Minireview Article

Mathematical Modelling of Wastewater Treatment

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ABSTRACT

Sewon domestic wastewater treatment plant (WWTP) plays an important role as a domestic wastewater treatment unit in a densely populated area of Yogyakarta. Domestic wastewater is mostly in the form of suspended solids, organic matter, pathogens, and nutrients. The environmental impact of contaminated and unregulated domestic wastewater treatment can cause various diseases and risk damaging the quality of water that will be reused. Sedimentation and eutrophication that occur can cause effects that impact the performance of wastewater treatment units. Phenomena and problems that occur in wastewater treatment plants can be modelled mathematically, especially with dynamic models. The finite difference method is studied as a numerical simulation to solve differential equations in models that are difficult to solve exactly. This article presents a review of observations in the field and a review of related scientific articles, to be able to identify various phenomena and problems in the scope of wastewater treatment which are seen as mathematical models and their simulations.

Keywords: wastewater treatment, mathematical models, WWTP, domestic wastewater

1. INTRODUCTION

Sustainable Development Goal No. 6 (SDG6) aims to achieve safe sanitation by 2030. However, inadequate household wastewater collection and treatment facilities still exist, especially in Southeast Asian countries (Arcowa, 2018; Bongartz et al., 2016; McConville, 2011; Rosemarin et al., 2016). Water contamination and pollution is a serious global environmental challenge as it can adversely affect human health, food production, economic growth, and the natural environment (Mian et al., 2023; Tauseef et al., 2023).

The concentration of settlements in an area raises new problems, especially in the urban sewerage system (Sudharto & Samekto, 2007). The development of cities characterized by economic growth, population increase, and expansion of urbanization (Smol, Adam, et al., 2020; Zhang et al., 2017) in urban settlements is one of the main factors causing environmental pollution. The denser an area is, the more complex the pollution problem, especially household wastewater or domestic wastewater from settlements. If pollution is not handled properly, it can cause serious problems. Uncontrolled domestic waste has caused pollution in almost all rivers in Indonesia, especially in Java (Sudharto & Samekto, 2007; Sunarsih & Sutrisno, 2023).

Water pollution occurs due to improper discharge of domestic and industrial wastewater effluents into the environment, either directly or indirectly, without adequate treatment. Numerous studies have shown the challenges associated with sewage treatment (Afolalu et al., 2022). Treatment of contaminated water (wastewater) is a common step to ensure it is safe before reuse in a variety of applications, including agricultural irrigation, road cleaning, firefighting, geothermal production, industrial processes, commercial washing, and construction (Eslami et al., 2018). One place to treat domestic wastewater is by using a centralized wastewater treatment plant (WWTP) unit (Sunarsih & Sutrisno, 2023).

2. STUDY RESULT

2.1 Domestic Liquid Waste Sources

Sewon WWTP presented in Figure 1 is one of the centralized domestic wastewater treatment units located in Bantul Regency. The Sewon WWTP service area is the Yogyakarta Urban Area (KPY) or called Kartamantul which includes almost all of Yogyakarta City, parts of Sleman regency (Mlati, Depok, Gamping and Ngaglik sub-districts) and Bantul regency (Kasihan, Sewon and Banguntapan sub-districts). Domestic wastewater from these areas is flowed by gravity through a network of service pipes from lateral pipes to main pipes with a total network length of 323.29 km (Dinas PUP-ESDM, 2022).

Figure 1. Sewon WWTP in October 2023

In line with the development of Yogyakarta City, several sectors have experienced an increase in activity which has an impact on increasing the amount of liquid waste produced. The home industry sector such as Micro, Small and Medium Enterprises (MSMEs) batik typical of Yogyakarta can be found in several places in Bantul Regency.

2.2 Components of Domestic Liquid Waste

Liquid waste is the residual material from industrial and domestic processes that utilize water as raw material. its physical, chemical, and biological properties determine its characteristics (Britton, 1994). Wastewater or liquid waste is separated into two categories. (1) Wastewater from industries resulting from the production process is referred to as industrial wastewater. (2) Domestic wastewater is wastewater originating from residences, hotels, restaurants, markets, cinemas, retail establishments, hospitals, and other places of worship, not from industry (Irianto, 2016).

Domestic waste consists of feces from bathrooms, toilets, and kitchens. Dirt is a complex mixture of mineral substances and organic materials in various forms, including large and small particles, solid objects, remnants of materials in a floating state in colloidal and semi-colloidal form (Kamilah et al., 2022; Martopo, 1987).

The main constituents of domestic wastewater are suspended solids, biodegradable organic matter, pathogens, and nutrients (Sudharto & Samekto, 2007). A prominent characteristic of urban wastewater is its high organic content which contains particulate components such as heavy metals (Zheng et al., 2021), nitrogen compounds, phosphorus (Li et al., 2012; Smol, Preisner, et al., 2020), bacteria, algae, zooplankton, detritus organic matter, NH3, Dissolved Oxygen (DO), total coliforms, fecal coliforms and Biochemical Oxygen Demand (BOD) (P. Sunarsih et al., 2015). Unregulated discharge of contaminated domestic and industrial wastewater in the environment (Pan et al., 2019) can lead to various waterborne diseases such as infectious hepatitis, cholera, and typhoid.

Heavy metals are persistent pollutants known for their inherent toxicity and non-biodegradability, leading to accumulation in living organisms (Hafidi et al., 2023). The presence of heavy metals poses a significant risk to water quality and human health, causing severe diseases and adverse effects (Hafidi et al., 2023; Lü et al., 2019). The main source of heavy metal contamination comes from direct discharge into sewage systems and rivers, thus posing a high risk of toxicity. Previous research conducted by several authors showed that most heavy metals are present in suspended particles in wastewater and surface water (Xu et al., 2022).

Sedimentation can occur in the WWTP. Sedimentation is a physical wastewater treatment process that uses gravity to separate suspended solid particles that have formed from the water (Kristijarti et al., 2013). In wastewater treatment ponds, sediment or sludge deposits can be used to reduce the concentration of suspended particles (Edzwald, 2010). However, if it occurs in excess, it can cause the inflow and outflow of the pond to be suboptimal which can result in reduced effluent treatment performance.

Eutrophication can also occur in the WWTP. The onset of the eutrophication process is due to the excessive entry of nutrient compounds such as nitrogen and/or phosphorus resulting from industrial, agricultural, domestic activities, surface runoff and others that enter wastewater treatment, causing blooms or “blooms” in phytoplankton (Ansari et al., 2011; González & Roldán, 2019).

2.3 Mathematical Model of Domestic Liquid Waste Volume Growth

The interaction between components in wastewater is widely studied as a mathematical model. In the optimization model approach, a model can be formed to optimize the pollutant reduction process in facultative stabilization ponds (S. Sunarsih et al., 2019). The model is developed to determine the optimal volume of wastewater to be treated in facultative stabilization ponds and to determine how long the wastewater storage time is so that the efficiency value of the pond will approach the reference efficiency value. Another optimization model is for the optimization of variables that affect the phosphorus content in the effluent stabilization pond (Sells et al., 2018). In addition, a multi-objective optimization model can represent the reduction of BOD waste load in WWTPs (Kamilah et al., 2022). This optimization model has three objective functions, namely maximizing the BOD waste load treated in the pond by minimizing the difference between the efficiency value of BOD reduction in the WWTP and the reference efficiency value and minimizing the power used by the aerator.

In addition to the optimization model approach, approaches with dynamic models are also studied such as a simple mathematical model of a system of nonlinear differential equations in biological wastewater treatment by considering inflows and outflows during the treatment process (Islami et al., 2022). The system of equations was constructed using the basic Monod equation. Another dynamic model can formulate chemical processes and biological processes that run simultaneously with the rate of change of substance concentration based on a system of non-linear differential equations (P. Sunarsih et al., 2015). This model applies the monod equation with a time correction factor at the maximum growth rate.

Mathematical models with control were also studied such as a mathematical model to minimize wastewater treatment time with aerator control (oxygen transfer coefficient) (Grigor'eva & Khailov, 2020). Another model is about the control of the aerator machine which is then analyzed for the limitation and continuity of the interval given in the mathematical model (Bondarenko et al., 2016). In addition, another mathematical model can be used to minimize the total use of electrical energy in the aeration process at WWTP (Chen et al., 2022). The model makes dissolved oxygen and sludge discharge as control variables.

2.4 Finite Difference Method for Numerical Simulation of Sedimentation

Numerical simulation is used to solve a differential equation that is difficult to solve exactly, especially if it contains nonlinear terms in the equation (Kartono, 2012). There are various methods in this numerical procedure, one of which is the finite difference method. The finite difference method can be used to simulate sedimentation and flocculation units and predict removal efficiency with the help of Matlab software (Naser & Abdulrazzaq, 2022). The finite difference method can also represent the simulation of the sedimentation process using the Mason-Weaver equation which can further be used to model the behavior of epithelial cells and the lung system on the chip (Nikolic et al., 2021). The finite difference scheme of this method can be combined with the Lattice Boltzmann Method (LBM) to simulate complex flow through a uniform lattice with low diffusivity so that it can facilitate coupling with finite difference methods (Lemus et al., 2021).

The finite difference method can also be used as a numerical simulation of the Burger equation in two-dimensional nonlinear steady-state mode to model the phenomenon of particle fall velocity in stagnant or nearly stagnant fluids, such as water settling behind a dam that indicates the presence of suspended particles at several positions and times (Rezaei & Rahideh, 2020). Finite difference numerical models can also be modified based on the conservative law of solid-fluid mixture hydrodynamics as an ongoing effort to reduce siltation due to interstitial clogging in sand-filled reservoirs so as to increase water storage potential (Olufayo et al., 2011).

3. CONCLUSION

The domestic wastewater treatment process at the Sewon WWTP has an important role as a domestic wastewater treatment unit. Domestic wastewater is mostly in the form of suspended solids, organic matter, pathogens, nutrients that come from the densely populated areas of Yogyakarta. The environmental impact of contaminated and unregulated domestic wastewater treatment can cause various diseases and risk damage to water quality. Sedimentation and eutrophication that occur if not controlled can cause effects that can impact the performance of wastewater treatment units. Various mathematical models examine the interaction between components in WWTPs to solve various problems such as optimization, dynamics, and control. The finite difference method is also studied as a numerical simulation to solve differential equations that are difficult to solve exactly.

The results of this research provide further opportunities to study and solve problems in WWTPs. Mathematical modeling, especially dynamic models, can play a role in controlling interacting components in the wastewater treatment process as well as monitoring and predicting populations so as to obtain strategies to reduce pathogens that can interfere with health and maintain the quality of wastewater before it can be reused.

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