

A Review of Mathematical Logical Reasoning Literacy in China

Formatted: Highlight

Formatted: Right

ABSTRACT:

Formatted: Highlight

Logical reasoning is an effective method to analyze and solve problems, obtain mathematical conclusions, and build a mathematical system. It is also the basis for forming a methodical, logical quality of thinking and rational spirit, so logical reasoning literacy has become a focus in educational studies. This paper summarizes the relevant research results and draws the following conclusions: Firstly, the previous research on logical reasoning literacy mainly focuses on four aspects: the connotation, level, influencing factors, and training strategies. Secondly, among them, level status and training strategies are hot topics in current research. Thirdly, regarding research methods, scholars mostly use test questions, questionnaires, and interviews to study the status. Fourthly, the single research method, lack of empirical research, unsystematic research on influencing factors, and unfeasible research conclusions on training strategies are deficiencies in previous research. Finally, the current research on logical reasoning literacy lacks theoretical research and balanced grade research. Therefore, it is necessary to further improve the research method in future studies and conduct a more in-depth and systematic study on the influencing factors from an empirical perspective, to find out more comprehensive influencing factors and more operable cultivation strategies.

Comment [NA1]:

Keywords: Middle School; Logical Reasoning; Influencing Factors.

Formatted: Font: Not Bold, Italic, Highlight

Formatted: Font: Italic, Highlight

Formatted: Font: Italic

1. INTRODUCTION

Logical reasoning literacy refers to literacy that starts from some facts and propositions and deduces other propositions according to the rules. It is a process of accumulating acquired knowledge by learning, which can help students to acquire unknown content by reasoning in the process of autonomous learning [1]. Cultivating students' logical reasoning ability can not only improve students' learning efficiency, but also enhance students' thinking ability, and it also plays an important role in the formation of students' innovative spirits and rational spirits. In 2018, the *General High School Mathematics Curriculum Standards (2017 Edition)* promulgated by the Ministry of Education of the People's Republic of China pointed out that logical reasoning literacy should be implemented in senior high schools [2]. In recent years, some scholars have paid attention to logical reasoning literacy, but there is currently a lack of sorting out relevant research results. Thus this paper intends to review and organize the existing relevant literature, and systematically analyze the current situation and shortcomings of relevant research. This study will not only provide references for strategies to improve the logical reasoning literacy of high school students but also help researchers grasp the characteristics and current situation of current research, so as to promote further research.

The research objective of this paper is to understand the research status of logical reasoning literacy. The research question of this paper is: What is the research status of logical reasoning literacy? The following specific questions will be included: (1) What previous researches have

- been done on logical reasoning literacy? (2) Which aspects have been researched more by scholars?
(3) What are the main research methods scholars have used to research logical reasoning literacy?
(4) What are the shortcomings in the current research on logical reasoning literacy?

2. LITERATURE SOURCE

2.1 Data Sources

This paper adopts the literature analysis method and systematically reviews the literature from China National Knowledge Infrastructure (CNKI). CNKI is the most authoritative literature search tool for national academic journals in China, which contains all the contents of journals and dissertations in China, and the selection of this database can ensure the persuasiveness and reliability of the study in this paper.

2.2 Data Collection

In the process of literature search, 428 articles were searched with the subject terms of *logical reasoning literacy*, 591 articles were searched with the subject term of *mathematical core literacy* and *middle school students*, and 43 articles were searched with the subject term of *logical reasoning literacy* and *middle school students*. Since this paper focuses on the logical reasoning literacy of high school students, 53 references were selected in consideration of the research questions and the number of citations in the literature.

3. ANALYSIS OF RESULTS

3.1 Mathematics Core Literacy

Core literacy does not originate in China. Japan has been committed to researching core literacy as early as the 1950s and has defined three characteristics for it. In the 1990s, core literacy appeared in the research reports of the OECD and the council of the European Union. Since then, developed countries, such as the United Kingdom and the United States, had successively conducted research on core literacy. They see core literacy as the new standard in education.

The exploration of core literacy has been carried out since the preliminary stage of the founding of New China. At this stage, the Ministry of Education announced the *Syllabus for Middle School Mathematics*, which was further revised in 1956. The syllabus and the revised syllabus propose to attach importance to the teaching of basic knowledge and skills and develop their logical thinking and spatial imagination. The syllabus does not accurately mention the concept of core literacy [3]. In 1963, the Ministry of Education compiled the 12-year Mathematics Syllabus (Draft) for Full-time Middle Schools, which for the first time put forward the requirement of *cultivating students' correct and rapid calculation ability, logical reasoning ability, and spatial imagination ability*. In 1978, the *Mathematics Syllabus for Secondary Schools in Full-time Schools* was formulated, which for the first time proposed *gradually cultivating students' ability to analyze and solve problems*. In 1982, the *Syllabus of Mathematics for Full-time Six-year Key Middle*

Schools clearly stated that *the ability to use mathematics to analyze and solve practical problems should be gradually formed*. In November 1986, the *Syllabus of Mathematics for Full-time Middle Schools* was formulated by the State Education Commission, which officially took *two basics and three major abilities* as the core content of mathematics teaching goals in middle schools [4]. The outline does not put forward the concept of *core literacy*, but the *three major abilities* proposed have already involved the essence of *core literacy*. Therefore, this period can be regarded as the embryonic stage of *core literacy*.

In 1992, the term of mathematical literacy appeared for the first time in the Mathematics Teaching Syllabus for Junior Middle Schools issued. *Full-time General Senior Middle School Mathematics Teaching Syllabus (Revised Edition)* in 2000, *Nine-year Compulsory Education Mathematics Curriculum Standard* in 2001, *General High School Mathematics Curriculum Standard (Experimental)* in 2003, *Mathematics Curriculum Standard* in 2011, the concept of mathematical literacy has been successively introduced in the *Compulsory Education Mathematics Curriculum Standard* [5]. During this period, the term of mathematical literacy was frequently submitted in important documents, and educational scholars began to have a preliminary concept of mathematical literacy.

In March 2014, the Ministry of Education issued the *Opinions on Comprehensively Deepening the Curriculum Reform and Implementing the Fundamental Tasks of Establishing Morality and Building People*. It clearly defined the concept of *core literacy*, that is, the necessary character and key abilities that students should possess to adapt to the needs of lifelong development and social development. It put forward the task of *organizing research and putting forward the core literacy system for students in each school stage*. So far, *core literacy* has officially appeared in the public's field of vision. It has triggered intense discussions among experts and scholars.

In February 2016, the Chinese Society of Education released the *Core Competencies for Chinese Students' Development* to solicit suggestions from the educational circles. Ma pointed out that mathematics core literacy is the comprehensive ability that mathematics learners should achieve in learning mathematics or learning mathematics in a certain field. Mathematical core literacy is the basic literacy that should be paid special attention to in the process of mathematics teaching and learning [6]. Hua believes that mathematical literacy is the inner literacy that people can use to observe the world, discover, propose, analyze and solve problems with mathematical eyes [7]. Professor Kong believes that mathematical literacy refers to the knowledge acquired in current or future life to meet the needs of an individual. You will become a caring and thinking citizen [8]. Professor Wang's point of view is: Mathematical literacy is an inherent feature of mathematics. It is a relatively stable state based on human innate physiology. Mathematical literacy covers five aspects: Mathematical thinking, mathematical awareness, mathematical application awareness, innovation awareness, and understanding and appreciation of the aesthetic value of mathematics [9]. Jie believes that core literacy includes not only learning ability and cultural knowledge but also skills to adapt to society and life. Core literacy refers to the ability that students can adapt to lifelong development and social development. It is an important part of literacy. And with the progress of the times and the advancement of education reform, the definition of core literacy is slightly different. The generally defined core literacy includes two aspects: one's ability and one's

ability to get along with society [10]. In 2017, the Ministry of Education gave the most formal statement in the latest version of the curriculum standards and finally divided the core literacy into six literacy: mathematical operation, logical reasoning, mathematical abstraction, mathematical modeling, intuitive imagination, and data analysis. So far, core literacy has officially entered the stage, which has sparked a heated discussion.

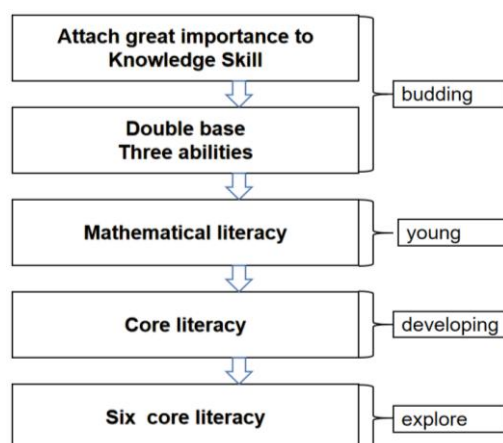


Fig. 1. Core literacy development map

3.2 The Connotation of Logical Reasoning Literacy

In Polya's *Mathematics and Conjecture*, mathematics has two forms: argumentative reasoning and plausible reasoning. Argumentative reasoning affirms mathematical knowledge and plausible reasoning provides the basis for conjecture. Logical reasoning has a positive relationship with mathematics achievement, and the core of mathematics is logical reasoning [11]. In 1989, the US *National Research Council (NRC)* discussed plausible reasoning in mathematics education in its *The Future of Mathematics Education — A Report to the Nation*, arguing that plausible reasoning is *exploring, guessing, experimenting, estimating, debating and other kinds of thinking training* [12]. In 1963, the basic definition of logical reasoning appeared in the curriculum syllabus formulated by our country. And then it was changed to reasoning and argumentation. Whether it is logical reasoning or reasoning argumentation, it essentially refers to a thinking process what conforms to the rules of reasoning in form and maintains the truth-value of reasoning [13]. The literacy of logical reasoning is one of the six core literacy in my country, and it is the basic literacy that it is beneficial to to personal life-long development and can meet the needs of society. For the connotation of logical reasoning, many domestic experts and scholars have expressed their views. Zhou believes that reasoning is the thinking process from propositional judgment to propositional judgment. Logical reasoning ensures the rigor of mathematics, and the development of mathematics requires logical reasoning [14]. Li expressed the following opinions on logical reasoning under the core literacy: Logical reasoning is based on the logical relationship of mathematical concepts, and it points to the development of students' logical reasoning ability. The process of students' logical reasoning is the process of building a building of mathematical knowledge; Students' formation of logical reasoning literacy is not limited to mathematics

Comment [NA2]: "Fig. 1", this should be referenced (in the text).

Comment [NA3]: In the text, do not use the first person "our".

Comment [NA4]: In the text, do not use the first person "my".

learning, and can be transferred to all aspects. Therefore, cultivating students' core literacy of logical reasoning can not only improve learning efficiency but also enhance students' thinking ability [15]. Jing believes that logical reasoning literacy is an important way for students to obtain mathematical conclusions and build a mathematical system. It is the basis of high school mathematics learning [16]. In the Curriculum Standard (2017 Edition), the Ministry of Education gave an accurate definition of logical reasoning literacy: Logical reasoning refers to the literacy of starting from some facts and propositions and deriving other propositions according to the rules, mainly including two forms, namely Inductive and analogical reasoning from particular to general, and deductive reasoning from general to particular [17]. The meanings of the nouns are shown in Table 1.

Table 1. Noun meaning [18]

Noun	Meaning
Propositions	Mainly represent sentences that determine a specific thing, including language, formulas, etc., sentences that can determine true or false. In mathematics courses, formulas and theorems are very typical mathematical propositions.
Rules	Mainly represent the scientific and reasonable guidelines formulated by people based on their own logical thinking and upholding certain logical laws. Among them, logical thinking follows rules such as the law of contradiction.
Inductive reasoning	Based on the properties exhibited by related objects in a particular class of things, judging all objects in it requires reasoning that exhibits such properties.
Analogical reasoning	Based on the fact that two objects are basically similar in related attributes, by means of comparative inference, the same inference is obtained for the rest of the attributes.
Deductive reasoning	From the original general reasoning to the special reasoning, start with the general premise, and then continue to deduce to obtain the final conclusion.

3.3 Situation of High School Students' Logical Reasoning Literacy

Yu et al. conducted research on the overall cognition of high school students' core literacy by using questionnaires and establishing a three-dimensional model. He found that the core literacy of high school students' logical reasoning is at a medium level in the overall literacy level, and there are few students with high logical reasoning abilities [19]. Dong used the method of questionnaire to study the core literacy cognition of more than 80,000 middle school students in Jiangsu Province. He found that middle school students have a high level of awareness of core literacy, but there is a severe bipolar phenomenon [20]. Zhu analyzed the cognition of core literacy among high school students in 16 cities in Hubei Province utilizing a questionnaire. It is concluded that there are obvious differences in the cognition of core literacy among high school students in different cities [21]. Liu conducted a study on the cognition of logical reasoning literacy among high school students in Tibetan areas. The study showed that most cognition of logical reasoning among high school students in Tibetan areas stayed at the first level, and the distribution of cognition at each level was uneven [22]. Yu used the questionnaire method to investigate and analyze the logical reasoning literacy level of the eighth grade students. The survey results show that the level of logical reasoning literacy in the eighth grade is at a medium

level, among which the sensible reasoning literacy is better than the deductive reasoning literacy, and there is a correlation between the various types of logical reasoning literacy levels [23]. Sun used the questionnaire method to study the current situation of the logical reasoning literacy level of the second-year senior high school students from the aspects of gender and subject. The results show that the level of logical reasoning literacy in the second grade of senior high school is generally not high, and gender and subject have little effect on logical reasoning literacy [24].

3.4 Influencing Factors of Logical Reasoning Literacy

3.4.1 Influence of Teachers

Formatted: Highlight

Wang summarized the influencing factors through the investigation and analysis of the logical reasoning literacy level of middle school students, and she believes that the logical reasoning literacy of students is affected by three aspects: family, teacher, and student. Among them, the teacher's teaching strategy is an important external motivation for students to form logical reasoning literacy [25]. Li used the questionnaire method to study the influencing factors of logical reasoning literacy of middle school students and analyzed them from three levels. The results showed that the display of mathematical inquiry results set up by teachers in teaching had a positive impact on level 1, and the negative impact is the teacher's ability to guide students' mathematical expression in teaching. Teachers instruct students to explain in precise language that their process of *finding mathematical problems* has a positive impact on level 2, and the negative effect is that the teacher guides the students themselves to explain the whole process of *solving mathematical problems*. Teachers attach importance to students' proficiency in mathematical theorems have a greater positive impact on level 3, and the negative impact is that teachers require students to clarify how to deduce conclusions [26].

3.4.2 The influence of Students Themselves

Comment [NA5]: This should be checked.

3.4.2.1 Mathematical Cognitive Structure Factors

Comment [NA6]: This should be checked.

The mathematical cognitive structure is an overall structure with internal rules composed of mathematical knowledge in students' minds according to their depth and breadth of understanding, combined with their cognitive characteristics such as feeling, perception, memory, thinking, and association. German mathematician Hilbert said: The knowledge base of students refers to the knowledge and experience that students have before the beginning of classroom teaching, which greatly affects the subsequent learning of students. The mathematical cognitive structure constructed by students according to their situation also greatly affects the literacy of logical reasoning. Bin once pointed out that students should learn to organize mathematical knowledge, and build exercises in the process of knowledge sorting to obtain a network of it [27]. The knowledge network structure has an important influence on the improvement of students' logical reasoning literacy. At present, students have a fixed mindset, and the way of thinking is too simple to build a network structure of mathematical knowledge, so it is difficult to improve the literacy of logical reasoning [28, 29]. Bai and Chen also mentioned that we must start with basic knowledge to improve students' logical reasoning literacy. Only with solid basic knowledge can students

Formatted: Highlight

Formatted: Highlight

Formatted: Highlight

Formatted: Highlight

Formatted: Highlight

Comment [NA7]: In the text, do not use the first person "we".

deepen their understanding of more complex knowledge in subsequent learning [30, 31].

3.4.2.2 Factors of Learning Strategies

Learning strategies usually consist of the following three dimensions: cognitive strategies, metacognitive strategies, and resource management strategies. Among them, cognitive strategies focus on information processing, metacognitive strategies refer to management and regulation strategies for it, and resource management strategies refer to strategies for selecting and making full use of various scientific resources in learning [32]. Hou has studied the impact of metacognitive monitoring on students' logical reasoning literacy. Metacognition is the cognition of cognition, and metacognitive monitoring is the ability to actively adjust and consciously optimize the process of logical reasoning. In the whole process of logical reasoning, students are required to have the ability to evaluate, integrate, and critically reflect on evidence and assumptions. Improving these abilities requires students to be more flexible in their thinking and transfer their thinking more quickly when they are carrying out logical reasoning. These abilities all rely on active monitoring of thinking quality [33]. Wang believes that the mastery of metacognitive strategies is a necessary means to improve their logical reasoning literacy [34]. From the perspective of students' field cognition, Pan studied the relationship between students' logical reasoning literacy and it. The logical reasoning literacy of students is slightly higher than that of the field-intermediate and field-dependent [35].

Formatted: Highlight

Formatted: Highlight

3.4.2.3 Learning Motivation

Learning motivation refers to a dynamic tendency to induce and maintain students' learning behavior and make it point to a certain academic goal. It contains two components: learning needs and learning expectations, which can be divided into different categories according to different standards. We usually understand students' motivation as students' learning interests and learning attitudes. Mathematical knowledge in high school has certain abstraction and difficulties. Teachers blindly flood students with the knowledge, so students will feel bored. In particular, the topics related to logical reasoning will make students feel more obscure and difficult to understand. Teachers should discover students' problems in time, stimulate students' interest in learning by teaching, and allow students to actively explore to achieve the purpose of improving logical reasoning literacy [36]. Qiao also expressed his opinions on the aspect of learning interest. He believes that the inability to feel a sense of participation in mathematics class is the main reason why students cannot study patiently. Only by stimulating students' interests, can teachers further develop the teaching of students' logical reasoning ability and enhance students' innovative ability. Only by allowing students to feel the joy of exploration in their learning, can they have the effect of exercising their logical reasoning ability [37]. Sun also pointed out that in the process of developing logical reasoning literacy, teachers should fully stimulate students' interest in mathematics learning, guide students to participate in the process of mathematical thinking and logic learning, help students build good and gradually infiltrate the importance and value of mathematical logical reasoning literacy for students, and enhance students' cognitive thinking in mathematics [38]. Lu investigated the relationship between students' learning attitudes and logical reasoning ability. The results showed that students' mathematics learning attitude and logical

Formatted: Highlight

Comment [NA8]: In the text, do not use the first person "we".

reasoning ability are positively correlated. Students' positive learning attitude will help students solve problems in learning and gain learning satisfaction [39]. Nunes tests the relationship between logical ability and mathematics performance, and the results show that there is a positive correlation between it and logical reasoning ability [40].

3.4.2.4 External Factors

Formatted: Highlight

The so-called external factors refer to the factors that cannot be changed by the students themselves, including gender factors and subject factors. Xian has surveyed the logical reasoning literacy of middle school students. From the analysis of gender factors, there is little difference in the logical reasoning literacy level of students, but boys are slightly more than girls. The reason is that girls have sensitive personalities and more traditional and rigorous thinking. They are more accustomed to passive learning, and it is uneasy to establish a connection between knowledge and knowledge. However, the boy's personality is more active and jumpy, more independent and adventurous. Although they are slightly lacking in learning rigor, they can take the initiative to learn, their thinking is easy to diverge, and they are better at guessing and summarizing [41]. Pan has researched the level of logical reasoning literacy of middle school students on gender and subject. The results show that although the data shows that boys are slightly better than girls, and science students are slightly better than liberal arts students, gender and subject have little effect on the level of logical reasoning literacy [42]. Erdem selected 409 middle and high school students from different provinces in Turkey to conduct a mathematical logic reasoning test to study the effect of gender on the ability of logic reasoning. Data shows that the mathematical logic reasoning ability of boys is significantly better than girls [43].

3.5 Cultivating Strategies for Students' Logical Reasoning Literacy

The training strategies for students' logical reasoning literacy can be roughly divided into two categories: 1. To cultivate students' logical reasoning literacy in general teaching; 2. To cultivate students' logical reasoning literacy in classroom teaching.

Regarding the cultivation of students' logical reasoning literacy in general teaching, Guo and others believe that the cultivation of students' logical reasoning literacy and innovative ability needs start from the problem, take the problem as the engine, and deeply explore the process of knowledge acquisition so that students can find similar problems, and ultimately form a *problem awareness* [44]. Shao researched the training path of logical reasoning literacy. The author divided the training path into three aspects: creating problem scenarios to guide students to think independently; using mind maps to deepen students' logical reasoning awareness; focusing on the combination of teaching and learning, guiding students' reason step by step [45]. Cao conducted a study on how to cultivate students' logical reasoning literacy in teaching from the perspective of teaching and learning. We should actively carry out logical reasoning mathematics activities, pay attention to the logical clarification of the writing process, and finally construct a relatively complete theoretical and practical system for cultivating students' logical reasoning ability [46]. Chen proposed a strategy for cultivating students' core literacy of logical reasoning by taking triangle stability as an example, and demonstrated it from the aspects of strategy connotation and

Comment [NA9]: In the text, do not use the first person "we".

specific practice. Finally, he concluded that teachers should pay attention to the penetration of mathematical thinking methods in teaching, so that students can understand logical reasoning in the situations created by teachers, and encourage students to conduct more knowledge exploration [47]. Ma suggested that logical reasoning literacy should be cultivated by advocating reflective learning, extending students' logical thinking space, introducing mathematical experiments, and enriching logical reasoning ability [48].

Regarding the cultivation of logical reasoning literacy in classroom teaching, Yao believes that the cultivation of logical reasoning ability in mathematics teaching must pay attention to students' classroom attention. Among them, efficient and orderly logical reasoning must be based on attention, and the satisfaction of logical requirements in the process of reasoning must be guaranteed by attention [49]. Li believes that cultivating logical reasoning literacy requires giving full play to the role of the classroom. Teachers should grasp every minute, divide the classroom time reasonably, add an appropriate number of practice questions for students, and lead students to train their thinking while answering questions [50]. Hong believes that high school mathematics teachers should train students in the foundation of logical reasoning by training in classroom teaching, and cultivate students' ability to apply it to analyze propositions. Students are required to apply a variety of mathematical ideas to analyze more complex problems [51].

4. LITREATURE ANALYSIS

Based on the data of CNKI resource, an advanced search was used to input the keywords *logical reasoning literacy* and *middle school students* to conduct a literature search. In addition, a visual analysis of the literature in the past four years was carried out from the number of published papers and the main themes. The results found that in 2018, the number of articles published was only 2, but in 2019, after the promulgation of the new curriculum reform, the number of articles published surged to 16. Although there are local fluctuations in the number of published papers from the prediction results of CKNI, literacy is still a research hot spot in 2022. The details are shown in Figure 2.

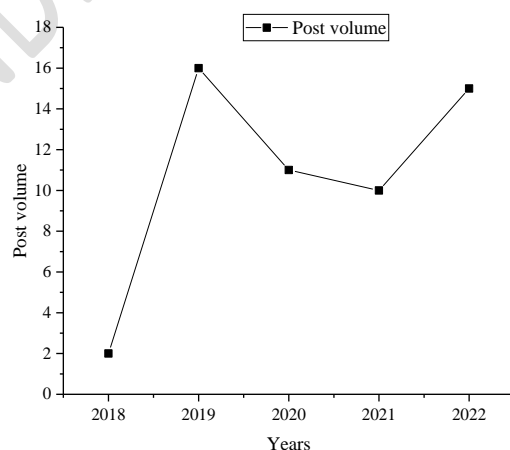


Figure 2. The Number of published papers on logical reasoning literacy

Formatted: Highlight

From the perspective of themes, the previous research topics mainly focused on the investigation of the level of logical reasoning literacy and the cultivation strategy, and the research on junior high school mathematics was significantly more than that of high school mathematics. It can be seen that there are still many deficiencies in the research on logical reasoning literacy, such as the lack of theoretical research, the lack of research on influencing factors, and the imbalance of research segments. The cultivation of logical reasoning literacy should attract enough attention in all school sections, so the research on logical reasoning literacy of high school students still needs to be strengthened. The details are shown in Table 2.

Table 2. Main topic distribution

Theme	Number of documents
logical reasoning	16
middle middle school student	14
mathematical Core Literacy	6
junior high school mathematics	4
high school mathematics	3
cultivation research	3
high school mathematics teaching	2
literacy level	2
cultivation strategy	2

Formatted: Font: Bold, Highlight

From the perspective of the authors, the people who study logical reasoning literacy are mainly concentrated in colleges and universities, and there are few studies of teachers in scientific research institutes and middle schools. This reflects that the current secondary school teachers' research initiative and enthusiasm for logical reasoning literacy are not enough, and theoretical researchers and practical workers have not established a good connection. The details are shown in Table 3.

Table 3. Distribution of authors' institutions

Author's organization	Number of documents
Northwest Normal University	4
Central China Normal University	2
Chongqing Normal University	2
The Third Middle School of Huining County, Gansu Province	1
Fuqing No.1 Middle School Fujian Province	1

In addition, we have sorted out the references mentioned in this study from three aspects: methodology, limitation, and issues or challenges of logical reasoning literacy technique. Among them, there are 12 references on methodology, and the methods involved mainly include the questionnaire, interview, and literature review methods. There are 14 references on the limitations of the study. Through summarizing, we conclude that there are shortcomings such as a relatively single research method, lack of empirical research, and the small number of survey samples. There are 15 references on issues or challenges of the logical reasoning literacy technique, which mainly

point out many deficiencies in the current development of students' logical reasoning literacy. The details are shown in Table 4.

Table 4. The methodology, limitation, issues or challenges of logical reasoning literacy technique

	Number of Documents	Point of View
Methodology	12	Questionnaire Interview Literature review
Limitation	14	The research method is relatively simple There is a lack of empirical research The sample size of the survey was small The overall level of logical reasoning literacy is low
Issues or challenges of logical reasoning literacy technique	15	The development of logical reasoning literacy is uneven in urban and rural The development of logical reasoning literacy in each section is unbalanced The training strategy of logical reasoning literacy lacks practical verification

5. DISCUSSION

It can be seen from the above studies that the study of logical reasoning literacy has become a hot issue in pedagogy research in recent years. At present, scholars mainly focus on the connotation of logical reasoning literacy, the level of middle school students' logical literacy, the influencing factors, and training strategies for logical reasoning literacy. Among them, the status and training strategies of middle school students' logical reasoning literacy are the current research hotspots.

First of all, scholars have different interpretations of the concept of logical reasoning literacy, but all emphasize the importance of thinking development and its fundamental role in mathematical learning. However, theoretical research on logical reasoning literacy is relatively lacking. The lack of theoretical research will lead to the in-depth construction of China's core literacy system, so some fundamental problems are difficult to solve, which is consistent with the conclusions reached by Gu in his review of core literacy research [52].

Scholars are more consistent with the results of the study on the current level of logical reasoning literacy in middle school students. Although there is an imbalance in the level of logical reasoning literacy of middle school students in different regions, the level of logical reasoning literacy of students in urban areas is generally slightly higher than that in rural areas, and the overall level of logical reasoning literacy is still not high. At present, scholars mostly use questionnaire methods and interview methods for the investigation and research of logical reasoning levels, and the research methods are too single, resulting in insufficient convincing of the research results. In addition, the research on logical reasoning literacy has the deficiency of unbalanced grades, and

the imbalance of research grades will lead to the difficulty of implementing logical reasoning literacy in various sections.

In the research of the influencing factors of logical reasoning literacy, the research of scholars mainly focuses on the two aspects of teachers and students, and their analysis of influencing factors is not comprehensive, systematic, and in-depth. From the perspective of research methods, scholars mostly use theoretical speculation to analyze the influencing factors, which leads to a strong subjectivity of the research results. Therefore, it is necessary to improve the research methods in the future and to conduct a more objective and comprehensive analysis of the factors affecting the logical reasoning literacy of middle school students from an empirical point of view.

Scholars mostly propose training strategies based on their experience with existing research. Most suggestions focus on improving teachers' behavior but are nonspecific enough, which leads to poor feasibility of relevant suggestions. Ma also pointed out that the research on the connotation of core mathematical literacy needs to be further refined, and the cultivation path needs to be further explored [53]. Therefore, follow-up research can be carried out from more perspectives. More feasible suggestions for the cultivation of logical reasoning literacy can be put forward. At the same time, we reviewed the recent article on logical reasoning literacy technique in the reputed journal IEEE Transaction, Springer. Probably because mathematical logical reasoning literacy is a proper noun in China, we found that there are many articles on *logical reasoning*, but there is a lack of articles related to *logical reasoning literacy*.

6. CONCLUSION

Summarizing the related research in the above four aspects, the following conclusions can be drawn: 1. The previous research on logical reasoning literacy mainly focused on the connotation, the level status, influencing factors, and training strategies of logical reasoning literacy. 2. Among them, the level status and training strategies of logical reasoning literacy are the hot issues of current research. 3. In terms of research methods, scholars mostly use test questions, questionnaires, or interviews to study the situation, and they use theoretical thinking to analyze the influencing factors and training measures. 4. The single research method, lack of empirical research, unsystematic research on influencing factors, and unfeasible research conclusions on training strategies are deficiencies in previous research. 5. The current research on logical reasoning literacy lacks theoretical research and balanced grade research.

Combining the above conclusions, we believe that the current research on mathematical logic reasoning literacy is relatively comprehensive, covering almost all aspects of it. However, the research in some aspects is not in-depth, systematic, and lacks feasible strategies. Therefore, it is necessary to further improve the research methods of influencing factors and training strategies in future research and to conduct more in-depth and systematic research on the influencing factors of logical reasoning literacy from an empirical point of view, in order to find out more comprehensive influencing factors and more actionable training strategies.

REFERENCES

Comment [NA10]: These references should be rearranged according to this example [Boateng R, Mbrokoh AS, Boateng L, Senyo PK, Ansong E. Determinants of elearning adoption among students of developing countries. *Int. J. Inf. Learn. Technol.* 2016;33(4):248–262.]

Formatted: Highlight

1. Li XL. On the cultivation of high school students' mathematical logical reasoning literacy and its path. *Journal of Lvliang Institute of Education*. 2021;38(03):113-114+119.
2. Ministry of Education of the People's Republic of China. *Mathematics Curriculum Standards for General High Schools (2017 Edition)*. Beijing: People's Education Press; 2018.
3. Wei Q, Zhang YX. *Historical data on the evolution of Chinese middle school mathematics curriculum textbooks*. Beijing: People's Education Press; 1996.
4. Yang GW. *Mathematics Curriculum Standard Training and Textbook Analysis*. Hangzhou: Zhejiang University Press; 2011.
5. Huang YC. Research and Analysis of Mathematical Literacy in my country. *Curriculum. Textbook. Teaching Method*. 2015;35(8):55-59.
Available: <https://doi.org/10.19877/j.cnki.kcjcjf.2015.08.010>
6. Ma YP. Several questions about the core competence of mathematics. *Curriculum. Textbook. Teaching Method*. 2015;35(9):36-39.
Available: <https://doi.org/10.19877/j.cnki.kcjcjf.2015.09.006>
7. Hua ZY. The connotation and composition of mathematics core literacy. *Educational Research and Review (Secondary Education and Teaching)*. 2016;7(05):41-44.
8. Kong FZ, Shi NZ. The definition of the concept of core mathematical literacy and the way to develop it in Chinese students' development. *Educational Science Research*. 2017;33(06): 5-11.
9. Wang ZX. On Mathematical Literacy. *Mathematical Bulletin*. 2002;67(01):6-9.
10. Jie XL. Reflections on the implementation of core literacy in high school mathematics textbooks. *Mathematics Learning and Research*. 2019;37(03):107.
11. Jansson LC. Logical Reasoning Hierarchies in Mathematics. *Journal for Research in Mathematics Education*. 1986;17(1):3-20.
12. National Committee of American Mathematics Teachers. The topic of American mathematics education in the 21st century. *Foreign Education Materials*. 1999;2.
13. Bai Y. Investigation and Research on the Status Quo of High School Students' Logical Reasoning Literacy. (Master's thesis, Harbin Normal University, Harbin); 2021.
14. Zhou YL. The reflection of high school mathematics core literacy in examination evaluation. *Mathematics World (Mid)*. 2017;24(10):7-8+13.
15. Li HQ. Logical Reasoning: A Detailed Study of the Implications of the Core Competencies of High School Mathematics Subjects. *Mathematics Teaching Communications*. 2021;43(21):45-46.
16. Jing H. Rediscovering the significance of logical reasoning in high school mathematics teaching. *Mathematics Teaching Communications*. 2021;43(24):42-43.
17. Ministry of Education of the People's Republic of China. *Mathematics Curriculum Standards for General High Schools (2017 Edition)*. Beijing: People's Education Press; 2018.
18. He SY. Research on Teaching Strategies to Improve High School Students' Mathematical Logical Reasoning Literacy under the Background of the New College Entrance Examination. (Master's thesis, Southwest University, Chongqing); 2021.
Available: <https://doi.org/10.27684/d.cnki.gxndx.2021.001028>
19. Yu C, Zhu XY, Wu N, Xu J, Wang CL, Liu QL, Qu Q. Investigation and Analysis of the Core Competence Level of High School Students in Mathematics. *Journal of Mathematics*

Comment [NA11]: This should be checked.

Formatted: Font color: Red, Highlight

- Education. 2018;27(02):59-64.
20. Dong LW, Yu P. Investigation on the Development of Junior High School Students' Mathematics Core Literacy Based on Academic Level Quality Monitoring. *Journal of Mathematics Education*. 2017;26(01):7-13.
 21. Zhu XY, Hu DS. An Empirical Study on the Influencing Factors of High School Students' Mathematical Core Literacy Development Level——Based on the Investigation and Analysis of 16 Cities and Prefectures in Hubei Province. *Educational Measurement and Evaluation*, 2020;13(02):50-58.
Available: <https://doi.org/10.16518/j.cnki.emae.2020.02.007>
 22. Liu XJ. Investigation and research on the status quo of logical reasoning literacy of high school students in Tibetan areas. (Master's thesis, Northwest Normal University, Lanzhou); 2020.
 23. Sun WB, Sun Y. Investigation and analysis of mathematical logical reasoning literacy of junior high school students. *Journal of Anshan Normal University*. 2021;23(06):10-13.
 24. Sun HF. Research on the status quo and teaching strategies of logical reasoning literacy of senior two students. (Master's thesis, Tianshui Normal University, Tianshui); 2020.
 25. Wang SL. Research on the Status Quo and Influencing Factors of Junior High School Students' Mathematical Logical Reasoning Ability. (Master's thesis, Tianjin Normal University, Tianjin); 2020.
 26. Li SH. Research on the influencing factors of high school students' logical reasoning literacy. (Master's thesis, Shandong Normal University, Jinan); 2019.
 27. Bin X, Li JJ. The main ways and strategies to implement "logical reasoning" literacy. *Mathematics Teaching Communications*. 2020;42(19):5-7.
 28. Shen WX. Research on the cultivation strategies of high school students' mathematical logic reasoning ability. (Master's thesis, Yanbian University, Yanji); 2021.
 29. Han X. Research on the level of mathematical logic reasoning ability of high school students. (Master's thesis, Guangxi University for Nationalities, Nanning); 2018.
 30. Bai TJ. Research on the status quo and training strategies of logical reasoning literacy of ordinary high school students. (Master's thesis, Northwest Normal University, Lanzhou); 2021.
 31. Chen XQ. An example of strategies for improving students' logical reasoning ability in mathematics teaching. *College Entrance Examination*. 2021;18(11):95-96.
 32. Wang ZL, Wang JP. Review and Reflection on the Research of Chinese Mathematical Logical Reasoning——Based on the Quantitative Analysis of "China HowNet" Literature. *Journal of Mathematics Education*. 2018;27(04):88-94.
 33. Hou BK. Influencing factors and evaluation indicators of high school students' mathematical logic reasoning ability. *Teaching and Management*. 2021;37(04):39-42.
 34. Wang YR. Investigation and countermeasure research on the current situation of algebraic reasoning ability of high school students. (Master's thesis, Tianjin Normal University, Tianjin); 2021.
 35. Pan JJ. Research on the influence of high school students' field cognitive style on the development of mathematical logic reasoning literacy. (Master's thesis, Guizhou Normal University, Guiyang); 2021.
 36. Zhang LJ. Exploration on the cultivation strategies of logical reasoning core literacy in high

- school mathematics teaching. *Exam Weekly*. 2020;14(49):69-70.
37. Qiao L. Cultivation strategies for high school mathematics logical reasoning ability under the background of core literacy. *Exam Questions and Research*. 2022;45(01):26-27.
 38. Sun XR. On the cultivation of high school students' mathematical logical reasoning literacy and its path. *Mathematics Learning and Research*. 2021;39(27):92-93.
 39. Lu YY. Investigation and Research on the Logical Reasoning Ability and Mathematics Learning Attitude of Junior High School Students. (Master's thesis, Guangxi Normal University, Guilin); 2021.
 40. Nunes T, Bryant P, Evans D. The contribution of logical reasoning to the learning of mathematics in primary school. *British Journal of Developmental Psychology*. 2011;25(1):147-166.
 41. Xian YN. Investigation and Research on the Status Quo of the Mathematical Logical Reasoning Literacy of High School Students. (Master's thesis, Chongqing Three Gorges University, Chongqing); 2019.
 42. Pan JJ. Research on the influence of high school students' field cognitive style on the development of mathematical logic reasoning literacy. (Master's thesis, Guizhou Normal University, Guiyang); 2021.
 43. Erdem EY. Age- and Gender-Related Change in Mathematical Reasoning Ability and Some Educational Suggestions. *Journal of Education & Practice*. 2017;8(7):116-127.
 44. Guo LX, Li HY. "Problem awareness" promotes the development of high school students' mathematical logical reasoning literacy. *Middle School Mathematics Research* (South China Normal University Edition). 2021; 67(20):5-7.
 45. Shao TY. On the Path to Cultivating High School Students' Mathematical Logical Reasoning Literacy. *Everyday Love Science* (Teaching Research). 2021;4(08):73-74.
 46. Cao HM. The cultivation of students' logical reasoning ability in senior high school mathematics under the core literacy. *Reading and Writing*. 2021;37(20):85-86.
 47. Chen Y. Teaching strategies and practice of mathematics in junior high school under the core literacy of logical reasoning: Taking "triangle stability" as an example. *Mathematics Teaching Communications*. 2021;43(20):16-17.
 48. Ma HZ. Strategies for cultivating students' logical thinking ability in junior high school mathematics teaching. *Intelligence*. 2021;39(35):1-3.
 49. Yao J. The relationship between students' logical reasoning ability and attention in junior high school mathematics teaching—taking the Zhejiang Education Edition "Determination of Congruence of Triangles" as an example. *Mathematics Teaching Communications*. 2020;42(35):51-52.
 50. Li JJ. Cultivation of logical reasoning ability in high school mathematics teaching. *Mathematics Learning and Research*. 2021;39(32):59-61.
 51. Hong HM. Cultivation of "logical reasoning" ability in high school mathematics teaching. *Mathematics World* (Mid). 2020;27(06):12.
 52. Gu QK, Li LJ. A Review of Research on Mathematical Core Literacy. *Science Consulting* (Science and Technology Management). 2021;4(08):4-6.
 53. Ma DT. A review of the research on core competencies in high school mathematics—Based on the analysis of the articles published in "Copying Newspaper Materials, High School Mathematics Teaching and Learning" by the National People's

Congress in 2020. Journal of Neijiang Normal University. 2020;36(06):19-23.

UNDER PEER REVIEW