Stochastic Analysis of the impact of Growth-Rates on Stock Market Prices

Abstract

A stochastic analysis of stock market expected returns and Growth-rates for investors are considered. The precise conditions for obtaining the drifts, volatilities and Growth-rates of four different stocks were also considered herein. From the stochastic analysis of the model; systems of non-linear stochastic differential equations were developed by means of covariance matrix solution on the stochastic part of the expected returns of investors while the deterministic part a function of the drift parameter as a mean column vector. We imposed a condition that multiplied drift parameter by one and solving simultaneously to obtain future stock prices.

Keywords: Stock market price, drift, volatility, Growth-rates, SDE and Stochastic analysis.

1 Introduction

Mathematical models grow out of equations of which stock price models cannot be left behind due to its numerous applications in our daily trading activities. Such applications include quantitative finance, Accountancy, Banking and finance etc. In many fields of science and engineering the accurate analysis, design and assessment of system subjected to realistic environment effects must take into account of the potential of "white noise" random forces that would affect the system or error measurements in the system. Randomness is intrinsic to the mathematical formulation of many phenomena, such as, fluctuations in the stock market, noise in population systems, communication networks or irregular fluctuation in observed signals .

The unstable nature of stock prices has kept our nation on economic crisis; investors, policy makers, families and federal Government do not predict the future due to uncertainty involve in stock trading. Unstable stock prices puts fear in lives of the people which results to lots of criminal activities in order to meet up individual demands, and results to panic buying. Even financial analysts who invest in stock market are usually not conscious of the stock market behavior. They also went through this problem of stock trading; without knowing which stocks to buy and sell in order to maximize profits. Both financial analysts and potential investors need regular information in forecasting the behavior of stock prices in Nigeria for the good of our country.

The unstable property and other considerable factors such as liquidity on stock return, since the sudden change in share prices occur randomly and frequently. Researchers are kin to study the behavior of the unstable market variable so as to enable investors and owners of cooperation make decisions on the level of their investment in stock market exchange,[1].

Nevertheless, the price evolution of a risky assets are usually modeled as the trajectory of a risky assets that are usually of a diffusion process defined on some underlying probability space ,with the geometric Brownian motion the paramount tool used as the established reference model, [7].

A lot of scholars have Modeled Stock market prices with different approaches and results obtained differently. For instance, [2] considered the unstable nature of stock market forces using proposed differential equation model. In the work of [6] studied stability analysis of stochastic model of price change at the floor of a stock market. In their research précised conditions are obtained which determines the equilibrium price and growth rate of stock shares.

[1] Considered stochastic analysis of the behavior of stock prices. Results reveal that the proposed model is efficient for the prediction of stock prices. In the same vain, [5] studied the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in their research the drift and volatility coefficients for the stochastic differential equations were determined and the Euler-Maruyama method for system of SDE'S was used to stimulate the stock prices.[3],built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data.

[12] Worked on stochastic modeling of stock prices; applied a method of Brownian motion model to explain the stock price time series. The result showed that as long as a model based upon the white noise is fitted to the market values, the two interpretations will provide different estimates of the parameters, but identical values concerning the predicted stock prices.

However [7] worked on stochastic model of the fluctuation of stock market price is considered. Here conditions for determining the equilibrium price, sufficient conditions for dynamic stability and convergence to equilibrium of the growth rate of the value function of shares. On the other hand, [9] considered a stochastic model of price changes at the floor of stock market. In their research the equilibrium price and the market growth rate of shares were determined. See [8] for considerable extensions and constrains subsequently in this particular area of study.

The aim of this paper is first, to present the unstable nature of stock market prices which aimed at determining the drifts, volatilities and Growth-rates of asset returns. From the stochastic analysis of the model; we considered the two parameters of the model and imposed a condition which the stock drift parameter (dt) was multiplied by one; this way, an accurate analytical solution of future stock prices were obtained. To the best of our knowledge this novel contribution have not seen particularly in financial mathematics as this will widen the area of applicability of problem of this nature.

This paper is arranged as follows: Section 2 presents mathematical modeling of asset returns, the formulation of the is seen in Subsection 2.1, Subsection 2.2 is Estimation of parameters, and the data analysis and graphical presentations are seen in Section 3, The paper is concluded in Section 4.

2 Mathematical modeling of Asset Returns

Let $S(t)$ be the price of some risky asset at time t ,and μ ,an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock price follows a random walk which is governed by a stochastic differential equation.

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \tag{1}$$

Where, μ is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp)$, ξ is a $\sigma -$algebra generated by $W_t, t \geq 0$.

Definition 1.1: A standard Brownian Motion is simply a stochastic process $\{B_t\}_{t\in\tau}$ with the following properties:

i) With probability 1, $B_0 = 0$.

ii) For all $0 \leq t_1 \leq t_2 \leq \cdots \leq t_n$, the increments $B_{t2-}B_{t1}, B_{t3-}B_{t2}, B_{t2}, \cdots, B_{tn} - B_{tn-1}$ are independent.

iii) For $t \geq s \geq 0$, $B_t - B_s \sim N(0, t-s)$.

iv) With probability 1, the function $\to B_i$ is continuous.

Stock Price Modelling

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \mu, \mathrm{F}(\beta)$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta)$ processing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t) \tag{2}$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \square))$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW(t) \tag{3}$$

Using theorem 1.1 and equation (3) comfortably solves the SDE with a solution given below:

$$S(t) = S_0 \exp\left\{\sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}, \forall t \in [0,1]$$

Following the properties of standard Brownian motion process for $n \geq 1$ such that any sequence time has it thus: $0 \leq t_0 \leq t_1 \leq \cdots \leq t_n$, .Hence, we have Euler's method for discretization of the SDE as follows:

$$\ln S_t - \ln S_{t-1} = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma(W_t - W_{t-1}) \tag{4}$$

Definition 1.2: A random variables say $W_t - W_{t-1}$ are functions, σ and therefore independent; which has a standard normal distribution with zero mean and variance one respectively.

From (4) if we let $y = \ln S_t - \ln S_{t-1}$, $\varepsilon = W_t - W_{t-1}$ and $\Delta t = 1$ (4) becomes

$$y_t = \mu - \frac{1}{2}\sigma^2 + \sigma\varepsilon_t \tag{5}$$

Linking (4) and (5) gives:

$$\ln S_t = \ln S_{t-1} + \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon(\sqrt{dt}) \tag{6}$$

Divide both sides by $\ln$ gives

$$S_t = S_{t-1} e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon(\sqrt{dt})} \tag{7}$$

2.1 Formulation of the problem

Let S_1, S_2, S_3 and S_4 represents daily prices in naira of four selected stocks. Time t is counted for trading days in multiples of fundamental unit, say days. Also ,let an $N \times n$ data matrix associated with $S_0(1), S_0(2), S_0(3)$ *and* $S_0(4)$ be X_{it} ,We consider N stocks over n trading days; time horizon. For each of four X_i ,we define the vector D_{it} as follows:

$$D1_{it} = \frac{1}{N}\sum_{i=1}^{N}\left(D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}\right) \tag{8}$$

$$D2_{it} = \frac{1}{N}\sum_{i=1}^{N}\left(D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}\right) \tag{9}$$

$$D3_{it} = \frac{1}{N}\sum_{i=1}^{N}\left(D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}\right) \tag{10}$$

$$D4_{it} = \frac{1}{N}\sum_{i=1}^{N}\left(D_{i1}, D_{i2}, D_{i3}, \cdots, D_{in}\right) \tag{11}$$

Thus, from where further statistics are derived we also define Growth-rates as:

$$Gr_i = \frac{\sigma_i - \mu_i}{\sigma_i} \tag{12}$$

Following the method of [5], we define covariance matrix as:

$$B(\mathrm{t}, \mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3, \mathrm{S}_4) = \begin{pmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_4 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{pmatrix}^{\frac{1}{2}} \tag{13}$$

Where covariance matrix represents the volatility coefficient of the stochastic differential equation. It is sufficient to know that covariance matrix is a positive definite symmetric matrix with a positive definite square-root. While stock drift is represented as a column vector form which we considered the mean of each of the companies represented below:

$$\mu(t, S_1, S_2, S_3, S_4) = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} \tag{14}$$

Combining (9) and (10) gives

$$dS(t) = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} dt + \begin{pmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_3 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{pmatrix}^{\frac{1}{2}} dW(t) \tag{15}$$

This invariably gives us system of stochastic differential equations.

$$dS(t) = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} dt + \begin{pmatrix} dS_1 & dS_1S_2 & dS_1S_3 & dS_1S_4 \\ dS_2S_1 & dS_2 & dS_2S_3 & dS_2S_3 \\ dS_3S_1 & dS_3S_2 & dS_3 & dS_3S_4 \\ dS_4S_1 & dS_4S_2 & dS_4S_3 & dS_4 \end{pmatrix} dW(t) \tag{16}$$

2.2 Estimation of parameters

Using data provided in [5] we estimates the following: volatility (σ) and the drift (μ) of the stock price for the four selected company such as S1-INTERBREW,S2-AP,S3-ASHAKACEM and S4-STANBIC. The daily stock prices is made of 60 observations where we partitioned each stock to be four (4) compartments which gave fifteen (15) Tables.

The formula for the volatility and drift of the stock price is defined as follows:

2.2.1 the volatility, σ : let S_i represent each of the initial stock prices at the end of $i-th$ trading period, $T = t, -t_{i-1}, i \geq 1.$ we define it as the logarithm of the daily return on each of four compartments of stock prices such that:

$$\mu_i = \ln\left[\frac{S_i}{S_{i-1}}\right] \tag{16}$$

The mean of each of stock prices are given as

$$\bar{u} = \frac{1}{N}\sum_{i=1}^{n} u_i \qquad (17)$$

The standard deviation is given as

$$v = \frac{\sqrt{\sum | u_i - \bar{u} |^2}}{n-1} \qquad (18)$$

The volatility of the daily stock return:

$$\sigma = \frac{v}{\sqrt{\tau}} \qquad (19)$$

2.2.2 the Drift Parameter, μ : this simply means expected annual rate of return which is given as

$$\mu = \frac{\bar{u}}{\tau} + \frac{1}{2}\sigma^2 \qquad (20)$$

3 Data Analysis and Result of relevant parameters

To demonstrate the unstable nature of stock market prices, we use daily prices in naira of four (4) selected stocks for sixty (60) trading days from Nigeria stock Exchange (NSE) extracted from [5].The stock prices were partitioned into four(4) compartments each to having a total of fifteen(15) compartments in sixty(60) trading days , see appendix 1.

The initial stock prices were taken from each of the respective compartments to give a total of fifteen (15) observations for different stocks (ie $S_0(1),\cdots,S_0(4)$). The volatility and drift coefficients were obtained from each of these four (4) different stocks (ie S_i 's, $\text{i} = 1,2,3,4$ using equations (16)-(20). The trading days were taken to be $\sqrt{60}$ .This computations were made using the stochastic part which is the function of stock volatility and stock drift as column vector while the Growth rates were obtained using (12) see Table 1 and 2 respectively.

Table 1: The Values of initial stock prices, volatility, Drift and Growth-rates for stock (1) and stock (2)

$S_0(1)$	Volatility (σ_1)	Drift (μ_1)	Growth-rates (Gr_1)	$S_0(2)$	Volatility (σ_2)	Drift (μ_2)	Growth-rates (Gr_2)
20.71	0.3986	0.07983	0.7997	22.52	0.0000	0.00036	0.0000
21.66	0.4895	0.12017	0.7545	22.52	0.0000	0.00036	0.0000
18.90	0.9177	0.42149	0.5407	22.52	0.0000	0.00036	0.0000
21.49	0.0000	0.00038	0.0000	21.49	0.0000	0.00036	0.0000

21.49	0.2591	0.03394	0.8690	22.52	0.0000	0.00036	0.0000
20.00	0.0000	0.00041	0.0000	20.00	0.2425	0.02979	0.8772
20.00	0.0000	0.00041	0.0000	21.00	0.2425	0.02940	0.8788
20.00	0.0000	0.00041	0.0000	20.38	0.155	0.01241	0.9199
20.00	0.3569	0.06409	0.8204	20.10	0.03465	0.000998	0.9712
20.01	0.0025	0.00041	0.836	20.02	0.0000	0.000406	0.0000
19.50	0.1184	0.00743	0.9372	21.35	0.2125	0.02296	0.8919
19.07	0.2734	0.03779	0.8618	20.50	0.0000	0.000396	0.0000
19.52	0.1301	0.00889	0.9317	20.50	0.2454	0.03050	0.8757
19.00	0.125	0.00824	0.9341	21.35	0.0000	0.000380	0.0000
19.50	0.0000	0.00042	0.0000	21.35	0.0000	0.000380	0.0000

Table 2: The Values of initial stock prices, volatility, Drift and Growth-rates for stock (3) and stock (4)

$S_0(3)$	Volatility (σ_3)	Drift (μ_3)	Growth-rates (Gr_3)	$S_0(4)$	Volatility (σ_4)	Drift (μ_4)	Growth-rates (Gr_4)
21.02	0.3501	0.061689	0.8238	21.01	0.1275	0.03289	0.7420
19.32	0.52	0.1356	0.7392	20.00	0.12	0.02921	0.7566
21.40	0.3882	0.07574	0.8049	18.50	0.6336	0.8032	-0.2677
19.95	0.0000	0.000407	0.0000	19.90	0.6062	0.7353	-0.2130
20.57	0.0000	0.000395	0.0000	20.00	0.4295	0.3693	0.1402
21.50	0.0000	0.00038	0.0000	19.90	0.0000	0.0004	0.0000
21.00	0.0000	0.00038	0.0000	19.00	0.0000	0.0004	0.0000
20.47	0.1325	0.00092	0.9931	19.90	0.2250	0.1017	0.548
21.50	0.2974	0.04461	0.85	19.90	0.0000	0.0004	0.0000
21.57	0.0175	0.00053	0.96971	19.90	0.0289	0.000207	0.9927
22.60	0.275	0.03822	0.8610	15.03	0.4660	0.4348	0.0670
22.00	0.125	0.00819	0.9345	17.00	0.2246	0.1014	0.5485
21.50	0.2264	0.02601	0.8851	19.90	1.4001	3.9207	-1.8003
22.15	0.0000	0.00037	0.0000	15.05	0.0000	0.0005	0.0000
21.10	0.3559	0.06370	0.8210	15.05	0.0000	0.0005	0.0000

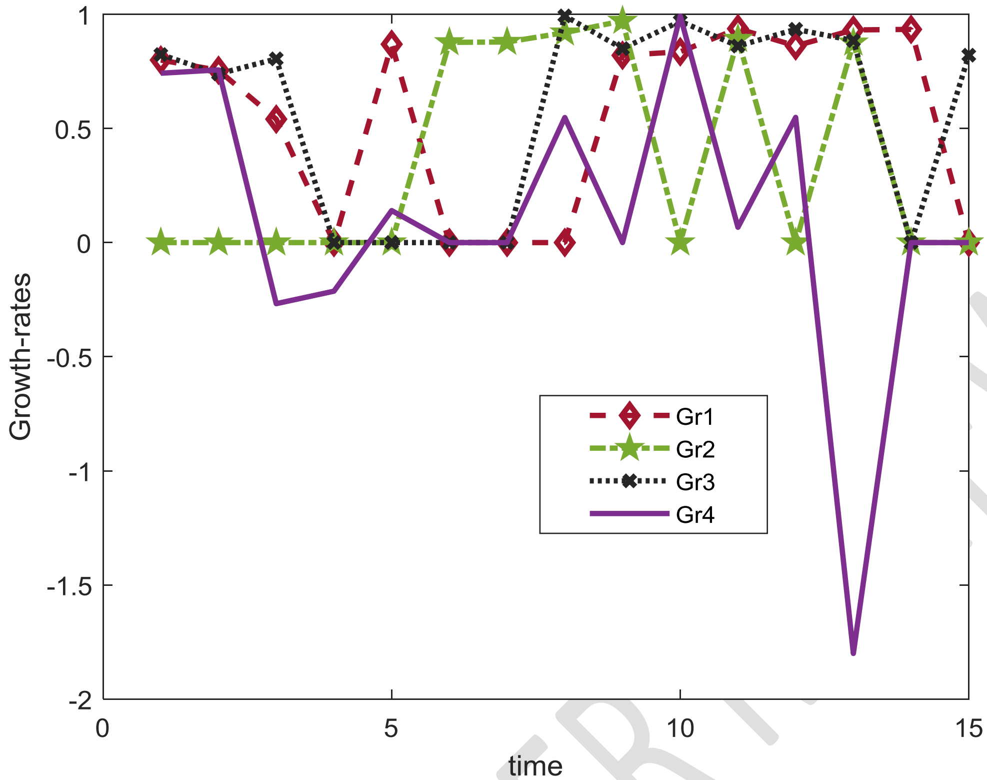

Figure 1: Intrinsic Growth-rates 0f four Securities

In the Figure 1, above shows stochastic nature of stock prices in respect to growth rates, they are correlated. This is highly reliable because; it describes the stock volatility as time dependent variable which is characterized by random features. There were lots of crashes in Growth-rate (4) this lead to a severe depletion of securities which are indexed by per thousands of naira leading to financial liquation.

In Table 3, Section 2.1 shows that stock (1) gives the best investment returns. This It implies that investors can eventually invest on Stock (1) in order to maximize profit and minimize loss; which is the expectation of every investor. This observation leads to a beneficial profit margin throughout the sixty trading days.

Using equation (14) gives a system of stochastic differential equation

$$dS(t) = \begin{pmatrix} 0.7843 \\ 0.1294 \\ 0.4654 \\ 6.5328 \end{pmatrix} dt + \begin{pmatrix} 0.2486 & 0.1122 & 0.1503 & 0.0624 \\ 0.1122 & 0.1039 & 0.0616 & 0.0849 \\ 0.1503 & 0.0616 & 0.0894 & 0.1323 \\ 0.0624 & 0.0849 & 0.1323 & 0.37 \end{pmatrix} dW(\mathrm{t})$$

This gives systems of non-linear stochastic differential equation below:

$dS_1 = dS_2 = dS_3 = dS_4 = 0 \;\; and\; dt = 1$

$$
\begin{aligned}
0.2486dW_1 + 0.1122dW_2 + 0.1503dW_3 + 0.0624dW_4 &= -0.7843 \\
0.1122dW_1 + 0.1039dW_2 + 0.0616dW_3 + 0.0849dW_4 &= -0.1294 \\
0.1503dW_1 + 0.0616dW_2 + 0.0894dW_3 + 0.1323dW_4 &= -0.4654 \\
0.0624dW_1 + 0.0849dW_2 + 0.1323dW_3 + 0.37dW_4 &= -6.5328
\end{aligned}
$$

Putting the above in matrix form gives

$$
dS(\mathrm{t}) = \begin{pmatrix} 0.2486 & 0.1122 & 0.1503 & 0.0624 \\ 0.1122 & 0.1039 & 0.0616 & 0.0849 \\ 0.1503 & 0.0616 & 0.0894 & 0.1323 \\ 0.0624 & 0.0849 & 0.1323 & 0.37 \end{pmatrix} \begin{pmatrix} dW_1 \\ dW_2 \\ dW_3 \\ dW_4 \end{pmatrix} = \begin{pmatrix} -0.7843 \\ -0.1294 \\ -0.4654 \\ -6.5328 \end{pmatrix}
$$

Solving simultaneously gives the following stock prices

$dW_1 = 35.5069, \;\; dW_2 = -19.8008, \;\; dW_3 = -62.0535, \; dW_4 = -1.00175$

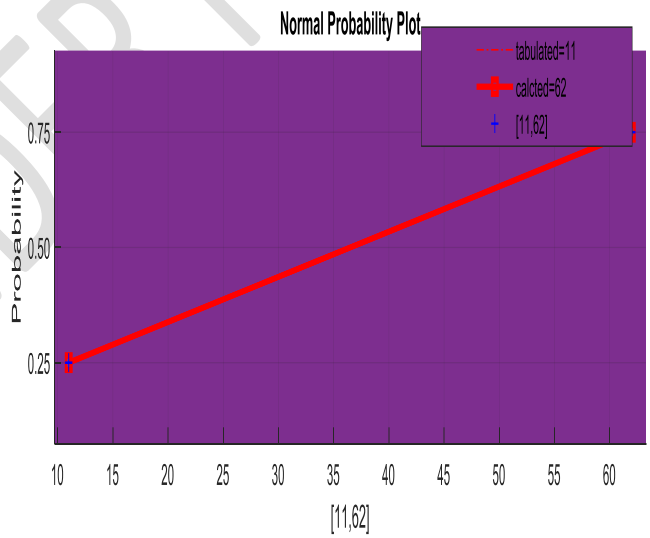

Figure 2: Graphical representation of normal distribution of stock prices

Figure 2 is just to attest for the adequacy of the propose model. The normal distribution is vital because it makes statistics a lot easier and more practicable in the life of every investor.

4. Conclusion

This paper considered the problem of stock market prices on expected returns for investors. The precise conditions for obtaining the drifts, volatilities and growth-rates of four different stocks were considered. From the stochastic analysis of the model; systems of non-linear stochastic differential equations were developed by means of covariance matrix on the stochastic part of the expected returns of investors while the deterministic part a function of the drift parameter as a column vector. We imposed a condition that multiplied the drift parameter (dt) by one and solving simultaneously we obtained future stock prices and stock prices follows a normal distribution. From the estimated growth-rates; there was severe depletion of securities which are indexed by per thousands of naira leading to financial liquation.

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Appendix 1

Table 1

S_1	20.71	21.60	20.63	19.65
S_2	22.52	22.52	22.52	22.52
S_3	21.02	20.02	20.00	19.32
S_4	21.01	21.52	21.52	21.52

Table 2

S_1	21.66	22.80	22.87	20.83
S_2	22.52	22.52	22.52	22.52
S_3	19.32	21.40	21.40	21.40
S_4	20.00	20.00	20.00	19.52

Table 3

S_1	18.70	18.00	21.49	21.49
S_2	22.52	22.52	22.52	22.52
S_3	21.40	20.39	20.47	19.50
S_4	18.50	17.27	19.90	19.90

Table 4

S_1	21.49	21.49	21.49	21.49
S_2	22.52	22.52	22.52	22.52
S_3	19.95	19.95	19.95	19.95
S_4	19.90	19.90	22.00	22.00

Table 5

S_1	21.49	21.70	20.71	20.71
S_2	22.52	22.52	22.52	22.52
S_3	20.57	20.57	20.57	20.57
S_4	20.00	19.20	19.20	21.01

Table 6

S_1	20.00	20.00	20.00	20.00
S_2	20.02	20.99	20.99	20.99
S_3	21.50	21.50	21.50	21.00
S_4	19.90	19.90	19.90	19.90

Table 7

S_1	20.00	20.00	20.00	20.00
S_2	20.02	20.99	20.99	20.99
S_3	21.50	21.50	21.50	21.00
S_4	19.90	19.90	19.90	19.90

Table 8

S_1	20.00	20.00	20.00	20.00
S_2	21.00	20.38	20.38	20.38
S_3	21.00	20.47	20.47	20.47
S_4	19.00	19.90	19.90	19.90

Table 9

S_1	20.00	20.42	19.01	19.01
S_2	20.38	20.38	20.50	20.50
S_3	20.47	20.47	21.50	21.50
S_4	19.90	19.90	19.90	19.90

Table 10

S_1	20.01	20.01	20.01	20.00
S_2	20.02	20.02	20.02	20.02
S_3	21.57	21.57	21.57	21.50
S_4	19.90	19.90	20.00	20.00

Table 11

S_1	19.50	19.01	19.01	19.07
S_2	21.35	21.35	21.35	20.50
S_3	22.60	22.60	22.60	21.50
S_4	15.03	16.50	17.00	17.00

Table 12

S_1	19.07	19.8	19.44	18.52
S_2	20.50	20.5	20.5	20.5
S_3	22.00	21.5	21.5	21.5
S_4	17.00	17	17.57	17.91

Table 13

S_1	19.52	19.01	18.99	19
S_2	20.5	20.5	21.35	21.35
S_3	21.50	21.50	21.05	22.15
S_4	19.90	19.90	15.05	15.05

Table 14

S_1	19	19	19	19.5
S_2	21.35	21.35	21.35	21.35
S_3	22.15	22.15	22.15	22.15
S_4	15.05	15.05	15.05	15.05

Table 15

S_1	19.5	19.5	19.5	19.5
S_2	21.35	21.35	21.35	21.35
S_3	21.1	21.5	22.5	22.5
S_4	15.05	15.05	15.05	15.05

Appendix 2: Matlab codes for solving systems of non linear SDE

```
>> eqn1=0.2486*dw1+0.1122*dw2+0.1503*dw3+0.0624*dw4==0.7843;

>> eqn2=0.1122*dw1+0.1039*dw2+0.0616*dw3+0.0849*dw4==0.1294;

>> eqn3=0.1503*dw1+0.0616*dw2+0.0894*dw3+0.1323*dw4==0.4654;

>> eqn4=0.0624*dw1+0.0849*dw2+0.1323*dw3+0.37*dw4==6.5328;

>> [A,B]=equationsToMatrix([eqn1,eqn2,eqn3,eqn4],[dw1,dw2,dw3,dw4]);

>> x=linsolve(A,B)
```