UNDER PEER REVIEW

Original Research Article

Rogue waves of the Lakshmanan-Porsezian-Daniel equation depending on multi-parameters

Abstract

Quasi-rational solutions to the Lakshmanan Porsezian Daniel equation are presented. We construct explicit expressions of these solutions for the first orders depending on real parameters.
We study the patterns of these configurations in the (x, t) plane in function of different parameters.

Key Words : Lakshmanan Porsezian Daniel equation, quasi-rational solutions.

PACS numbers :
33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

1 Introduction

We consider the Lakshmanan Porsezian Daniel (LPD) equation in the following normalization

$$iu_t + u_{xxxx} + 8|u|^2u_{xx} + 2u^2\overline{u}_{xx} + 6u_x^2\overline{u} + 4u|u_x|^2 + 6|u|^4u = 0, \qquad (1)$$

where the subscripts mean the partial derivatives.
There are different models that describe the dynamics of soliton propagation through optical wave guides. One of the most important models is the nonlinear Lakshmanan Porsezian Daniel (LPD) equation. This model was introduced originally in the context of Heisenberg spin chain [1].
Among the different methods used to get the soliton solutions of the LPD equation, we can quote the following: the Riccati equation method has been applied

to get dark and bright soliton solutions of (1) in [2]; the sine-Gordon expansion method has been used to get analytical solutions of the LPD equation in [3]; we can also cite, the undetermined coefficients method [4], the semi-inverse variational principle [5], a multipliers method [6], the modified extended direct algebraic method [7] and expansion method [8].
More precisely, the LPD equation has been used in the polarization-preserving fibers context [9]; it has been applied to the study of the propagation of periodic ultrashort pulses in the optical fibers [10]

2 Quasi-rational solutions of order 1 to the Lakshmanan Porsezian Daniel equation

Theorem 2.1 *The function* $v(x,t)$ *defined by*

$$v(x,t) = \left(1 - 4\,\frac{1 + 24\,it}{1 + 4\,x^2 + 576\,t^2}\right) e^{6\,it} \tag{2}$$

is a solution to the Lakshmanan Porsezian Daniel equation (1)*.*

$$\tag{3}$$

Proof: We have to replace the expression of the solution given by (2) and check that (1) is verified.

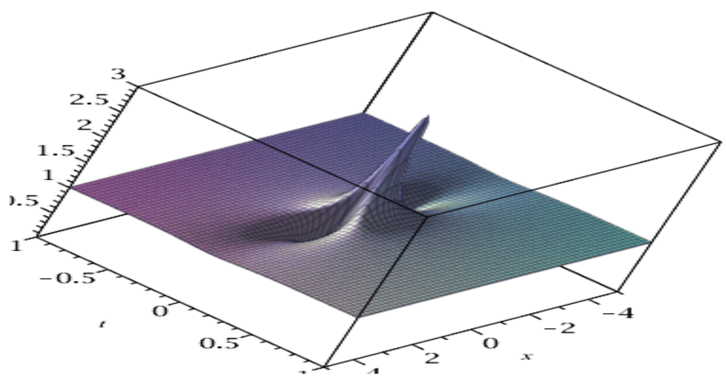

Figure 1. Solution of order 1 to the (1) equation.

3 Quasi-rational solutions of order 2 to the Lakshmanan Porsezian Daniel equation

Theorem 3.1 *The function $v(x,t)$ defined by*

$$v(x,t) = \left(1 - 12\frac{n(x,t)}{d(x,t)}\right)\exp(6it + 2a_1) \tag{4}$$

with
$n(x,t) = (2\,x - 12\,b_1)^4 + 6\,((4\,a_1 + 24\,t)^2 + 1)(2\,x - 12\,b_1)^2 - 192\,b_1(2\,x - 12\,b_1) + 5\,(4\,a_1 + 24\,t)^4 + 18\,(4\,a_1 + 24\,t)^2 + 384\,t(4\,a_1 + 24\,t) - 3 + i((4\,a_1 + 24\,t)(2\,x - 12\,b_1)^4 + 2\,((4\,a_1 + 24\,t)^3 - 12\,a_1 - 168\,t)(2\,x - 12\,b_1)^2 - 192\,(4\,a_1 + 24\,t)b_1(2\,x - 12\,b_1) + (4\,a_1 + 24\,t)^5 + 2\,(4\,a_1 + 24\,t)^3 + 192\,t(4\,a_1 + 24\,t)^2 - 60\,a_1 - 552\,t)$
and
$d(x,t) = ((2\,x - 12\,b_1)^2 + (4\,a_1 + 24\,t)^2 + 1)^3 + 192\,b_1(2\,x - 12\,b_1)^3 - 24\,((4\,a_1 + 24\,t)^2 + 48\,t(4\,a_1 + 24\,t) - 1)(2\,x - 12\,b_1)^2 - 576\,((4\,a_1 + 24\,t)^2 + 1)b_1(2\,x - 12\,b_1) + 24\,(4\,a_1 + 24\,t)^4 + 384\,t(4\,a_1 + 24\,t)^3 + 96\,(4\,a_1 + 24\,t)^2 + 3456\,t(4\,a_1 + 24\,t) + 36864\,t^2 + 9216\,{b_1}^2 + 8$
is a solution to the Lakshmanan Porsezian Daniel equation (1).

Proof: It is sufficient to replace the expression of the solution given by (4) and check that the relation (1) is verified.

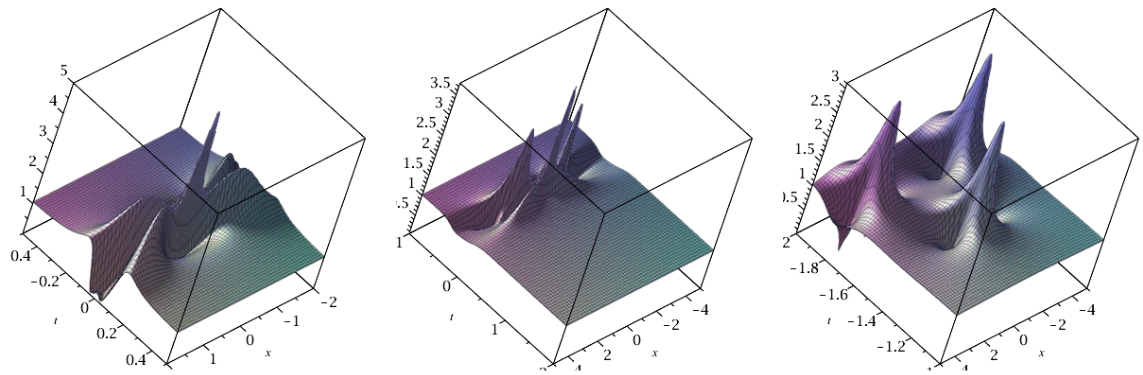

Figure 2. Solution of order 1 to (1); to the left $a_1 = 0$, $b_1 = 0$; in the center $a_1 = 1$, $b_1 = 0$; to the right $a_1 = 10$, $b_1 = 0$.

A very fast evolution of the structure of the solution is observed when the parameter a_1 grows; it evolves toward three peaks.

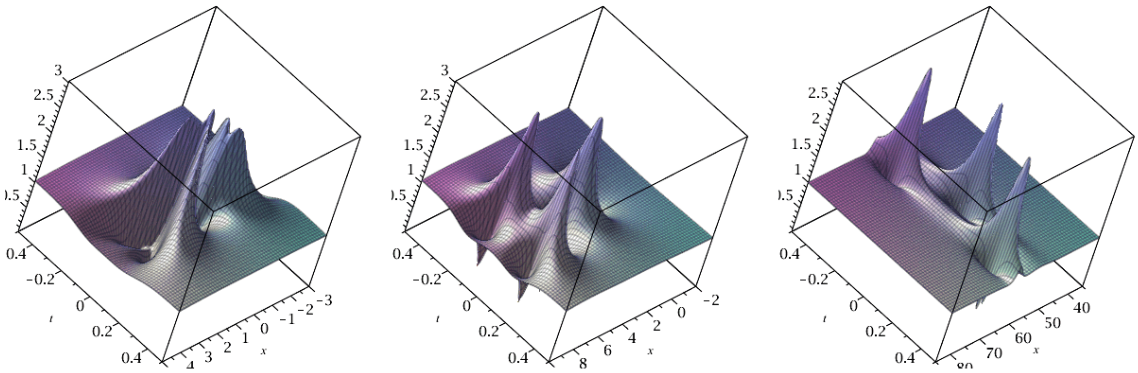

Figure 3. Solution of order 1 to (1); to the left $a_1 = 0$, $b_1 = 0.1$; in the center $a_1 = 0$, $b_1 = 1$; to the right $a_1 = 0$, $b_1 = 10$.

As in the case of the parameter a_1, an evolution of the structure of the solution toward three peaks is observed when the parameter b_1 grows.

4 Quasi-rational solutions of order 3 to the Lakshmanan Porsezian Daniel equation

Theorem 4.1 *The function $v(x,t)$ defined by*

$$v(x,t) = \left(1 - 24\frac{n(x,t)}{d(x,t)}\right) \mathrm{e}^{6\,i\alpha\,t} \qquad (5)$$

with

$n(x,t) = 1024\,x^{10} + 3840\,(1+576\,t^2)x^8 + 64\,(210 - 218880\,t^2 + 16588800\,t^4)x^6 + 675 + 16\,(-1584000\,t^2 - 315187200\,t^4 + 13377208320\,t^6 - 450)x^4 + 4\,(5424537600\,t^4 + 80263249920\,t^6 + 4953389137920\,t^8 - 1555200\,t^2(5-3072\,t^2) - 675 - 19353600\,t^2)x^2 + 1977915801600\,t^6 + 107323431321600\,t^8 + 697437190619136\,t^{10} + 149299200\,t^4(-17 + 3072\,t^2) + 2388787200\,t^4 + 259200\,t(16\,t - 16384\,t^3) + 388800\,t^2(-3 + 4096\,t^2) + 23500800\,t^2 + i(24576\,tx^{10} + 256\,(-840\,t + 69120\,t^3)x^8 + 64\,(240\,t - 4147200\,t^3 + 79626240\,t^5)x^6 + 16\,(-22809600\,t^3 - 3264675840\,t^5 + 45864714240\,t^7 + 10800\,t(-3 - 1024\,t^2) + 14400\,t)x^4 + 4\,(51836682240\,t^5 - 275188285440\,t^7 + 13209037701120\,t^9 - 12441600\,t^3(7 - 3072\,t^2) + 16200\,t(7 + 4096\,t^2) + 345600\,t + 88473600\,t^3)x^2 + 716636160\,t^5(-107 + 3072\,t^2) + 3110400\,t^2(-176\,t - 16384\,t^3) - 3110400\,t^3(11 + 20480\,t^2) - 16200\,t(-7 + 22528\,t^2) - 265420800\,t^3 - 138549657600\,t^5 - 2155641569280\,t^7 + 224553640919040\,t^9 + 1521681143169024\,t^{11} + 151200\,t)$

and

$d(x,t) = 2024 + 256\,(-207360\,t^2 + 120)x^8 + 64\,(3778560\,t^2 - 291962880\,t^4 + 2320)x^6 + 16\,(-5255331840\,t^4 - 91729428480\,t^6 + 138240\,t^2(56 - 11520\,t^2) - 26265600\,t^2 + 3360)x^4 + 4\,(5870683422720\,t^6 + 26418075402240\,t^8 - 79626240\,t^4(-326 - 34560\,t^2) + 276480\,t^2(-76 + 311040\,t^2) + 57330892800\,t^4 - 1555200\,t(96\,t - 16384\,t^3) +$

$12144 + 8294400\,t^2)x^2 + 15288238080\,t^6(191 + 6912\,t^2) + 298598400\,t^3(368\,t - 16384\,t^3) + 79626240\,t^4(599 - 57600\,t^2) - 388800\,t(-496\,t - 65536\,t^3) + 13824\,t^2(3881 + 7257600\,t^2) + (1 + 4\,x^2 + 576\,t^2)^6 + 223948800\,t^2 + 33973862400\,t^4 + 4127824281600\,t^6 + 937841676779520\,t^8 + 12680676193075200\,t^{10}$
is a solution to the (LPD) equation (1).

Proof: We check that the relation (1) is verified when we replace the expression of the solution given by (6).

We can give also the solutions to the Lakshmanan Porsezian Daniel equation depending on 4 real parameters. But, because of the length of the expression, we only give it in the appendix.
We give patterns of the modulus of the solutions in function of the parameters a_1, a_2, b_1, b_2.

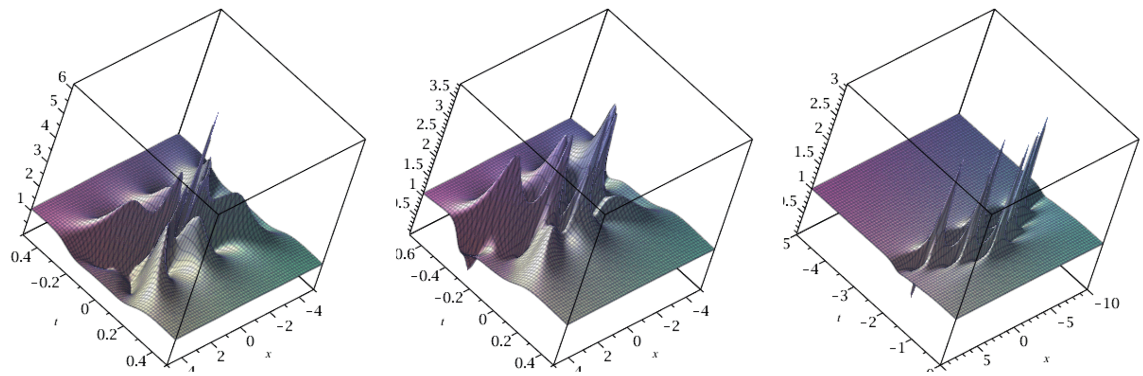

Figure 4. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$; in the center $a_1 = 1$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$; to the right $a_1 = 10$, $b_1 = 0$, $a_2 = 10$, $b_2 = 0$; with $\alpha = \beta = 1$.

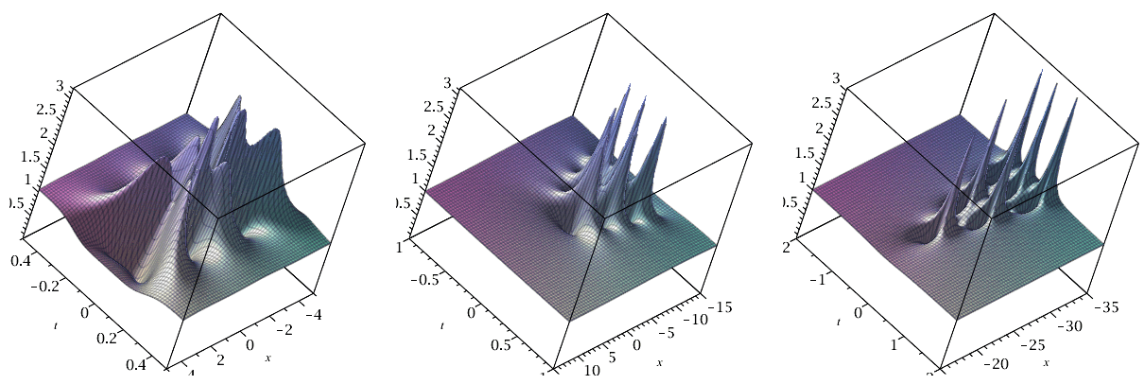

Figure 5. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0.1$, $a_2 = 0$, $b_2 = 0$; in the center $a_1 = 0$, $b_1 = 1$, $a_2 = 0$, $b_2 = 0$; to the right $a_1 = 0$, $b_1 = 5$, $a_2 = 0$, $b_2 = 0$; with $\alpha = \beta = 1$.

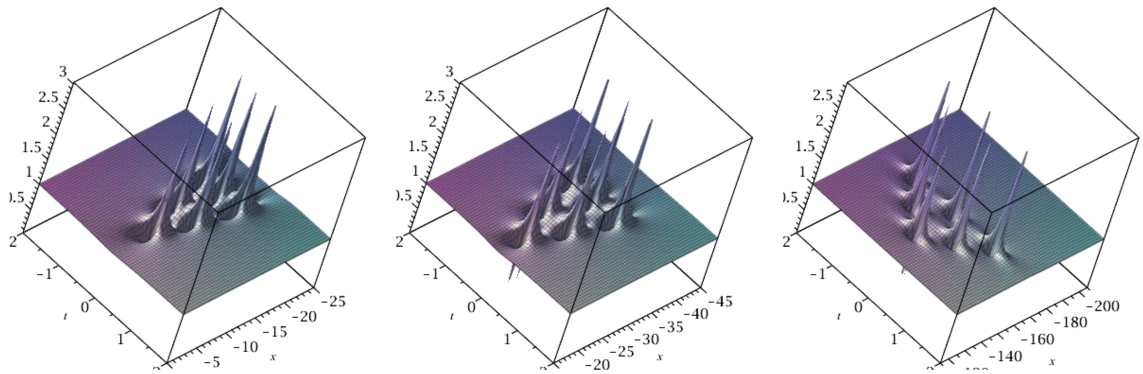

Figure 6. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0.5$, $b_2 = 0$; in the center $a_1 = 0$, $b_1 = 0$, $a_2 = 1$, $b_2 = 0$; to the right $a_1 = 2$, $b_1 = 0$, $a_2 = 5$, $b_2 = 0$; with $\alpha = \beta = 1$.

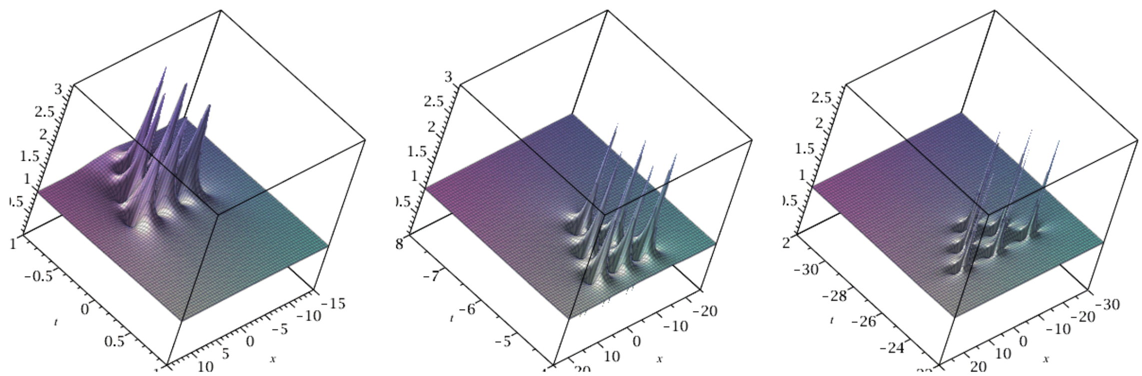

Figure 7. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0.1$; in the center $a_1 = 0$, $b_1 = 10$, $a_2 = 0$, $b_2 = 1$; to the right $a_1 = 10$, $b_1 = 0$, $a_2 = 0$, $b_2 = 5$; with $\alpha = \beta = 1$.

In the evolution of the structure of the solutions according to the different parameters, we note the appearance of triangles with six peaks with more at least speed according to the parameters.

5 Conclusion

Quasi-rational solutions to the Lakshmanan Porsezian Daniel equation have been given for the first orders depending on several real parameters.
In the case of order 2, the evolution of the structure of the solutions toward three peaks is observed when parameters a_1 or b_1 grow.
In the same way, in the case of order 3, the evolution of the structure of the solutions toward six peaks is observed when parameters a_1, a_2, b_1, b_2 grow.
These solutions have to be compared with these constructed by the author in

the case of the NLS or mKdV equations [11, 12, 13, 14, 15].
We can cite some recent works about this equation. In [16], new travelling wave solutions to the (LPD) equation with Kerr nonlinearity are constructed. In [17], bright, dark, singular, kink and periodic optical solitons solutions of the LPD equation are constructed. In [18], three images of nonlinearity to the fractional LPD equation in birefringent fibers are investigated. In [19], New analytical solutions to the LPD equation is presented by Jacobi elliptic functions.

References

[1] M. Lakshmanan, K. Porsezian, M. Daniel, Effect of discreteness on the continuum limit of the Heisenberg spin chain, Phys. Lett. A, V. 133, N. 9, 483488, 1998

[2] A.A. AlQarni et al., Optical solitons for Lakshmanan Porsezian Daniel model by Riccati equation approach, Optik, V. 182, 922-929, 2019

[3] H. Rezazadeh et al., Optical solitons of Lakshmanan Porsezian Daniel model with a couple of nonlinearities, Optik, V. 164, 414-423, 2018

[4] J. Vega-Guzman et al., Optical solitons for Lakshmanan Porsezian Daniel model with spatiotemporal dispersion using the method of undetermined coefficients, Optik, V. 144, 115-123, 2017

[5] R.T. Alqahtani, M.M. Babatin, A. Biswas, Bright optical solitons for Lakshmanan- Porsezian-Daniel model by semi-inverse variational principle, Optik, V. 154, 109-114, 2018

[6] A. Biswas et al., Conservation laws for optical solitons of Lakshmanan-Porsezian-Daniel model, Proc. Rom. Acad. Ser. A Math. Phys. Tech. Sci. Inf. Sci., V. 19, 39-44, 2018

[7] M.B. Hubert, et al., Optical solitons with Lakshmanan Porsezian Daniel model by modified extended direct algebraic method, Optik, V. 162, 228-236, 2018

[8] S. Arshed et al., Optical solitons in birefringent fibers for Lakshmanan Porsezian Daniel model using exp(-iϕ)-expansion method, Optik, V. 172, 651-656, 2018

[9] Y. Yildirim et al., Cubicquartic optical soliton perturbation with LakshmananPorsezianDaniel model by sine-Gordon equation approach, J. Opt., V. 50, N. 2, 322329, 2021

[10] Y.L. Ye et al., Rogue wave solutions of the vector LakshmananPorsezian-Daniel equation, Phys. Lett. A, V. 384, N. 11, 126226, 2020

[11] P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, J. Phys. A : Meth. Theor., V. 44, 435204-1-15, 2011

[12] P. Gaillard, Degenerate determinant representation of solution of the NLS equation, higher Peregrine breathers and multi-rogue waves, J. Math. Phys., V. 54, 013504-1-32, 2013

[13] P. Gaillard, Deformations of third order Peregrine breather solutions of the NLS equation with four parameters, Phys. Rev. E, V. 88, 042903-1-9, 2013

[14] P. Gaillard, The mKdV equation and multi-parameters rational solutions, Wave Motion, V. 100, 102667-1-9, 2021

[15] P. Gaillard, Rational solutions to the mKdV equation associated to particular polynomials, Wave Motion, V. 107, 102824-1-11, 2021

[16] H. Rezazadeh, D. Kumar, A. Neirameh, M. Eslami, M. Mirzazadeh, Applications of three methods for obtaining optical soliton solutions for the LPD model with Kerr law nonlinearity, Pramana J?Phys., V. 89, 1-11, 2020

[17] G. Akram, M. Sadaf, M. Dawood, D. Baleanu, Optical solitons for LakshmananPorsezianDaniel equation with Kerr law non-linearity using improved tan expansion technique, Res. In Phys., V. 29, 104758-1-13, 2021

[18] H.F. Ismael, H.M. Baskonius, H. Bulut, Abundant novel solutions of the conformable Lakshmanan-Porsezian-Daniel model, Disc. And Cont. Dun. Syst. Ser. S, V. 14, N. 7, 2311-2333, 2021

[19] H. Yepez-Martinez, H. Rezazadeh, M. Inc, M.Ali-Akinlar, J.F. Gomez-Aguilar, Analytical solutions to the fractional LakshmananPorsezianDaniel model, Opt. And Quant. Elec., V. 54, 1-41, 2021

Appendix

Solution of order 3 to the (LPD) equation depending on 4 real parameters :
The function $v(x,t)$ defined by

$$v(x,t) = \left(1 - 24\frac{n(x,t)}{d(x,t)}\right) \mathrm{e}^{i(2\,a_1+6\,t+20\,b_2)} \tag{6}$$

with

$n(x,t) = 675+(2\,x+12\,b_1+60\,a_2)^{10}+2190\,(4\,a_1+24\,t+120\,b_2)^6+27000\,(-8\,b_1-80\,a_2)^2+91800\,(-16\,t-160\,b_2)^2+495\,(4\,a_1+24\,t+120\,b_2)^8+11\,(4\,a_1+24\,t+120\,b_2)^{10}+88473600\,{a_2}^2+353894400\,{b_2}^2+15\,(1+(4\,a_1+24\,t+120\,b_2)^2)(2\,x+12\,b_1+60\,a_2)^8+(210-60\,(4\,a_1+24\,t+120\,b_2)^2+50\,(4\,a_1+24\,t+120\,b_2)^4+480\,(4\,a_1+24\,t+120\,b_2)(-16\,t-160\,b_2))(2\,x+12\,b_1+60\,a_2)^6+(-720\,(4\,a_1+24\,t+120\,b_2)^2(-8\,b_1-80\,a_2)+5760\,b_1-11520\,a_2)(2\,x+12\,b_1+60\,a_2)^5+(450\,(4\,a_1+24\,t+120\,b_2)^2-150\,(4\,a_1+24\,t+120\,b_2)^4+70\,(4\,a_1+24\,t+120\,b_2)^6+1200\,(4\,a_1+24\,t+$

$120\,b_2)^3(-16\,t-160\,b_2)-450+5400\,(-8\,b_1-80\,a_2)^2-1800\,(-16\,t-160\,b_2)^2+$
$3600\,(4\,a_1+24\,t+120\,b_2)(-16\,t-224\,b_2))(2\,x+12\,b_1+60\,a_2)^4+(-2400\,(4\,a_1+$
$24\,t+120\,b_2)^4(-8\,b_1-80\,a_2)+28800\,(4\,a_1+24\,t+120\,b_2)(-8\,b_1-80\,a_2)(-16\,t-$
$160\,b_2)-57600\,b_1-806400\,a_2-7200\,(4\,a_1+24\,t+120\,b_2)^2(-16\,b_1-128\,a_2))(2\,x+$
$12\,b_1+60\,a_2)^3+(6750\,(4\,a_1+24\,t+120\,b_2)^4+420\,(4\,a_1+24\,t+120\,b_2)^6+45\,(4\,a_1+$
$24\,t+120\,b_2)^8-2700\,(4\,a_1+24\,t+120\,b_2)^2(5+4\,(-8\,b_1-80\,a_2)^2-12\,(-16\,t-$
$160\,b_2)^2)-675-10800\,(-8\,b_1-80\,a_2)^2-10800\,(-16\,t-160\,b_2)^2+21600\,(4\,a_1+$
$24\,t+120\,b_2)(-32\,t-384\,b_2)-7200\,(4\,a_1+24\,t+120\,b_2)^3(-32\,t-128\,b_2))(2\,x+$
$12\,b_1+60\,a_2)^2+(-1680\,(4\,a_1+24\,t+120\,b_2)^6(-8\,b_1-80\,a_2)-28800\,(4\,a_1+$
$24\,t+120\,b_2)^3(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)-10800\,(4\,a_1+24\,t+120\,b_2)^2(-8\,b_1-$
$272\,a_2)-86400\,b_1-1209600\,a_2+43200\,(-8\,b_1-80\,a_2)^3+43200\,(-8\,b_1-80\,a_2)(-16\,t-$
$160\,b_2)^2+3600\,(4\,a_1+24\,t+120\,b_2)^4(-8\,b_1+80\,a_2)-86400\,(4\,a_1+24\,t+120\,b_2)((-8\,b_1-$
$80\,a_2)(-16\,t-160\,b_2)+32\,(-16\,t-160\,b_2)a_2-64\,(-8\,b_1-80\,a_2)b_2))(2\,x+12\,b_1+$
$60\,a_2)-720\,(4\,a_1+24\,t+120\,b_2)^7(-16\,t-160\,b_2)+450\,(4\,a_1+24\,t+120\,b_2)^4(-17+$
$28\,(-8\,b_1-80\,a_2)^2+12\,(-16\,t-160\,b_2)^2)-3600\,(4\,a_1+24\,t+120\,b_2)^3(-48\,t-$
$1376\,b_2)-720\,(4\,a_1+24\,t+120\,b_2)^5(-272\,t-3168\,b_2)+10800\,(4\,a_1+24\,t+$
$120\,b_2)(16\,t+224\,b_2+4\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+4\,(-16\,t-160\,b_2)^3)+$
$675\,(4\,a_1+24\,t+120\,b_2)^2(-3+16\,(-8\,b_1-80\,a_2)^2+16\,(-16\,t-160\,b_2)^2-$
$4096\,(-8\,b_1-80\,a_2)a_2-8192\,(-16\,t-160\,b_2)b_2)-2764800\,(-8\,b_1-80\,a_2)a_2-$
$11059200\,(-16\,t-160\,b_2)b_2+i(151200\,t-5529600\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)a_2+$
$64800\,(-16\,t-160\,b_2)^3-870\,(4\,a_1+24\,t+120\,b_2)^7+25\,(4\,a_1+24\,t+120\,b_2)^9+(4\,a_1+$
$24\,t+120\,b_2)^{11}-21600\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+(4\,a_1+24\,t+120\,b_2)(2\,x+$
$12\,b_1+60\,a_2)^{10}+(-60\,a_1-840\,t-6600\,b_2+5\,(4\,a_1+24\,t+120\,b_2)^3)(2\,x+12\,b_1+$
$60\,a_2)^8+(-600\,a_1+240\,t+58800\,b_2-140\,(4\,a_1+24\,t+120\,b_2)^3+10\,(4\,a_1+$
$24\,t+120\,b_2)^5+240\,(4\,a_1+24\,t+120\,b_2)^2(-16\,t-160\,b_2))(2\,x+12\,b_1+60\,a_2)^6+$
$(-240\,(4\,a_1+24\,t+120\,b_2)^3(-8\,b_1-80\,a_2)-1440\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+$
$720\,(4\,a_1+24\,t+120\,b_2)(-8\,b_1-176\,a_2))(2\,x+12\,b_1+60\,a_2)^5+(-450\,(4\,a_1+$
$24\,t+120\,b_2)^3-210\,(4\,a_1+24\,t+120\,b_2)^5+10\,(4\,a_1+24\,t+120\,b_2)^7+300\,(4\,a_1+$
$24\,t+120\,b_2)^4(-16\,t-160\,b_2)+450\,(4\,a_1+24\,t+120\,b_2)(-3+12\,(-8\,b_1-80\,a_2)^2-$
$4\,(-16\,t-160\,b_2)^2)+14400\,t+259200\,b_2+1800\,(4\,a_1+24\,t+120\,b_2)^2(-16\,t-$
$224\,b_2))(2\,x+12\,b_1+60\,a_2)^4+(-480\,(4\,a_1+24\,t+120\,b_2)^5(-8\,b_1-80\,a_2)+$
$14400\,(4\,a_1+24\,t+120\,b_2)^2(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+7200\,(4\,a_1+24\,t+$
$120\,b_2)(-8\,b_1-48\,a_2)-2400\,(4\,a_1+24\,t+120\,b_2)^3(-16\,b_1-128\,a_2)-14400\,(-8\,b_1-$
$80\,a_2)(-16\,t-160\,b_2)-460800\,(-16\,t-160\,b_2)a_2+921600\,(-8\,b_1-80\,a_2)b_2)(2\,x+$
$12\,b_1+60\,a_2)^3+(1710\,(4\,a_1+24\,t+120\,b_2)^5-60\,(4\,a_1+24\,t+120\,b_2)^7+5\,(4\,a_1+$
$24\,t+120\,b_2)^9-900\,(4\,a_1+24\,t+120\,b_2)^3(7+4\,(-8\,b_1-80\,a_2)^2-12\,(-16\,t-$
$160\,b_2)^2)+675\,(4\,a_1+24\,t+120\,b_2)(7+16\,(-8\,b_1-80\,a_2)^2+16\,(-16\,t-160\,b_2)^2)+$
$345600\,t+4492800\,b_2-21600\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)-21600\,(-16\,t-$
$160\,b_2)^3+691200\,(4\,a_1+24\,t+120\,b_2)^2b_2-1800\,(4\,a_1+24\,t+120\,b_2)^4(-64\,t-$
$448\,b_2))(2\,x+12\,b_1+60\,a_2)^2+(-240\,(4\,a_1+24\,t+120\,b_2)^7(-8\,b_1-80\,a_2)-$
$7200\,(4\,a_1+24\,t+120\,b_2)^4(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+10800\,(4\,a_1+24\,t+$
$120\,b_2)(-24\,b_1-400\,a_2+4\,(-8\,b_1-80\,a_2)^3+4\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)^2)+$
$3600\,(4\,a_1+24\,t+120\,b_2)^3(-24\,b_1-176\,a_2)+720\,(4\,a_1+24\,t+120\,b_2)^5(-56\,b_1-$
$400\,a_2)+21600\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+1382400\,(-16\,t-160\,b_2)a_2-$
$2764800\,(-8\,b_1-80\,a_2)b_2-43200\,(4\,a_1+24\,t+120\,b_2)^2((-8\,b_1-80\,a_2)(-16\,t-$
$160\,b_2)+32\,(-16\,t-160\,b_2)a_2-64\,(-8\,b_1-80\,a_2)b_2))(2\,x+12\,b_1+60\,a_2)-$

$$90\,(4\,a_1+24\,t+120\,b_2)^8(-16\,t-160\,b_2)+90\,(4\,a_1+24\,t+120\,b_2)^5(-107+28\,(-8\,b_1-80\,a_2)^2+12\,(-16\,t-160\,b_2)^2)+5400\,(4\,a_1+24\,t+120\,b_2)^2(-176\,t-2464\,b_2+4\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+4\,(-16\,t-160\,b_2)^3)-120\,(4\,a_1+24\,t+120\,b_2)^6(-80\,t-1248\,b_2)+900\,(4\,a_1+24\,t+120\,b_2)^4(-464\,t-4000\,b_2)-225\,(4\,a_1+24\,t+120\,b_2)^3(11+80\,(-8\,b_1-80\,a_2)^2+80\,(-16\,t-160\,b_2)^2+4096\,(-8\,b_1-80\,a_2)a_2+8192\,(-16\,t-160\,b_2)b_2)-675\,(4\,a_1+24\,t+120\,b_2)(-7+56\,(-8\,b_1-80\,a_2)^2+88\,(-16\,t-160\,b_2)^2-4096\,(-8\,b_1-80\,a_2)a_2-131072\,{a_2}^2-524288\,{b_2}^2)+5529600\,(-8\,b_1-80\,a_2)^2b_2-5529600\,(-16\,t-160\,b_2)^2b_2+1857600\,b_2)$$

and

$$d(x,t)=2024+518400\,(-16\,t-160\,b_2)^4+356400\,(-8\,b_1-80\,a_2)^2+874800\,(-16\,t-160\,b_2)^2+3720\,(4\,a_1+24\,t+120\,b_2)^8+120\,(4\,a_1+24\,t+120\,b_2)^{10}+530841600\,{a_2}^2+2123366400\,{b_2}^2+(1+(2\,x+12\,b_1+60\,a_2)^2+(4\,a_1+24\,t+120\,b_2)^2)^6+518400\,(-8\,b_1-80\,a_2)^4+(-1440\,(4\,a_1+24\,t+120\,b_2)^4+720\,(4\,a_1+24\,t+120\,b_2)^5(-16\,t-160\,b_2)+240\,(4\,a_1+24\,t+120\,b_2)^2(56+135\,(-8\,b_1-80\,a_2)^2-45\,(-16\,t-160\,b_2)^2)+32400\,(4\,a_1+24\,t+120\,b_2)(-16\,t-288\,b_2)+7200\,(4\,a_1+24\,t+120\,b_2)^3(-48\,t-544\,b_2)+3360+32400\,(-8\,b_1-80\,a_2)^2-54000\,(-16\,t-160\,b_2)^2+2764800\,(-8\,b_1-80\,a_2)a_2+5529600\,(-16\,t-160\,b_2)b_2)(2\,x+12\,b_1+60\,a_2)^4+(-960\,(4\,a_1+24\,t+120\,b_2)^6(-8\,b_1-80\,a_2)+57600\,(4\,a_1+24\,t+120\,b_2)^3(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)-43200\,(4\,a_1+24\,t+120\,b_2)^2(-24\,b_1-272\,a_2)-7200\,(4\,a_1+24\,t+120\,b_2)^4(-48\,b_1-448\,a_2)-345600\,b_1-5529600\,a_2-86400\,(-8\,b_1-80\,a_2)^3-86400\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)^2+172800\,(4\,a_1+24\,t+120\,b_2)((-8\,b_1-80\,a_2)(-16\,t-160\,b_2)-32\,(-16\,t-160\,b_2)a_2+64\,(-8\,b_1-80\,a_2)b_2))(2\,x+12\,b_1+60\,a_2)^3+(13440\,(4\,a_1+24\,t+120\,b_2)^6+240\,(4\,a_1+24\,t+120\,b_2)^8-240\,(4\,a_1+24\,t+120\,b_2)^4(-326+45\,(-8\,b_1-80\,a_2)^2-135\,(-16\,t-160\,b_2)^2)+480\,(4\,a_1+24\,t+120\,b_2)^2(-76+135\,(-8\,b_1-80\,a_2)^2+1215\,(-16\,t-160\,b_2)^2)-129600\,(4\,a_1+24\,t+120\,b_2)^3(-32\,t-256\,b_2)-12960\,(4\,a_1+24\,t+120\,b_2)^5(-32\,t-256\,b_2)-64800\,(4\,a_1+24\,t+120\,b_2)(96\,t+1280\,b_2+4\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+4\,(-16\,t-160\,b_2)^3)+12144-97200\,(-8\,b_1-80\,a_2)^2+32400\,(-16\,t-160\,b_2)^2+530841600\,{a_2}^2-33177600\,(-16\,t-160\,b_2)b_2+2123366400\,{b_2}^2)(2\,x+12\,b_1+60\,a_2)^2+(-360\,(4\,a_1+24\,t+120\,b_2)^8(-8\,b_1-80\,a_2)-17280\,(4\,a_1+24\,t+120\,b_2)^5(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)-1440\,(4\,a_1+24\,t+120\,b_2)^6(-8\,b_1-240\,a_2)+32400\,(4\,a_1+24\,t+120\,b_2)^4(-8\,b_1+112\,a_2)+64800\,(4\,a_1+24\,t+120\,b_2)^2(40\,b_1+752\,a_2+4\,(-8\,b_1-80\,a_2)^3+4\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)^2)-777600\,(4\,a_1+24\,t+120\,b_2)((-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+64\,(-16\,t-160\,b_2)a_2-128\,(-8\,b_1-80\,a_2)b_2)-172800\,(4\,a_1+24\,t+120\,b_2)^3(3\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+32\,(-16\,t-160\,b_2)a_2-64\,(-8\,b_1-80\,a_2)b_2)+648000\,b_1+8553600\,a_2+259200\,(-8\,b_1-80\,a_2)^3+1296000\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)^2-33177600\,(-8\,b_1-80\,a_2)^2a_2+33177600\,(-16\,t-160\,b_2)^2a_2-132710400\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)b_2)(2\,x+12\,b_1+60\,a_2)+120\,(-8\,b_1-80\,a_2)(2\,x+12\,b_1+60\,a_2)^9-120\,(4\,a_1+24\,t+120\,b_2)^9(-16\,t-160\,b_2)+80\,(4\,a_1+24\,t+120\,b_2)^6(191+63\,(-8\,b_1-80\,a_2)^2+27\,(-16\,t-160\,b_2)^2)-2160\,(4\,a_1+24\,t+120\,b_2)^5(-240\,t-4576\,b_2)+21600\,(4\,a_1+24\,t+120\,b_2)^3(368\,t+3488\,b_2+4\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+4\,(-16\,t-160\,b_2)^3)-1440\,(4\,a_1+24\,t+120\,b_2)^7(-80\,t-864\,b_2)+240\,(4\,a_1+24\,t+120\,b_2)^4(599+135\,(-8\,b_1-80\,a_2)^2-225\,(-16\,t-160\,b_2)^2-11520\,(-8\,b_1-80\,a_2)a_2-23040\,(-16\,t-160\,b_2)b_2)-16200\,(4\,a_1+24\,t+120\,b_2)(-496\,t-6240\,b_2+80\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+16\,(-16\,t-160\,b_2)^3+4096\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)a_2-4096\,(-8\,b_1-$$

$80\,a_2)^2 b_2+4096\,(-16\,t-160\,b_2)^2 b_2)+24\,(4\,a_1+24\,t+120\,b_2)^2(3881+12150\,(-8\,b_1-80\,a_2)^2+28350\,(-16\,t-160\,b_2)^2+691200\,(-8\,b_1-80\,a_2)a_2+22118400\,{a_2}^2+88473600\,{b_2}^2)+1036800\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)^2+46080\,a_2(2\,x+12\,b_1+60\,a_2)^7-24883200\,(-8\,b_1-80\,a_2)a_2-82944000\,(-16\,t-160\,b_2)b_2+(-120\,(4\,a_1+24\,t+120\,b_2)^2+360\,(4\,a_1+24\,t+120\,b_2)(-16\,t-160\,b_2)+120)(2\,x+12\,b_1+60\,a_2)^8+(480\,(4\,a_1+24\,t+120\,b_2)^2-240\,(4\,a_1+24\,t+120\,b_2)^4+960\,(4\,a_1+24\,t+120\,b_2)^3(-16\,t-160\,b_2)+2320+2160\,(-8\,b_1-80\,a_2)^2+5040\,(-16\,t-160\,b_2)^2-1440\,(4\,a_1+24\,t+120\,b_2)(-64\,t-960\,b_2))(2\,x+12\,b_1+60\,a_2)^6+(-720\,(4\,a_1+24\,t+120\,b_2)^4(-8\,b_1-80\,a_2)-17280\,(4\,a_1+24\,t+120\,b_2)(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+4320\,(4\,a_1+24\,t+120\,b_2)^2(-8\,b_1-176\,a_2)+51840\,b_1+103680\,a_2)(2\,x+12\,b_1+60\,a_2)^5$

is a solution to the Lakshmanan Porsezian Daniel equation (1).