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JournalName:	<a href="#">AsianResearchJournalofMathematics</a>
ManuscriptNumber:	Ms_ARJOM_130005
TitleoftheManuscript:	UltrafilterinGraphTheory:RelationshipptoTree-decomposition
TypeoftheArticle	ShortResearchArticle

**PART 1:**Comments

	Reviewer'scomment	Author'sFeedback(Pleasecorrectthemanuscriptand highlightthatpartinthemanuscript.Itismandatory thatauthorsshouldwritehis/herfeedbackhere)
Pleasewriteafewsentencesregarding theimportance ofthismanuscriptfor thescientificcommunity.Aminimumof 3-4sentencesmayberequiredforthis part.	Ultrafilterconstructionsindifferentcategoriesofmathematicalobjectssuchas partialsets,Booleanalgebras,graphs,etcareanactiveresearchtopicwhere themainchallengeiswhetheranultrafilterofacertainypeexistsornot.The articleunderreviewaimstodiscussthestructureofultrafilteronthe category ofgraphsintermsofadualrelationshipwithtreewidthandextensionof propertiesfromBooleansettingtographsetting.	
Isthetitleofthearticlesuitable? (If notpleasesuggestanalternative title)	No.Irecommend  "UltrafilterinGraphTheoryviathesettingoftree-decomposition"	

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<p>Istheabstractofthearticle comprehensive? Doyou suggestthe addition(ordeletion)ofsomepointsin thissection?Pleasewriteyour suggestionshere.</p>	<p>No.TherearepointsinAbstractwhichareNotrelatedtothecurrentarticle. Thesentence“Ourgoalistoextendthisresearchinthefuturebydelvingdeeper intothepropertiesofUltrafiltersongraphs,particularlyfocusingonuncovering uniquecharacteristicsthatmanifestinvariousgraphclasses” shouldberemoved.Becauseithasnothingtodowiththisresearch.</p>	
<p>Isthemanuscriptscientifically,correct? Pleasewritehere.</p>	<p>Severalmathematicaltoolsandconceptsareappliedwithoutanyappropriate explanationswhichmadedifficulttoreadthisarticle.Severalclaimsare addressedwithoutprovidingenoughmathematicalreasoning.Thereforethe currentpresentationofthisarticleneedstobere-writtenintermsofthe followingremarks:</p> <p>CommentstoAuthor:</p> <ol style="list-style-type: none"> <li>1.Oneimportantissueisusingthephrase“metric”forthe parameter “tree width”inAbstractandIntroduction.Iftthisisametricstructureonthespaceof graphs,thenanexplicitdefinitionneedstobeaddedintheSection:Definitions andNotations.Otherwise,usingthephrase“metric”foraparameter doesnot makesense.</li> <li>2.Whatisthemeaningof“anaturaldefinitionofultrafiltersongraphs”in Abstract?Doesexistanun-naturaldefinition?Theadjective“natural”shouldbe removed.</li> <li>3.Replace“Ultrafilters”with“ultrafilters”inthewholepaper.</li> <li>4.Thesentence“Ourgoalistoextendthisresearchinthefuturebydelving deeperintothepropertiesofUltrafiltersongraphs,particularlyfocusingon uncoveringuniquecharacteristicsthatmanifestinvariousgraphclasses.”in Abstracthasnothingtodowiththecurrentwork.Soitshouldberemovedor replacedtoConclusionpart.</li> <li>5.InthefirstparagraphinIntroduction,page1,thesentence“Width parameterspertaintometricsderivedfromtree-likestructures,commonly referredtoasgraphdecompositions.”isunclearandgenerates misunderstanding.Itisliketheauthorsaystheexistenceofacorrespondence betweenacertainclassofmetricsandgraphdecompositions.Isthatright?I recommendrewritingthissentenceandprovidinganappropriatereferencefor it.</li> </ol>	

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6. An explicit definition for the concept "treewidth" should be added as a sentence before the concept "Tangle" in the first paragraph in Introduction, page 1.

7. In the second paragraph in Introduction, page 1, the sentence "Simply put, a filter can be interpreted as a collection of sets containing a specific element..." is meaningless. It should be replaced by an explicit definition for the concept "Filter".

8. An appropriate reference for the statement "In the domain of Boolean algebra, maximal filters are referred to as ultrafilters" is essential to be provided.

9. Replace the phrase "natural definition" with "definition" in whole paper, and, remove the phrase "It is truly remarkable" in the last paragraph in Introduction.

10. Remove the sentence "Our goal is to extend this research in the future by delving deeper..." from Introduction or replace it to the Conclusion.

11. The first paragraph in Section 2, namely, "A graph  $G$  is a mathematical structure composed of nodes (vertices) connected by edges, representing relationships or....." should be rewritten as a Definition.

12. Having two short subsections for the Section is necessary? I recommend removing these divisions.

13. The sentence "As mentioned in the introduction, Filters and Ultrafilters are fundamental concepts in mathematics." in page 2 is not necessary and should be deleted.

14. Appropriate references for the sentence "The complement of a filter in a Boolean algebra  $(X, \cup, \cap)$  is referred to as an ideal in a Boolean algebra  $(X, \cup, \cap)$ ." should be added.

15. Appropriate references for Definition 1 should be added.

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	<p>16. Appropriate references for the sentence "at a angle in a graph <math>G</math> is a way to describe how the vertices can be separated into distinct groups based on certain conditions." in subsection 2.2 should be added.</p> <p>17. The paragraph "A separation of a graph <math>G</math> is a pair of subgraphs ..." in subsection 2.2 should be rewritten as a Definition.</p> <p>18. The sentence "in the field of graph width parameters, duality theorems are frequently discussed." is unclear and it needs to be rewritten together with appropriate references.</p> <p>19. The sentence "The definition of path tangle, which has a deep connection with path-decompositions, are outlined below." is not necessary and it should be deleted.</p> <p>20. Appropriate references for the sentence "A path-decomposition is a tree structure that restricts tree-decompositions to a path-like structure." should be added.</p> <p>21. Appropriate references for Definition 3 should be added.</p> <p>22. An explicit proof for Theorem 2 should be added. This theorem is the main tool for the rest of this work, therefore it is essential to have its proof together with appropriate references.</p> <p>23. The importance of Theorem 3 for the rest of this work should be explained in a paragraph at the end of the subsection 2.2.</p> <p>24. I recommend to double check the reference [35] and its relation to Theorem 3? I think Theorem 3 needs another appropriate references or its proof should be added in this paper.</p> <p>25. The sentence "We naturally extend the definition from Boolean algebras to graphs." in the first paragraph in Section 3 is incorrect. How is that possible? An explicit proof for the existence of a functor from Category of Boolean algebras to the Category of graphs (or algebras of graphs) and the structure of this functor is essential to be added. Otherwise it does not make sense to extend concepts for Boolean algebras to the space of graphs.</p>	
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	<p>26. Without providing an explicit relation between Category of Boolean algebras and Category of graphs, Definition 4 is Not well-defined. For which class of graphs does work this definition? Is it valid for infinite graphs, and multi-edge graphs?</p> <p>27. The validity of Theorem 4 can be evaluated when the author could first introduce a well-defined functor from Category of Boolean algebras to Category of graphs or algebras of graphs, and then the author should check which type of properties can be extended by that functor.</p> <p>28. The proof of Theorem 4 is Not working for infinite graphs and graphs with multi-edges.</p> <p>29. A proof for Theorem 5 should be added.</p> <p>30. A proof for Theorem 6 should be added.</p> <p>31. In Section 5, the paragraph "Two subgraphs A and B touch if either their vertex sets have a non-empty intersection (<math>V(A) \cap V(B) \neq \emptyset</math>), or there exists an edge <math>e \in E(G)</math> that connects..." should be rewritten as a Definition with appropriate references.</p> <p>32. In Section 5, the paragraph "Now, we define a G-bramble of G as a family B consisting of connected subgraphs of G in which any two subgraphs within..." should be rewritten as a Definition with appropriate references.</p> <p>33. Proof of Theorem 8 is related to the validity of Theorem 4. I recommend the author could fix a certain class of graphs and then work on the construction of a suitable functor to extend properties from Boolean algebras to that class of graphs.</p> <p>34. In Section 6, the sentence "We investigate the characteristics of ultrafilters on graphs across various graph classes." is very general and it needs to be revised.</p> <p>35. In Section 6, the sentence "...explore topics such as the Ultrafilter Axiom of Choice and the finite intersection property within the framework of ultrafilters on graphs." needs a double check. Because the author is already using Axiom of Choice by extending properties from Boolean algebras to Graph Theory.</p>	
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Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.	There are several papers in the list of References which are not related to this article. References are not applied appropriately in the text. The authors should rigorously double check this list and then keep only related ones and remove the rest.	
Is the language/English quality of the articles suitable for scholarly communications?	Average	
<u>Optional/General</u> comments	<p>Theorems 2, 3, 5, 6 are Not proved. The author has the idea of extending properties from Boolean algebras to the space of graphs which is incorrect in general and it needs fundamental conditions and several mathematical structures to have the possibility of extending some properties from Boolean algebras to graphs. The author does Not provide an explicit proof for the existence and the structure of a required functor from Category of Boolean algebras to the Category of graphs (or algebras of graphs). Otherwise, Theorems 4, 8 as the main result of this article cannot be proved.</p> <p>More explanations are provided as Comments to Author in the previous parts.</p>	

**PART 2:**

	Reviewer's comment	Author's comment (if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)
Are there ethical issues in this manuscript?	<i>(If yes, Kindly please write down the ethical issues here in details)</i>	

**Reviewer Details:**

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