Original Research Article

NEW APPROACH: TABULAR FUZZY ARITHMETIC OF THE LR TYPE BY JOMATOPFE

Abstract

This paper presents a fuzzy approach of LR types, also called LR-type tabular fuzzy arithmetic capable of handling or computing simultaneously the kernels and supports of trapezoidal fuzzy numbers, instead of doing it separately (Advantage of this approach) in order to minimize the tedious steps of alpha-cut based approaches.
On the theoretical level, the aim of this article is to explain in a concise and clear manner some basic concepts of fuzzy logic that seem to continue to complicate authors and readers (researchers) in this field, given the important place of this theory in Artificial Intelligence today.

Some user interfaces have been created in this article, with the python language, in order to automatically calculate certain results; and especially to minimize the calculation time, in particular of membership degrees, kernels and supports.
Trapezoidal fuzzy numbers are transformed into LR form in order to allow a comparative study between the alpha-cut approach with the Jomatopfe LR-type tabular fuzzy arithmetic presented in this paper.

We successfully demonstrated the transition from the trapezoidal fuzzy form to the LR type form and vice versa.
After a comparative study between the alpha-cut approach and the tabular fuzzy arithmetic of the LR type, Jomatopfe's LR type arithmetic significantly reduced the complexity of the calculation processes compared to classical methods, which require distinct steps for each element.

This fuzzy tabular altimetry is not to be confused with other fuzzy tables operations on membership degrees.

Keywords : Fuzzy tabular arithmetic, membership function, trapezoidal fuzzy number and alpha-cut.

1. Introduction

This paper is not the first to address the issue of fuzzy arithmetic in fuzzy subset theory.

The literature informs us, for several decades and through numerous works in fuzzy environment, that fuzzy arithmetic based on alpha-cuts and intervals have been applied in several models using the theory of uncertainty to gradually capture alpha-cuts and modal values. These fuzzy arithmetics have been developed in the following way:

- In 1965 the theory of fuzzy subsets by Lofti Zadeh ,
- GD (left-right) fuzzy number arithmetic based on kernel and support by Dubois and Prade,
- Fuzzy arithmetic decomposed by Kaufmann and Gupta (early 90s) to present the nuance between fuzzy arithmetic and interval arithmetic with the aim of representing a fuzzy number by its different alpha-cuts as intervals whose membership degree is greater than alpha (fixed level) (Kaufmann and Gupta, 1988) and (Kaufmann and Gupta, 1991)
- Fuzzy arithmetic of alpha-cuts and intervals by Jean Christophe Buisson (2004).
- The arithmetic of discretized fuzzy numbers by M. Hanss (2005),
- Fuzzy arithmetic of alpha-cuts and intervals by Jean pierreMukeba (2018).

Of all the fuzzy arithmetics mentioned above, we note, unfortunately, that these fuzzy approaches require very heavy calculations to have the kernels and supports of the fuzzy numbers. Beyond this problem, these arithmetics only calculate, separately, the kernels with the kernels, then the supports between them; and finally it is necessary to recompose the kernels with the supports of the results in order to have a single complete fuzzy number. This is a tedious process with several steps.

How can we build a flexible fuzzy approach capable of simultaneously calculating the kernels and supports of fuzzy numbers, in order to reduce the computation time?
Hence the importance of this work.

2. Elements of Fuzzy Logic [1,2,9,10,11,12]

We recall that the theory of fuzzy logic has the task of managing fluctuations, uncertainties, imprecisions or ambiguities that taint any linguistic variable defined in an inexact or vague way, in a Universe of discourse X. For example, "a little warmer", "most", "about 7", "high size", "average speed", "about 2 meters".

From this, two main approaches emerge in the theory of fuzzy subsets, namely:

2.1. Dubois and Prade approach [1,2, 9, 10, 11,12]

Zadeh extension principle with fuzzy operators Min (denoted AND) indicating the intersection and Max (denoted OR) indicating the union of fuzzy subsets.
In practice in artificial intelligence, this approach is based on the degrees and membership function that are most often used in the definition of fuzzy inference systems (FIS) of fuzzy control models or fuzzy command such as that of Mamdani ,Sugeno , Takagi , having the following steps:

- Fuzzification
- Evaluation of rules

- Aggregation of rule outputs
- Defuzzification .

Among the fuzzy control models using the fuzzy inference-based approach, it is worth mentioning: NEFCON, PEDRYCZ, ANFIS, Fuzzy SVM, Fuzzy Decision Tree, FCM, etc.

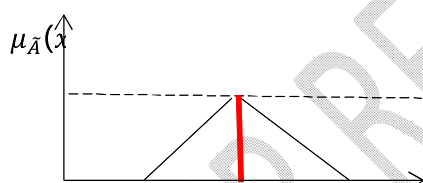
The most commonly used membership functions are as follows and in the following manner:

2.1.1. Triangular fuzzy number and membership function [3, 4, 5, 6, 7,8]

A fuzzy subset $\tilde{A}$with membership function $\mu_{\tilde{A}}(x)$is called a triangular fuzzy number if there exist three real numbers $a < b < c$such that:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} \text{ if } a \le x \le b \\ \frac{c-x}{c-b} \text{ if } b < x \le c \\ 0 \quad \text{elsewhere} \end{cases}$$

Graphically, this is represented as follows:



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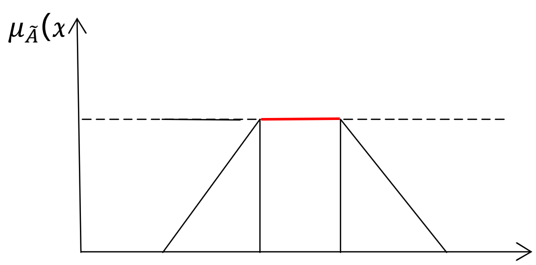
2.1.2. Trapezoidal fuzzy number and membership function

A fuzzy subset $\tilde{A}$with membership function $\mu_{\tilde{A}}(x)$is called a trapezoidal fuzzy number if there exist four real numbers $a < b < c < d$such that:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} \text{ if } a \le x \le b \\ 1 \quad \text{ if } b < x \le c \\ \frac{d-x}{d-c} \text{ if } c < x \le d \\ 0 \quad \text{elsewhere} \end{cases}$$

Graphically, this is represented as follows:



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2.1.3. Search for membership degrees (Fuzzification)

It consists of assigning to a real number its degree of membership corresponding to a fuzzy subset considered.

With the triangular membership function, we can fuzzifythe elements of the fuzzy number $A = (1, 2, 3)$as follows:

For $x = 1$; $\mu_A(1) = \frac{x-a}{b-a} = \frac{1-1}{2-1} = 0$

For $x = 1{,}5$; $\mu_A(1{,}5) = \frac{x-a}{b-a} = \frac{1{,}5-1}{2-1} = 0.5$

For $x = 2$; $\mu_A(2) = \frac{x-a}{b-a} = \frac{2-1}{2-1} = 1$

For $x = 2{,}5$; $\mu_A(2{,}5) = \frac{c-x}{c-b} = \frac{3-2{,}5}{3-2} = 0.5$

For $x = 3$; $\mu_A(3) = \frac{c-x}{c-b} = \frac{3-3}{3-2} = 0$

Hence the triangular fuzzy number $A = (1, 2, 3)$can be written, after fuzzification , as follows:

A= {1/0; 1.5/0.5; 2/1; 2.5/0.5; 3/0} after fuzzification .

To reduce the calculation time, the following interface can be automatically applied:

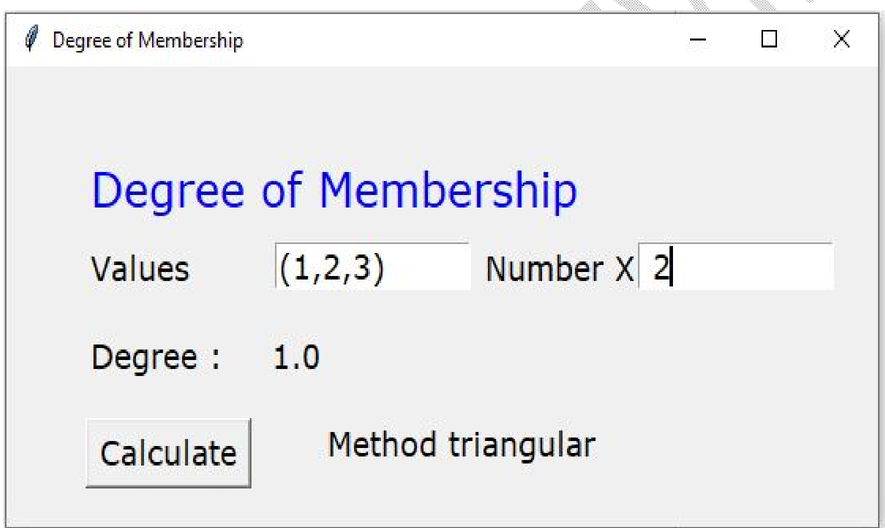


Figure 1: Calculation of Membership Degree Search (Fuzzification) 1

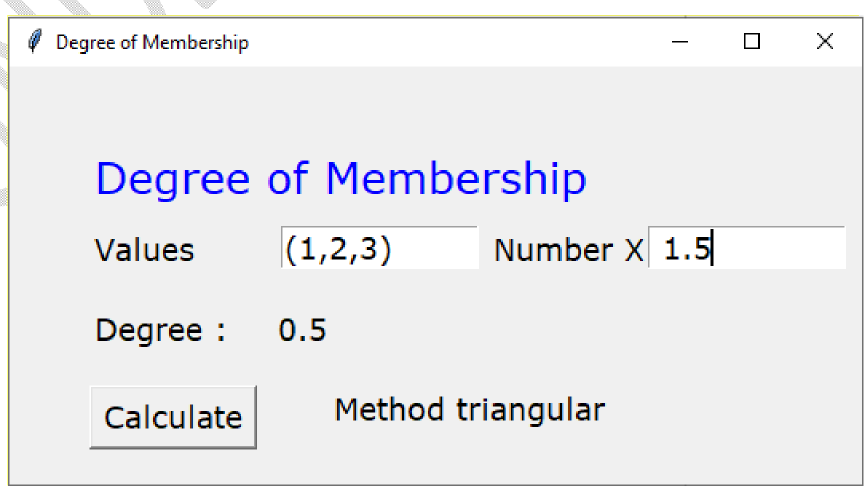


Figure 2: Calculation of Membership Degree Search (Fuzzification) 2

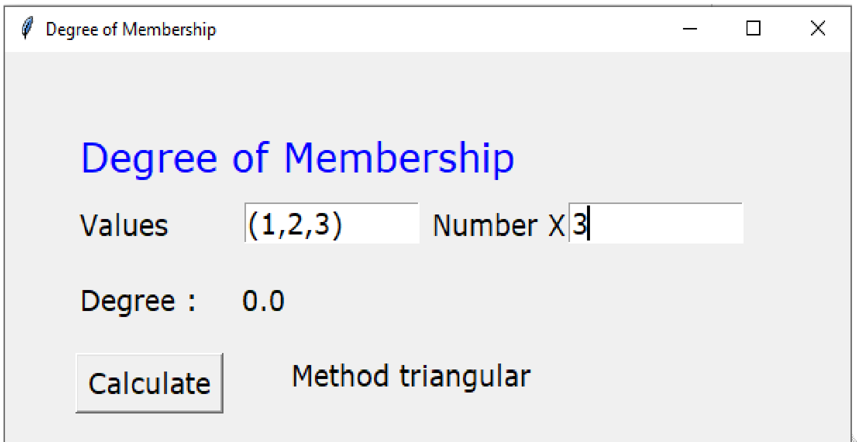


Figure 3: Calculation of Membership Degree Search (Fuzzification) 3

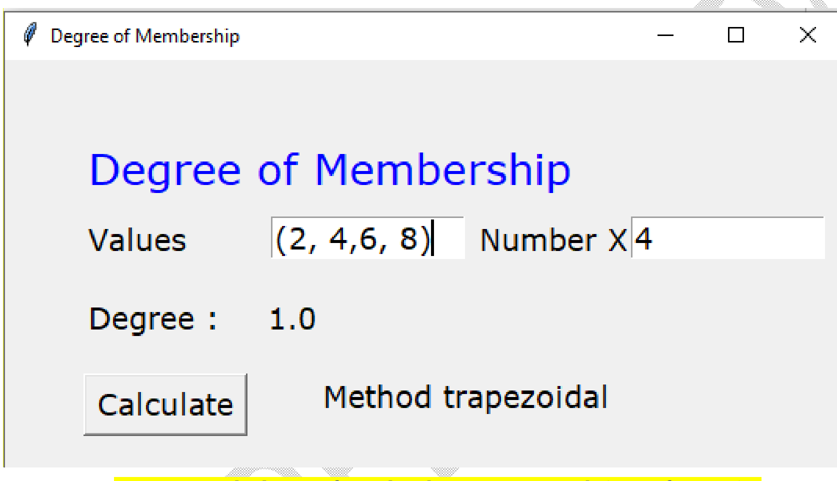


Figure 4: Calculation of Membership Degree Search (Fuzzification) 4

2.2. Approach based on alpha-cuts and intervals [13, 14, 15, 16]

This approach is applied in the determination or search for the kernels and supports of fuzzy numbers, in order to define the following fuzzy arithmetics:

2.2.1. Operations on the$\alpha -$ coupes

and $\tilde{B} = (a_2, c_2, b_2)$be $\tilde{A} = (a_1, c_1, b_1)$two triangular fuzzy numbers, defined by their $\alpha -$ coupesrespective values: $\tilde{A} = [a_1, b_1]$and $\tilde{B} = [a_2, b_2]$. We can then perform the following operations:

(i)Addition

$$\tilde{A} \oplus \tilde{B} = [a_1, b_1] \oplus [a_2, b_2] = [a_1 + a_2, b_1 + b_2]$$

(ii)Subtraction

$$\tilde{A} \ominus \tilde{B} = [a_1, b_1] \ominus [a_2, b_2] = [a_1 - b_2, b_1 - a_2]$$

(iii)Multiplication

$$\tilde{A} \otimes \tilde{B} = [a_1, b_1] \otimes [a_2, b_2] = [Min\ G, Max\ G]$$

where G is defined by $G = \{a_1 a_2, a_1 b_2, b_1 a_2, b_1 b_2\}$

(iv)Multiplication by a scalar
Leave $\lambda \in \mathbb{R}$and$\tilde{A} = [a_1, b_1]$
If $\lambda > 0, \lambda \otimes [a_1, b_1] = [\lambda a_1, \lambda b_1]$
If $\lambda < 0, \lambda \otimes [a_1, b_1] = [\lambda b_1, \lambda a_1]$
(v)Division

$$\frac{\tilde{A}}{\tilde{B}} = \frac{[a_1,b_1]}{[a_2,b_2]} = \left[\text{Min}\left(\frac{a_1}{a_2}, \frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{b_1}{b_2}\right), \text{Max}\left(\frac{a_1}{a_2}, \frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{b_1}{b_2}\right)\right], \text{ with } \tilde{B} \neq 0$$

2.2.2. Finding the α − coupeskernel and support of a triangular fuzzy number [13, 14, 15, 16, 17, 18]

Definition 2.2.2.1. Let be $\tilde{A} = (a, b, c)$a triangular fuzzy number such that $a < b < c$.The α − coupesare $\tilde{A}$defined by the relation:

$$\tilde{A} = [A_\alpha^- ;\ A_\alpha^+] = [(b-a)\alpha + a;\ (b-c)\alpha + c], \qquad \alpha \in [0,1].$$

Definition 2.2.2.2. We call the kernel of $\tilde{A}$and we denote it by $N(\tilde{A})$, the set:

$$N(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) = 1\}$$

Definition 2.2.2.3. We call support of $\tilde{A}$and we denote by $supp(\tilde{A})$, the set:

$$supp(\tilde{A}) = \{x \in X : 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$$

Or$\mu_{\tilde{A}}(x)$represents the degree of membership of the $x$subset$\tilde{A}$.
Note 2.2.2.4. In practice, for a triangular fuzzy number $\tilde{A} = (a, b, c)$:
(i)if $\alpha = 0$,so$[A_0^-, A_0^+] = [a, c] = supp(\tilde{A})$.
(ii)if $\alpha = 1$,so$[A_1^-, A_1^+] = \{b\} = N(\tilde{A})$.

2.2.3. Finding the α − coupeskernel and support of a trapezoidal fuzzy number

If$L_\alpha^-$and$L_\alpha^+$are the lower and upper limits respectively of the α− cuts d of the trapezoidal fuzzy number ($b_i - r$; b_i; c_i; $c_i + t$), we will apply the following formula:

$$[\ L_\alpha^- ;\ L_\alpha^+] = [\ r_i\alpha + (b_i - r_i)\ ;\ -t_i\alpha + (c_i + t_i)]\ , \forall\alpha \in [0.1]$$

If the fuzzy number is A, we will directly note:

$$[\ A_\alpha^- ;\ A_\alpha^+] = [\ r_i\alpha + (b_i - r_i)\ ;\ -t_i\alpha + (c_i + t_i)]$$

2.2.3.1. Numerical example of the LR type

It should be noted that:
A= (3; 4; 1; 1) **LR** is its LR-like shape and A= (2; 3; 4; 5) **TPZ** is its trapezoidal shape
Also, B= (5; 7; 2; 2) **LR** is its LR-like shape and B= (3; 5; 7; 9) **TPZ** is its trapezoidal shape
For A = (2; 3; 4; 5), the α – cuts are:

$$[\ A_\alpha^- ;\ A_\alpha^+] = [\ 1.\alpha + (3 - 1)\ ;\ -1.\alpha + (4 + 1)]\ , \forall\alpha \in [0.1]$$

$$[\ A_\alpha^- ;\ A_\alpha^+] = [\ \alpha + 2\ ;\ -\alpha + 5]$$

Core of A= [1 + 2 ; − 1 + 5] = [3 ; 4] with α =1

Support of A =[0 + 2 ; 0 + 5] = [2 ; 5] with α = 0

For B = (3, 5, 7, 9), the α – cuts are:
$[\, B_{\alpha}^{-} \,;\, B_{\alpha}^{+}] = [\, 2.\alpha + (5 - 2\,) \,;\, -2.\alpha + (7 + 2)]$,∀α ∈[0.1]

$[\, B_{\alpha}^{-} \,;\, B_{\alpha}^{+}] = [\, 2\alpha + 3 \,;\, -\, 2\alpha + 9]$

Core of B= [2.1 + 3 ; – 2.1 + 9] = [5 ; 7] with α =1

B support=[2.0 + 3 ; –2.0 + 9] = [3 ; 9] with α = 0

Finally, **Core of A**+ **Core of B**= [3; 4] + [5; 7] = [8; 11] the core of the response.
Support of A + **Support of B**= [2 ; 5] + [3 ; 9] = = [5 ; 14] the support for the response.
So the final answer is (5; 8; 11; 14)

3. Tabular fuzzy approach of the LR type

Matondo'sLR-type Fuzzy Tabular ArithmeticJonathanOpfointshiENGOMBANGI (JOMATOPFE)

- *Transition from Trapezoidal Form to LR Type Form*

Let $A = (\, a, b, c, d\,)$ TPZ, be the trapezoidal fuzzy form of A.

By defining $r = b - a$ as the left spread relative to the core of A and $t = d - c$ as the right spread relative to the core of A, the LR type form of A will be denoted as follows:
$A = (\; b, c, \;\; r, t\;)$ LR .

- *Transition from LR Type Form to Trapezoidal Form*

Let $A = (\; b, c, \;\; r, t\;)$ be the LR type form of A.

The trapezoidal form of A will be obtained as follows:
$A = (\, b - r \;;\; b;\; c\,;\;\; c + t\;\;)$ TPZ .

Knowing that $a = b - r$ represents the lower limit of A and $d = c + t$ represents the upper limit of A, we have:
$A = (\, a, b, c, d\,)$ TPZ .

Consider A= (a_1; b_1; r_1 ; t_1) and B= (a_2; b_2; r_2; t_2) two LR trapezoidal fuzzy numbers defined as 4-tuples where $r_{1,2}$and$t_{1,2}$indicate the left and right deviation respectively, then the results can be summarized in the following tables:

3.1.1.Fuzzy addition of the LR type.

Table 1: LR type fuzzy addition.

NFTPZ1	a_1	b_1	r_1	t_1
NFTPZ2	a_2	b_2	r_2	t_2
Result	$a_1 + a_2$	$b_1 + b_2$	$r_1 + r_2$	$t_1 + t_2$

Example : Let A = (3 ; 4 ; 1 ; 1) **LR** and B = (5 ; 7 ; 2 ; 2) **LR be** two trapezoidal fuzzy numbers of the LR type in 4-tuple form. In the table, we will have:

- **Example of fuzzy addition**

Table 2: Example of fuzzy addition

NFTPZ1	3	4	1	1
NFTPZ2	5	7	2	2
Result	8	11	3	3

Or this result can be written (8, 11, 3, 3) **LR**

3 .1.2.LR type fuzzy subtraction

Table 3: LR type fuzzy subtraction

NFTPZ1	a_1	b_1	r_1	t_1
NFTPZ2	a_2	b_2	r_2	t_2
Result	$a_1 - b_2$	$b_1 - a_2$	$r_1 + t_2$	$t_1 + r_2$

3.1.3.LR type fuzzy multiplication

Table 4: LR type fuzzy multiplication

NFTPZ1	a_1	b_1	r_1	t_1
NFTPZ2	a_2	b_2	r_2	t_2
Result	$a_1 . a_2$	$b_1 . b_2$	$\lvert a_1 r_2 + a_2 r_1 - r_1 r_2 \rvert$	$\lvert b_1 t_2 + b_2 t_1 + t_1 t_2 \rvert$

3.1.4. LR type fuzzy division

Table 5: Fuzzy division of LR type

NFTPZ1	a_1	b_1	r_1	t_1
NFTPZ2	a_2	b_2	r_2	t_2
Result	a_1 / b_2	b_1 / a_2	$(a_1 / b_2) - (a_1 - r_1) / (b_2 + b_2)$	$(b_1 + r_1) / (a_2 - r_2) - (b_1 / b_2)$

3.1.5. Square of a trapezoidal fuzzy number LR

Table 6: Square of a trapezoidal fuzzy number LR

NFTPZ1	$-a_1$	b_1	r_1	t_1
NFTPZ2	$-a_1$	b_1	r_1	t_1
Result	b_1^2	a_1^2	$\lvert(a_1 - r_1)(b_1 + t_1) - (a_1)^2\rvert$	$\lvert(b_1 + t_1)^2 - (a_1)^2\rvert$

Table 7: Fuzzy multiplication

	a_1	b_1	r_1	t_1
	a_1	b_1	r_1	t_1
Result	a_1^2	b_1^2	$\lvert(a_1 - r_1)(b_1 + t_1) - (a_1)^2\rvert$	$\lvert(b_1 + t_1)^2 - (a_1)^2\rvert$

To reduce the calculation time, the following interface can be automatically applied:

3. 2. Automatic calculation interface of the LR type

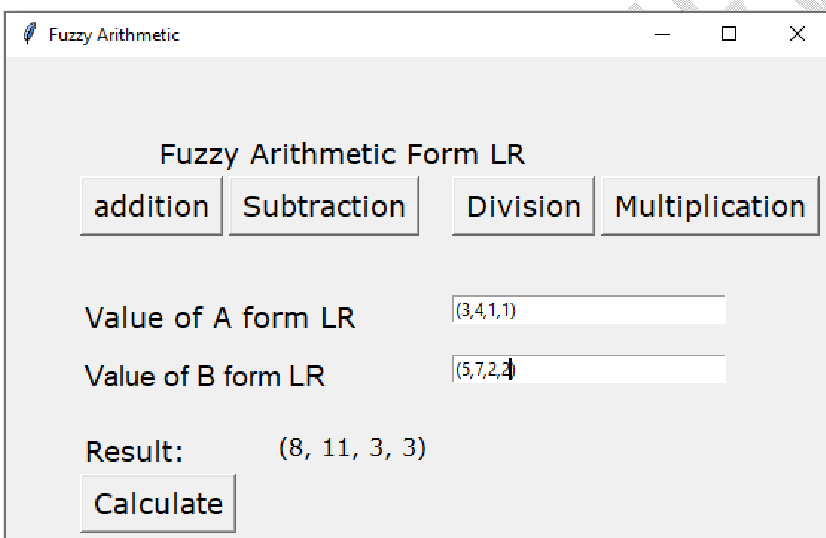


Figure 5: LR type automatic calculation interface (1)

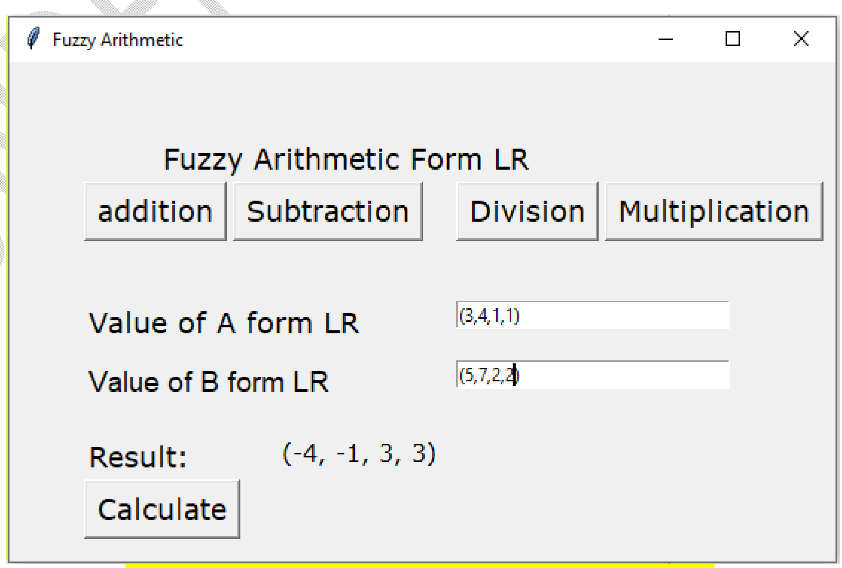


Figure 6: LR type automatic calculation interface (2)

3.3. Comparative study of alpha-cut approaches and LR-type tabular arithmetic

Let us multiply A = (2; 3; 4; 5) **TPZ** and **B** = (3; 5; 7; 9) **TPZ** in normal trapezoidal form
3.2.1. Alpha-cut approach

For A= (2; 3; 4; 5) **TPZ** , the α – cuts are:

$[A_\alpha^- ; A_\alpha^+] = [1.\alpha + (3-1) ; -1.\alpha + (4+1)]$,∀α ∈[0.1]

$[A_\alpha^- ; A_\alpha^+] = [\alpha + 2 ; -\alpha + 5]$

Core of A= $[1+2 ; -1+5] = [3 ; 4]$ with α =1

Support of A =$[0+2 ; 0+5] = [2 ; 5]$ with α = 0

For B = (3; 5; 7; 9) **TPZ** , the α – cuts are:

$[B_\alpha^- ; B_\alpha^+] = [2.\alpha + (5-2) ; -2.\alpha + (7+2)]$,∀α ∈[0.1]

$[B_\alpha^- ; B_\alpha^+] = [2\alpha + 3 ; -2\alpha + 9]$

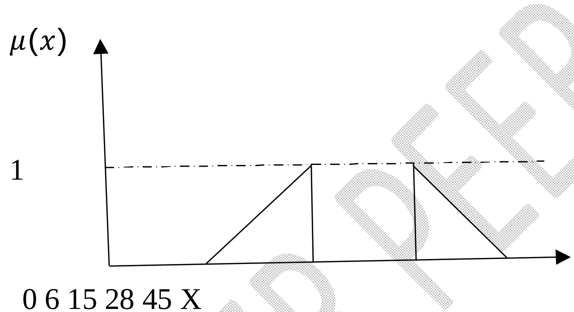
Core of B= $[2.1+3 ; -2.1+9] = [5 ; 7]$ with α =1

B support=$[2.0+3 ; -2.0+9] = [3 ; 9]$ with α = 0

Finally, **Core of A**x **Core of B**= $[3 ; 4]$ x $[5 ; 7] = [15 ; 28]$ and

Support of Ax **Support B**= $[2 ; 5]$ x $[3 ; 9] = [6 ; 45]$

Graphically we have:



3.2.1. Approach to JOMATOPFE LR-type tabular fuzzy arithmetic

Table 8: Example of fuzzy multiplication

HAS	3	4	1	1
B	5	7	2	2
Result	15	28	6+5-2	8+7+2

This result can be written (15; 28; 9; 17) **LR: Form of the LR type** or
(15-9; 15; 28; 28+ 17) = (6; 15; 28; 45) **TPZ: Trapezoidal blur shape.**

This result confirms that the two fuzzy approaches thus applied converge

Conclusion

Jomatopfe LR -type tabular fuzzy arithmetic, capable of computing simultaneously the kernels and supports of fuzzy numbers, instead of doing it separately, in order to minimize the tedious steps of the alpha-cut based approaches proposed by other authors.

A comparative study was made through numerical examples in order to show the reduction of calculation times with the new fuzzy approach of the **LR type.**Jomatopfe.

In this paper, we created some user interfaces to automatically calculate membership degrees and results of **LR-type fuzzy tabular arithmetic.**

This fuzzy tabular altimetry is not to be confused with other fuzzy tables operations on membership degrees.

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Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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Author(s) hereby declare that generative AI technologies such as Large Language Models, etc. have been used during the writing or editing of manuscripts. This explanation will include the name, version, model, and source of the generative AI technology and as well as all input prompts provided to the generative AI technology

Details of the AI usage are given below:

1.

2.

3.

References

1. Dubois D. and Prade H, operations on fuzzy numbers . Int. J. systems science vol. D; pp.613-626, 1978;
2. GradiKamingu , fuzzy set theory, properties and operations, lareg , 2016.
3. BaaliSabeur&MahmoudiMessaoud, Temperature and humidity regulator for a chicken coop, 2022
4. Gabriel Cormier, Fuzzy Logic 9 GELE 5313
5. Temperature Control using Fuzzy Logic P. Singhala1, DN Shah2, B. Patel, 2014.
6. Dominique Longin, Reasoning and Uncertainty, 2016
7. LK Wong, FHF Leung, PKS Tam, Takagi– Sugeno fuzzy model based system with guaranteed performance, 2001
8. Kamyar Mehran , Takagi- Sugeno Fuzzy Modeling for Process Control, *2008*

9. Sinzinkayo A. Application of fuzzy logic to the choice of an assembly method, 2000

10. Zadeh , LA **-** "Fuzzy Logic = Computing with Words" (2018).

11. Pedrycz, W., & Gomaa, A. **-** "Fuzzy Systems: A Comprehensive Introduction"(2020).

12. Cox, E.J. **-** "The Role of Fuzzy Logic in Decision Making"**.** (2017)

13. Mazoni, G. N., et al. (2024). Analysis of functional properties of fuzzy $\tilde{L}p$ (Ω) spaces. Asian Research Journal of Mathematics, 20(9), 140-150

14. Mazoni. G.N.et al, On Fuzzy Sobolev Spaces $\widetilde{W}^{1,p}(\boldsymbol{\Omega})$: Analysis of Dot Product and Fuzzy Norm

15. Lukezu .AMet al, Comparative study of fuzzy logic operators, (2024)

16. Mukeba .JP, Doctoral Thesis on Fuzzy Markov Chains, (2O16)

17. M.A. Albahar et al., *FuzzyArithmetic Operations with Alpha-Cuts:Algorithms and Applications*, (2023).

18. Zhongzhi et al., *Alpha-Cut BasedFuzzyArithmetic for DecisionMaking Under Uncertainty*, (2024).