic features in the volatile crude oil prices in Nigeria.

In addition, [3] studied the linear and non-linear modeling of Nigerian crude oil prices between 2006 and 2019 by applying ARIMA and Markov Switching Autogressive Model (MS-AR). Their work showed that MS-AR (1) outperformed the ARIMA models.

[4], studied modeling of fluctuations in the prices of crude oil in Nigeria using ARCH and ARCH-M models using monthly data from Central Bank of Nigeria from May, 1989 to April, 2019. They found that an ARCH-M model fits the volatility of crude oil price in Nigeria.

Furthermore, [5], in their work, modeling and forecasting volatility of crude oil returns in Nigeria based on six error innovations used daily data from Central Bank of Nigeria from October 23, 2009 to March 23, 2020 to capture the dynamics of crude oil price volatility in Nigeria using GARCH models. They found that asymmetric power AP-ARCH (1, 1) performed better than other models.

Likewise, [6], in their work, modeling price volatility of Nigerian crude oil markets using GARCH model with monthly data from Central Bank of Nigeria, January 1987 to June, 2017 showed that Student's-t GARCH (1,1) model fitted better than the other models.

Consequently, this research paper – ARIMA-GARCH Modeling of Monthly Crude Oil Prices Volatility from Nigeria has two major objectives. First, to fit an adequate model for monthly crude oil price volatility of Nigeria and secondly, to use the fitted model to forecast the monthly crude oil price behavior for the next 10 months ahead.

2.0 Methodology

The hybrid of Autoregressive Integrated Moving Averages and Generalized Autoregressive Conditional Heteroscedasticity (ARIMA-GARCH) models was adopted for this study and more so, monthly data for this study were obtained from the quarterly Statistical Bulletin of Central Bank of Nigeria for the period of January, 2010 to March, 2021.

2.1 Model Estimation

The ARIMA (p, d, q) to be estimated in this research study is:

$$Y_t = c + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q} \quad (1)$$

Equation (1) can be transformed using a backshift notation as:

$$(1 - \phi_1 B - \dots - \phi_p B^p) Y_t = c + (1 - \theta_1 B - \dots - \theta_q B^q) \varepsilon_t \quad [7]. \quad (2)$$

Where c = constant term, ϕ = k th autoregressive parameter, θ = j th moving average parameter, p = order of the autoregressive (AR) fragment, q = order of the moving average (MA) fragment and d = degree of first differencing involved. The maximum combination of p and q for the modeling of the data shall be $p = q = 5$.

2.2 The Monthly Crude Oil Price Returns

The monthly crude oil price returns within the period under consideration shall be computed as:

$$Y_t = \text{Monthly Crude Oil Price Returns (CPR}_t) = (\ln COP_t - \ln COP_{t-1}) * 100\%$$

Where COP = monthly crude oil price (Brent) in Nigeria.

2.3 Unit Root Test

The monthly crude oil price returns is a time series data. The error terms may be correlated; therefore the augmented Dickey Fuller test was applied to the model to check for unit root in the time series data using [8]:

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum \alpha_i \Delta Y_{t-1} + \varepsilon_t \quad (3)$$

Where, ε_t is a pure white noise error term and where $\Delta Y_{t-1} = Y_{t-1} - Y_{t-2}$, $\Delta Y_{t-2} = Y_{t-2} - Y_{t-3}$. The unit root test is carried out under the null hypothesis $r=0$ against $r < 0$. The Dickey Fuller test statistic is:

$$DF_T = \frac{\hat{r}}{SE(\hat{r})}$$

(4)

2.4 Model Selection

The best ARIMA (p, d, q) model for forecasting monthly crude oil price returns shall be selected using the Mean Squared Error (MSE). The model with minimum MSE shall be selected.

The MSE = $\frac{1}{n} \sum_{t=1}^n e_t^2$, where n = number of periods and e_t = error.

2.5 Model Adequacy

The adequacy of the best ARIMA (p, d, q) model selected was conducted on the residuals of the model by checking the PACF and ACF of the residuals. More so, a portmanteau test of Ljung-Box Q^* Statistic was equally conducted on the residuals to check if they were white noise.

Ljung-Box Q^* Statistic = $n(n+2)\sum_{k=1}^h(n-k)^{-1}r_k^2$ where n = number observation, h = maximum

number of lag being considered and r = residuals. The data are white noise if the value of Q^* lies in the extreme 5% of the right hand tail of Chi-Square distribution; otherwise they are not white noise [9].

2.6 Tests for ARCH Effect

This is a joint null hypothesis that all q lags of the squared residuals have coefficient values that are not significantly different from zero.

$$\hat{u}_t^2 = \gamma_0 + \gamma_1\hat{u}_{t-1}^2 + \gamma_2\hat{u}_{t-2}^2 + \dots + \gamma_q\hat{u}_{t-q}^2 + v_t \quad (5)$$

The test statistic is defined as TR^2 (the number of observations multiplied by the coefficient of multiple correlation) and is distributed as a $\chi^2(q)$. If the value of the test statistic is greater than the critical value from the χ^2 distribution, then reject the null hypothesis of no ARCH effect.

2.7 GARCH (p, q) Model

The generalized GARCH model adopted for this study is:

$$\sigma_t^2 = \alpha_0 + \alpha_1u_{t-1}^2 + \alpha_2u_{t-2}^2 + \dots + \alpha_qu_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2 + \dots + \beta_p\sigma_{t-p}^2 \quad (6)$$

The maximum combination of p and q for the modeling of the data shall be $p = q = 4$.

If $\alpha_1 + \beta_1 < 1$ it shows that there is a low volatility, $\alpha_1 + \beta_1 > 1$ shows persistence and high volatility. The ARIMA-GARCH model with minimum MSE and AIC shall be selected.

The AIC = $n(1 + \log(2\pi)) + n \log \sigma^2 + 2m$, where n = number of periods, σ^2 = variance of the residuals and m = number of terms estimated in the model

3.0 Results and Analysis

3.1 Descriptive Analysis of the Variable

Table 3.1: The Descriptive Statistics of the Monthly Crude Oil Price Returns.

Mean	Median	Standard Deviation	Skewness	Kurtosis
-0.13%	0.92%	13.63%	-1.33	15.33

Source: Author's computation

This study considered data on monthly crude oil price returns from January, 2010 to March, 2021. Table 3.1 depicts that average monthly crude oil price return was a decrease of -0.13%. The median was 0.92% while, standard deviation from the mean for was 13.63%. Distribution of the monthly crude oil price return is negatively skewed to the left. The kurtosis value of 15.33 means that the curve of the monthly crude oil price returns is leptokurtic in nature.

3.2 Unit Root Test

Table 3.2: The Unit Root Test of the Monthly Crude Oil Price Returns.

Dickey-Fuller test (ADF(stationary) / k: 5 / Crude Oil Price Returns):	
Tau (Observed value)	-4.9555
Tau (Critical value)	-0.8170
p-value (one-tailed)	0.0004
alpha	0.05

Source: Author's computation

Test interpretation:

H_0 : There is a unit root for the series (CPR).

H_1 : There is no unit root for the series (CPR).

As the computed p-value is lower than the significance level $\alpha=0.05$, one should reject the null hypothesis H_0 , and accept the alternative hypothesis H_1 . The risk to reject the null hypothesis H_0 while it is true is lower than 0.04%. Therefore, the monthly crude oil price return is stationary at level and does not need differencing.

3.3 Model Estimation and Selection

Table 3.3: The Mean Squared Error Values of Estimated ARMA Models.

Mean Squared Errors (MSE)						
	q = 0	q = 1	q = 2	q = 3	q = 4	q = 5
p = 0		181.2	173.0	171.8	169.1	170.0
p = 1	177.9	177.0	170.0	175.4	171.0	171.3
P = 2	174.3	171.4	170.9	172.2	173.0	172.6
P = 3	172.7	172.5	171.2	171.5	172.9	174.0
P = 4	170.4	171.7	171.3	169.4	168.5	175.4
P = 5	171.7	172.6	168.2	169.1	169.2	-

Source: Author's computation

Table 3.3 shows the mean squared error values of the estimated ARIMA (p, d, q) models. The ARIMA (2, 0, 5) model is the best since it has the least mean squared errors of 168.2. Some models were not able to be estimated by the computer program used; this means those models are not suitable for this analysis.

3.4 Parameters of the Best Selected Model

Table 3.4: The Estimated Parameters of the Selected Best ARIMA Model.

Type	Coef.	SE Coef.	T	P
AR 1	-0.3764	0.1271	-2.96	0.0004
AR 2	-0.8030	0.0806	-9.96	0.0000
MA 1	-0.5452	0.1657	-3.29	0.0001
MA 2	-0.6719	0.1118	-6.01	0.0000
MA 3	0.0936	0.1102	0.85	0.3970
MA 4	0.4055	0.1073	3.78	0.0000
MA 5	0.1378	0.1212	1.14	0.2570
Constant	-0.5180	1.7490	-0.30	0.7680

Source: Author's computation

The ARIMA (p, d, q) model with smallest values of MSE is ARIMA (2, 0, 5). It is the ARIMA (p, d, q) model that fitted the crude oil price returns from January, 2010 to March, 2021. Table 3.4 shows that estimated parameters of ARIMA (2, 0, 5) model. Based on the table, parameters of the autoregressive (AR) are all significant; also some parameters of the moving average (MA) part of the model are equally significant since all their p-values are less than the significance level of 0.05. The constant is not significant; its p-value is more than the significance value of 0.05.

The ARIMA (2, 0, 5) model can be represented as:

$$Y_t = -0.3764Y_{t-1} - 0.8030Y_{t-2} + \varepsilon_t + 0.5452\varepsilon_{t-1} + 0.6719\varepsilon_{t-2} - 0.0936\varepsilon_{t-3} - 0.4055\varepsilon_{t-4} - 0.1378\varepsilon_{t-5}$$

Or

$$(1 + 0.3764B + 0.8030B^2)Y_t = (1 + 0.5452B + 0.6719B^2 - 0.0936B^3 - 0.4055B^4 - 0.1378B^5)\varepsilon_t$$

(7)

3.5 Model Adequacy of the Best Selected Model

The residuals from the best selected model, ARIMA (2, 0, 5) were checked if they are white noise.

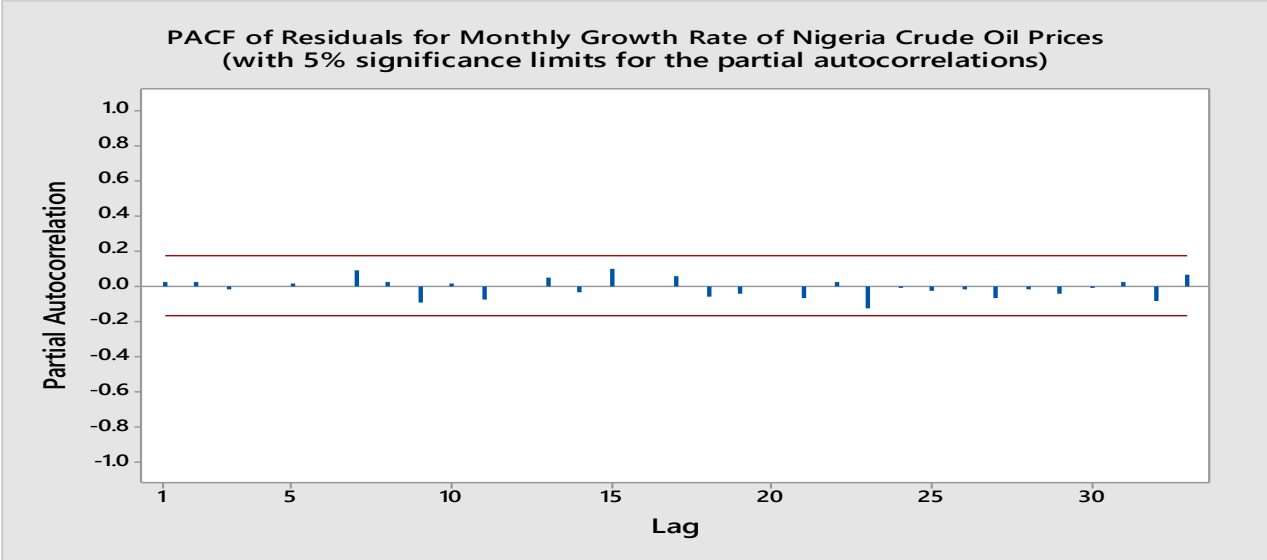
Table 3.5: Ljung-Box Q* Statistic Test for the Residuals of ARIMA (2, 0, 5) Model

Lag	12	24	36	48
Chi-Square	3.7	10.7	13.7	24.2
DF	4	16	28	40
P-Value	0.446	0.828	0.989	0.977

Source: Author's computation

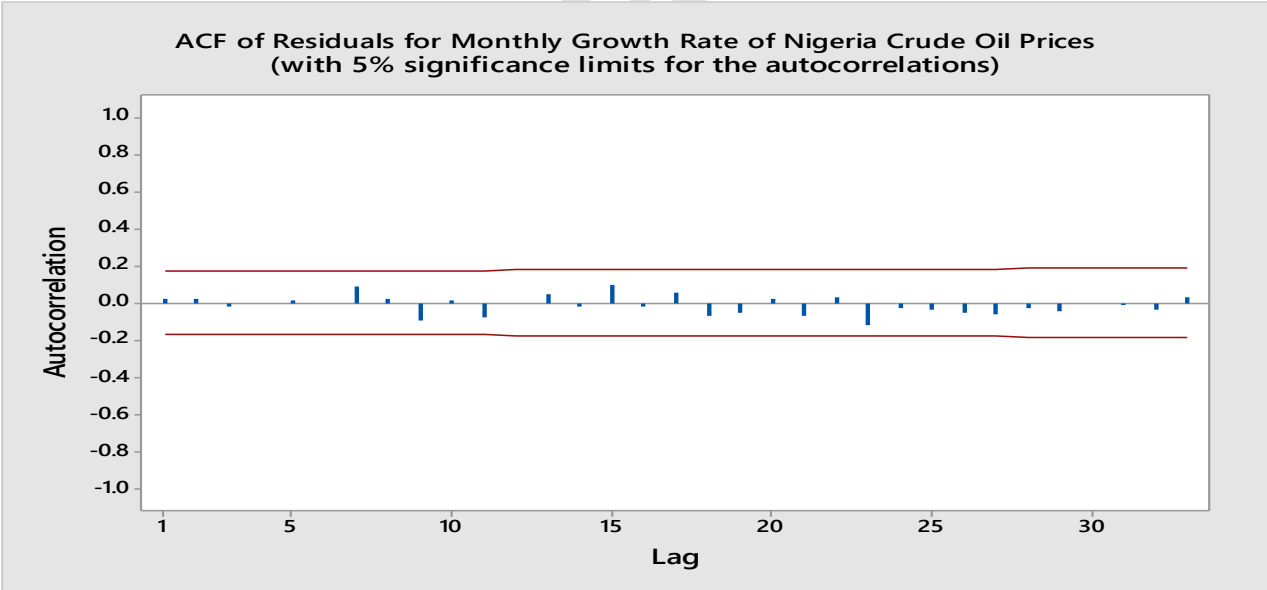
Table 3.5 shows the Ljung-Box Q Statistic test on residuals from ARIMA (2, 0, 5) model. Since the P-values at different lags are more than the significance level of 0.05, it means the residual values are not significantly different from zero. Therefore, they are white noise.

In addition, spikes of PACF and ACF of the residuals are within the 5% significance limits as shown in Fig. 3.1 and Fig. 3.2. This equally means that the residuals are white noise.



Source: Author’s computation

Fig. 3.1: PACF for the Residuals of ARIMA (2, 0, 5) Model



Source: Author’s computation and XLSTAT Output

Fig. 3.2: ACF for the Residuals of ARIMA (2, 0, 5) Model

Table 3.6: Test for ARCH Effect

Heteroskedasticity Test: ARCH			
F-statistic	18.97491	Prob. F(5,122)	0.0000
Obs*R-squared	55.99524	Prob. Chi-Square(5)	0.0000

Source: Author's computation

Table 3.6 shows that both the F-statistic and TR^2 are very significant, suggesting the presence of ARCH effect in crude oil price return series.

Table 3.7: ARIMA-GARCH Modeling of the Crude Oil Price Return

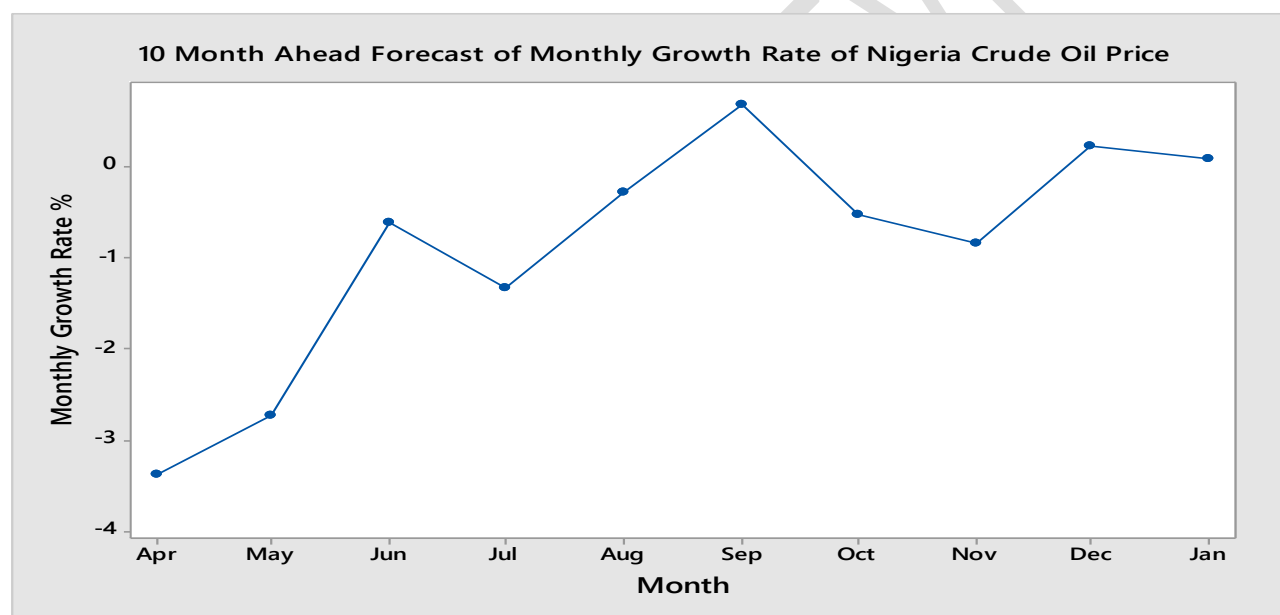
ARIMA-GARCH	α_1	β_1	$\alpha_1 + \beta_1$	AIC
ARIMA(2,0,5)-GARCH(1,1)	1.004	0.2957	1.2961	7.2439
ARIMA(2,0,5)-GARCH(1,2)	0.9308	0.1506	1.0814	7.2489
ARIMA(2,0,5)-GARCH(1,3)	0.9115	0.7745	1.6860	7.2591
ARIMA(2,0,5)-GARCH(1,4)	0.5657	1.0745	1.6402	7.1855
ARIMA(2,0,5)-GARCH(2,1)	0.9793	0.4179	1.3972	7.2470
ARIMA(2,0,5)-GARCH(2,2)	0.9332	0.3553	1.2885	7.2610
ARIMA(2,0,5)-GARCH(2,3)	0.5464	1.0038	1.5502	7.1901
ARIMA(2,0,5)-GARCH(2,4)	-	-	-	-
ARIMA(2,0,5)-GARCH(3,1)	0.8436	0.6166	1.4602	7.2558
ARIMA(2,0,5)-GARCH(3,2)	0.1735	0.3027	0.4762	7.6047
ARIMA(2,0,5)-GARCH(3,3)	0.8947	1.0920	1.9867	7.2801
ARIMA(2,0,5)-GARCH(3,4)	0.8670	-0.3036	0.5634	7.2890
ARIMA(2,0,5)-GARCH(4,1)	0.8557	0.5175	1.3732	7.2635

ARIMA(2,0,5)-GARCH(4,2)	0.6876	1.4320	2.1196	7.2919
ARIMA(2,0,5)-GARCH(4,3)	0.5032	0.7881	1.2913	7.2354
ARIMA(2,0,5)-GARCH(1,1)	0.8789	0.2773	1.1562	7.3042

Source: Author's computation

Table 3.7 shows that ARIMA (2, 0, 5)-GARCH (1, 4) outperformed other ARIMA-GARCH models since it has the minimum AIC value of 7.1855. The crude oil price return showed persistence and high volatility because $\alpha_1 + \beta_1 > 1$.

3.6 Forecasts



Source: Author's computation and XLSTAT Output

Fig. 3.3: 10 Month Ahead Forecast of the Crude Oil Price Return using ARIMA (2, 0, 5)-GARCH (1, 4) Model

The 10 month ahead forecast of monthly crude oil price returns from Nigeria shows there will be a rise in monthly crude oil price returns from April, 2021 to June, 2021. The monthly return is expected to fluctuate from July, 2021 to January, 2022.

4.7 Discussion of Results

This work concentrated on ARIMA-GARCH modeling of monthly crude oil price returns of Nigeria for the period ranging from January, 2010 to March, 2021. The data used for this research were obtained from the Quarterly Statistical Bulletin of the Central Bank of Nigeria, June 2021. Results were obtained from analyzing the data; the descriptive analysis revealed that mean monthly crude oil price return was a loss of 0.13%. While, standard deviation from the mean for was 0.92%, this shows riskiness of depending oil as means of revenue for the Nigerian government. Distribution of the monthly crude oil price return is negatively skewed to the left. The kurtosis value of 15.33 means that the curve of the monthly crude oil price returns is leptokurtic in nature.

Also, the Augmented Dickey Fuller test for unit root was performed on the monthly crude oil price returns to evaluate for unit root. The series of the variable were stationary at significance level of 0.05. Additionally, several ARIMA (p, d, q) models were applied to the series, with maximum $p = q = 5$ to select the best ARIMA (p, d, q) that fits the series using the Mean Squared Error (MSE). ARIMA (2, 0, 5) model best fitted the monthly crude oil price returns of Nigeria with minimum MSE of 168.2.

More so, the model adequacy of ARIMA (2, 0, 5) was assessed using the Ljung-Box Q Statistic test on residuals, partial autocorrelation function (PACF) and autocorrelation function (ACF) of the residuals as well. The Ljung-Box Q Statistic test on residuals from ARIMA (2, 0, 5) at different lags revealed that the residual values are not significantly different from zero. Therefore, the residuals are white noise. The spikes of PACF and ACF were within the 5% significance limit showing equally that the residuals are white noise.

Consequently, the ARIMA (2, 0, 5) was used as the conditional mean for the GARCH modeling of Nigeria crude oil price volatility within the period under consideration. It was observed that ARIMA (2, 0, 5)-GARCH (1, 4) performed better than other ARIMA (2,0,5)-GARCH models with minimum AIC of 7.1855. Also, the crude oil price return showed a persistent high volatility, this makes it difficult for Nigerian government to predict and plan her revenue from the sale of crude oil as a result of high fluctuations in prices.

Furthermore, the 10 month ahead forecast of the monthly crude oil price returns using ARIMA (2, 0, 5)-GARCH (1, 4) showed an oil with an unstable monthly price movement. Fig. 3.3 showed an upward price trend movement from April, 2021 to June, 2021, while from July, 2021 to January, 2022 shall experience an unstable monthly crude oil price in Nigeria. Therefore, Nigerian government is advised to be cautious in planning with revenue from crude oil sales due to instability in monthly prices.

5.0 Conclusion

This research work focused on ARIMA-GARCH modeling monthly crude oil price volatility from Nigeria for the period of January, 2010 to March, 2021. Consequently, ARIMA (2, 0, 5)-GARCH (1, 4) model was selected to be the best fit model from other competing ARIMA-GARCH models. The selected model was found to be adequate for short-term forecast of the monthly crude oil price returns from Nigeria. The forecast for the future crude oil price showed an unstable price movement that will persist. Therefore, the Nigerian government should promote policies that will create confidence in the economy thereby diversifying for other sources of revenue, and also be conservative with planning based on revenue sales from future crude oil sales.

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