Original Research Article

Bayesian Estimation of the Parameters of the Odd Generalized Exponentiated - Inverse Exponential Distribution (OGE -IED)

ABSTRACT

The Odd Generalized Exponentiated-Inverse Exponential Distribution, a three parameter distribution, is a hybrid of the Generalized Exponential distribution. Each of the parameters were assigned a gamma prior independently resulting to a posterior distribution that is mathematically intractable impossible to obtain marginal posterior distribution for two of the parameters, and a likelihood function that is not known traditionally to R or other statistical software. Resort was made to STAN in order to obtain Bayesian estimates - leveraging on STAN's provision for user-defined distribution functions. Two datasets were used; remission times (in months) of bladder cancer patients and COVID-19 Survey data in Andalusia, Spain. In the end, the Maximum Likelihood estimates maximized the likelihood more than the Bayesian estimates - though with a slight margin of no more than 0.77. On the other hand, the Bayesian estimates proved to be more stable yielding very negligible standard errors compared to the Maximum Likelihood estimates.

Keywords

Generalized Exponential Distribution, Odd Generalized Exponentiated-Inverse Exponential Distribution, STAN, COVID-19 Survey

1 INTRODUCTION

Parameter Estimation is one of the areas in Statistical Inference that draws the attention of researchers with the aim of getting the estimate of parameters in order to make inference about a population. Several methods can be used to estimate any parameter of interest but the most used methods are; the method of moments, the method of maximum likelihood and the Bayesian method of estimation.

In 1992 R. A. Fisher introduced the method of maximum likelihood, the making of maximum likelihood was one of the most important development in the 20th century Statistics (Aldrich, 1997). Also, the method of maximum likelihood have advantages of analyzing moderate or large sample sizes and widely applied along with availability of computer technology (Phoong and Ismail, 2015). Kim and Han (2015) also used the method of maximum likelihood to estimate censored data. Several researchers also used the method of maximum likelihood to estimate parameters, some of them are Singh et al. (2014), Guure and Bosomprah (2013), Javadkhani et al. (2014) and Yahaya and Abba (2017). Bayesian Estimation was proposed by Laplace in 1986. Laplace highlighted the three pillars of Bayesian estimation which are; prior distribution, the likelihood and the posterior distribution. The prior distribution reflects the researcher's prior belief about the nature of the parameter of interest. A prior distribution could be informative or non-informative, improper or conjugate. For a demonstration on the use of priors, see Nawaz and Aslam (2015), Chandra and Rathavr (2016), Aslam et al. (2011), Guure and Bosomprah (2013), Yin (2012), Yin and Zhao (2013), Yin and Li (2014), Goltong and Doguwa (2018) and Goltong and Doguwa (2019).

The posterior distribution is derived approximately as the product of the prior distribution and the likelihood function. Some posterior distributions end up to be any of the familiar probability distribution functions, said to be in closed form, while at other times, they may not exist in closed form and therefore, cannot be solved analytically. Some methods can be used such as Lindley's approximation as used by Guure and Bosomprah (2013), and the Gibbs sampler as used by Singh et al. (2014).

The parameters of the hybrid distribution presented by Yahaya and Abba (2017) have not been estimated using the Bayesian estimation method which we intend to do in this paper.

2 LITERATURE REVIEW

Several researches have been conducted on the hybridization of distributions as well as parameter estimation for such distributions. Farahani and Khorrom (2014) used Azzadini's method to derive the weighted exponential distribution, whose bayesian parameter estimates were obtained using Lindley's approximation. In the study, comparison be- tween the classical and Bayesian estimation methods showed that the Bayesian approach would be the best method for the estimation.

Chandra and Rathavr (2016) derived the augmented strength reliability models by assuming that the inverse Gaussian stress is subjected to equipment having exponential strength and are independent of each other.

Other researchers who compared the Bayesian and Maximum Likelihood estimates include Simbolon et al. (2016), who compared the Bayesian and maximum likelihood method of estimating the shape parameter of the Kumaraswamy distribution, Singh et al. (2014), who considered the classical and Bayesian estimation of the parameter and reliability characteristic of extension of the exponential distribution, Kim and Han (2015) who considered the maximum likelihood estimation and Bayes estimation of the parameters of the Generalized Exponential Distribution based on progressive first failure censored samples and Ariza-Hernandez et al. (2017), using different loss functions, different criteria for comparison and arriving at different conclusions under the various prevailing circumstances. Authors interested in Bayesian estimation of parameters include Nawaz and Aslam (2015), Guure and Bosomprah (2013), Javadkhani et al. (2014), Pradhan and Kundu (2011), Aslam et al. (2011), Dikko and Isaac (2018) and Rasheed and Khalifa (2016) using varry- ing choices of priors, loss functions and different criteria to compare the estimates ranging from posterior risk to Mean Square Error (MSE).

Yahaya and Abba (2017) proposed a new life time distribution called the Odd General- ized Exponential Inverse-Exponential Distribution (OGE-IED) which is the hybrid of the Generalized Exponentiated Exponential distribution. In hybridization, they made use of the new OGE family generator. The Odd Generalized Exponential Inverse-Exponential Distribution is a three parameter distribution, with parameter vector $\bar{\omega} = (\alpha, \beta, \lambda)$ where $\alpha>0$, $\beta>0$, and $\lambda>0$. Some Statistical properties of the asymptotic behavior and the moments of the random variable X of OGE are estimated. The method of maximum likelihood estimation was used to estimate the parameter of the OGE-IED. More so, the mathematical expression for some reliability functions were also determined.

3 METHODOLOGY

3.1 Maximum Likelihood Estimation

Yahaya and Abba (2017) used Maximum Likelihood Estimation (MLE) to estimate the parameters of the OGE-IE distribution. Let $x = (x_1, x_2, \ldots, x_n)'$ be a sample of size n from the OGE-IE distribution with parameter vector $\bar{\omega} = (\alpha, \beta, \lambda)'$. Then the log-likelihood function of $\bar{\omega}$ is given by

$$
\log \prod_{i=1}^{n} f(x_i|\bar{\omega}) = n\, ln\alpha + n\, ln\beta + n \ln \lambda - \lambda \sum_{i=1}^{n} \frac{1}{x_i} + 2 \sum_{i=1}^{n} \frac{1}{x_i} - 2 \sum_{i=1}^{n} \ln\left(1 - e^{\frac{-\lambda}{x_i}}\right) - \alpha \sum_{i=1}^{n} \left(\frac{e^{\frac{-\lambda}{x_i}}}{1 - e^{\frac{-\lambda}{x_i}}} \right) + (\beta - 1) \sum_{i=1}^{n} ln\left(1 - exp\left[-\alpha \left(\frac{e^{\frac{-\lambda}{x_i}}}{1 - e^{\frac{-\lambda}{x_i}}} \right)\right]\right) \quad (1)
$$

The partial derivatives of the log-likelihood with respect to α, β and λ are given below as:

$$
\frac{\partial l(\bar{\omega})}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} \left(\frac{e^{\frac{-\lambda}{x_i}}}{1 - e^{\frac{-\lambda}{x_i}}} \right) + (\beta - 1) \sum_{i=1}^{n} \left\{ \frac{C}{\left(1 - exp\left[-\alpha \left(\frac{e^{\frac{-\lambda}{x_i}}}{1 - e^{\frac{-\lambda}{x_i}}} \right)\right]\right)} \right\} \quad (2)
$$

where

$$
C = \left(\frac{e^{\frac{-\lambda}{x_i}}}{1 - e^{\frac{-\lambda}{x_i}}} \right) exp\left[-\alpha \left(\frac{e^{\frac{-\lambda}{x_i}}}{1 - e^{\frac{-\lambda}{x_i}}} \right)\right]
$$

$$
\frac{\partial l(\bar{\omega})}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} ln\left(1 - exp\left[-\alpha \left(\frac{e^{\frac{-\lambda}{x_i}}}{1 - e^{\frac{-\lambda}{x_i}}} \right)\right]\right) \quad (3)
$$

and

$$
\frac{\partial l(\bar{\omega})}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} \frac{1}{x_i} + \alpha \sum_{i=1}^{n} \left(\frac{e^{\frac{-\lambda}{x_i}}}{x_i \left[1 - e^{\frac{-\lambda}{x_i}}\right]^2} \right) - 2 \sum_{i=1}^{n} \left(\frac{e^{\frac{-\lambda}{x_i}}}{x_i \left[1 - e^{\frac{-\lambda}{x_i}}\right]} \right) + (\beta - 1) \sum_{i=1}^{n} \left\{ \frac{\alpha\, exp\left[-\frac{\lambda}{x_i} - \alpha \left(\frac{e^{\frac{-\lambda}{x_i}}}{x_i \left[1 - e^{\frac{-\lambda}{x_i}}\right]^2} \right)\right]}{x_i \left[1 - e^{\frac{-\lambda}{x_i}}\right]^2 \left(1 - exp\left[-\alpha \left(\frac{e^{\frac{-\lambda}{x_i}}}{1 - e^{\frac{-\lambda}{x_i}}} \right)\right]\right)} \right\} \quad (4)
$$

It becomes obvious from equations (2), (3) and (4) that the Maximum Likelihood Estimates of the three parameters cannot be obtained in close form. Therefore, we resort to the use of the R package *maxlik*.

3.2 Bayesian Estimation

Since the parameters are viewed as random rather than fixed, we assign the Gamma prior to each of the three parameters. The Gamma Prior is chosen for two basic reasons. The first is due to the fact that the parameters to be estimated are all positive and hence the need to stick to a distribution that is only defined for positive values. Secondly, the Gamma distribution is chosen for easier computation. Therefore, we now assign Gamma priors to the parameters in the following manner

$\pi(\alpha) \sim Gamma(a_1, b_1);\ \pi(\beta) \sim Gamma(a_2, b_2);\ \pi(\lambda) \sim Gamma(a_3, b_3)$ (5)

Note that the likelihood function can be expressed as

$$L(\varpi|x) \propto (\alpha\beta\lambda)^n e^{-\sum \frac{\lambda}{x_i}} e^{-\alpha \sum_{i=1}^{n} \left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}} \right)} \prod_{i=1}^{n} \left(1 - e^{\frac{-\lambda}{x_i}}\right)^{-2} \prod_{i=1}^{n} \left(1 - e^{-\alpha\left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}}\right)}\right)^{\beta-1} \quad (6)$$

And the posterior as

$$\pi(\varpi|x) \propto L(\varpi|x)\pi(\varpi) \quad (7)$$

By further simplification, equation (7) becomes

$$\pi(\varpi|x) \propto \alpha^{n+a_1-1}\ \beta^{n+a_2-1}\ \lambda^{n+a_3-1}\ exp\left\{-\left[b_1\alpha + b_2\beta + \lambda\left(b_3 + \sum \frac{1}{x_i}\right)\right]\right\} \times exp\left\{-\alpha\left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}}\right)\right\} \prod_{i=1}^{n}\left(1 - e^{\frac{-\lambda}{x_i}}\right)^{-2} \times \prod_{i=1}^{n}\left(1 - exp\left\{-\alpha\left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}}\right)\right\}\right)^{\beta-1} \quad (8)$$

By taking a critical look at equation (8), it can be seen that the posterior distribution so obtained is not in closed form. The marginal posterior distributions of parameters are given as:

$$\pi(\alpha|\beta,\lambda) \propto \alpha^{n+a_1-1}\, exp\left\{-\alpha\left[b_1 + \sum_{i=1}^{n}\left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}}\right)\right]\right\} \times exp\left\{(\beta-1)\sum_{i=1}^{n} log\left[1 - exp\left\{-\alpha\left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}}\right)\right\}\right]\right\} \quad (9)$$

and equation (9) does not look like the kernel of any known distribution.

$$\pi(\beta|\alpha,\lambda) \propto \beta^{n+a_2-1}\, exp\left\{-\beta\left[b_2 - \sum_{i=1}^{n} log\left[1 - exp\left\{-\alpha\left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}}\right)\right\}\right]\right]\right\} \quad (10)$$

which is the kernel of the Gamma distribution. Furthermore,

$$\pi(\lambda|\alpha,\beta) \propto \lambda^{n+a_3-1} exp\left\{(\beta-1)\sum_{i=1}^{n} log\left[1-exp\left\{-\alpha\left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}}\right)\right\}\right]\right\}$$

$$\times exp\left\{-\lambda\left(b_3+\sum\frac{1}{x_i}\right)-\alpha\sum_{i=1}^{n}\left(\frac{e^{\frac{-\lambda}{x_i}}}{1-e^{\frac{-\lambda}{x_i}}}\right)-2\sum_{i=1}^{n} log\left(1-e^{\frac{-\lambda}{x_i}}\right)\right\} \quad (11)$$

We have only succeeded in obtaining the marginal posterior distribution of β in closed form whereas the marginal posterior distributions of α and λ could not be obtained in closed form. We resort to Markov Chain Monte Carlo Techniques to estimate the three parameters. One such environment that provides for user defined probability functions is STAN developed by Stan Development Team (2020). With STAN, one can either specify a user defined probability density function, distribution function, or the probability density function in the log space depending on one's choice.

4 RESULTS AND DISCUSSION

In this research, two different datasets are used. The first is data on Remission times found in Lee and Wang (2003) and used by Yahaya and Abba (2017) and many other researchers before them. The second data is on COVID-19 Survey in Andalusia, Spain as used by Berihuete et al. (2021) and can be found here: https://cnecovid.isciii.es/covid19/#documentacin-y-datos and https://www.ine.es/up/9Gq4uzeUiT. The data on COVID-19 Survey in Andalusia consists of number of infected persons per day per 100,000 persons in Andalusia which we have scaled down to represent the average number of infected persons per day per 1,000 persons. For both datasets, we used the Maximum Likelihood and Bayesian estimation methods to fit the Odd Generalized Exponentiated Inverse Exponential Distribution.

The histogram of Remission times (in months) of 128 patients of Bladder cancer and COVID-19 Survey shown in figures 1 and 4 reveal that both datasets are right-tailed. Hence, the Odd Generalized Exponentiated Inverse Exponential Distribution can be used to fit the data.

In STAN, four (4) independent chains were run (i.e. from the probability distributions of each of the three parameters and from the posterior distribution of the data). The length of each of the chains is 6,000 iterations (4,000 iterations in the case of the COVID-19 Survey data) with the first 1,000 iterations discarded as warm-up or burn-in.

Figures 2 and 5 show the distributions of the parameters and the posterior distribution of the data where α is denoted by "alph", β by "bet", λ by "lam" and the posterior distribution of the data by "y_ppc". These notations where used in order to not cause software issues by conflicting with reserved words in R. Figures 3 and 6 show the histogram of the distribution of the parameters and the posterior distribution of the data. Tables 3 and 4 display the summary of estimates using Bayesian and Maximum Likelihood Estimation methods respectively for the remission times data while tables 7 and 8 does same for the COVID-19 Survey data. Tables 5 and 9 display the summary of both methods side by side for easy comparison.

Table 1: Remission times (in months) of 128 patients of Bladder cancer (Lee and Wang, 2003)

0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.20	2.23
3.52	4.98	6.97	9.02	13.29	0.40	2.26	3.57	5.06	7.09
9.22	13.80	25.74	0.50	2.46	3.64	5.09	7.26	9.47	14.24
25.82	0.51	2.54	3.70	5.17	7.28	9.74	14.76	26.31	0.81
2.62	3.82	5.32	7.32	10.06	14.77	32.15	2.64	3.88	5.32
7.39	10.34	14.83	34.26	0.90	2.69	4.18	5.34	7.59	10.66
15.96	36.66	1.05	2.69	4.23	5.41	7.62	10.75	16.62	43.01
1.19	2.75	4.26	5.41	7.63	17.12	46.12	1.26	2.83	4.33
5.49	7.66	11.25	17.14	79.05	1.35	2.87	5.62	7.87	11.64
17.36	1.40	3.02	4.34	5.71	7.93	11.79	18.10	1.46	4.40
5.85	8.26	11.98	19.13	1.76	3.25	4.50	6.25	8.37	12.02
2.02	3.31	4.51	6.54	8.53	12.03	20.28	2.02	3.36	6.76
12.07	21.73	2.07	3.36	6.93	8.65	12.63	22.69		

Table 2: Summary of Remission Times

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.080	3.348	6.395	9.366	11.838	79.050

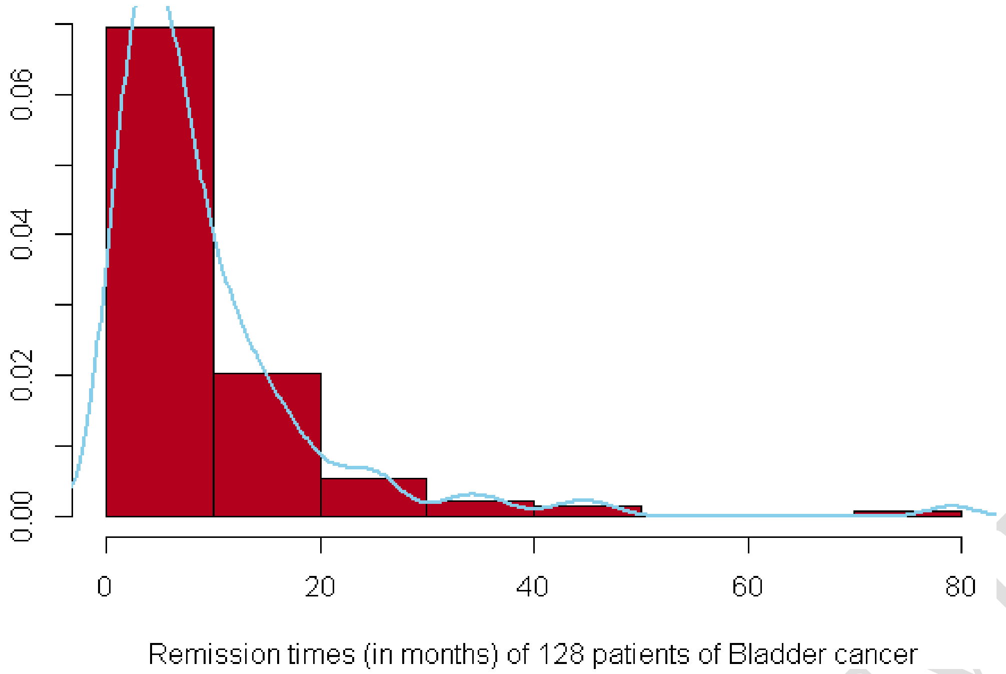

Figure 1: Histogram of Remission times (in months) of 128 patients of Bladder cancer (Lee and Wang, 2003)

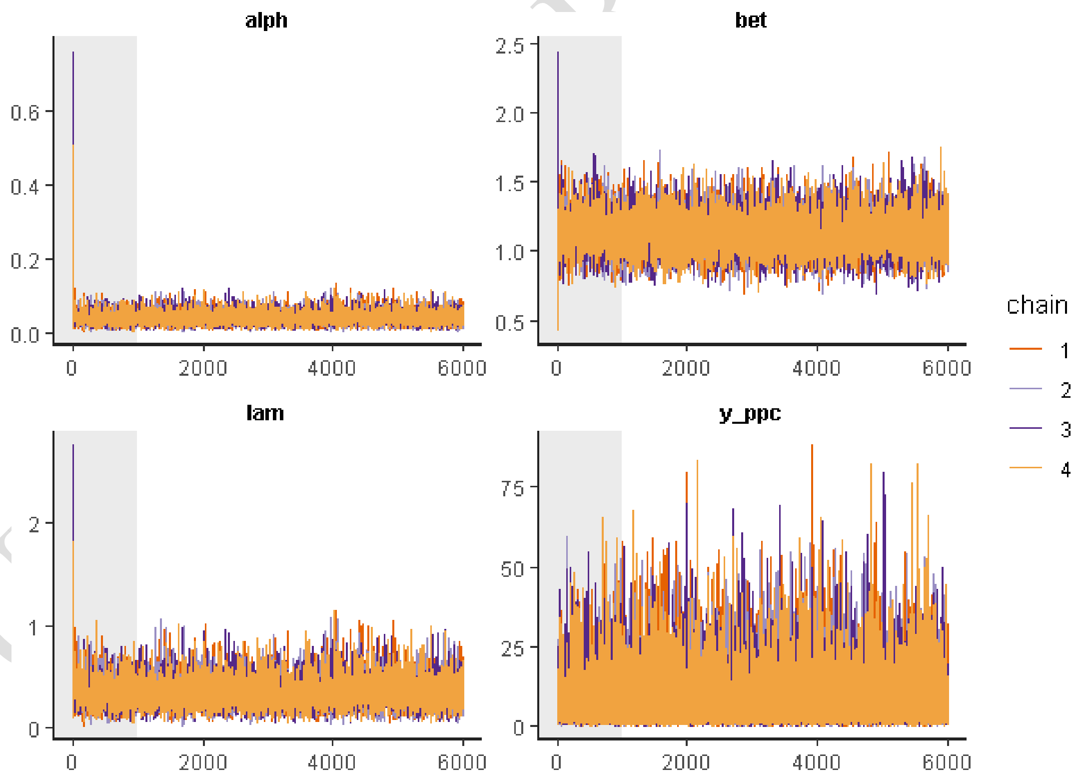

Figure 2: Traceplot of the 4 chains sampled from the posterior distribution of the parameters and the data (Remission Times)

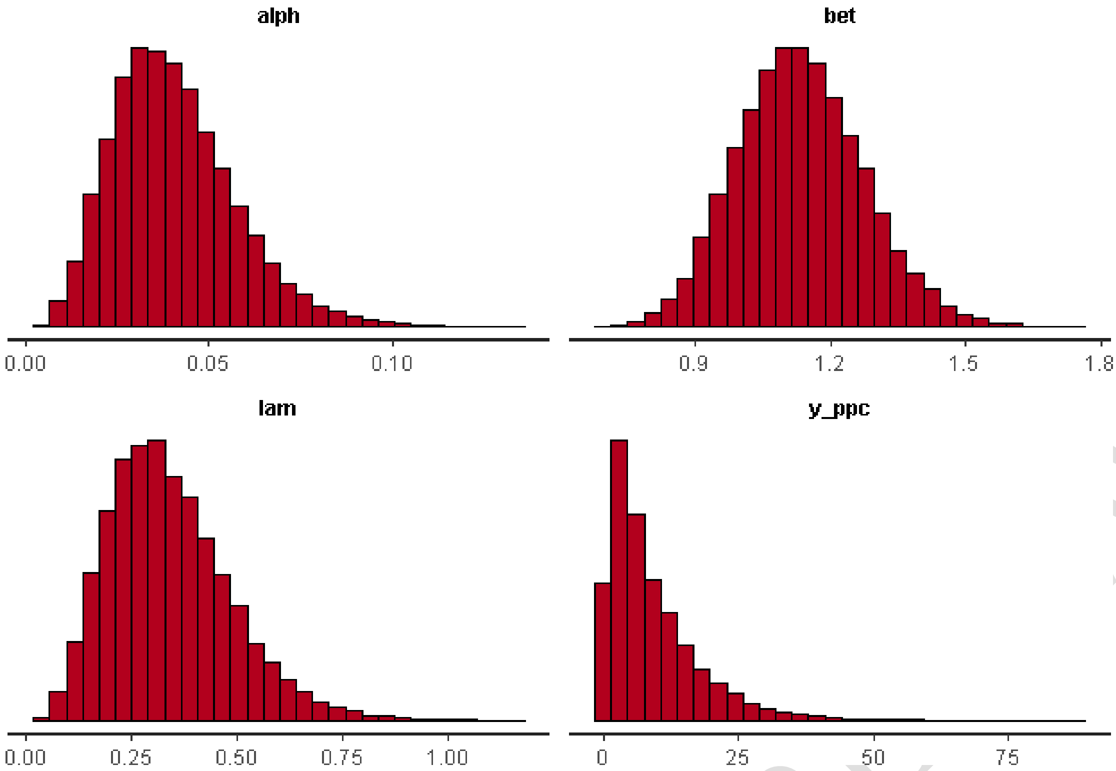

Figure 3: Histogram of the 4 chains sampled from the posterior distribution of the parameters and the data (Remission Times)

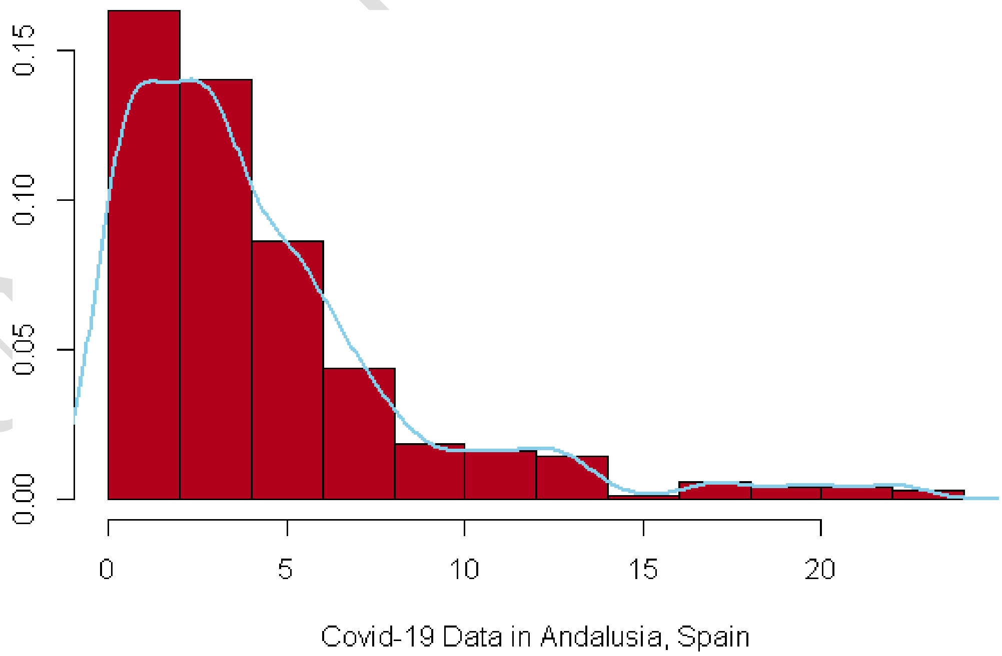

Figure 4: Histogram of COVID-19 Survey in Andalusia, Spain

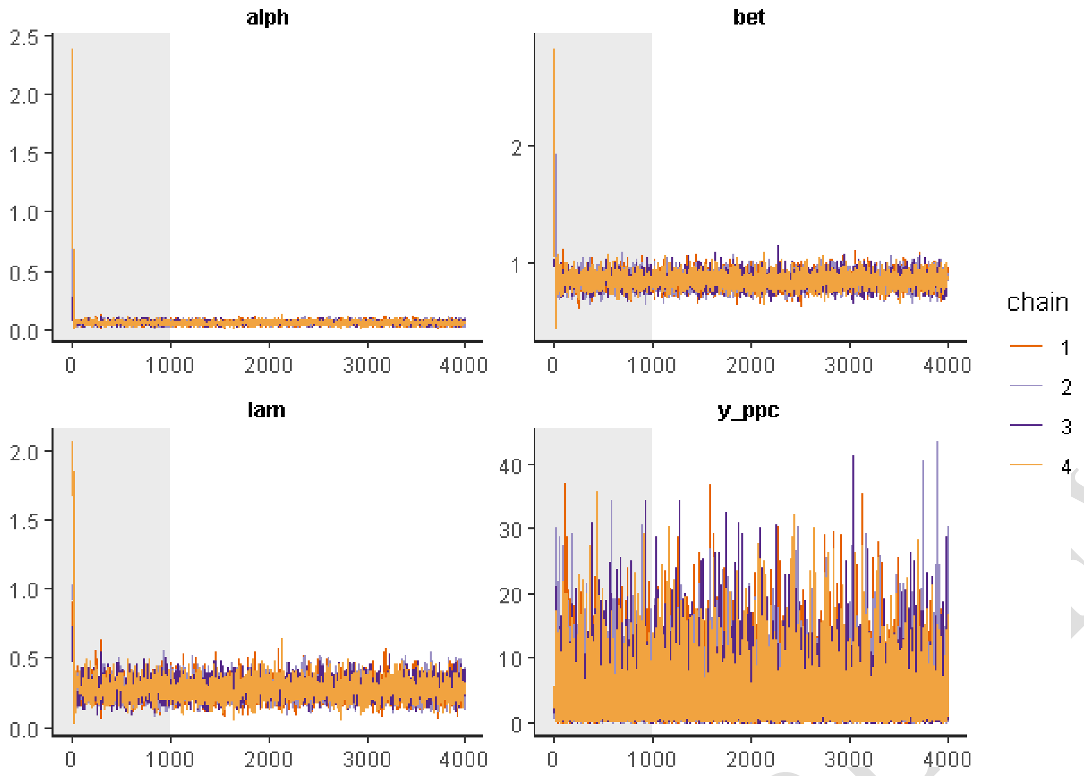

Figure 5: Traceplot of the 4 chains sampled from the posterior distribution of the parameters and the data (COVID-19 Survey in Andalusia)

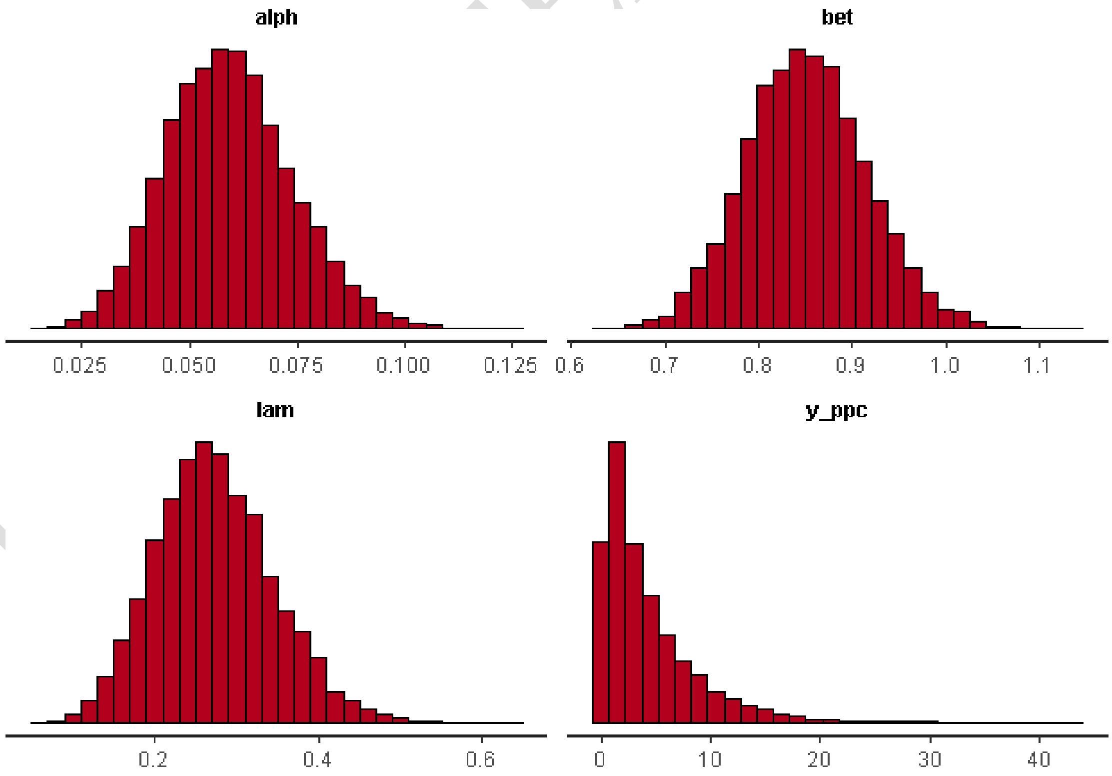

Figure 6: Histogram of the 4 chains sampled from the posterior distribution of the parameters and the data (COVID-19 Survey in Andalusia)

Table 3: Summary of the Bayesian Estimation using Stan

	mean	standard error	standard deviation	2.5%	50%	97.5%	n eff	Rhat
alpha	0.04	0.00	0.02	0.01	0.04	0.08	6404	1
beta	1.13	0.00	0.14	0.87	1.13	1.43	7970	1
lambda	0.34	0.00	0.15	0.11	0.32	0.68	6173	1
y ppc	9.17	0.06	8.72	0.44	6.53	32.57	19466	1

Log-Likelihood: 138.1963
*y ppc is the posterior predictive distribution
*n eff is a crude measure of effective sample size
*Rhat is the potential scale reduction factor on split chains (at convergence, Rhat=1)

Table 4: Summary of Maximum Likelihood Estimation

	Estimate	standard error	t value	Pr(¿ t)
alpha	0.01121	0.02357	0.476	0.634
beta	1.17986	0.16441	7.176	7.16e-13 ***
lambda	0.09389	0.20022	0.469	0.639

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Log-Likelihood: 138.966

4.1 Discussion

The traceplots for parameters α, β, λ and the posterior distribution of the data, comparison made between the summary of the actual data and the summary of the data sampled from the posterior distribution, values of Rhat for all the samples, strongly indicate that convergence has been achieved.

Comparison based on table 5 shows that the Maximum Likelihood Estimates (MLEs) really maximize the likelihood though with a small margin of 0.7697 compared to the Bayesian estimates. This is not surprising since method of Maximum Likelihood Estimation is hinged on the concept of maximization of the likelihood. On the other hand, the Bayesian estimates give smaller standard error compared to the MLEs. This implies that the Bayesian method give more stable estimates than Maximum Likelihood. Surprisingly, though the MLEs maximize the likelihood more than the Bayesian estimates, it results in producing two non-significant parameters out of the three.

Similarly, comparison based on table 9 shows that the Maximum Likelihood Estimates (MLEs) really maximize the likelihood, again with a small margin of 0.2117 compared to the Bayesian estimates. This is not surprising since method of Maximum Likelihood Estimation is hinged on the concept of maximization of the likelihood. On the other hand, the Bayesian estimates give smaller standard error compared to the MLEs. This implies that the Bayesian method give more stable estimates than Maximum Likelihood. In this case however, in addition to the MLEs maximizing the likelihood more than the Bayesian estimates, all parameters were significant.

Table 5: Summary of Model Fitting for Remission times

		Maximum Likelihood Estimation (MLE)	Bayesian
a	Estimates	0.01121	0.04
	Standard Error	0.02342	0.00
	95% Confidence/Credible Interval	(-0.0347,0.0571)	(0.01,0.08)
β	Estimates	1.17986	1.14
	Standard Error	0.16440	0.00
	95% Confidence/Credible Interval	(0.8576,1.5021)	(0.88,1.44)
λ	Estimates	0.09388	0.33
	Standard Error	0.19898	0.00
	95% Confidence/Credible Interval	(-0.2961,0.4839)	(0.10,0.67)
Log-likelihood		138.966	138.1963

Table 6: Summary of COVID-19 Survey in Andalusia, Spain

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.0747	1.3953	3.0868	4.2847	5.5721	22.5174

4.2 Conclusions

In this research, we used the Maximum Likelihood and Bayesian Estimation methods to estimate the parameters of the Odd Generalized Exponentiated Inverse Exponential Distribution using two datasets i.e. Remission times of cancer patients and COVID-19 Survey in Andalusia, Spain.

Using the Remission times (in months) of Bladder cancer patients data with n = 128, Maximum Likelihood Estimates maximized the likelihood more than the Bayesian estimates even though two (2) out of the three (3) parameter estimates based on Maximum Likelihood estimation method were insignificant. This suggests that even if Maximum Likelihood Estimation results to some insignificant parameters in the course of model fitting, the likelihood can still be maximized. Consequently, the Bayesian estimates had very negligible standard errors implying that Bayesian method produced more stable estimates.

Table 7: Summary of the Bayesian Estimation using Stan for COVID-19 Survey in Andalusia, Spain

	mean	standard error	standard deviation	2.5%	50%	97.5%	n eff	Rhat
alpha	0.06	0.00	0.01	0.03	0.06	0.09	3112	1
beta	0.85	0.00	0.06	0.73	0.85	0.98	3226	1
lambda	0.27	0.00	0.07	0.14	0.27	0.43	2924	1
y ppc	4.29	0.04	4.47	0.16	2.82	16.31	11576	1

Log-Likelihood: 704.6799
*y ppc is the posterior predictive distribution
*n eff is a crude measure of effective sample size
*Rhat is the potential scale reduction factor on split chains (at convergence, Rhat=1)

Table 8: Summary of Maximum Likelihood Estimation for COVID-19 Survey in Andalusia, Spain

	Estimate	standard error	t value	Pr(¿ t)
alpha	0.0534	0.0157	3.415	0.000638 ***
beta	0.8557	0.0667	12.824	¡ 2e-16 ***
lambda	0.2466	0.0772	3.195	0.001399 **

Signif. codes: 0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
Log-Likelihood: 704.8916

Table 9: Summary of Model Fitting for COVID-19 Survey in Andalusia, Spain

		Maximum Likelihood Estimation (MLE)	Bayesian
α	Estimates	0.0535	0.06
	Standard Error	0.0157	0.00
	95% Confidence/Credible Interval	(0.0228, 0.0841)	(0.03,0.09)
β	Estimates	0.8557	0.85
	Standard Error	0.0667	0.00
	95% Confidence/Credible Interval	(0.7249 ,0.9865)	(0.73,0.98)
λ	Estimates	0.2466	0.27
	Standard Error	0.0772	0.00
	95% Confidence/Credible Interval	(0.09532 ,0.3979)	(0.14,0.43)
Log-likelihood		704.8916	704.6799

Where the COVID-19 Survey in Andalusia data with n = 493 was used, Maximum Likelihood Estimates still maximized the likelihood more than the Bayesian estimates with all three (3) parameter estimates significant. However, the Bayesian estimates still had very negligible standard errors implying that Bayesian method produced more stable estimates even with increased sample size. This is to say, the change in sample size has not affected the trend of results.

In a nutshell, while Maximum Likelihood Estimation gives the maximum likelihood of the data and consequently small values of information criteria, Bayesian estimation ensures stability of estimates i.e. estimates with very negligible standard errors.

5 Recommendation

Based on the foregoing results as well as discussions, the following recommendations are considered valuable when using the Odd Generalized Expenontiated - Inverse Exponential Distribution to fit a given data:

i. Where the desire or purpose of research is to produce estimates that maximize the likelihood of the data, Maximum Likelihood Estimation method is to be preferred to the Bayesian method.

ii. However, if on the other hand, the purpose is hinged on stable parameters, the Bayesian estimates have proven to perform better than the Maximum Likelihood estimates and as such, should be given preference.

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